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Identifying and Ranking the Effective Financial Factors on Rural Entrepreneurial Small Businesses

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Abstract

Rural entrepreneurship is one of the most important goals of policy-makers to develop villages and prevent migration from rural to urban areas. Significant challenges at the rural entrepreneurship level have reduced the entrepreneurship development process. Recognizing these challenges can be constructive in identifying the various dimensions related to rural entrepreneurship and removing barriers to rural business development. This study aims to identify the effective financial factors on entrepreneurship in rural small businesses and rank these factors. The research method applied based on purpose and data collection is a semi-structured interview done through interviews with academic experts, implementers of rural employment and entrepreneurship projects, and entrepreneurs. Data analysis and validation of financial factors on rural entrepreneurship in both service and agriculture sectors were performed using the fitting of a time-series regression model. The results indicate a positive and significant effect of rural population ratio, financing through financial institutions, economic growth, openness rate in business space, gender composition, and inflation rate in agriculture sectors, as well as a significant reverse effect of official employment rate, job-seeking population ratio, exchange rate uncertainty, government employment rate and graduation ratio on entrepreneurship in both service and agriculture sectors. Finally, the results of the Friedman test showed that the financing factor through financial institutions has the highest average rating due to the bank-oriented financial resources of entrepreneurial enterprises in the rural service and agriculture sectors. Therefore, policy-makers seeking to develop rural entrepreneurship should prioritize policies related to the ease of access of rural businesses to financial resources.

Keywords: Effective financial factors on rural businesses, Rural entrepreneurial factors, Rural small businesses.

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1|Introduction

In today's world, rural areas are regarded as one of the most important economic hubs, not only in less developed countries but also in the developed world, and play a significant role in increasing GDP, food security, raw materials, and ultimately the growth and prosperity of countries [1]. However, it should be noted that rural development is not achieved only by expanding civil infrastructure and providing social services. Still, it also requires income-generating job creation for villagers and accelerating their cultural development (Compared to socio-cultural attractions of urban areas) [2]. In this regard, expanding small businesses has been considered one of the essential strategies for job creation and increasing income in rural areas in recent years, so establishing micro-enterprises is a vital way to empower the rural economy [3].

There are many studies on the impact of financial factors on entrepreneurship development. The results show that the low-income level, especially in rural areas in developing countries, is mainly due to a lack of resources. In many developing countries, small and medium-sized enterprises suffer from insufficient funding opportunities. On the other hand, owing to the inefficiency of capital markets and the lack of venture capital companies, investment value in small businesses, particularly in rural service and agriculture sectors, faces significant challenges. Finally, these financial constraints and other challenges, such as inflation, prevent entrepreneurs from entering the business and make new businesses enter the market in sub-optimal comparisons.

Without efficient financial development in rural areas, all policies to reduce migration from rural to urban areas will be ineffective. The development of entrepreneurship in villages due to technological changes is not limited to agriculture; it also includes other sectors such as communication services, production, etc. [4]. Following these changes, innovative usage of rural resources and facilities is significant in developing rural businesses to take advantage of entrepreneurial opportunities [3]. Today, strengthening entrepreneurship and creating a suitable environment for its development is one of the essential tools for economic and business progress. Highly effective entrepreneurial activities lead to economic development (job creation, innovation activities, and competitiveness). However, no research has comprehensively investigated the most crucial reason for rural development, i.e., effective financial factors in developing rural small businesses [5]. Rural development is linked to entrepreneurship more than ever before; entrepreneurship promotes rural communities' development by creating job opportunities in villages. Rural job creation has led to quantitative growth of enterprises by investing entrepreneurs in the rural and agriculture sectors, which is improved by productive activity. Therefore, the identified financial factors in developing rural small businesses are crucial for policy-makers in rural growth and ultimately lead to the reverse migration of the population from cities to villages [2]. So, the study of financial factors on the creation of rural small jobs should be considered more than ever to create productive employment for sustainable development and poverty reduction, improve the living conditions of villagers, and create a consistent income for them. By the way, the effective financial factors of entrepreneurship that have been less considered in previous research should be identified among rural business owners. Thus, the research begins with this question: What are the effective financial factors on entrepreneurship in rural small businesses?

2|Theoretical Foundations and Research Backgrounds

Today, there is no doubt that rural development, as one of the priorities and essential challenges of macro-development programs, requires something beyond a simple strategy, especially in non-industrial countries, As it would be possible only with careful and systematic planning and only through the proper functioning of an efficient and pluralistic system [6]. As the most critical element of the rural economy, rural businesses create employment, reduce unemployment, and increase income and productivity [7]. Entrepreneurship can play an essential role in rural economic development in areas such as access to rural goods and services, rural economic growth, reducing the phenomenon of migration to cities, encouraging villagers to start new businesses, and improving the level of social security and welfare in rural areas [8]. Meanwhile, according to many thinkers, entrepreneurship development in various forms and contexts in rural areas, which indicates human orientation as the primary source of development, is one of the critical elements of rural and even national development by creating employment, generating capital, helping to distribute income more equitably in society and reducing poverty [9].

There are two research approaches to the impact of financial factors on developing rural small businesses. The first approach focuses on entrepreneurs' access to financial resources, and the second one concentrates on other financial factors that emphasize the willingness of small business entrepreneurs to start a new job. Relating to the first approach, studying the factors affecting business formation shows that access to financial resources can positively impact entrepreneurship. Regarding the second one, research demonstrates that the lack of government support, graduation ratio, population composition, entrepreneurial knowledge, etc., has a negative impact on real entrepreneurship [10]–[12]. Rural entrepreneurship can lead to the creation of a small business in the village by investment, trust, risk, and providing sales and marketing services [13]. Rural entrepreneurship is: "Creating a new enterprise that introduces a new product or service, creates a new market, or uses new technology in the rural environment" [14]. The development of small businesses, especially in villages, is one of the suitable areas for job creation; it also has basic elements whose main focus is on providing the village's infrastructure, accelerating economic growth, and reducing poverty [15]. Research shows that paying attention to financial factors affecting the development of small businesses has a greater impact on rural entrepreneurship in developing countries than in developed ones. This is because the GDP in developing countries is lower than the production capacity curve, so after improving the quality of financial indexes affecting the development of small businesses in developing countries, the entrepreneurial situation improves, and the production level approaches the optimal value.

3 | Research Methodology

The present study is applied based on purpose and exploratory-descriptive based on implementation strategy. According to the data collection method, this research has been conducted in two forms: library and field research (using semi-structured interview tools and questionnaires, as well as documents and records). In terms of the research period, this study is one cross-sectional and time series in qualitative and quantitative parts, respectively. Also, due to the non-experimental nature of the study, the experimenter's intervention in the research process is minimal. The statistical population includes academic experts in rural entrepreneurship and small businesses, implementers of rural employment projects, and officials and policy-makers of employment in the villages. Qualitative sampling was performed with a purposive and judgmental approach to the extent of theoretical saturation of data, and 16 people (6 academic experts, 4 implementers of rural employment projects, and 6 officials and policy-makers of employment in rural areas) were identified and participated in the interview process. In the quantitative part of the research, considering that the time series information and figures related to the expert variables have been used, the statistical population of the research is Iran, where the information required for the variables has been collected and studied for 20 years. The period studied in this research is from 1379 to 1398. Also, in the quantitative approach, the measurement model presented for each factor identified in the research is evaluated and validated. Effective factors on entrepreneurship in rural small businesses in the expertise process have been identified through expert analysis. After extracting the components and indexes related to the effective factors on entrepreneurship in rural small businesses based on the thematic analysis method, their effectiveness or ineffectiveness on small businesses was discussed by fitting the time series regression model according to the identified variables. The mana tests of the variables, time series regression using the least square method, and the tests of the initial regression assumptions were used to analyze the data. Finally, the factors were ranked. Data analysis was performed in EViews 10.0.

4 | Research Findings

The Delphi technique was used to identify the factors affecting entrepreneurship in rural businesses. First, the effective factors for developing the new businesses were identified based on the theoretical literature and then reviewed by experts through an expertise process. This process was performed in 3 steps; some of the proposed indexes were removed or added in each. The final results obtained from evaluating the importance of these indexes, emphasizing the country's macro and economic factors, and using the coefficient and content validity index criteria are presented in *Table 1*.

Table 1. Factors affecting entrepreneurship in rural small businesses.

Factors	Evaluation Method	Content Validity Index (CVI)	Content Validity Ratio (CVR)
Proportion of rural population to the total population	Statistics center	1	0.67
Official employment rate	Statistics center	0.83	0.67
The proportion of the job-seeking population (over 18 years) to the total population	Statistics center	0.83	0.83
Economic growth	Percentage of growth/decrease of GDP in each period compared to the previous period	0.916	0.83
Openness rate in business space	The growth rate of the number of business licenses issued each year compared to the previous year	1	1
Exchange rate uncertainty	Annual standard deviation of the exchange rate (US dollar) in the free market	0.83	0.67
Government employment rate	Proportion of the population employed in the public sector to the total employed population	1	0.67
Gender composition of the population	The proportion of men to women	0.83	0.67
The inflation rate	Consumer price index	1	0.83
Graduation ratio	The proportion of the educated population to the total population	1	1
Financing rural small businesses	Reports of the Central Bank of the Islamic Republic of Iran	1	0.84

Due to the estimated results for CVI and CVR values for research factors, which is greater than the acceptable value of 0.49 according to the number of research experts (16 people), the validity of these indexes in explaining entrepreneurship in small businesses is confirmed. After identifying these factors, their effectiveness or ineffectiveness on entrepreneurship in rural businesses in the service and agriculture sectors was tested.

4.1|Models and Variables

According to the results of the research expertise process and the factors identified in *Table 1*, regression models and research variables are designed and defined as follows:

Influence model on entrepreneurship in the service sector:

$$\mathrm{Srv.Ent}_t = \beta_0 + \beta_1\mathrm{Rur.Pop}_t + \beta_2\mathrm{Form.Emp}_t + \beta_3\mathrm{Empd.Pop}_t + \beta_4\mathrm{Eco.Grw}_t + \beta_5\mathrm{Bus.Opn}_t + \beta_6\mathrm{Ex.Unc}_t + \beta_7\mathrm{Gov.Emp}_t + \beta_8\mathrm{Gen.pop}_t + \beta_9\mathrm{Inf}_t + \beta_{10}\mathrm{Edu.pop}_t + \beta_{11}\mathrm{Fin.Ins}_t + \varepsilon_t\,. \tag{1}$$

Influence model on entrepreneurship in the agriculture sector:

$$\mathrm{Agr.Ent}_t = \beta_0 + \beta_1 \mathrm{Rur.Pop}_t + \beta_2 \mathrm{Form.Emp}_t + \beta_3 \mathrm{Empd.Pop}_t + \beta_4 \mathrm{Eco.Grw}_t + \beta_5 \mathrm{Bus.Opn}_t + \beta_6 \mathrm{Ex.Unc}_t + \beta_7 \mathrm{Gov.Emp}_t + \beta_8 \mathrm{Gen.pop}_t + \beta_9 \mathrm{Inf}_t + \beta_{10} \mathrm{Edu.pop}_t + \beta_{11} \mathrm{Fin.Ins}_t + \varepsilon_t. \quad (2)$$

So, to measure entrepreneurship in rural businesses in these models, the employment growth rate in each of the studied sub-sectors was used to measure the entrepreneurial content in the form of creating new jobs. The variables studied in these models are:

I. $\mathrm{Srv.Ent}_t$: growth rate of employment in the rural services sector, calculated from the percentage increase/decrease in employment in the rural service sector in each period compared to the previous period.
II. $\mathrm{Agr.Ent}_t$: growth rate of employment in the agricultural sector in the rural area, calculated from the percentage increase/decrease in employment in the agricultural sector in each period compared to the previous period.
III. $\mathrm{Rur.Pop}_t$: proportion of the rural population to the total population in year t.
IV. $\mathrm{Form.Emp}_t$: official employment rate of the country in year t.
V. $\mathrm{Empd.Pop}_t$: proportion of the job-seeking population (over 18 years) to the total population in year t.
VI. $\mathrm{Eco.Grw}_t$: percentage increase/decrease of the country's GDP in year t compared to the previous period.
VII. $\mathrm{Bus.Opn}_t$: increase/decrease rate of business licenses issued in year t compared to the previous year.
VIII. $\mathrm{Ex.Unc}_t$: the natural logarithm of the standard deviation of the exchange rate (US dollar) in the free market based on the values of the final dollar price on the last trading day of each month.
IX. $\mathrm{Gov.Emp}_t$: proportion of the population employed in the public sector to the total employed population in year t.
X. $\mathrm{Gen.pop}_t$: proportion of male to female population in year t.
XI. Inf_t: the inflation rate in year t.
XII. $\mathrm{Edu.pop}_t$: proportion of the educated population to the total population in year t.
XIII. $\mathrm{Fin.Int}_t$: proportion of the financing of rural small businesses by financial institutions to the total financing of small businesses in year t.

The results of the descriptive evaluation of the values of each variable are presented in *Table 2*.

Table 2. Descriptive evaluation of variables.

Variable	Average	Median	Standard Deviation	Minimum	Maximum
Entrepreneurship in the service sector	-0.1396	-0.1503	0.1660	-0.3554	0.1554
Entrepreneurship in the agricultural sector	-0.0651	-0.1149	0.1838	-0.3192	0.2173
Rural population ratio	0.2339	0.2242	0.0472	0.1516	0.3028
Official employment rate	0.3771	0.3617	0.0747	0.2737	0.4829
Proportion of job seekers	0.1740	0.1769	0.0507	0.0986	0.2591
Economic growth	0.0126	0.0167	0.0489	-0.0926	0.0834
Openness rate in business space	0.1381	0.1785	0.1368	-0.1351	0.3220
Exchange rate uncertainty	3.5149	3.5377	0.2106	3.2146	3.9916
Government employment rate	0.2374	0.2455	0.0508	0.1568	0.3179
Gender composition of the population	1.1122	1.0942	0.1573	0.8801	1.4237
The inflation rate	0.7687	0.6444	0.4296	0.2361	1.5824
Graduation ratio	0.2155	0.2160	0.0496	0.1310	0.2825
Financing rural businesses	0.5892	0.6325	0.0936	-0.0125	0.7536

According to the findings of this table, it can be seen that the average rate of entrepreneurship in the service and agricultural sectors during the research period is equal to 0.1396 and -0.0651, respectively, which indicates a moderate decrease in employment in each of these sections during each period compared to the previous period. On average, the ratio of the rural population to the total population is 0.2339, and the official employment rate of the country is estimated to be 0.3771. The ratio of job seekers (over 18 years old) to the total population equals 0.1740, and the economic growth index is estimated to be 0.0126 on average. The average openness of business space is equal to 0.3881, which displays the increase in the business license numbers issued in the country during each period compared to the previous period. The exchange rate uncertainty and the government employment rate averaged 3.5149 and 0.2374, respectively. The gender composition of the population shows that the ratio of men to women was 1.1122 on average. The average inflation rate and graduation ratio are equal to 0.7677 and 0.2155 sequentially. Finally, the financing of rural businesses averaged 0.59292.

Then, before fitting the regression models of the research, the hypothesis of the durability of the variables was investigated using the Augmented Dickey-Fuller test, and the results are described in *Table 3*.

Table 3. Durability test of variables.

Variable	ADF Statistic	Significance
Entrepreneurship in the service sector	-5.4243	0.0000
Entrepreneurship in the agricultural sector	-6.3773	0.0001
Rural population ratio	-4.9328	0.0001
Official employment rate	-7.3355	0.0000
Proportion of job seekers	-4.1981	0.0064
Economic growth	-6.9379	0.0000
Openness rate in business space	-3.9045	0.0086
Exchange rate uncertainty	-2.3162	0.0246
Government employment rate	-3.9762	0.0007
Gender composition of the population	-9.6340	0.0001
The inflation rate	-4.270	0.0003
Graduation ratio	-6.0501	0.0002
Financing rural businesses	-5.356	0.0002

According to the results obtained from *Table 3*, it can be seen that the significance level of the test for all variables is less than 0.05, which confirms the significance of the research variables.

Table 4 indicates the results of fitting the regression model of *Eq. (1)* to confirm the impact of identified factors on entrepreneurship in the service sector in rural areas.

Table 4. Factors influencing test on entrepreneurship in the service sector.

Variable	Impact Factor	Test Statistics	Significance	Collinearity
Rural population ratio	1.1706	13.3154	0.0000	1.4658
Official employment rate	-0.0582	-3.7676	0.0000	1.4399
Proportion of job seekers	-0.0449	-5.3188	0.0000	1.5967
Economic growth	1.1432	11.0724	0.0000	2.1935
Openness rate of business space	0.2677	7.0760	0.0000	2.3834
Exchange rate uncertainty	-0.0115	-5.8875	0.0000	1.4213
Government employment rate	-1.7905	-20.1031	0.0000	1.7128
Gender composition of the population	0.1683	5.8741	0.0000	1.7860
The inflation rate	0.0858	8.4674	0.0000	1.6311
Graduation ratio	-0.5787	-6.7772	0.0000	1.5909
Financing rural businesses	1.6895	8.1890	0.0000	3.4589
Constant value	-0.1789	-1.9933	0.0000	-
The Goodness of the Fit Index				
F statistics likelihood ratio	14.54417			
Model significance	0.029232			
Adjusted coefficient of determination	0.593006			
Breusch-Pagan-Godfrey test (significance)	(0.9455) 0.328604			
Breusch-Godfrey test (significance)	(0.1811) 2.202705			
Jarque-Bera test (significance)	(0.4768) 1.481265			

The overall significance of the research regression model is confirmed by considering the significance level of the model (p-value = 0.029232), which is less than the error (0.05). Also, the significance levels of Pagan-Godfrey (p-value = 0.9455), Godfrey (p-value = 0.1811), and Jarque-Bera test (p-value = 0.4768) with values greater than 0.05 indicate the homoscedasticity and independent and normal distribution of model error terms. It could be expected that the factors identified in the research explain up to 59.3006% of the changes in entrepreneurship in the service sector, based on the determination coefficient of the model. The results of evaluating the impact of factors on entrepreneurship in the service sector show that official employment rate (beta = -0.0582), job-seeking population ratio (beta = -0.0449), exchange rate uncertainty (beta = -0.0115), government employment rate (beta = -1.7905) and graduation ratio (beta = -0.5787) had a significant reverse effect on entrepreneurship in service sector among the studied factors, while rural population ratio (beta = 1/1706), economic growth (beta = 1.1432), openness rate of business space (beta = 0.2677), gender composition of the population (beta = 0.1683), inflation rate (beta = 0.0858) and financing rate of rural businesses (beta = 1.6895), had a direct and significant impact on it. The results of measuring the effect of these factors on entrepreneurship in the agricultural sector have been described in *Table 5*.

Table 5. Factors influencing test on entrepreneurship in the agriculture sector.

Variable	Impact Factor	Test Statistics	Significance	Collinearity
Rural population ratio	1.5647	9.3754	0.0000	1.4658
Official employment rate	-1.0244	-5.1622	0.0000	1.4399
Proportion of job seekers	-2.9094	-7.8951	0.0000	1.5967
Economic growth	2.6883	5.7235	0.0000	2.1935
Openness rate of business space	1.5708	10.2078	0.0000	2.3834
Exchange rate uncertainty	-0.0825	-1.8094	0.0000	1.4213
Government employment rate	-1.2270	-4.2014	0.0000	1.7128
Gender composition of the population	0.4884	5.5151	0.0000	1.7860
The inflation rate	0.0453	3.1552	0.0000	1.6311
Graduation ratio	-3.1288	-9.5752	0.0000	1.5909
Financing rural businesses	1.6985	9.3698	0.0000	2.1945
Constant value	-0.3744	-2.8692	0.0000	-
The Goodness of the Fit Index				
F statistics likelihood ratio	43.09811			
Model significance	0.035960			
Adjusted coefficient of determination	0.750556			
Breusch-Pagan-Godfrey test (significance)	(0.4678) 1.476729			
Breusch-Godfrey test (significance)	(0.7117) 0.357206			
Jarque-Bera test (significance)	(0.4888) 1.431579			

According to *Table 5*, the significance level of the model (p-value = 0.035960) is less than the error, which was 0.05, and it shows that the overall significance of the research regression model is confirmed. A significance level of Pagan-Godfrey (p-value = 4678), Godfrey (p-value = 0.7117), and Jarque-Bera test (p-value = 0.4888) with values greater than 0.05 Confirms the initial assumptions of regression in this model. It could be expected that the factors identified in the research can explain up to 75,056.75% of the changes in entrepreneurship in the agricultural sector, based on the determination coefficient of the model. The results of evaluating the impact of factors on entrepreneurship in the agricultural sector demonstrate that the official employment rate (beta = -1.0244), job-seekers ratio (beta = -2.09494), exchange rate uncertainty (beta = -0.025), the government employment rate (beta = -1.2270) and graduation ratio (beta = -3.288) had a significant reverse effect on entrepreneurship in the agricultural sector, while rural population ratio (beta = 1.5647), economic growth (beta = 2.6883), openness rate of business space (beta = 1.5708), population gender composition (beta = 0.4884), inflation rate (beta = 0.0453) and financing rate of rural businesses (beta = 1.6895) had a direct and significant impact on entrepreneurship in this sector. These results are similar to those in the service sector and illustrate that the factors identified in the research in both sectors had the same effect regarding the types of impact on entrepreneurship.

5 | Friedman Test

Table 6 displays the results of the Friedman test to examine the financial factors on entrepreneurship in agriculture and service sectors.

Table 6. Results of the Friedman test to examine the financial factors on entrepreneurship in agriculture and service sectors.

Test Statistics Value	Degrees of Freedom	Test Error	Test Level	Test Result
503.911	4	0.01>	0.05	Validate

Table 6 shows that the value of Chi-square test statistics is 503/911 with a test error of less than 0.01, which concludes that the error level is less than 0.05. The significance of the Friedman test means that the ranking of financial factors affecting entrepreneurship in the agriculture and service sector is significant according to the research sample. *Table 7* displays the results of ranking the effective financial factors.

Table 7. Friedman results for investigating financial factors on entrepreneurship in agriculture and service sectors.

Prioritization	Factors	Ranking Average
First	Financing rural businesses	4.01
Second	The inflation rate	3.86
Third	Openness rate of the business space	3.45
Forth	Rural population ratio	2.332
Fifth	Economic growth	2.14
Sixth	Gender composition of the population	2.052

A comparison of the ranking average of financial factors affecting the entrepreneurship of rural businesses in the above table illustrates that the highest ranking average (4.01) is related to the financing of rural businesses. In other words, the growth of entrepreneurial activities in rural service and agriculture depends on design and using new financing methods for villagers.

6 | Conclusions

This study aims to explain the financial factors affecting entrepreneurship in small rural businesses. First, these factors, which include the rural population ratio, official employment rate, job-seeking population ratio, economic growth, openness rate of business, exchange rate uncertainty, government employment rate, gender composition of the population, inflation rate, graduation ratios, and financing rate, identified by interviewing and using previous research. To validate the effect of these factors, an evaluation of their impact in both the service and agriculture sectors and fitting regression models from 1379 to 1398 were used. The results showed that the official employment rate, job-seekers ratio, exchange rate uncertainty, government employment rate, and graduation ratio significantly affected entrepreneurship in the service and agriculture sectors. In contrast, the rural population ratio, economic growth, openness rate of business, gender composition, and inflation and financing rates directly and significantly impacted it.

According to the research findings, the increase in the official employment rate and the proportion of job-seekers has decreased rural entrepreneurship. It also can be concluded that the increase in official employment in the country and the job-seeking population (over 18 years old) does not necessarily cause an increase in rural entrepreneurship. Still, this employment seems more prevalent in the private and public sectors. The increase in exchange rate uncertainty will also reduce the reliability and stability of business plans, so it is expected to have a reverse effect on rural entrepreneurship because the income sources of rural businesses are not dependent on the exchange rate. In contrast, the costs of starting businesses and entrepreneurship in all areas of the country are directly related to it. Correspondingly, according to the results, the increase in government employment rates has decreased rural entrepreneurship, which shows that the employed population's acceptance of government employment has led to the loss of rural jobs and entrepreneurship in this area. Likewise, the reverse effect of the graduation ratio on rural entrepreneurship demonstrates that increasing the proportion of the educated population has reduced the tendency toward rural entrepreneurship. Therefore, with increasing job expectations, a decrease in rural entrepreneurship is also expected.

On the other hand, among the factors that have a direct impact on rural entrepreneurship, financing is the most important. Policy-makers contribute notably to developing rural entrepreneurial activities by creating financial incentives and fair distribution of resources throughout the country. In addition, the nature of the sources of financial facilities for small and medium-sized enterprises differs from that of large industries. Particular financial institutions should be defined to finance small businesses, and large institutions should be financed through the capital market.

After financing, the inflation rate is one of the most critical factors. It seems that the entrepreneur's attitude towards entrepreneurship and creating private careers is increasing with rising inflation. This impact can be attributed to the effects of inflation on public expenses, which in turn leads to a significant proportion of the population turning to entrepreneurship. Rural entrepreneurship will also be affected. These findings confirm the results of the research of Faraji Sabokbar et al. [16], Alavizade [17], and Rezaei et al. [18]. So, following the research findings, it is suggested that controlling inflation by stabilizing prices, facilitating the legal processes of registering and launching new businesses, reducing exchange rate fluctuations, educational policies to increase the capabilities of graduates, and increasing the proportion of the rural population by creating incentive schemes (to migrate from city to village) should be prioritized at the macro level.

Despite the importance of these results, there are also limitations in this study. This article focuses only on the financial factors affecting the development of small rural businesses in the agriculture and service sectors. In contrast, other economic factors, such as the degree of development of countries and the role of government, can be investigated in the future.

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A Mathematical Model of the Location Problem for Central and Secondary Warehouses in the Multi-Level Supply Chain Network of Perishable Products

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Abstract

A supply chain is a network that creates and delivers products and services to customers. In this research, the four-level supply chain in the field of food products, including the levels of supplier, customer, and central and secondary warehouses, has been investigated. A mixed integer mathematical model of the research problem is presented to minimize chain costs, including the setting up and preparation of warehouses, transportation between transfer levels, and holding products. The proposed model is developed based on constraints such as inventory, warehouse capacity, vehicle capacity, and multi-period multi-product. The decision variables are determined after solving the model, which includes the optimal number of central and secondary warehouses, the optimal amount of product transferred between the factory and central warehouse, the central warehouse and secondary warehouse, and the secondary warehouse and customer, the optimal amount of product storage in secondary warehouses, the type of vehicle for transportation between levels, and the capacity level of each product in each central warehouse. To validate the proposed model, experiments were conducted using the Kaleh company's real data in GAMS software. Finally, the sensitivity analysis of the model was carried out on two critical parameters influencing decision-making: demand and the cost of increasing the capacity of the central warehouse. The output results confirmed the validity and efficiency of the proposed model.

Keywords: Mathematical model, Multi-product supply chain, Secondary warehouses, Transportation.

1 | Introduction

Applying the Supply Chain Management (SCM) approach in organizations has led to high profits, remarkable financial savings, and significant progress in the last 40 years. In addition, customers have benefited too. That is why this approach has been especially accepted among different organizations and countries, which itself demands more popularity every day. SCM is the management of supply chain activities to maximize customer value and achieve sustainable competitive advantage. Supply chain activities cover everything from product

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development, sourcing, manufacturing, and logistics, as well as the information systems needed to coordinate these activities. SCM offers opportunities for the product with a positive approach to integration and management within and between companies. Based on the advantages of commercial processes, SCM provides a new method for managing commercial activities and communicating with the other members of the supply chain. Therefore, the concept of SCM has been redefined from the integration of logistic services to the integration and management of key commercial processes along the supply chain.

One of the most fundamental strategic decisions is the design of the supply chain network. The research aims to design a four-level supply chain, including supplier, customer, and central and secondary warehouses. Transportation is the most expensive distribution factor in many industries. Transportation costs include moving costs from factories to warehouses and from warehouses to customers. Transportation costs comprise an average of 45% of total costs in all industries, and another essential factor in this regard is holding costs, which include approximately 19% of total costs. Therefore, the supply chain design of this research should be performed in a multi-period, multi-product environment to minimize the costs of transportation, preparation or setup of multi-level warehouses, and holding products.

A schematic representation of the research problem is shown in *Fig. 1*. As can be seen, the suppliers transport the products by vehicles to the central warehouses. For the transportation of products, four different types of vehicles with predetermined capacity have been used. The products are then transferred from central warehouses to secondary warehouses and finally to customers. One of the objectives of this research is to determine the type and number of vehicles required to transport products from the factory to the central warehouse, from the central warehouse to the secondary warehouse, and from the secondary warehouse to the customers (movement levels).

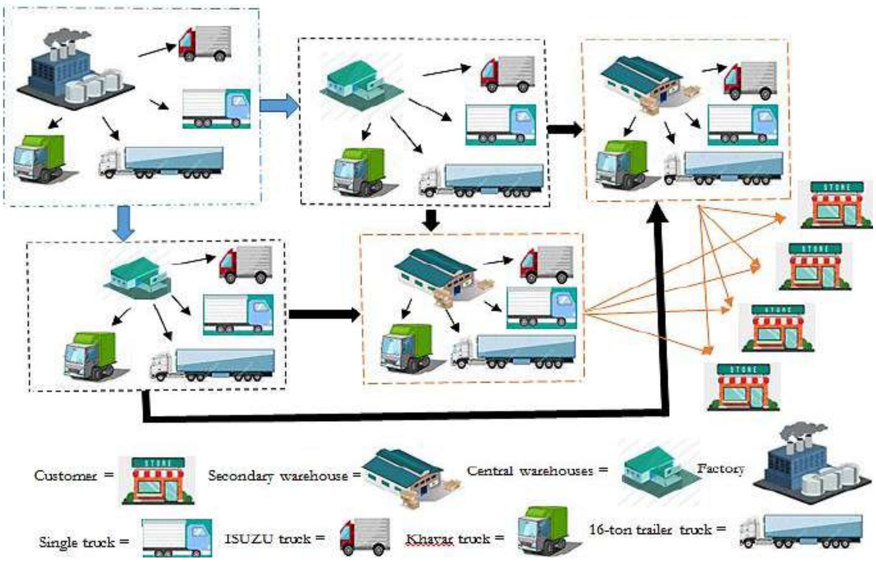

Fig. 1. Research problem schematic.

One of the strategic and vital decisions in supply chain design is determining the optimal number and location of both central and secondary warehouses because it improves processes, reduces costs, and increases customer service. In the research model, it is assumed that there are already warehouses in some places and that it is possible to re-select and change their capacity levels. It should be noted that several capacity levels have been considered for central warehouses, and the optimal capacity level is determined by solving the model. In addition, some new locations have the potential to be warehouses that can be built by investing. Therefore, we have the costs of increasing the capacity level or establishing warehouses. A further goal of this research is to determine the optimal number and location of warehouses (central and secondary) among the previously specified options that minimize the cost of increasing the capacity level, construction, and transportation.

Therefore, the main research question is how to design a multi-period, multi-product supply chain with multi-level warehouses and different vehicles to transfer products from the supplier to the warehouse and finally to the customers to minimize the total chain costs. In this study, we attempted to control the cost components in the supply chain by providing a mathematical model and determining optimal policies to design an efficient and effective supply chain network.

The remainder of the paper is organized as follows. Section 2 provides a review of relevant literature, and at the end, the gap, research model, and assumptions are described. In Section 3, the proposed mathematical model of the research is presented. In Section 4, a case study, implementation, and sensitivity analysis are presented. Finally, Section 5 presents the conclusions and future studies.

2|Literature Review

The scientific and academic attitude toward the subject of SCM has been very positive, and many papers and books have been written in this field in recent years. The literature review section refers to several instances of papers on multi-level chains with the title of Location Problem (LP) in various fields.

Mayer and Wanger [1] studied an unbounded multiple distribution of LP. They presented a new branch-and-bound method called HubLocator to generate optimal solutions. The lower bound, which was the main component of their method, was determined in two steps. The upper bounds were calculated by considering complementary slackness conditions. Marianov and Serra [2] developed a distribution center location model to minimize total costs, including fixed and transportation costs. Due to the computational complexity of the formulation, the model was solved using a heuristic based on tabu search. Martin and Roman [3] developed a mathematical model that analyzed the competition for hub location when airlines operate in illegal intercontinental markets. The entire process was considered to be dynamically a complete game. Topcuoglu et al. [4] presented a novel and robust solution based on a genetic search framework for the uncapacitated single allocation hub LP.

Yaman et al. [5] presented a 0-1 Mixed Integer Programming (MIP) model for ground-transportation-based parcel-delivery services with stopovers for the latest arrival hub LP. Azadeh et al. [6] presented an integrated hierarchical approach for the location of solar plants using principal component analysis, data envelopment analysis, and numerical taxonomy. The implementation of their proposed approach led to the selection of the best possible location to build a solar power plant at the lowest cost. Contreras et al. [7] presented a hub tree LP that combined several aspects of the investigated problem, including location, network design, and routing. They proposed an MIP formulation to minimize transportation costs through the network. Sender and Clausen [8] presented a new model for the hub LP to design a wagon load traffic network. The strategic objective was to determine the location, size, and function of the formation yards and the connections between them.

Gelareh and Nickel [9] investigated an uncapacitated multiple allocation hub LP for urban transport and liner shipping networks. They described the problem in the form of a mathematical model and proposed an efficient greedy heuristic to solve the NP-hard problem under consideration. Alumur et al. [10] studied hub LPs under uncertainty in setting costs and demands. They presented generic models representing these different sources of uncertainty for the single and multiple allocation versions of the problems. Vera et al. [11] proposed an MIP formulation to determine the location of a fixed number of facilities to maximize their utilization. They applied the heuristic concentration integer procedure to solve larger instances of the problem. Chang et al. [12] investigated the LP of Taiwanese service apartments. The fuzzy Delphi method, analytic network process, and technique for order preference by similarity to the ideal solution were integrated to select the optimal locations effectively.

Darestani and Hemmati [13] designed a supply chain network for perishable products under uncertain conditions. The objective function considered for the proposed model included minimizing total network costs and greenhouse gas emissions. Three multi-criteria decision-making methods were used to solve a two-objective model. Aazami and Saidi-Mehrabad [14] developed a new multi-period production–distribution planning method for fixed-life perishable products in a seller-buyer system. A hierarchical heuristic approach based on the benders' decomposition algorithm and genetic algorithm was proposed to maximize the seller's profit subject to the buyer's optimality in a three-level supply chain, including factories, distribution centers, and retailers. Tavana et al. [15] developed a bi-objective MILP model to solve location-inventory-routing problems in green supply chains with low-carbon emissions under uncertainty. The proposed model was solved using a weighted fuzzy multi-objective solution approach coupled with an intelligent simulation algorithm to ensure the feasibility of the solution space. Shavarani et al. [16] developed a multi-objective mathematical model to calculate the optimal number and location of facilities among a set of candidate locations to simultaneously minimize the total travel distance, costs, and lost demand.

Liu et al. [17] presented a comprehensive study aimed at providing a combined framework for the location of emergency facilities in transportation networks. Ghasemi and Khalili Damghani [18] proposed a robust simulation-optimization approach for multi-period location-allocation-inventory planning before a disaster that determined the location of distribution centers and suppliers and how to allocate them to affected areas. Liu et al. [19] proposed an integrated location-inventory-routing model for perishable products, which considered carbon emission factors and product freshness. A multi-objective planning model was developed to achieve the lowest economic cost and carbon emissions and the highest product freshness. Constraints were established regarding the actual location-inventory-routing situation. Azami et al. [20] developed a bi-objective optimization model for integrated production–distribution planning of perishable products under uncertainty. The objective functions were to maximize profit in a specific supply chain with three levels of plants, distribution centers, and customers and to minimize gas emissions. Robust optimization was used to deal with the operational uncertainty of some cost parameters. Acevedo-Chedid et al. [21] presented a transport model incorporating a cross-dock system to deliver goods from production plants to markets efficiently. The model incorporated a vehicle routing model that considered time windows for pick-ups and deliveries, optimal cross-dock center locations, a heterogeneous vehicle fleet of limited capacity, and scheduling product collections, arrivals, and departures. They proposed a mixed-integer non-linear optimization model to effectively minimize logistics costs and environmental impacts by considering various parameters such as speed, waiting times, loading and unloading times, and costs associated with the entire operation.

The literature review is summarized in *Table 1*. As recently reviewed, the supply chain problem has received more attention because of its academic importance and wide application in the real world. Although there have been studies on the problem of the supply chain with characteristics such as multi-period, multi-product, the number and location of central and secondary warehouses with limited capacity, and holding products individually or a combination of several characteristics mentioned in these years, the advantage of the present research is to consider the mentioned characteristics simultaneously. In addition, several capacity levels are defined for each product in each central warehouse, and the optimal capacity level for each product must be determined. In addition, different types of vehicles are considered between movement levels. To reduce transportation costs, the right vehicle should be chosen according to the amount of product movement. Therefore, the problem is considered more realistically and becomes an integrated SCM problem. The assumptions of the research problem are presented as follows.

The supply chain consists of four levels, including the supplier, central warehouse, secondary warehouse, and customer. In addition, this multi-product and multi-period supply chain is considered. The geographic locations of customer and supplier locations are determined outside the scope of the research model. Still, the locations of the central and secondary warehouses are determined by the model from among the previously specified options. The central warehouse is considered to have multiple capacities, and the model is only allowed to use a maximum of one capacity level. There are several transportation modes for moving products between levels. The capacity of the vehicles is known, but the model determines the number of vehicles needed. In this study, the possibility of a vehicle breakdown is not considered. In addition, there is no shortage of products in this model. It is possible to store products for future periods. The proposed model is single-objective and minimizes costs.

One of the essential issues in the supply chain is how to communicate optimally between the levels of the chain and transfer products between them. To design a supply chain with maximum efficiency, the optimal values of the system parameters at the macro level and the optimal policies at the lower levels of the chain must be achievable. In the following section, the operational research model of the above system is described using mathematical modeling techniques.

Table 1. Summary of the reviewed papers.

First Author and the Year	Problem Statements Period							Objectives		Modeling Approach	Case Study
	Single	Multi	Multi-Product	Holding Products	Warehouse with Multi-Level Capacity	Different Types of Vehicles	Secondary Warehouse	Single	Bi/ Multi		
Mayer and Wagner [1]	×					×		×		Linear programming	
Marianov and Serra [2]	×				×			×		Integer programming optimization	
Topcuoglu et al. [4]	×							×		Mixed integer linear programming	
Yaman et al. [5]	×					×		×		Zero-one MIP	×
Azadeh et al. [6]	×							×		Data envelopment analysis	×
Gelareh et al. [9]	×							×		MIP	×
Alumur et al. [10]	×								×	Robust optimization	×
Aros-Vera et al. [11]	×							×		Mixed linear programming	
Avakh Darestani et al. [13]		×	×	×	×				×	Multi-objective optimization problems	
Tavana et al. [15]		×	×	×					×	Bi-objective mixed integer linear programming	
Aazami et al. [14]		×		×	×			×		Mixed-integer non-linear bi-level programming	×
Shavarani et al. [16]	×								×	Mixed-integer non-linear programming	×
Ghasemi and Khalili-Damghani [18]		×		×	×			×		Multi-period location–allocation-inventory problem	×
Aazami et al. [20]		×	×	×	×				×	Bi-objective optimization model	×
Liu et al. [17]		×	×	×	×	×			×	Multi-objective problem	×
Acevedo-Chedid et al. [21]						×			×	Mixed-integer non-linear optimization	×
This research		×	×	×	×	×	×	×		Mixed integer mathematical model	×

3 | Proposed Mathematical Model

A 0-1 MIP model has been presented to describe the problem in more detail. The purpose of presenting the MIP formulation of the research problem is not to use the formulation in developing a solution procedure but to precisely describe the problem. The indices, input parameters, decision variables, and mathematical model are as follows.

Indices

Indices used to model our problem are listed below:

i	Index for products, $i = 1, 2, 3, \dots, I$.
d	Index for central warehouses, $d = 1, 2, 3, \dots, D$.
s	Index for secondary warehouses $s = 1, 2, 3, \dots, S$.
c	Index for customers, $c = 1, 2, 3, \dots, C$.
m	Index for the mode of transportation, $m = 1, 2, 3, \dots, M$.
k	Index for capacity levels, $k = 1, 2, 3, \dots, K$.
t	Index for periods, $t = 1, 2, 3, \dots, T$.

Input parameters

$cost_{ikd}^{dist}$	Cost of setting up/increasing capacity for central warehouse d at capacity level k for product i.
$cost_{s}^{strg}$	Cost of setting up a secondary warehouse s.
cap_m	Transport mode capacity m.
cap_{is}^{strg}	Secondary warehouse capacity s for product i.
cap_{ikd}^{max}	Maximum capacity of the central warehouse d for product i at capacity level k.
h_{it}	Cost of holding product i in period t.
dem_{ict}	Demand for product i by customer c in period t.
$trn_{imdt}^{sup-dist}$	Cost of transporting each unit of product i from the factory to the central warehouse d with the mode of transportation m in period t.
$trn_{imdst}^{dist-strg}$	Cost of transporting each unit of product i from the central warehouse d to the secondary warehouse s with transportation mode m in period t.

Decision Variables

X_{ikd}^{dist}	1 if the capacity level k of the central warehouse d is set up for product i and 0 otherwise.
X_{s}^{strg}	1 if secondary warehouse s is set up and 0 otherwise.
$x_{imdt}^{sup-dist}$	Number of product i transferred from factory to central warehouse d by transportation mode m in period t.
$x_{imdst}^{dist-strg}$	Number of product i transferred from central warehouse d to secondary warehouse s by transportation mode m in period t.
$x_{imsct}^{strg-cus}$	Number of product i transferred from secondary warehouse s to customer c by transport mode m in period t.
inv_{ist}	Number of product i stored in secondary warehouse s in period t.
f_m	Number of transport modes m to move the product from the factory to the central warehouse.
z_m	Number of transport modes m to move the product from the central warehouse to the secondary warehouse.
y_m	Number of transport modes m to move the product from the secondary warehouse to the customer.

3.1 | Mathematical Formulation

The problem can now be formulated as

$$\text{Min } Z=\sum_{i,k,d} cost_{ikd}^{dist}\times x_{ikd}^{dist}+\sum_{s} cost_{s}^{strg}\times x_{s}^{strg}+\sum_{i,m,d,t} trn_{imdt}^{sup-dist}\times x_{imdt}^{sup-dist}+\sum_{i,m,d,s,t} trn_{imdst}^{dist-strg}\times x_{imdst}^{dist-strg}$$
$$+\sum_{i,m,s,c,t} trn_{imsct}^{strg-cus}\times x_{imsct}^{strg-cus}+\sum_{i,s,t} inv_{ist}\times h_{it}.$$

$$\sum_{m} x_{imdt}^{sup-dist}+bigm\times(1-x_{ikd}^{dist})\geq cap_{ikd}^{max},\qquad \text{for all } i,k,d,t. \quad (1)$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq cap_{i(k+1)d}^{max}+bigm\times(1-x_{ikd}^{dist}),\qquad \text{for all } i,k,d,t. \quad (2)$$

$$\sum_{k} x_{ikd}^{dist}\leq 1,\qquad \text{for all } i,d. \quad (3)$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq \sum_{k} cap_{ikd}^{max},\qquad \text{for all } i,d,t. \quad (4)$$

$$\sum_{m,s} x_{imdst}^{dist-strg}\leq \sum_{k} cap_{ikd}^{max},\qquad \text{for all } i,d,t. \quad (5)$$

$$\sum_{m,d} x_{imdst}^{dist-strg}\leq cap_{is}^{strg},\qquad \text{for all } i,s,t. \quad (6)$$

$$inv_{ist}=inv_{is(t-1)}+\sum_{m,d} x_{imdst}^{dist-strg}-\sum_{m,c} x_{imsct}^{strg-cus},\quad \text{for all } i,s,t>1. \quad (7)$$

$$inv_{is1}=\sum_{m,d} x_{imds1}^{dist-strg}-\sum_{m,c} x_{imsc1}^{strg-cus},\qquad \text{for all } i,s. \quad (8)$$

$$\sum_{m} x_{imdt}^{sup-dist}\geq \sum_{m,s} x_{imdst}^{dist-strg},\qquad \text{for all } i,d,t. \quad (9)$$

$$\sum_{m,s} x_{imsct}^{strg-cus}\geq dem_{ict},\qquad \text{for all } i,c,t. \quad (10)$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq bigm\times\sum_{k} x_{ikd}^{dist},\qquad \text{for all } i,d,t. \quad (11)$$

$$\sum_{m} x_{imdst}^{dist-strg}\leq bigm\times x_{s}^{strg},\qquad \text{for all } i,d,s,t. \quad (12)$$

$$\sum_{i,m,d} x_{imdt}^{sup-dist}\leq \sum_{m} cap_{m}\times f_{m},\qquad \text{for all } t. \quad (13)$$

$$\sum_{i,m,d,s} x_{imdst}^{dist-strg}\leq \sum_{m} cap_{m}\times z_{m},\qquad \text{for all } t. \quad (14)$$

$$\sum_{i,m,s,c} x_{imsct}^{strg-cus}\leq \sum_{m} cap_{m}\times y_{m},\qquad \text{for all } t. \quad (15)$$

The objective function is formulated in the form of cost minimization, which includes the cost of setting up both the central and secondary warehouses, the cost of transferring products between levels, and the cost of holding. *Constraint Sets (1)* and *(2)* determine the level of central warehouse capacity that should be used for each product. *Constraint Set (3)* ensures that approximately one level of central warehouse capacity is used. *Constraint Sets (4)-(6)* ensure that the capacity of the central and secondary warehouses is not exceeded. *Constraint Sets (7)* and *(8)* are related to the inventory balance and the determination of the inventory amount of each product in each period. *Constraint Set (9)* states that the output of each central warehouse should not exceed its input. *Constraint Set (10)* guarantees that customers' demands are satisfied. The condition of product transfer between levels is that the central and secondary warehouses have been set up, which are given in *Constraint Sets (11)* and *(12)*, respectively. Non-exceeding the capacity of the transportation mode (vehicles) and determining the number of each of them between the levels of the supplier to the central warehouse, the central warehouse to the secondary warehouse, and the secondary warehouse to the customer are given in *Constraint Sets (13)-(15)*, respectively.

4 | Numerical Result

For the evaluation of the proposed model, a numerical experiment was performed on the data set of the Kaleh company. The dataset related to Kaleh company is as follows: five categories of frozen products, including dairy, sauces, drinks, ice creams, and meats; four modes of transportation, including 16-ton trailer truck, single truck, Khavar truck, and ISUZU truck; three potential places for central warehouses in the cities of Tehran, Karaj, and Qazvin; four potential places for secondary warehouses in East Tehran, West Tehran, Karaj, and Qazvin; three levels of capacity for the central warehouse; and finally 7 time periods (information about one week). Based on the case study data, the model was coded and solved using GAMS optimization software.

4.1 | Results Based on the Model Output

The output of the model for the objective function is equal to 239092847620 Iranian currency units, and the values of the problem decision variables for this value of the objective function are as follows:

According to the output of the model, the value of decision variables $X_{1.1.1}^{dist}$, $X_{2.2.1}^{dist}$, $X_{3.2.1}^{dist}$, $X_{4.2.1}^{dist}$, and $X_{5.2.1}^{dist}$ is equal to one. These results show that first, the central warehouse located in Tehran (d=1) should be set up. Second, product 1 should use the capacity of level 1, and products 2, 3, 4, and 5 should use the capacity of level 2 of this central warehouse. The value of decision variables X_1^{strg} and X_2^{strg} is equal to one, which means that out of four potential secondary warehouses in East Tehran, West Tehran, Karaj, and Qazvin, two secondary warehouses should be established in East Tehran and West Tehran.

Table 2 shows the amount of each product transferred between the factory and central warehouse levels ($X_{imdt}^{sup-dist}$). From this table, it can be seen that only the central warehouse of Tehran (d=1) should be set up, and only mode 1 of transportation (m=1), i.e., a 16-ton trailer, should be used to transport products from the factory to the central warehouse of Tehran. The number of 16-ton trailers needed to move the products is obtained based on the variable output f_m, which is equal to 29 here ($f_1 = 29$).

Table 3 shows the amount of each product transferred between the levels of the central and secondary warehouses ($X_{imdst}^{sup-dist}$). From this table, it can be seen that two potential secondary warehouses in Tehran East and Tehran West (s=1 and 2) should be set up. Only mode 1 of transportation (m=1), i.e., a 16-ton trailer, should be used to transport products from the central warehouse of Tehran to the secondary warehouses in Tehran East and Tehran West. The number of 16-ton trailers needed to move the products is obtained based on the variable output z_m, which is equal to 27 here ($z_1 = 27$).

Table 4 shows the storage amount of each product transferred in each period in each of the secondary warehouses (inv_{ist}). *Table 5* shows the amount of each product between the secondary warehouse and the customer ($X_{imsct}^{strg-cus}$). From this table, it can be seen that modes 3 and 4 of transportation (m=3 and 4), i.e., Khavar and ISUZU trucks, have been used to transport products from the secondary warehouses in Tehran East and Tehran West to the customer. The number of Khavar trucks (m=3) and ISUZU trucks (m=4) needed to move the products are obtained based on the output of variable y_m, which are equal to 136 and 269, respectively ($y_3 = 136$ and $y_4 = 269$).

Table 2. The amount of product transferred between the factory and the central warehouse.

i	m	d	t						
			1	2	3	4	5	6	7
1	1	1	400000	400000	400000	400000	400000	400000	400000
2	1	1	11000	11000	11000	11000	11000	11000	11000
3	1	1	12200	12200	12200	12200	12200	12200	12200
4	1	1	15100	15100	15100	15100	15100	15100	15100
5	1	1	18200	18200	18200	18200	18200	18200	18200

Table 3. The amount of product transferred between the levels of central and secondary warehouses.

i	m	d	s	t						
				1	**2**	**3**	**4**	**5**	**6**	**7**
1	1	1	1	200000	200000	200000	200000	200000	144881	200000
1	1	1	2	181353	137804	166466	181353	160265	0	0
2	1	1	1	5500	5500	5500	5500	5500	5456	0
2	1	1	2	4225	3227	2566	2051	0	0	0
3	1	1	1	6500	6500	6500	6500	6500	4411	6500
3	1	1	2	4946	4770	4946	4946	4946	0	0
4	1	1	1	7000	7000	7000	7000	7000	7000	65
4	1	1	2	4909	5358	0	0	0	0	0
5	1	1	1	8800	8800	8800	8800	8800	7899	7177
5	1	1	2	6037	5661	6933	4730	8041	0	0

Table 4. The storage amount of products in secondary warehouses in each period.

i	s	t					
		1	**2**	**3**	**4**	**5**	**6**
1	1	2226	0	0	27693	0	2368
2	1	0	892	1880	527	0	3688
2	2	0	0	0	835	0	0
3	1	0	2346	2585	1280	0	318
4	1	2650	515	0	2123	1217	0
4	2	0	5320	5044	1085	0	0
5	1	0	966	2568	0	0	0

Table 5. The amount of product transferred between the secondary warehouse and the customer.

i	m	s	c	t						
				1	**2**	**3**	**4**	**5**	**6**	**7**
1	3	1	1	0	0	0	0	0	13820	0
1	3	1	2	10517	0	79065	0	0	0	0
1	3	1	3	0	104083	4281	976	65114	0	95113
1	3	1	4	0	11632	103506	130681	144042	25586	0
1	3	2	1	0	10796	0	0	11856	0	0
1	3	2	2	0	0	0	0	75130	0	0
1	3	2	3	0	0	0	124581	0	0	0
1	3	2	4	0	114584	0	0	0	0	0
1	4	1	1	2904	2300	13148	13059	2527	0	0
1	4	1	2	0	84211	0	27591	16010	79958	0
1	4	1	3	39808	0	0	0	0	23149	0
1	4	1	4	144545	0	0	0	0	0	107255
1	4	2	1	13625	0	0	0	0	0	0
1	4	2	2	79354	0	0	56772	0	0	0
1	4	2	3	88374	12424	110088	0	73279	0	0
1	4	2	4	0	0	6378	0	0	0	0
2	3	1	1	44	0	198	0	0	0	0
2	3	1	2	0	1457	1386	0	178	1033	0
2	3	1	3	0	2455	0	3066	2592	0	1770
2	3	1	4	4109	0	291	0	3043	317	1918
2	3	2	1	207	0	0	0	0	0	0
2	3	2	2	0	0	0	1216	0	0	0
2	3	2	4	0	0	2566	0	0	0	0
2	4	1	1	0	232	0	337	214	137	0
2	4	1	2	232	0	0	259	0	0	0
2	4	1	3	1115	0	2637	0	0	281	0
2	4	1	4	0	464	0	3191	0	0	0
2	4	2	2	1489	0	0	0	835	0	0

Table 5. Continued.

i	m	s	c	t						
				1	2	3	4	5	6	7
2	4	2	3	2529	249	0	0	0	0	0
2	4	2	4	0	2978	0	0	0	0	0
3	3	1	1	0	0	0	293	0	399	0
3	3	1	2	0	0	0	1762	1435	1629	0
3	3	1	3	750	1822	555	883	4972	0	0
3	3	1	4	0	483	0	4867		0	0
3	3	2	1	0	0	115	0	200	0	0
3	3	2	2	1281	0	0	0	0	0	0
3	3	2	3	0	1503	4105	4144	0	0	0
3	3	2	4	0	3267	0	0	4746	0	0
3	4	1	1	303	167	24	0	43	0	0
3	4	1	2	166	1682	1558	0	0	0	0
3	4	1	3	0	0	0	0	0	991	2932
3	4	1	4	5281	0	4124	0	1330	1074	3886
3	4	2	3	3516	0	0	0	0	0	0
3	4	2	4	149	0	726	802	0	0	0
4	3	1	1	554	1085	82	0	373	274	0
4	3	1	2	0	0	39	0	52	0	0
4	3	1	3	0	4182	0	0	4454	3874	31
4	3	1	4	0	0	0	644	3027	4032	34
4	3	2	1	0	0	0	337	0	0	0
4	3	2	2	80	38	0	0	0	0	0
4	3	2	3	3619	0	0	0	0	0	0
4	3	2	4	0	0	0	0	1085	0	0
4	4	1	1	0	0	0	72	0	0	0
4	4	1	2	17	8	0	62	0	37	0
4	4	1	3	771	0	3401	4099	0	0	0
4	4	1	4	3008	3860	3993	0	0	0	0
4	4	2	1	0	0	276	0	0	0	0
4	4	2	4	1210	0	0	3622	0	0	0
5	3	1	1	0	414	0	0	0	0	0
5	3	1	2	0	0	249	0	333	1394	0
5	3	1	3	0	930	0	6265	0	2919	0
5	3	1	4	0	6213	6360	0	7440	3162	0
5	3	2	1	0		0	0	295	0	0
5	3	2	2	0	1298	0	0	1562	0	0
5	3	2	3	0	4363	5367	0	6184	0	0
5	3	2	4	0	0	0	4730	0	0	0
5	4	1	1	583	0	85	493	63	424	0
5	4	1	2	112	277	0	1367	0	0	0
5	4	1	3	1292	0	504	0	964	0	3445
5	4	1	4	6813	0	0	3243	0	0	3732
5	4	2	1	0	0	399	0	0	0	0
5	4	2	2	1525	0	1167	0	0	0	0
5	4	2	3	4512	0	0	0	0	0	0

4.2 | Sensitivity Analysis of the Demand Parameter

Demand is one of the most important parameters whose changes affect supply chain decisions and costs. In this section, a sensitivity analysis is performed on the product demand parameter to examine the validity of the proposed model more precisely. Two categories of scenarios for demand, one based on demand reduction (demands less than the initial value) and another based on demand increase (demands greater than the initial value), are considered for sensitivity analysis.

In the first scenario, it is expected that the value of the objective function will not deteriorate (that is, it will not increase) as demand decreases. The justification for cost reduction is that a decrease in demand decreases the movement of products in the chain. Therefore, the costs of transporting products are reduced accordingly.

For the construction of the first problem of this scenario, the demand for the products is reduced to 95% of the initial value ($0.95 \times \text{dem}_{ict}$). The model with new demand information was implemented in the software, and the output of its objective function reached 227189486340. The demand for products is reduced to 90, 85, 80, 75, and 70% of the initial value, respectively, due to the construction of the problems 2, 3, 4, 5, and 6 of the first scenario. The output of the objective function has reached the numbers 215278812940, 203369943320, 191458924220, 179548583840, and 167640015280, respectively.

The change trend in the objective function for the demand reduction scenario is shown in *Fig. 2*. It can be seen that as demand decreases, the value of the objective function decreases, too. Therefore, the results of the model obtained in the sensitivity analysis were consistent with reasonable and logical expectations.

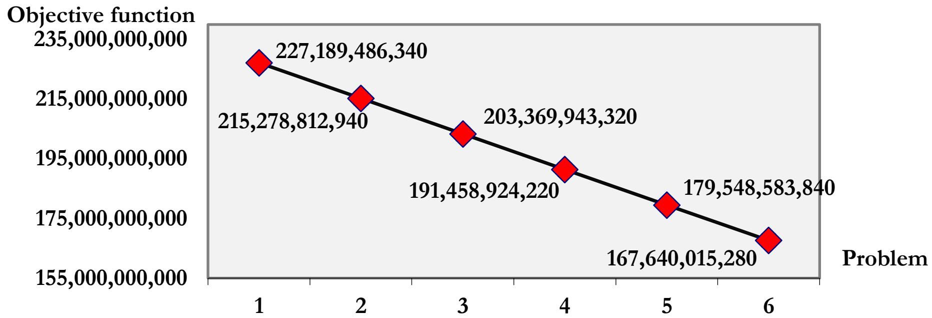

Fig. 2. The value of the objective function in the demand reduction scenario.

In the second scenario, it is expected that the value of the objective function will not improve (that is, it will not decrease) as demand increases. The justification for the rise in cost is that the increase in demand increases the movement of products in the chain. Therefore, the costs of transporting products are increased accordingly.

For the construction of the first problem of this scenario, the demand for the products is increased by 2% of the initial value ($1.02 \times \text{dem}_{ict}$). The model with new demand information was implemented in the software, and the output of its objective function reached 243862966880. The demand for products is increased by 4, 6, and 8% of the initial value, respectively, due to the construction of problems 2, 3, and 4 of the second scenario, and the output of the objective function has reached the numbers 248626575100, 253391115640, and 258156712520, respectively. The equivalent model with an increase in demand by 10% of the initial value was recognized as infeasible by the software.

The trend of change in the objective function for the demand increase scenario is shown in *Fig. 3*. It can be seen that as demand increases, the value of the objective function increases, too. Therefore, the results of the model obtained in the sensitivity analysis were consistent with reasonable expectations.

Based on the results of the two categories of proposed scenarios, it was found that by changing the parameters, the model changes logically and correctly. This change shows the effectiveness and guarantees the validity of the proposed model.

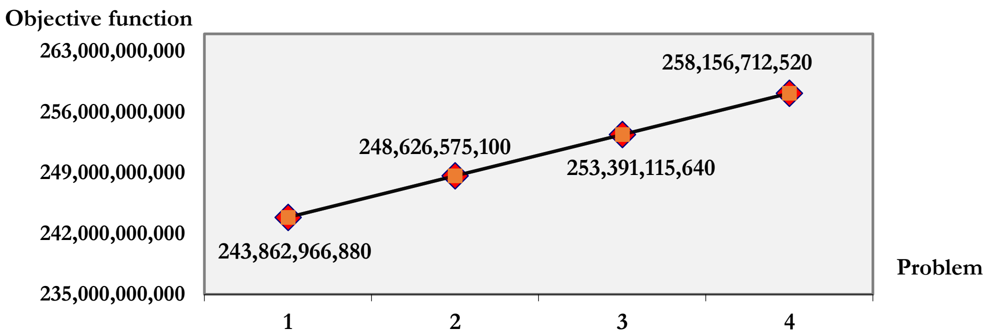

Fig. 3. The value of the objective function in the scenario of increasing demand.

4.3 | Sensitivity Analysis of the Cost Parameter

In this section, sensitivity analysis is performed based on the cost of increasing the capacity of the central warehouse to examine the validity of the proposed model through the cost parameter. To construct problems, the central warehouse capacity costs are increased by 50, 75, 100, 125, 150, 175, and 200% of the initial value to observe how the model behaves with regard to these changes. The model with new cost information was implemented in the software, and the output of the objective function reached 251047490001, 260611203905, 282129560191, 298866259525, 310820701906, and 327557201239, respectively. It can be seen that the increase in the cost of the central warehouse has increased the value of the objective function of the system.

Because the value of the objective function is the sum of several costs, the part related to the cost of increasing the capacity level in the central warehouse has been reduced (due to the higher cost of doing this than before). Instead, the sections related to the costs of setting up the secondary warehouse (due to the construction of a new warehouse), the cost of holding products in the secondary warehouse (due to the completion of the capacity of the central warehouse and the rapid transfer of products from the central warehouse to the secondary warehouse), and the cost of transportation (due to the rapid transfer of products from the central warehouse to the secondary warehouse with vehicles whose capacity level has not been completed) have increased. The changes in costs have been such that the increase in costs was more than the decrease in costs; therefore, the total cost of the whole system has increased.

5 | Conclusions

Supply chain network design is one of the most fundamental strategic decisions. This research aimed to design a four-level supply chain, including supplier, customer, and central and secondary warehouses. The advantage of the present research is that it simultaneously considers characteristics such as multi-period, multi-product, the number and location of central and secondary warehouses with limited capacity, and holding products. In addition, several capacity levels were defined for each product in each central warehouse, and the optimal capacity level for each product in the central warehouse was determined. In addition, different types of vehicles were considered for different movement levels. For reducing transportation costs, the best vehicle should be determined according to the amount of product movement. Therefore, the supply chain design of this research should be performed in a multi-period, multi-product environment to minimize the costs of transportation, preparation, and setting up the multi-level warehouses and holding products. To design a supply chain with maximum efficiency, the optimal values of the system parameters at the macro level and the optimal policies at the lower levels of the chain must be achievable. The set of assumptions, constraints, and objectives of the problem were developed and integrated to achieve the aim of the research. It has been attempted to control the cost components in the supply chain by providing a mathematical model and determining optimal policies so that an efficient and effective supply chain network can be designed. The decision variables were determined after solving the model with the real data of Kaleh company, which include the optimal number of central and secondary warehouses, the optimal amount of product transferred between the factory and central warehouse, the central warehouse and secondary warehouse, and the secondary warehouse and customer, the optimal amount of product storage in secondary warehouses, the type of vehicle for transportation between levels, and the capacity level of each product in each central warehouse. Finally, a sensitivity analysis was performed on the demand and the cost of increasing the capacity of the central warehouse parameters, which is essential in decision-making. In fact, this research has developed a decision support tool for the Kaleh company. Many companies and organizations make critical decisions using the experiences of their managers, some of which may fail and harm the organization. Nowadays, organizations cannot rely only on the genius and experience of managers to make decisions; having support tools can significantly help organizations. In the developed model, it is sufficient to provide updated information about the company as input to the system and to receive the results as output. For future research, it is suggested to consider indicators related to green criteria and sustainable development criteria using multi-objective optimization techniques.

Conflicts of Interest

All co-authors have read and agreed with the contents of the manuscript, and there is no financial interest to report. We certify that the submission is original work and is not under review by any other publication.

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Development of a Lean Maturity Assessment Model Using Interval-Valued Spherical Fuzzy AHP Method

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Abstract

Continuously adding value to a company's products and services is inevitable in adapting to this evolving and challenging global market. That is why lean philosophy is becoming increasingly important and popular among companies, and they are relying more and more on it. It not only assists in increasing profitability and quality by eliminating all processes that provide no value to the customer but also enables increased flexibility in production and productivity. In this study, the criteria affecting the Lean Maturity Level (LML) were determined, and a lean maturity measurement model, which helps companies define and understand the level of lean maturity and lean effectiveness, was developed. A recently completed case study included data from an online survey with 116 questions, which were conducted on 187 middle to senior-level professionals in Türkiye from different industries. In this model, 9 main and 14 sub-lean criteria were generated to determine LML , and each criterion was weighted based on the assessments of experts. In this paper, the interval-valued spherical fuzzy AHP method is applied for the very first time to the weighting of the criteria of a lean maturity assessment model. After collecting data through an online survey study, Confirmatory Factor Analysis (CFA) in the IBM SPSS AMOS V26 program was applied to test the model fit, validity, and reliability. To determine the LMLs, the leveling scale (understanding, implementation, improvement, and sustainability) was used from the model for LMLs in manufacturing cells. As a result of the analysis of the survey results obtained from the participating companies, the overall LML was calculated as 2.55 out of 4. This result corresponds to the level 3 - improvement range on the leveling scale. The lean maturity success rate of surveyed companies was set at 64%.

Keywords: Lean manufacturing, Lean maturity assessment model, Interval-valued spherical fuzzy analytic hierarchy process, Confirmatory factor analysis.

1 | Introduction

In today's world, companies and other organizations always aim to perform better to survive and become more competitive. This goal leads them to seek more efficient production and management systems in the constantly changing conditions and today's competitive environment. To help companies achieve these goals, a series of tools, methodologies, and models are offered designed to improve organizations and achieve higher

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business performance. The Lean Management approach seems to be the most effective in terms of achieving significant productivity improvement in a relatively fast manner [1], considering that lean thinking and lean concepts are of great importance in a constantly evolving global market where information technologies are at the forefront. The main goal of lean thinking is to eliminate waste and continuously create more value for resources and processes. In today's competitive environment, companies must address the concepts of lean maturity in detail to make themselves stand out and assure continuity by increasing the level of lean maturity. They can only extend their quality and life cycle in these competitive conditions by measuring their performance through the methods they determine their Lean Maturity Levels (LMLs). In addition, an increase in quality levels, process and production improvement, customer satisfaction, and inventory control were the most frequently expressed benefits of the introduction of Lean Production [2]. Therefore, it is evident that the importance of finding a way to ensure a reliable and sustainable implementation of Lean is on the rise. However, implementation is not a simple process. According to the research of these authors, even before the 90s, there were some concerns about "how" to implement Lean practices. As also stated by Wan and Chen [3], the primary demand to provide information is 'how to become Lean'. Hence, the need to find a way to ensure that Lean implementations are practical and efficient at the same time is clear.

Lean Maturity Assessment Models (LMAM) are developed to define and track the lean journey of the enterprises easily. LMLs indicate the degree to which an organization is following lean practices [4], [5]. With an accurate and reliable implementation, based on the results, strengths and weaknesses in lean practices can be identified, the organization's lean progress can be observed, and actions can be defined to achieve the objectives. Thus, companies have well-defined roadmaps to move to the next level of lean maturity. As the uncertainties are eliminated, positive improvements in outputs such as lower complexity level, higher profitability, shorter delivery time, or higher consumer satisfaction rate can be more easily observed.

This study is based on a Maturity Model (MM), which consists of a self-assessment tool for defining the LML, the production areas, and warehouses. In this study, the primary criteria affecting the LML were determined in a broad scope, and then questions were formed under each criterion by feeding on the literature. After that, methods of measuring the LML were investigated. Based on the knowledge gap in this field, the purpose of this research is to help the following statements through the development of a dynamic, multi-dimensional Lean Maturity Model (LMM) tailored for management and operational level:

I. A company can assess its overall leanness through using the LMAM.

II. A company can easily define improvement points according to its LML.

III. A company can set the target for the next level of lean maturity.

IV. A company can have other outcome variables that are positively influenced by lean implementations.

Such as complexity level, profitability (in%), delivery time, consumer satisfaction rate, organizational performance, agility, and sustainability. Using this tool would be beneficial to managers, lean practitioners, and engineers in many ways, primarily in understanding current gaps in Lean adoption and in identifying further transformation opportunities. Since there is no one-best-way recipe for lean implementation [6], this study only intends to guide the firms with a detailed inspection opportunity of LML for mainly production and logistic areas.

2|Literature Review

2.1|Lean History

In 1978, Ohno published "Toyota Production System" (TPS) in Japan and credited Ford Production System and American supermarkets behind his Just-In-Time (JIT) thinking [7]. TPS is targeted at removing any waste and inconsistency in the production system. TPS consists of two pillars that are JIT and Jidoka [8]. At the start, because of its concept of reducing inventory and tangible benefits, few researchers [9] focused only on JIT. The success of TPS resulted in its wide acceptance by the manufacturing industries globally; later on, it was disseminated into other non-conventional industries, and the TPS philosophy only preceded the foundation of the more widely recognized term of 'Lean Production' (LP) [10]. The concept of LP was formally introduced in the paper 'triumph of the lean production system' by Krafcik in 1988 [11]. In 1990, the book the machine that changed the world was published by Womack and Jones [12], and the term LP gained more popularity. LP addresses the elimination of waste and makes the process flow more streamlined and efficient [13]. Today, in this current era of global competitiveness, lean principles have been applied in all sectors of manufacturing, banking, healthcare, and even non-profit organizations [14].

2.2 | Maturity Models

Based on the assumption of predictable patterns of evolution and change, MMs usually include a sequence of levels or stages that together form an anticipated, desired, or logical path from an initial state to maturity [15]–[18]. In this context, Maturity Levels (MLs) indicate an organization's current or targeted capabilities concerning a particular type of asset [19]. MMs are commonly applied to assess the as-is situation, to derive and prioritize improvement measures, and to control progress [20]. As for their application in practice, MMs are expected to disclose current and desirable MLs and to include respective improvement measures [15]. The intention is to diagnose and eliminate deficient capabilities. MMs are such tools as engines for continuously improving systems, roadmaps for guiding organizations, and blueprints for designing new entities [21]. On the other hand, in the development of any world-class manufacturing principles, performing an assessment is critical for the successful implementation process [22]. The organization needs a well-defined MM along with an evaluation that consists of multiple checklists to monitor the level of lean maturity over time and evaluate the progress of the process throughout the lean journey. Lean implementation is also a gradual process to shape the organizational culture. Therefore, the maturity assessment models need to be implemented gradually and step-by-step, following the evolution of lean change, to achieve the next level of lean status [23]. According to the literature review of the LML tools by Cetnarski et. al. [24], between the years 1996 and 2015, 51 models have been evaluated. It is undoubtedly an indication that there is an increasing academic interest in MMs [25]. In this study, the leveling scale in the model for LML in MC, which was developed by Maasouman and Demirli [14], has been used to determine the LML of the organization. LML used in this study is given in *Table 1*. *Table 2* was created to show the most recent lean MM, which is mentioned in the relevant articles, and maintained to collect different models with their different levels of lean maturity.

Table 1. Four levels of LMM in production cells.

Focus of the Level	Expected Level of Perception /Implementation	Expected Level of Results	Description
Capability of the people	Understanding (training, standardization, not applicable/lack of implementation)	Quantitative progression of standardization	Quantitative progress in deploying the tools/concepts to raise awareness of the issue
		Qualitative Progression of standardization	Qualitative progress in deploying the tools/concepts to deepen understanding of the issue
Results and performance	Implementation	Effectiveness	Deployment of tools/concepts in a way that is conducive to the achievement of expected results.
	Improvement	Efficiency	Deployment tools/concepts in a way that achieves the expected results and simultaneously uses resources efficiently.
Autonomy and flexibility	Sustainability	Daily excellence	Deployment tools/concepts and improve results continuously and autonomously

3 | Methodology

This study aims to offer a standardized LMM that is developed using the very first-time Interval-Valued Spherical Fuzzy Analytic Hierarchy Process (IVSF-AHP) as a decision-making method and validated by using Confirmatory Factor Analysis (CFA). Also, the proposed model that meets the requirements of production areas and warehouses aims to help managers, lean practitioners, and engineers in the sector significantly. Therefore, a conceptual model is developed based on the review of the literature.

Firstly, extensive research is conducted not only in the area of lean concepts, principles, tools, and objectives but also in MMs. In the design phase, LMLs and criteria are defined. In this model, 9 main 14 sub-lean criteria (management and leadership, quality, JIT, lean methods (gemba-kaizen, ergonomics and 5S, VSD, waste), facility management TPM and OEE, supply chain management, production processes, working conditions, people) were used. There are many more lean criteria and tools in the literature. *Table A3* shows a detailed list of criteria considered under different studies. All the criteria used in the study can be shown in appendix *Table A4* with their codes and literature references. It is apparent that to assess the ML accurately, a broader perspective is required. Hence, when determining the criteria under lean production, human-oriented criteria such as management factors, working conditions, and people are among the factors that define the LML. The leveling scale in the model for LML in MC is used as a leveling scale. As a second task of the design phase, the design of the lean maturity checklists and finalization of the survey instrument are completed. The group of experts, who consisted of 5 people, supported this study with their know-how and experiences in the LP field. They provided data for the criteria weightings and pre-testing of the assessment model. Each criterion is weighted based on the assessments of experts and IVSF-AHP calculations in the EXCEL to define their degree of importance.

Then, several modifications and refinements are done to get the best and final version of the LMAM using IVSF-AHP. After collecting data through a survey study, CFA in the IBM SPSS AMOS V26 program was applied to test the model fit, validity, and reliability. Several iterations are completed to reach an adequate model with good model fit, validity, and reliability. *Fig. A1* shows the overall framework of the research methodology in the appendix section.

3.1 | Measurement Phase

Interval-values spherical fuzzy AHP

Kutlu Gündogdu and Kahraman [26] have recently introduced the Spherical Fuzzy Sets (SFS). These sets are based on the fact that the hesitancy of a decision-maker can be defined independently from membership and non-membership degrees, satisfying the following conditions:

$$0\leq \mu_{\tilde{A}}^2(u) + v_{\tilde{A}}^2(u) + \pi_{\tilde{A}}^2(u) \leq 1, \text{for all } u \in U, \tag{1}$$

where μAu, vAu, and πAu are the degrees of membership, non-membership, and hesitancy of u to ~A for each u, respectively. On the surface of the sphere, *Eq. (1)* becomes

$$\mu_{\tilde{A}}^2(u) + v_{\tilde{A}}^2(u) + \pi_{\tilde{A}}^2(u) = 1, \text{for all } u \in U. \tag{2}$$

The idea behind SFS is to let decision-makers generalize other extensions of fuzzy sets by defining a membership function on a spherical surface and independently assigning the parameters of that membership function with a larger domain. SFS is a synthesis of PFS and NS. The proposed IVSF-AHP method consists of several steps, as given in this section.

Step 1. Form the hierarchical structure based on four levels. In this step, a hierarchical structure consisting of at least three levels is developed. Level 1 shows an objective that means selecting the best alternative based on the score index. The scoring index is estimated based on a finite set of criteria C ={C1, C2, . . . , Cn}, which are shown at Level 2. Many sub-criteria are at Level 3 defined for any criterion C in this hierarchical structure. Therefore, at Level 4, a discrete set of m feasible alternatives X={x1, x2, . . . , xm} (m ≥ 2) is defined, and also, there is a discrete set of K feasible decision-makers for each level [27].

Step 2. Construct pairwise comparison matrices. Pairwise comparisons using interval-valued spherical fuzzy evaluation matrices are constructed based on linguistic terms of importance. The CR of each pairwise comparison matrix is calculated. For this purpose, switch the linguistic terms in the pairwise comparison matrix to their corresponding score indices given in *Table A1* in the Appendix. Then, apply the classical consistency check ratio formula [28]. It can be said that pairwise comparison matrices are consistent when the CR is less than 10%. Otherwise, decision-makers must consider their judgments once again.

Step 3. Aggregate the individual evaluator groups' interval-valued spherical fuzzy weights. In real-life problems, there can be many different types of evaluators. Firstly, to get individual evaluator groups' weights ($\tilde{\omega}_j^{S_k}$), each criterion and alternative pairwise comparison matrices taken from different types of evaluators are aggregated by using the IVSWAM operator.

Step 4. Constitute the interval-valued spherical fuzzy local weights of each criterion. Then, to obtain the interval-valued spherical fuzzy local weights ($\tilde{\omega}_j^{S}$), $\tilde{\omega}_j^{S_k}$ values formed according to the evaluations of different types of evaluators are aggregated with the help of an Interval-Valued Spherical Weighted Geometric Mean (IVSWGM).

Step 5. Construct the hierarchical form to obtain global weights. *Eq. (3)* de-fuzzified the criteria weights by using a modified score function. 1.0 is added to the previous definition of score function since a positive score value may be more beneficial for spherical calculations.

$$\text{Defuzz } (\tilde{\omega}_j^{S}) = \tilde{\omega}_j^{\text{lokal}} = \frac{(\mu^-)^2+(\mu^+)^2-(v^-)^2-(v^+)^2-(I^-/2)^2-(I^+/2)^2}{2}+1. \quad (3)$$

Step 6. The local weights at each level are multiplied by each related sub-criterion local weight to estimate the final global weights ($\overline{\omega}_j^{\text{global}}$)for each criterion and sub-criterion. After necessary multiplication, *Eq. (4)* can be used to normalize the global criteria weights.

$$\overline{\omega}_j^{\text{final}} = \frac{\overline{\omega}_j^{\text{global}}}{\sum_{j=1}^{n} \overline{\omega}_j^{\text{global}}}. \quad (4)$$

After this calculation, normalized global weights of each criterion and sub-criterion are obtained. If alternatives exist in the problem, the algorithm must continue with Step 7.

Step 7. Compute the weighted decision matrix and find the global preference weights ($\tilde{s}_{S_{ij}}$) in terms of alternatives. The normalized global criteria weights ($\overline{\omega}_j^{\text{final}}$) are multiplied by the decision matrix utilizing *Eq. (5)*.

$$\tilde{s}_{S_{ij}} = \overline{\omega}_j^{S}.\ \tilde{s}_{S_i}. \quad (5)$$

Step 8. Defuzzify the final score of each alternative and normalize the de-fuzzified values.

Step 9. Determine the rank among alternatives with respect to the normalized and defuzzified final scores. The best alternative has the largest final score value.

Table 2. Literature review on determining lean maturity levels.

Model	Author	Maturity Levels						
		0	1	2	3	4	5	6
Jorgensen	[51]		Sporadic production optimization	Basic lean understanding and implementation	Strategic lean interventions	Proactive lean culture	Lean in the extended manufacturing enterprise	
LCMM	[52]	Uncertain	Awakening	Systematic	Integrated	Challenging		
BPI	[53]	The Lean approach is not described or poorly described	The Lean approach is described but not structured with a procedure model	The Lean approach is structured with a procedure model	The Lean approach is structured with a procedure model, and one or more techniques are described for some or all the activities of the procedure model	The Lean approach is structured with a procedure model, with techniques and the results of some of the activities or all the activities described	The Lean approach is structured as in level 4, but for each activity, the roles are defined	The Lean approach is structured as in level 5, but an information model is also provided
Lesat Lai	[54]		Some awareness	General Awareness	A systematic approach/methodology deployed in varying stages across most areas	Continuous improvement across the enterprise	Exceptional, well-defined, innovative approach is fully deployed across the extended enterprise.	
Model for lean maturity level in MC	[14]		Understanding (training, standardization)	Implementation	Improvement	Sustainability		
IDEAL	[55]		Initial	Defined	Enhanced	Advanced integrated	Long-term optimized	
Lean manufacturing maturity model	[50]		Inconsistent and unstable results	Department-level management, local efficiency improvement	Well-defined company-wide processes, standardization, and best practices	Continuous improvement	Optimized	
A conceptual model for LM maturity level	[56]		Understanding	Implementation	Success			

3.2 | Analysis and Verification Phase

Scale development and validation: CFA

The process of theory building relies on the existence of solid proof based on a rigorous research methodology so that researchers can develop reliable, valid, and realistic diagnostic instruments [29]. Professionals can successfully use such tools for the development and advancement of any theory, including the definition of LML. Questionnaire surveys are widely acknowledged as a method of measuring the perceptions of various groups of experts and practitioners on a particular topic. CFA is a type of Structural Equation Modeling (SEM) that deals specifically with measurement models, that is, the relationships between observed measures or in dicators (e.g., test items, test scores, behavioral observation ratings) and latent variables or factors [30]. CFA requires that it should be based on logic and/or theory, and hence, the researcher should have a good knowledge of the latent factors that explain the variation in the observed variables [31]. The adequacy of the CFA model is based on acceptable measures of model fit, reliability, and construct validity for scales [32]. Since there are multiple benefits of the CFA approach, in this study, CFA is used to perform factor analysis in defining lean maturity measurement models and their associated latent factors and observed variables.

Model fit

In the literature, there are several statistical methods to measure the model fit in CFA, such as $\chi 2$, Incremental Fit Indices, Absolute Fit Indices, and many more. All aim to evaluate different facets of a model fit.

The Chi-Square value is the traditional measure for evaluating overall model fit and 'assesses the magnitude of discrepancy between the sample and fitted covariances matrices' [33]. Incremental fit indices, also known as comparative [34] or relative fit indices [35], are a group of indices that do not use the chi-square in its raw form but compare the chi-square value to a baseline model. An incremental fit index is used to assess the improvement in fit between default and baseline models. A null model in which no items covary is the most generally used baseline model [31]. Commonly used incremental fit indexes include, among other things, the Normed Fit Index (NFI), the Tucker–Lewis Index (TLI), the Relative Noncentrality Index (RNI), and the Comparative Fit Index (CFI) [33]. Absolute fit indices determine how well a priori model fits the sample data [35]. There is no reference model used in this index; however, an implicit or explicit comparison is made to a saturated model that reflects a perfectly fitting model [32]. The absolute fit index category includes the Chi-Square test, RMSEA, Goodness-of-Fit Index (GFI), Adjusted Goodness-of-Fit Index (AGFI), RMR, and SRMR. To check whether the CFA model is adequate, there are multiple theories and philosophies concerning how many indices/statistics should be reported and what combinations of these indices are appropriate. Some of these statistics/indices are influenced by sample sizes or the ratio of indicators per factor and may not provide an adequate representation of the model fit [36]. For instance, the Chi-Square statistic is theoretically expected to be non-significant ($p > 0.05$) for a good model fit. However, studies have shown that the Chi-Square Statistic is very sensitive to sample size. For a large sample size that is typically required for CFA and SEM models, the Chi-Square statistic and its associated probability value will invariably turn out to be significant ($p < 0.05$). Therefore, it was suggested to use the Chi-Square/df measure, which is required to fall between 1 and 3 for an acceptable fit. Similarly, GFI, which is a measure of absolute fit, is also largely influenced by sample size. Some of these indices work better in certain scenarios, while others perform well in other scenarios [32]. To address these problems, researchers have suggested the use of multiple fit indices to provide a more holistic view of Goodness of Fit, addressing issues related to sample size and model complexity [37]–[39]. Given such varied suggestions on the use of the right set of Indices, researchers have suggested specific key indices that must be reported in research findings [33], [36], [40], [41]. In light of the aforementioned explanations on the indices, *Table 3* shows that the following required set of indices and cutoff criteria will be evaluated in this study.

Characteristic of good measurement–validity and reliability

The characteristics of a suitable measurement instrument should address the ability of the tool to measure what it intends to measure [42] adequately. Scale reliability and validity are the two primary criteria that are used to ensure whether the measurement instrument is good enough to do the measurement. Without ensuring reliability and validity, measurement scales cannot be standardized and will not be able to measure the required construct [29].

Table 3. Set of indices and cutoff criteria.

Model Fit	The Goodness of Fit Measure	Acceptable Range	Required Indices and Cutoff Criteria
Incremental fit	NFI	0 to 1	No
	TLI	0 to 1	No
	Relative non-centrality index	0 to 1	No
	CFI	>0.95 – Excellent 0.9 to 0.95 - Good	Yes
Absolute fit	Chi-square/df	1 to 3	Yes
	Root mean square error of approximation	<0.06 – Excellent 0.06 to 0.08 – Good	Yes
	Standardized root mean square residual	<0.08 – Excellent 0.08 to 0.10 - Good	Yes
Classic goodness of fit	$\chi 2$ goodness-of-fit statistic		No

The reliability of a scale is the ability of the scale to provide consistent results [43]. Even though reliability and validity are analytically distinguishable, they are related as reliability is a prerequisite to ensure validity [44]. Equivalent forms, split halves method, test–retest method, internal consistency method using Cronbach's alpha, and Composite Reliability (CR) can be given as examples as a few of many applications. In CFA, researchers have argued that CR is a better measure to ensure internal consistency [45]. Unfortunately, there is no consensus in the methodological literature on scale validity. Commonly used validity types include content, convergent, discriminant, and criterion-related validity [32]. Thestandardization of the instrument can be carried out by tests of unidimensionality, reliability, and construct validity (including content, convergent, discriminant, and criterion-related validities) using a CFA approach [32]. *Table 4* shows a set of reliability and validity measures, as well as their description and cutoff criteria, that are needed for CFA.

Table 4. Cutoff criteria for reliability and validity measures in CFA.

Purpose	Measure	Description	Acceptable Values
Reliability	CR	An indicator of the shared variance among the observed variables used as an indicator of a latent construct.	CR>0.7 CR>AVE
Content validity	Judgemental	The degree to which the content of these items adequately represents the universe of all relevant items under survey.	NA
Convergent validity	Average variance extracted	AVE refers to the amount of variance extracted by a latent factor as compared to its measurement error.	>0.5
Discriminant validity	Max. Shared Squared Variance Average	Maximum amount of squared variance shared by a latent factor with any other latent factor.	AVE>MSV
	shared squared variance	Average of all the squared variances of the latent factor and other latent factors.	AVE>ASV
Criterion validity	Concurrent	Typically, regression analysis of an output criterion with the latent factors is used to check for significance.	ρ<0.01 R^2 (above 80%)

4 | Implementation

4.1 | IVSF-AHP Implementation

The algorithmic steps of the IVSF-AHP method are as follows.

Step 1. The problem is defined. The criteria required for the decision and model itself are determined; these are management and leadership, quality, JIT, lean methods (gemba-kaizen, ergonomics, and 5S, VSD, waste), facility management TPM and OEE, supply chain management, production processes, working conditions, people and then the priorities criteria are defined. The importance ranking is prepared in line with the opinions of experts.

Step 2. A hierarchical structure is created. *Fig. 1* shows a hierarchical structure of evaluation of LMM. At the top is the main goal to be achieved. Below are the main criteria and sub-criteria. At the bottom of the hierarchy are the alternatives. The stage of the hierarchy number depends on the complexity of the problem and the degree of detail. When creating the hierarchy, the same options in the plane are considered to be completely independent of each other.

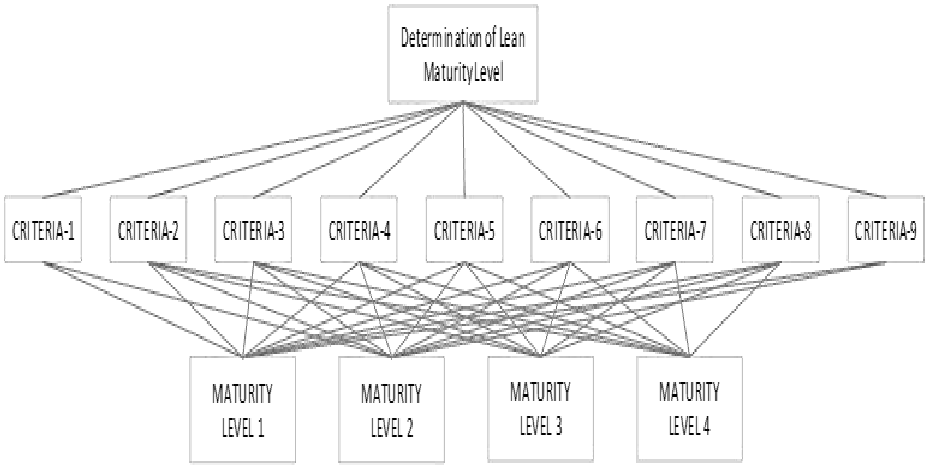

Fig. 1. Hierarchical structure of evaluation of LMM.

Step 3. A matrix of pairwise comparisons is created. Using a scale of 1 to 9, matrices are created comparing the decision options according to the criteria, first considering the main criteria, then the sub-criteria, if any, and finally all criteria. As described in *Eq. (6)*, comparison matrices are square matrices with diagonal elements of 1.

$$A = \begin{bmatrix} 1 & a_{12} & \cdot & \cdot & \cdot & a_{1n} \\ a_{21} = 1/a_{12} & 1 & \cdot & \cdot & \cdot & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} = 1/a_{1n} & a_{n2} = 1/a_{2n} & \cdot & \cdot & \cdot & 1 \end{bmatrix}. \quad (6)$$

α_{ij} is the pairwise comparison value of criterion i and criterion j and the value of α_{ji} is obtained from $1/\alpha_{ij}$. This is known as the correspondence function. α_{ij} value is the answer to the 'In which ratio should the criterion i be preferred over the criterion j?' question. Decision alternatives are compared separately according to each criterion. Decision matrices are created using the 1/9-9 comparison scale. The comparison scale with abbreviation code and score indices is shown above in *Table 3*. To calculate the weights of the criteria listed according to the degree of importance, pairwise comparisons were made between the criteria in the light of experts' views and knowledge *Table 5*.

Table 5. LMAM criteria pairwise comparison of Expert 5.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	EI	EI	HI	SMI	SMI	HI	SMI	SMI	EI
C2		EI	HI	EI	SMI	HI	EI	HI	SLI
C3			EI	LI	LI	EI	LI	SLI	LI
C4				EI	EI	HI	SLI	SMI	LI
C5					EI	SMI	LI	EI	LI
C6						EI	LI	EI	LI
C7							EI	VHI	EI
C8								EI	LI
C9									EI

Step 4. Normalize the Pairwise Comparison (PC) matrices. Each element in the matrix is normalized by dividing by the sum of its columns. The sum of each column of the normalized matrix equals 1. The calculation phases have been shown for expert 5 in *Table 6* and *Table 7*.

Table 6. LMAM normalization of PC – step 1.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	1.00	1.00	5.00	3.00	3.00	5.00	3.00	3.00	1.00
C2	1.00	1.00	5.00	1.00	3.00	5.00	1.00	5.00	0.33
C3	0.20	0.20	1.00	0.20	0.20	1.00	0.20	0.33	0.20
C4	0.33	1.00	5.00	1.00	1.00	5.00	0.33	3.00	0.20
C5	0.33	0.33	5.00	1.00	1.00	3.00	0.20	1.00	0.20
C6	0.20	0.20	1.00	0.20	0.33	1.00	0.20	1.00	0.20
C7	0.33	1.00	5.00	3.00	5.00	5.00	1.00	7.00	1.00
C8	0.33	0.20	3.00	0.33	1.00	1.00	0.14	1.00	0.20
C9	1.00	3.00	5.00	5.00	5.00	5.00	1.00	5.00	1.00
Total	4.73	7.93	35.00	14.73	19.53	31.00	7.08	26.33	4.33

Table 7. LMAM normalization of PC – step 2.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	0.211	0.126	0.143	0.204	0.154	0.161	0.424	0.114	0.231
C2	0.211	0.126	0.143	0.068	0.154	0.161	0.141	0.190	0.077
C3	0.042	0.025	0.029	0.014	0.010	0.032	0.028	0.013	0.046
C4	0.070	0.126	0.143	0.068	0.051	0.161	0.047	0.114	0.046
C5	0.070	0.042	0.143	0.068	0.051	0.097	0.028	0.038	0.046
C6	0.042	0.025	0.029	0.014	0.017	0.032	0.028	0.038	0.046
C7	0.070	0.126	0.143	0.204	0.256	0.161	0.141	0.266	0.231
C8	0.070	0.025	0.086	0.023	0.051	0.032	0.020	0.038	0.046
C9	0.211	0.378	0.143	0.339	0.256	0.161	0.141	0.190	0.231

Step 5. The priority vector is calculated. The sum of each row of the normalized matrix is divided by the dimension of the matrix and averaged. These values are the importance weights calculated for each criterion. These weights form the priority vector.

$$w_i = \left(\frac{1}{n}\right) \sum_{i=1}^{n} a'_{ij} \quad , \text{i, j= 1,2,.....,n.} \tag{7}$$

Eq. (7) is used. Thus, percentage importance distributions showing the importance values of the criteria relative to each other are obtained. *Table 8* shows the calculations.

Table 8. Calculations of priority vector.

Criteria % Weights (w)	D=A*w	ei=(A*w)/w
0.20	1.98	10.10
0.14	1.38	9.76
0.03	0.25	9.45
0.09	0.88	9.60
0.06	0.62	9.52
0.03	0.29	9.58
0.18	1.80	10.14
0.04	0.41	9.50
0.23	2.31	10.14
	Total	87.79

Step 6. CR is calculated. After pairwise comparisons and prioritization, the consistency of the comparison matrices is calculated.

To determine whether a matrix resulting from pairwise comparison judgment is consistent or not, it is necessary to calculate the coefficient called "Consistency Index (CI)," which is one of many methods. CI is calculated with *Eq. 8*.

$$CI = \frac{\lambda_{max} - n}{n - 1}. \quad \textbf{(8)}$$

$$\lambda_{max} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\sum_{j=1}^{n} a_{ij} w_j}{w_i}\right). \quad \textbf{(9)}$$

To evaluate consistency, the "Random Index (RI)" value should be known. RI values defined for n-dimensional comparison matrices are given in *Table 9*. After determining the CI and RI values, the CR is calculated. It has been shown in *Table 10*.

Table 9. RI values according to the size of comparison matrices.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.53	1.56	1.57	1.59

Table 10. Consistency ratio.

CI	CR	λmax	RI (Randomness index)
0.094	0.065	9.755	1.450

$$CR = CI/RI^{*}. \quad \textbf{(10)}$$

If the CR defined by *Eq. (10)* is less than 0.10, the comparison matrix is considered to be consistent. Steps 3-6 were repeated for each of the experts.

Step 7. A pairwise comparison matrix for the criteria is created, and the priority vector of the decision options is calculated. This priority vector can also be defined as a weight vector for the criteria. Comparisons of the criteria in the pairwise comparison matrix were made with the help of the IVSF-AHP method, which consists of 9 categories. These comparisons are based on the difference in the attractiveness of the criteria and the degree of impact of the criteria on lean maturity.

Step 8. Decision alternatives are ranked. Priority vectors obtained for the criteria are combined to create the complete priorities matrix. The resulting vector is obtained by multiplying and summing the complete priorities matrix and the priority vectors of the decision alternatives. The decision alternative with the highest weight in this vector is determined as the decision alternative that should be preferred for the solution of the problem. *Table 11* shows fuzzy weights, and *Table 12* illustrates the de-fuzzification of them.

Table 11. Fuzzy weights for each criterion.

	C1	C2	C3	C4	C5	C6	C7	C8	C9
C1	1.00	1.32	1.57	1.44	1.40	1.44	1.34	1.53	1.47
C2	0.76	1.00	1.27	1.04	1.09	1.01	0.73	1.15	1.12
C3	0.64	0.79	1.00	0.78	0.92	0.91	0.77	0.78	0.96
C4	0.69	0.96	1.29	1.00	0.97	1.15	0.78	1.19	1.14
C5	0.71	0.92	1.09	1.03	1.00	0.95	0.61	1.02	0.98
C6	0.69	0.99	1.10	0.87	1.05	1.00	0.76	1.20	1.12
C7	0.75	1.37	1.30	1.28	1.65	1.32	1.00	1.57	1.42
C8	0.65	0.87	1.29	0.84	0.98	0.83	0.64	1.00	0.96
C9	0.68	0.89	1.04	0.87	1.02	0.89	0.71	1.03	1.00
Total	6.6	9.1	10.9	9.1	10.1	9.5	7.3	10.5	10.2

Table 12. De-fuzzification and normalization of fuzzy weights.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	Priority Index	Rank
C1	0.15	0.14	0.14	0.16	0.14	0.15	0.18	0.15	0.14	0.151	1
C2	0.12	0.11	0.12	0.11	0.11	0.11	0.10	0.11	0.11	0.110	3
C3	0.10	0.09	0.09	0.08	0.09	0.10	0.10	0.07	0.09	0.091	9
C4	0.11	0.11	0.12	0.11	0.10	0.12	0.11	0.11	0.11	0.110	4
C5	0.11	0.10	0.10	0.11	0.10	0.10	0.08	0.10	0.10	0.100	6
C6	0.11	0.11	0.10	0.09	0.10	0.11	0.10	0.11	0.11	0.105	5
C7	0.11	0.15	0.12	0.14	0.16	0.14	0.14	0.15	0.14	0.139	2
C8	0.10	0.10	0.12	0.09	0.10	0.09	0.09	0.10	0.09	0.096	8
C9	0.10	0.10	0.09	0.10	0.10	0.09	0.10	0.10	0.10	0.098	7
Total	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.00	

4.2 | CFA Implementation

A total of 116 survey items were created for the empirical validation of the 9 main 14 sub-axes LMM. The questionnaire survey is based on a 5-point Likert scale, where 0 is strongly disagree or lack of implementation and 4 is strongly agree. This instrument was developed based on extensive research of lean literature (theoretical, conceptual, experimental, and practical). To ensure the content validity of the survey instrument and identify and calculate the accurate criteria weightings, a pilot study was conducted with a group of 5 experts (academics, researchers, lean engineers, managers, and practitioners). Several iterations of the tool were created based on comments and suggestions of experts and calculations of the IVSF-AHP method, and the final version was designed to maximize all aspects of Lean maturity as they relate to the 9 main 14 sub-axes. The results were gathered through a broad online survey of 26 medium and large-sized companies from different sectors, with responses from 187 respondents who are mostly lean engineers, lean managers, and practitioners. As a result of the analysis of the survey results obtained from the participating companies, the overall LML was calculated as 2.55 out of 4. This result corresponds to the Level 3 - Improvement range on the leveling scale from the Model for LML, which is used in the study. As an example of the practical use of the developed model for assessing the maturity of lean manufacturing, *Table 13* shows the average of the answers given by Company 1 to the questions under each criterion and the score obtained by multiplying these averages by the criterion weights.

Table 13. The practical use of the developed model.

Criteria	C1	C2.1	C2.2	C2.3	C3	C4.1	C4.2	C4.3	C4.4	C5	C6	C7	C8	C9
Company 1	2.333	2.222	2.167	2.400	1.889	2.333	2.182	2.000	2.111	1.818	1.889	2.000	2.167	2.200
Priority Index	0.151	0.110	0.110	0.110	0.091	0.110	0.110	0.110	0.110	0.100	0.105	0.139	0.096	0.098
Weighting	0.352	0.249	0.249	0.249	0.172	0.237	0.237	0.237	0.237	0.182	0.198	0.278	0.208	0.216
Overall Score	2.092													
Rate	52%													
LML	Level 3: improvement - efficiency													

Deployment tools and concepts in a way that achieves the expected results and simultaneously uses resources efficiently. The LML overall rate of surveyed companies was set at 64%. Moreover, all items on the survey instruments were randomized to prevent bias. An example of a checklist that is used for Lean maturity measurement of leanness indicators is presented in the Appendix in *Table A2*.

CFA–results and discussion

LMAM has been constructed upon the identified 9 main 14 Sub-criteria. In order to determine whether the goodness of the model fits well overall, the following hypothesis has been proposed: H0 Lean maturity measurement model is a multi-dimensional construct consisting of the previously mentioned 9 main 14 Sub-axes.

CFA analysis was conducted using IBM SPSS AMOS V26 software. A graphical display can be shown for LMM axes in *Fig. 3*. The results which are derived from CFA analysis are in *Table 14* and *Table 15*. The required model fit indices and cutoff criteria specify that all the indices remain in an acceptable range. *Table 14* shows strong evidence of model fit because all mandatory indices such as CFI, RMSEA, and SRMR belong to the results of the group "excellent," and also, Chi-square/df is in the acceptable range. The CR values shown in *Table 15* indicate that for all the axes of LMAM, they are either above the requirement of 0.7. These results display a strong CR of the criteria. The values of Average Variance Extracted (AVE) shown in *Table 15* are higher than the required value of 0.5, providing strong evidence of convergent validity. Measures of discriminant validity, like Maximum Shared Squared Variance (MSV) and Average Shared Squared Variance (ASV), also meet the necessary criteria to provide strong evidence of discriminant validity. The respondents were asked to rate the ML of their organization in percentage terms, and this outcome was used to test concurrent validity. *Table 15* shows the results of the regression analysis between the 14 sub-axes and ML. The results demonstrate the existence of strong concurrent validity, with a high R2 of 86.79%, confirming the statistical significance of the MM and the 14 axes individually. Given the strong evidence of model fit and the reliability and validity measures, it can be concluded that the hypothesis "H0 Lean maturity measurement model is a multi-dimensional construct consisting of the previously mentioned 9 main 14 Sub-axes." is acceptable.

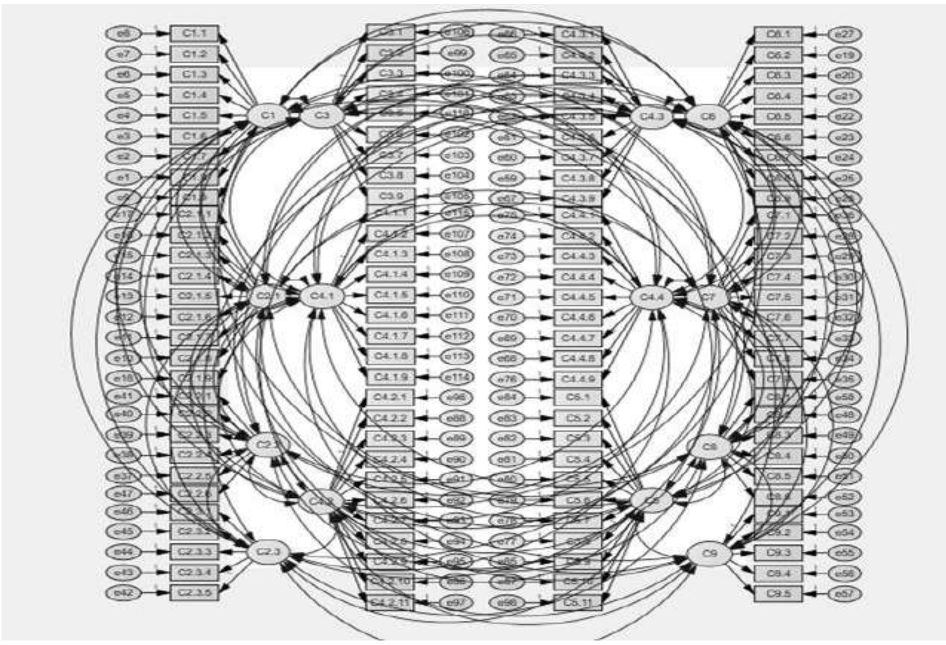

Fig. 2. A graphical display in IBM SPSS AMOS V26.

Table 14. Model fit results model.

Model fit	The goodness of fit measure	Acceptable range	Value	Review of results
Incremental fit	CFI	>0.95 – Excellent 0.9 to 0.95 - Good	0.985	Excellent
Absolute fit	Chi-square/df	1 to 3	1.719	Good
	Root mean square error of approximation	<0.06 – Excellent 0.06 to 0.08 - Good	0.040	Excellent
	Standardized root mean square residual	<0.08 – Excellent 0.08 to 0.10 - Good	0.059	Excellent

Table 15. Results of CFA–reliability and validity (convergent and discriminant).

#	Code	Criteria	AVE	CR	ASV	MSV	Criteria
1	C1	Management and leadership	0.568	0.896	0.024	0.051	Reliability CR > 0.7 CR > AVE Convergent validity AVE > 0.5 Discriminant validity AVE > MSV AVE > ASV
2	C2	Quality	0.594	0.828	0.015	0.114	
2.1	C2.1	Total quality management	0.635	0.811	0.033	0.064	
2.2	C2.2	Standardization and standard work	0.587	0.836	0.019	0.098	
2.3	C2.2	Jidoka	0.688	0.874	0.025	0.099	
3	C3	Just in time	0.576	0.792	0.043	0.105	
4	C4	Lean techniques	0.659	0.839	0.027	0.102	
4.1	C4.1	Gemba and kaizen	0.607	0.901	0.038	0.078	
4.2	C4.2	Ergonomy and 5S	0.572	0.873	0.046	0.057	
4.3	C4.3	Value stream mapping	0.678	0.914	0.032	0.124	
4.4	C4.4	Waste and loss management	0.602	0.875	0.030	0.120	
5	C5	Facility management	0.553	0.905	0.026	0.081	
6	C6	Supplier relations management	0.579	0.797	0.031	0.471	
7	C7	Production processes	0.551	0.858	0.044	0.063	
8	C8	Working conditions	0.624	0.926	0.041	0.117	
9	C9	People	0.545	0.867	0.034	0.089	

Table 16. Result of regression analysis between 1 axes and ML (concurrent validity).

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	14	2.33538	0.166813	370.05	0.000
Management and leadership	1	0.03259	0.032589	72.29	0.000
Quality					
Total quality management	1	0.01048	0.010480	23.25	0.000
Standardization and standard work	1	0.01766	0.017657	39.17	0.000
Jidoka	1	0.02018	0.020181	44.77	0.000
Just in time	1	0.01454	0.014535	32.24	0.000
Lean Techniques					
Gemba and Kaizen	1	0.02175	0.021748	48.24	0.000
Ergonomy and 5S	1	0.01783	0.017833	39.56	0.000
Value stream mapping	1	0.01561	0.015612	34.63	0.000
Waste and loss management	1	0.01472	0.014718	32.65	0.000
Facility Management	1	0.01328	0.013282	29.46	0.000
Supplier relations management	1	0.01164	0.011637	25.82	0.000
Production processes	1	0.01535	0.015353	34.06	0.000
Working conditions	1	0.02021	0.020209	44.83	0.000
People	1	0.02049	0.020494	45.46	0.000
Error	172	0.07753	0.000451	-	-
Total	186	2.41292	-	-	-
Model summary	S	R-Sq	R-Sq(adj)	R-Sq(pred)	
	0.0212317	86.79%	96.53%	96.24%	

5 | Conclusion

This research and the proposed MM aim to help industry managers, engineers, researchers, and practitioners evaluate the leanness of companies. The model and suggested methodology are a framework to understand

and develop lean philosophy progressively, especially in the production areas. In the light of generated knowledge, the model can be tweaked in detail by lean practitioners.

It has been used for the very first time in the IVSF-AHP method in a study of evaluating LML during calculations of the weighting of the criteria. CFA in the IBM SPSS AMOS V26 program has been applied to test the model's fit, validity, and reliability. The CFA approach successfully validated the proposed model, which can be used as a standardized measurement instrument by lean practitioners.

This study verifies that the Lean maturity measurement model consists of multiple dimensions, which were identified previously as 9 main axes (14 sub-axes). In theory, this paper contributes to creating a new way of thinking and better comprehending the dimensions of the Lean maturity measurement model by validating the identified axes. It has been shown that there is a need for an all-inclusive approach while measuring Lean maturity. One of the essential findings of the defined criteria for the lean maturity measurement model is the human factor. All criteria that are directly related to the human factor, such as Management and Leadership, Working Conditions, and People, show that the success of lean implementations is directly related to the motivation of the employees, adoption of lean methods, and involvement of the management team.

A model to measure the LML of the companies has been designed and used, and it has been validated with the CFA approach by examining model fit, reliability, and validity. This paper outlines a high-level model of lean maturity for companies. Since each organization is distinctive and unique, it is highly recommended to personalize a lean maturity measurement model that is tailored to its specific circumstances and constraints by taking into account the industry, the scale of the company, product type, product volume, production type and other particular requirements and strategies of the company. The essential factor for the successful implementation and achievement of the highest level (sustainability level) of Lean maturity in organizations is to draw a roadmap that represents a detailed transformation plan for the corporation.

To adopt the lean philosophy and to properly implement and develop lean tools, it is helpful to conduct a lean assessment using the lean checklist at regular intervals. As a recommendation, a soft assessment can be done twice a year with an internal auditor and a comprehensive check with an external auditor once a year. On the other hand, lean methods should be part of daily shop floor management. It is also recommended to develop a dynamic assessment system in line with changing needs over time by using the feedback of the previous assessments and by reviewing leanness results in comparison with performance.

5.1 | Implications for Research and Practice

Determining LMLs allows businesses to improve efficiency by examining their processes. Compliance with lean principles can increase effectiveness and efficiency by reducing waste and optimizing business processes. Having a lean manufacturing approach can provide businesses with a competitive advantage. Being able to respond quickly to customer demands and reducing costs allows us to get ahead in the market. Determining LMLs is essential to maintaining this competitive advantage. Lean principles can make business processes more flexible and adaptable. Businesses feel the need to assess their LML to quickly adapt to changing market conditions and increase their resilience.

In a dynamic production environment, there is a need to adapt to constantly changing conditions. Lean models offer the opportunity to adapt and optimize business processes by focusing on continuous improvement principles. Lean models often take a modular approach, which helps businesses focus on priority areas and respond quickly to changes. Modularity makes the application more manageable in a dynamic environment. Nowadays, technological advances can make it easier to implement lean models. Automation, data analysis, and other technological tools can support businesses in determining and improving their LMLs. Implementing models based on lean principles can provide a return on investment in the long term. As businesses see advantages such as cost savings, increased customer satisfaction, and operational efficiency, they can be rewarded for the time and resources they devote to these models.

As a result, implementing lean models in a dynamic manufacturing environment can be challenging. Still, the flexibility and continuous improvement-oriented nature of these models offer the potential to provide businesses with a competitive advantage.

The following essential findings of this study are of great value to researchers and practitioners by proposing and validating a measurement instrument to measure LML.

I. This research and the proposed MM are to help industry managers, engineers, researchers, and practitioners evaluate the leanness of companies.

II. The research contributes to developing theories in the new area of measuring LML. It aims to enhance comprehension of the different facets of a lean maturity evaluation model.

III. In the future, upcoming studies can provide a broader analysis of the literature on LMMs to provide a comprehensive account of research applications in this area.

IV. Practitioners can use the proposed instrument to measure the level of Lean maturity concerning the 9 main 14 sub-axes. This allows practitioners to develop a holistic approach to deploying the Lean maturity evaluating model, resulting in practical implementation.

V. The study's proposed measuring model can help other researchers use SEM to explore causal relationships between the axes of Lean maturity assessment.

VI. According to the specific requirements of organizations, the axes of the proposed LMM described in this study can be prioritized according to their relative importance in influencing organizational performance using different multi-criteria decision-making techniques.

VII. Since each organization is distinctive and unique, it is highly recommended to personalize a lean maturity measurement model that is tailored to its specific circumstances and constraints by taking into account the industry, the scale of the company, product type, product volume, production type and other specific requirements and strategies of the company.

VIII. Researchers and practitioners can use this work along with the developed instrument to study the effect of the axes of Lean maturity in influencing outcome variables such as complexity level, profitability (in %), delivery time, consumer satisfaction rate, organizational performance, agility, and sustainability.

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Appendix

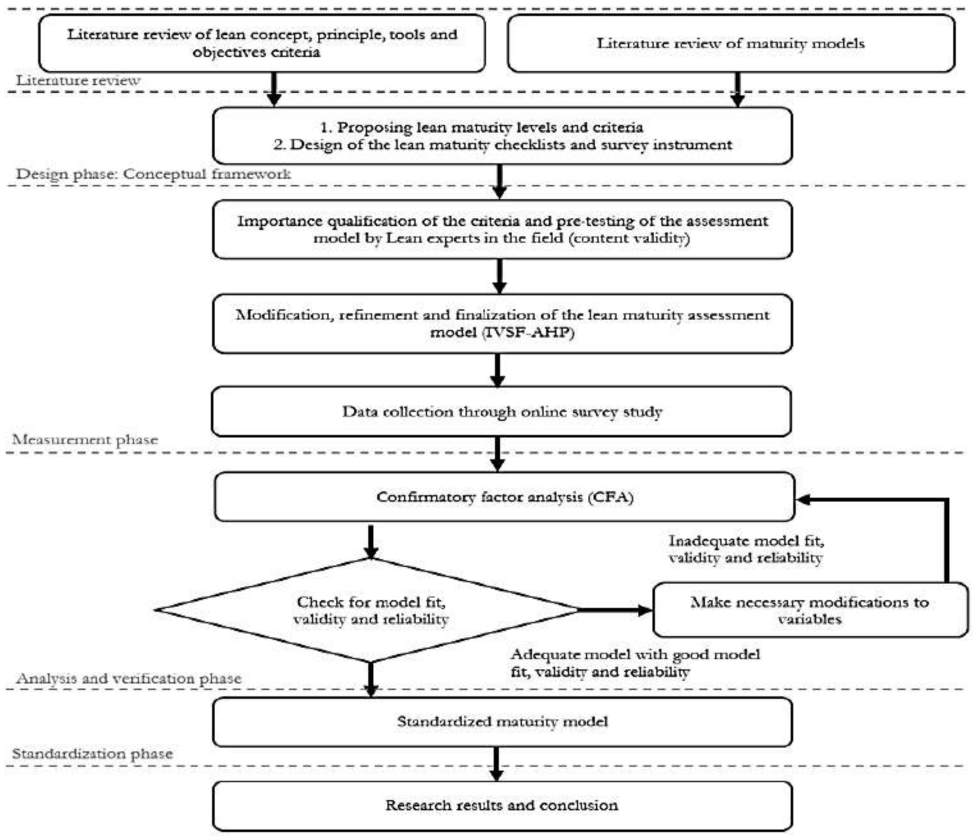

Fig. A1. General framework of the research methodology.

Table A1. Linguistic terms used for pairwise comparisons.

Linguistic terms	s˜ = ([μ−(x), μ+(x)], [ν−(x), ν+(x)], [I −(x), I +(x)])	Score Index
Absolutely more importance	([0.85, 0.95], [0.10, 0.15], [0.05, 0.15])	9
Very high importance	([0.75, 0.85], [0.15, 0.20], [0.15, 0.20])	7
High importance	([0.65, 0.75], [0.20, 0.25], [0.20, 0.25])	5
Slightly more importance	([0.55.0.65], [0.25, 0.30], [0.25, 0.30])	3
Equal importance	([0.50, 0.55], [0.45, 0.55], [0.30, 0.40])	1
Slightly low importance	([0.25, 0.30], [0.55, 0.65], [0.25, 0.30])	1/3
Low importance	([0.20, 0.25], [0.65, 0.75], [0.20, 0.25])	1/5

Table A2. An Example Survey Instrument (Checklist) to Measure LM and Scale.

Control item: Standard operating procedures Axis: 1 – Management and Leadership								
Checklist		Score						Evidence
		0	1	2	3	4	N/A	
1	Lean production systems coordinators/ lean production systems transformation leaders have been identified to manage lean production processes and provide support for lean practices.							
2	In order to implement lean thinking in the organization, goals are set with the support of leaders, process plans are made, and efforts are made to implement these plans. Leaders take an active role in this process.							
3	Management provides the necessary opportunities for employees to adopt lean thinking and to apply lean methods. Studies and training are organized on the subject.							
4	Management has developed a lean transformation strategy and is planning for it. Management aims to use resources efficiently and eliminate waste.							
5	Long-term vision, mission, goals, and responsibilities have been determined for lean production as the ultimate goal and for lean production.							
6	Leaders apply the Gemba walk, one of the lean guiding principles, to observe and identify the current situation on the ground and identify risk factors.							
7	In these audits, the lean expert acts as an external auditor.							
8	Management systematically identifies and monitors lean needs in products, processes, and operations.							
9	A lean production systems (PS) department was established to implement and execute lean production processes properly.							

Table A3. A literature review of Lean Maturity criteria.

Subject of Study	Used Criteria	Number of Considered Criteria	Author
A model for evaluating the degree of leanness of manufacturing firms	Elimination of waste, continuous improvement, zero defects, just-in-time deliveries, the pull of raw materials, multifunctional teams, decentralization, integration of functions, vertical information systems	9	[46]
A field study on measuring the lean maturity level in manufacturing firms in Turkey	Kanban, production planning and scheduling, setup reduction, industrial housekeeping (5S), reduction of work-in-process and inventory, visual controls, poka yoke, cellular manufacturing, supplier relations management, TPM, flexible workers, elimination of wastes	12	[2]
Examining the Association Between Leadership Styles and an Organization's Lean Manufacturing Maturity Level	Leadership style and management	1	[47]
A maturity assessment of lean development practices in manufacturing industry	Kanban, 5S, Kaizen, energy efficiency program, cellular manufacturing, poke-yoke, standardized work, visual stream mapping, plan do check action, statistical process control, SMED, JIT, total productive maintenance	13	[22]
Assessment of Lean Maturity Level in Manufacturing Cells	People, facility management, working conditions, production processes, quality, just-in-time, leadership	7	[48]
A literature review on lean maturity level tools	Continuous improvement (Kaizen), workload leveling (Heijunka), pull production (Kanban), visual management, single-minute exchange of die, 5S, total preventive maintenance, just in time, standardized work, value stream mapping, continuous production flow, supplier development, autonomation (Jidoka), cellular manufacturing, poka yoke, multifunctional teams, total quality management, training people, commitment of employees and management, challenging customers and suppliers, reduction of supply base, unit lots/reduction of production batches, empowerment, hoshin-kanri, root cause analysis, zero defects, reliable and tested technology, process mapping, radical improvement (Kaikaku), flexible information system, stocks replacement point, simulation	32	[24]
Developing an instrument to measure lean manufacturing maturity and its relationship with operational performance	Strategic planning, quality at sources, processes and tools, problem-solving, people, supplier integration, continuous improvement, customer focus	8	[49]
Lean manufacturing maturity model	Leadership, people, process, results	4	[50]

Table A4. Criteria and literature references.

#	Code	Criteria	Author	Model
1	C1	Management and leadership	[47]	MLQ and LESAT
2	C2	Quality		
2.1	C2.1	total quality management	[24]	Analysis of lean maturity level tools
2.2	C2.2	Standardization and standard work	[24]	Analysis of lean maturity level tools
2.3	C2.2	Jidoka	[24]	Analysis of lean maturity level tools
3	C3	JIT	[14]	A lean maturity model
4	C4	Lean techniques		
4.1	C4.1	Gemba and Kaizen	[22]	Maturity assessment of lean management tools
4.2	C4.2	Ergonomy and 5S	[22]	Maturity assessment of lean management tools
4.3	C4.3	Value stream mapping	[24]	Analysis of lean maturity level tools
4.4	C4.4	Waste and loss management	[46]	A lean maturity model
5	C5	Facility management	[14]	A lean maturity model
6	C6	Supplier relations management	[2]	Measurement of lean maturity level
7	C7	Production processes	[49]	Lean production maturity level measurement tool
8	C8	Working conditions	[14]	A lean maturity model
9	C9	People	[14]	A lean maturity model

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Modelling and Optimization of Residential Electricity Load under Stochastic Demand

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Abstract

The paper considers a modelling framework for a set of households in residential areas using electricity as a form of energy for domestic consumption. Considering the demand and availability of units for electricity consumption, optimal decisions for electricity load allocation are paramount to sustain energy management. We formulate this problem as a stochastic decision-making process model where electricity demand is characterized by Markovian demand. The demand and supply phenomena govern the loading and operational framework, where shortage costs are realized when demand exceeds supply. Empirical data for electricity consumption was collected from fifty households in two residential areas within the suburbs of Kampala in Uganda. Data collection was made at hourly intervals over a period of four months. The major problem focussed on determining an optimal electricity loading decision to minimize consumption costs as demand changes from one state to another. Considering a multi-period planning horizon, an optimal decision was determined for loading or not loading additional electricity units using the Markov decision process approach. The model was tested, and the results demonstrated the existence of optimal state-dependent decision and consumption costs considering the case study used in this study. The proposed model can be cost-effective for managers in the electricity industry. Improved efficiency and utilization of resources for electricity distribution systems to residential areas were realized, with subsequently enhanced service reliability to essential energy market customers.

Keywords: Demand, Electricity load, Modelling, Optimization, Stochastic.

1 | Introduction

Residential electricity load optimization has become one of the key issues in solving energy crisis problems in the past few years. In several nations worldwide, residential buildings constitute a large energy consumption.

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In the European Union, for instance, the residential sector accounted for 26.1% of the total energy consumption in 2018, and this catered to space, water heating, and electric end users such as lighting or appliances. In residential areas, variations of social, economic and technical characteristics among consumer groups influenced electricity consumption. This was based on the timing, location, peak and distribution of electric power. Considering the socioeconomic aspects and technical equipment used in Uganda, residential electricity load significantly influenced the type of dwelling and location. Indirect influences on electricity load were also attributed to the number of occupants [1], the number of bedrooms [2], the dwelling area [3], the floor area [4], incomes [5] and the household ownership of physical appliances.

Electricity consumption in residential areas was also affected by several social-economic factors worldwide. In some households, it was noted how much electricity appliances consumed to enable households to acquire knowledge about the expenditure patterns associated with such appliances. It was, therefore, prudent to devise appropriate methods for understanding electricity consumption based on household appliances. The degree to which such appliances were used in residential areas was of paramount importance.

In practical situations, the usage of an appliance and the related operational cost calculations considered estimates of the daily hours run by appliances that determined the product's wattage, daily consumption, annual energy consumption and the annual cost to run the appliance. The estimated daily run time of appliances was made through a rough estimate by keeping a log. Through a rough estimate, the household predicted the usage rates of an appliance daily, and such a household determined the hourly usage rate. As the number of power appliances consumed varied considerably depending on the setting, realistic estimates of current in residential areas were obtained considering the current and voltage used by the household appliance. Determining the daily consumption, annual consumption, and annual cost to run the appliance were critical factors influencing electricity loading decisions and consumption patterns of households in residential areas.

1.1 | Residential Electricity Load Background

The goal of meeting people's energy needs has become a crucial research topic of global concern in recent years. In modern society today, however, electric and electronic devices have increased tremendously, contributing to energy consumption for smartphones, televisions, appliances and various related devices. It had been widely believed that load demand for electricity did not vary significantly among households regardless of socioeconomic circumstances, considering the inhabitants of a family or an apartment building. However, residential electricity loading lacked predictability guidelines for modelling purposes since a solid understanding of the residential load profile and its prevailing state was needed. Despite this challenge, residential electricity load profiles had a big role in capacity planning by improving the efficiency of system operations, electricity grids, generation investment energy market, electricity tariffs, price structures, incentives, customer satisfaction, and other economic considerations.

1.2 | Research Objectives

I. To develop and optimize the residential electricity load model under stochastic demand.

II. To optimize electricity consumption costs under stochastic demand.

III. To test the residential electricity load model.

1.3 | Research Questions

I. What is the optimal residential electricity loading decision under stochastic demand?

II. What are the electricity consumption costs under stochastic demand?

1.4 | Methodology

1.4.1 | The survey instrument

Questionnaires were developed and pre-tested for the field survey, then administered to the residential areas and a sample of households within the residential areas. The first questionnaire established the demand transitions, electricity demand and electricity units available under the decision of loading additional units versus not loading additional units.

1.4.2 | Research participant

Field participants were trained to administer the questionnaire in this study and collected relevant data from households of a given residence.

1.4.3 | Study population/sample

The study was conducted on an accessible population that comprised two residential areas and one hundred households.

1.4.4 | Data analysis

To reduce the data collected to usable dimensions, the raw data collected was edited, processed and analyzed so that the data generated was organized and interpreted. An electronic database was created from the database generated. Data was analyzed from frequencies generated and presented to show the relationship between state transitions, the number of households, demand, available electricity units and the respective electricity loading decisions.

1.5 | Residential Load Management under Stochastic Demand

The study's theoretical foundation [6] emphasized the great challenge encountered while considering uncertainty in residential load demands. Considerations to load uncertainty of each residential customer were modelled using the simplex method with fuzzy numbers. The input for electricity consumers was used to optimize this fuzzified demand using the price vector as the input of the optimization problem. The effectiveness of the proposed stochastic load management scheme was validated by solving a two-demand problem. Each demand expected a minimum level of power contribution defined by a fuzzy constraint. In our approach, the residential electricity load under stochastic demand considered the nature of demand using a two-state Markov chain. The states of residential electricity demand represented possible states of demand. The optimal electricity loading decision and associated consumption costs were determined using the Markov decision process methodology over a designated finite period planning horizon.

The paper was organized as follows. After reviewing the literature in Section 2, the model was formulated in Section 3, where consideration was given to the process of estimating model parameters. The model was solved in Section 4, and a case study was presented in Section 5. The study showed the practical application of the proposed model, where analysis/discussion of the results and limitations of the study were presented. Lastly, conclusions followed in Section 6, with prospects for future research.

2 | Related Literature

In a recent study, an electrical system framework that measured the accessibility of electrical power [7] was considered. The author examined stochastic residential load management using fuzzy-based optimization approaches. A novel stochastic optimization framework to model the day-ahead load profile of a residential energy hub [8] was suggested using an incentive-based DR program. That was done through a distributed approach, where the load profile became smoother by considering the related aggregator's desirable load profile limits. Previous work on residential load considered three steps where an independent system operation ISO day-ahead RTP to a Residential Load Aggregator (RLA) was considered [9]: 1) the RLA predicted individual household loads, 2) aggregated the loads that minimized the costs, and 3) in the second layer, the RLA announced incentives to homes, and more Energy Management Systems (EMS) controlled the loads and maximized the reward in real-time.

A recent study of the load combination of power sales companies was based on various power values [10], where demand response data was extracted by load characteristics index and power consumption index. The proposed method reduced the power purchase cost and increased the power company's revenue. A systematic literature review, however, pointed to a diversity of modelling techniques and associated algorithms on short-term load forecasting [11]. The authors concluded that having a unified data set, a set of benchmarks, and well-defined metrics was desirable to compare all the modelling techniques and the corresponding algorithms clearly. A related approach that used a stochastic bottom-up model for generating electrical loads for residential buildings in Canada [12] was presented. The model investigated the impact of different household characteristics, appliance stock and energy behaviour on the timing and magnitude of non-HVAC energy loads at multiple houses and yielded significant results. The stochastic perturbation method and the transformed random variable method [13] produced important results, where energy-demand analysis was performed for the representative single house in Poland. The expanded polystyrene thermal conductivity and external temperature were considered uncertain. The stochastic perturbation method determined energy consumption's expected value and central moments.

In contrast, the transformed random variable method obtained the explicit form of the energy consumption probability density function. However, the highly resolved electricity consumption data of Austria, German and UK households [14] and the proposed applicable data-driven load model made critical awareness to model developers. Based on time series data, the average demand profiles were disentangled from the demand fluctuations. A stochastic model was then introduced to capture the intermittent demand fluctuations. A related study assigned pre-generated electricity and heat demand curves to georeferenced residential buildings in Germany [15]. That provided a large variety of residential load profiles that spatially corresponded to official social–demographical data. Results were validated on different aggregation values. The forecasting performance of models based on functional data analysis [16] gave important insights. The demand time series was first treated for the extreme values. The filtered series was then divided into deterministic and stochastic components. The additive modelling technique was used to model the deterministic component, whereas the functional autoregressive was used to forecast the stochastic component.

The literature cited showed important insights by current scholars that were crucial in studying the residential electricity load problem. However, the optimality of electricity loading decisions with associated consumption costs was not fully considered under demand uncertainty. The Markov decision process model provided a powerful framework for optimizing electricity loading decisions and electricity consumption costs under demand uncertainty considering several households in residential areas.

The major contributions of this paper to residential electricity load under stochastic demand highlighted the following:

I. The state-transition matrices that characterized the demand and consumption cost were computed under the prevailing electricity loading decisions.

II. The computation procedure calculated the expected consumption costs and accumulated consumption costs for the electricity loading decisions.

III. The Markov decision process formulation allowed the decision maker to load or not load extra units of electricity under different states of demand.

3 | Model Formulation

A discrete-time finite horizon MDP model was developed with decision epochs $t \epsilon T = \{1, 2, \ldots, E\}$. At each decision epoch t, the decision maker (i.e., electricity regulator) observed the electricity demand states by conducting some observatory tests concerning the electricity demand levels; when the available electricity exceeded demand, loading additional units was stopped, and the decision process was terminated. Otherwise, the decision maker decided (based on residential electricity demand) the optimal loading decision that had to be taken. The decision continued until the loading exercise ended for each decision maker's action. Therefore, an immediate reward emerged representing the total electricity consumption costs based on the decision taken.

Our goal was to solve the trade-off problem between loading additional electricity units with the associated consumption costs versus not loading additional electricity units. A formal definition of the core components of our MDP model follows.

3.1 | States

The demand state i was composed of two variables: favourable (state F) and unfavourable (state U). The favourable state was defined by the presence of customer N^S, with demand D^S observed by the decision maker at each decision epoch t within residential area r; where S∈{0,1}, r ∈{1,2}, t=1,2,…………T.

3.2 | Actions

We denoted the action space by A= $\{a_0, a_1, \dots \dots a_k\}$ where $a_i = 0$ represented not loading, and $a_i = 1$ represented loading additional units. We assumed that if $a_i = 0$ was chosen, additional electricity units were not loaded when customers in residential areas were fully supplied, while additional units needed to be loaded whenever electricity demand exceeded available electricity.

3.3 | Transition Probabilities

When the decision maker chooses action $s_t \epsilon S$ at decision epoch t when demand was in state s_t, the demand state moved to s_{t+1} at t+1 with probability P_t (S_t /S_{t+1}, a_t). We assumed that

$$P_t(S_{t+1}/S_t\ , a_t) = P_t^{\lambda}(c_t\ , a_t\ ) \text{ x } P_t^{\alpha}(\alpha_{t+1}\ \ P_t^{\alpha}\ (\alpha_{t+1}\ , \alpha_t), \tag{1}$$

where

$$P_t^{\lambda}\ (c_t\ , a_t\ ) \text{ and } P_t^{\alpha}\ (\alpha_{t+1}\ /\ \alpha_t\ ,a_t\ ),$$

where the transition probabilities for the favourable and unfavourable demand states, respectively. This assumption was consistent with our proposition that favourable and unfavourable demand depended not on each other but on the decision maker's action. More specifically, we assumed that

$$P_t^{\lambda}\ (c_{t+1}/c_t\ ,a_1) > P_t^{\lambda}\ (c_{t+1}\ /c_t\ , a_2)\ \ > \dots\dots\dots\dots > P_t^{\lambda}(c_{t+1}\ /\ c_t, a_k), \tag{2}$$

where c_{t+1} represented a favourable state.

$$P_t^{\lambda}(\lambda_{t+1}/\lambda_t\ ,a_1) < P_t^{\lambda}\ (\lambda_{t+1}/\lambda_t\ , a_t) < \dots\dots\dots < P_t^{\lambda}(\lambda_{t+1}\ /\lambda_t\ , a_k), \tag{3}$$

where λ_{t+1} represented unfavourable state than λ_t.

3.4 | Reward Functions

Our model included a reward function $\lambda_t(s_t\ , a_t)$ that reflected the utility/disutility of the decision maker as realized demand state s_t with action a_t which was taken at decision epoch t. This was defined as

$$\alpha_t(s_t/a_t) = \sum_{i=1}^{N} \alpha_{t,i}\ (c_i, a_t) + \alpha_t\ (\alpha_t\ , a_t), \tag{4}$$

where $\alpha_{t,i}$ (c_i, a_t) represented the immediate reward for favourable demand state c_i and $\alpha_t(\alpha_t, a_i)$ was the immediate reward for unfavourable demand state α_t.

Hence, the corresponding reward functions were assumed to follow the following inequality for all t:

$$\alpha_t\ (c_i\ , a_t) < \alpha_t(c_i'\ , a_t), \alpha_t\ (\alpha,a_t) < \alpha_t\ (\alpha'\ , a_t). \tag{5}$$

3.5 | Value Function

Our MDP model aimed to find the optimal strategy for loading electricity units. Therefore, a rule was sought to take action at each state to minimize the expected total electricity consumption costs over the planning period. This could be achieved by solving Bellmann's recursive equations for all $s_t \in S$ and t=1, 2, …, T.

$$V_t(s_t) = \min_{a_t \in A}\{\alpha_t\ (s_t\ , a_t) + \sum_{s_{t+1} \in S} P_t(s_{t+1}/s_t, a_t)\ v_{t+1}(s_{t+1})/s_t, a_t)v_{t+1}(s_{t+1})\}, \tag{6}$$

where $v_t(s_t)$ represented the minimum expected total reward at the decision epoch t when demand was in state s_t with the boundary condition

$$V_{\alpha+1}(s) = \alpha_{T+1}(s).$$

3.6 | Formulating the Finite-period Dynamic Programming Problem

Since demand was considered a favourable state (state F) or unfavourable state (state U), the problem was considered an optimal electricity loading decision, and this was modelled as a dynamic programming problem over a finite period planning horizon. We denoted $g_n(i, r)$ as the expected total consumption costs accumulated by residential area r during the periods n, n+1,….., N given that the system's state at the beginning of period n was i∊{F, U}. The recursive equation relating g_n and g_{n+1} became

$$g_N(i, r) = \min_S[e_i^S\ (r) + Q_{iF}^S(r)\ g_{n+1}(F,r) + Q_{iU}^S\ (r)\ g_{n+1}(U,r)]. \tag{7}$$

The following condition was sufficient:

$$g_{N+1}(F,r) = g_{N+1}(U,r) = 0. \tag{8}$$

The consumption costs C^S_{ij} (r) + g_{n+1} (j) resulting from reaching state j∊{F, U}at the start of period n+1 from state i ∊{F, U} at the start of period n occurred with probability $Q^S_{ij}(r)$. Clearly,

$$e^S(r) = [Q^S(r)]\ [C^S(r)]^{T,}\quad S\varepsilon\ [0, 1],\quad r = [1,2]. \tag{9}$$

The corresponding dynamic programming recursive equations were thus obtained.

$$g_N(i,r) = \min_S[e_i^S(r) + Q_{iF}^S(r)\ g_{n+1}(F,r) + Q_{iU}^S(r)\ g_{n+1}(U,r)]. \tag{10}$$

$$g_N(i,r) = \min_S\ [e_i^S(r)\ ]. \tag{11}$$

Electricity demand in excess of supply yielded the consumption cost matrix.

$$C^S(r) = (c_l + c_s)\ [D^S(r) - A^S(r)]. \tag{12}$$

Otherwise,

$$C^S(r) = c_o[A^S(r) - D^S(r)], \tag{13}$$

when supply exceeds demand. Clearly,

$$C_{ij}^S\ (r) = \begin{cases} (c_l\ +\ c_s\ +\ c_o)\left[D_{ij}^S(r) -\ A_{ij}^S(r)\right],\ \text{if}\ \ D_{ij}^S(r) >\ A_{ij}^S(r), \\ c_o\left[A_{ij}^S(r) -\ D_{ij}^S(r)\right],\ \text{if}\ \ D_{ij}^S(r) \leq\ A_{ij}^S(r). \end{cases} \tag{14}$$

For i,j ∊{F,U}, r = {1,2}, S∊{1,0).

The justification for *Eq. (13)* and *Eq. (14)* was that $D^S_{ij}(r) - A^S_{ij}(r)$ units had to be loaded to meet excess demand. Otherwise, loading was cancelled when demand was less than or equal to supply. The following conditions were, therefore, sufficient to execute the model:

I. S=1 when cl > 0 otherwise S=0 when cl = 0.

II. cs > 0 when shortages were allowed; otherwise, cs=0 when shortages were not allowed.

4 | Optimization

The electricity loading decision/consumption costs were optimized for periods 1 and 2 in residential area r.

4.1 | Optimization-Period 1

Considering favourable (state F) demand, the optimal loading decision was determined as

$$S=\begin{cases}1, & \text{if } e_F^1(r) < e_F^0(r),\\ 0, & \text{if } e_F^1(r) \geq e_F^0(r),\end{cases} \quad (15)$$

with expected consumption costs

$$g_1(F,r)=\begin{cases}e_F^1(r), & \text{if } S = 1,\\ e_F^0(r), & \text{if } S = 0.\end{cases} \quad (16)$$

When demand was unfavourable (i.e., in state U), the optimal loading decision was determined as

$$S=\begin{cases}1, & \text{if } e_U^1(r) < e_U^0(r),\\ 0, & \text{if } e_U^1(r) \geq e_U^0(r),\end{cases} \quad (17)$$

with expected consumption costs

$$g_1(U,r)=\begin{cases}e_U^1(r), & \text{if } S = 1,\\ e_U^0(r), & \text{if } S = 0.\end{cases} \quad (18)$$

4.2 | Optimization-Period 2

Using *Eq. (10)* and *Eq. (11)* and recalling that $a^S_i(r)$ denoted the already accumulated consumption costs at the end of period 1 as a result of decisions made during that period.

$$a_i^S(r) = e_i^S(r) + Q_{iF}^S(r)\min[e_F^1(r), e_F^0(r)] + Q_{iU}^S(r)\min[e_U^1(r), e_U^0(r)]. \quad (19)$$

$$a_i^S(r) = e_i^S(r) + Q_{iF}^S(r)\, g_2(F,r) + Q_{iU}^S(r) g_2(U,r). \quad (20)$$

Therefore, for favourable demand (i.e. in state F), the optimal loading decision during period 2 was determined as

$$S=\begin{cases}1, & \text{if } a_F^1(r) < a_F^0(r),\\ 0, & \text{if } a_F^1(r) \geq a_F^0(r).\end{cases} \quad (21)$$

While the associated accumulated consumption costs were

$$g_2(F,r)=\begin{cases}a_F^1(r), & \text{if } S = 1,\\ a_F^0(r), & \text{if } S = 0.\end{cases} \quad (22)$$

Similarly, when demand was unfavourable (i.e. in state U), the optimal loading decision during period 2 was determined as

$$S=\begin{cases}1, & \text{if } a_U^1(r) < a_U^0(r),\\ 0, & \text{if } a_U^1(r) \geq a_U^0(r).\end{cases} \quad (23)$$

In this case, the associated accumulated consumption costs were

$$g_2(U,r)=\begin{cases}a_U^1(r), & \text{if } S = 1,\\ a_U^0(r), & \text{if } S = 0.\end{cases} \quad (24)$$

5 | A Case Study of Uganda Electricity Distribution Company

The model developed was presented using a case study of Uganda Electricity Distribution Company (UEDC) in Uganda, which experienced random electricity demand in residential areas. The UEDC sought the elimination of excess electricity supply under unfavourable demand (state U) or avoiding shortages when demand was favourable (state F), and hence, UEDC sought an optimal electricity loading decision and consumption costs considering a planning horizon of two week.

5.1 | Data Collection

A number of households, demand and electricity units available (in kwh) were observed and recorded from two residential areas. The states of demand under electricity loading decisions were considered over ten weeks for favourable demand (state F) and unfavourable demand (state U). The data was captured in *Tables 1-3*.

Table 1. Households versus state-transitions for electricity loading decisions.

		Load Additional Units (S=1)		Do not Load Units (S=0)	
Residential Area (r)	States of Demand	F	U	F	U
1	F	91	71	82	30
	U	63	13	55	25
2	F	45	59	64	40
	U	59	13	45	11

Table 2. Demand (in kwh) versus state-transitions in residential areas for electricity loading decisions.

		Load Additional Units (S=1)		Do not Load Units (S= 0)	
Residential Area (r)	States of Demand	F	U	F	U
1	F	78	38	62	39
	U	60	65	39	40
2	F	100	30	36	39
	U	30	70	69	60

Table 3. Available electricity (in kwh) versus state-transitions in residential areas for electricity loading decisions.

		Load Additional Units (S=1)		Do not Load Units (S= 0)	
Residential Area (r)	States of Demand	F	U	F	U
1	F	95	80	34	45
	U	54	75	47	55
2	F	47	40	81	79
	U	36	56	38	72

For either loading decision taken, unit loading cost (c_l) = 1.20 USD per kwh, unit operational cost(c_o) = 0.80 USD per week and unit shortage cost(c_s) = 0.32 USD per week.

5.2 | Computation of Model Parameters

We illustrated how the demand transition matrices and consumption cost matrices were determined from empirical data. For example, considering matrix $Q^1(1)$ for residential area 1 and electricity loading decision 1 and referring to *Eq. (1)*.

$$Q^1_{FF}(1) = \frac{N^1_{FF}(1)}{N^1_{FF}(1)+ N^1_{FU}(1)} = \frac{91}{91+71} = 0.5617,$$

$$Q^1_{FU}(1) = \frac{N^1_{FU}(1)}{N^1_{FF}(1)+ N^1_{FU}(1)} = \frac{71}{91+71} = 0.4383,$$

$$Q^1_{UF}(1) = \frac{N^1_{UF}(1)}{N^1_{UF}(1)+ N^1_{UU}(1)} = \frac{53}{63+13} = 0.8289,$$

$$Q^1_{UU}(1) = \frac{N^1_{UU}(1)}{N^1_{UF}(1)+ N^1_{UU}(1)} = \frac{13}{63+13} = 0.1711.$$

Hence, $Q^1(1) = \begin{pmatrix} Q^1_{FF} & Q^1_{FU} \\ Q^1_{UF} & Q^1_{UU} \end{pmatrix} = \begin{pmatrix} 0.5617 & 0.4383 \\ 0.8289 & 0.1711 \end{pmatrix}.$

We note that $\sum_{i\in F,U} Q^1_{iF}(1) + Q^1_{iU}(1) = 1$ and $Q_{iF}(1) \leq 0$ for all i $\in$\{F,U\}.

Considering matrix $C^1(1)$ for residential area 1 given electricity loading decision 1, from *Eqs. (12)-(14)*.

$c^1_{FF}(1) = (95 - 78)(0.80) = 18.6,$

$c^1_{FU}(1) = (80 - 38)(0.80) = 33.6,$

$c^1_{UF}(1) = (60 - 54)(12+0.80+0.32) = 13.9,$

$c^1_{UU}(1) = (75 - 65)(0.80) = 8.0.$

Hence, $C^1(1)=\begin{pmatrix} c^1_{FF} & c^1_{FU} \\ c^1_{UF} & c^1_{UU} \end{pmatrix} = \begin{pmatrix} 18 & 33.6 \\ 13.9 & 8.0 \end{pmatrix}$.

Using a similar approach, the remaining matrices were calculated.

Using *Eq. (19)* and *Eq. (20)*, the expected consumption costs (in USD) and accumulated consumption costs (in USD) for the two residential areas were computed under favourable demand (state F) and unfavourable demand (state U); whose results were presented in *Table 4*.

Table 4. Expected and accumulated consumption costs (in USD) for residential areas.

Residential Are (r)	State of Demand (i)	Expected Consumption Costs $e^S(r)$		Accumulated Consumption Costs $a^S(r)$	
		Load Additional Units (S=1)	Do not Load Units (S=0)	Load Additional Units (S=1)	Do not Load Units (S=0)
1	F	22.37	48.80	37.03	66.46
	U	12.80	4.78	32.25	21.65
2	F	57.71	33.84	78.05	57.45
	U	9.80	52.73	35.30	79.75

5.3 | Analysis of Results

Week 1 (Residential area 1)

Considering residential area 1, when demand was favourable and noting that 22.37 < 48.80, S=1 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 22.37 USD for the case of favourable demand. Since 4.78<12.89, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 4.78 USD for the case of unfavourable demand.

Week 1 (Residential area 2)

Considering residential area 2, since 33.84<57.71, then S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 33.84 USD for the case of favourable demand. Since 9.80<52.73, S=1 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 9.80 USD for the case of unfavourable demand.

Week 2 (Residential area 1)

Considering residential area 1, since 37.03<66.46, S=1 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 37.03 USD for the case of favourable demand. Since 21.65<32.25, S=0 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 21.65 USD for the case of unfavourable demand.

Week 2 (Residential area 2)

Considering residential area 2, since 57.45<78.05, S=0 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 57.45 USD for the case of favourable demand.

Since 35.30<79.75, S=1 was chosen as an optimal electricity loading decision for week 2 with associated accumulated consumption costs of 35.30 USD for the case of unfavourable demand.

5.4 | Model Validation

In this section, we considered out-of-sample data in two residential areas to demonstrate the proposed model's predictive ability. A sample of 100 customers was considered in each residential area, and considerations on demand and available electricity units were captured in *Table 5* and *Table 6* below.

Table 5. Households versus state-transitions for electricity loading decisions.

Residential Area (r)	States of Demand (F/U)	Load Electricity Units (S=1)		Do not Load Electricity Units (S=0)	
		F	**U**	**F**	**U**
1	F	70	30	80	20
	U	40	60	25	75
2	F	90	10	55	45
	U	35	65	15	85

Table 6. Demand (in kwh) versus state-transitions for electricity loading decisions in residential areas.

Residential Area (r)	States of Demand (F/U)	Load Electricity Units (S=1)		Do not Load Electricity Units (S=0)	
		F	**U**	**F**	**U**
	F	80	60	10	30
1	U	30	60	30	40
	F	30	20	70	60
2	U	20	40	20	60

Table 7. Available electricity (in kwh) versus state-transitions for electricity loading decisions in residential areas.

Residential Area (r)	States of Demand (F/U)	Load Electricity Units (S=1)		Do not Load Electricity Units (S=0)	
		F	**U**	**F**	**U**
1	F	70	60	20	20
	U	45	70	40	50
2	F	40	10	60	70
	U	40	30	0	50

For either loading decision taken, unit loading cost (c_l) = 1.20 USD per kwh, unit operational cost (c_o) = 0.80 USD per week and unit shortage cost (c_s) = 0.32 USD per week.

5.5 | Computation of Model Parameters

5.5.1 | Demand transition matrices

Residential area 1 Residential area 2

$Q^1(1) = \begin{pmatrix} 0.700 & 0.300 \\ 0.400 & 0.600 \end{pmatrix}$, $Q^1(2) = \begin{pmatrix} 0.900 & 0.100 \\ 0.350 & 0.650 \end{pmatrix}$.

$Q^0(1) = \begin{pmatrix} 0.800 & 0.200 \\ 0.250 & 0.750 \end{pmatrix}$, $Q^0(2) = \begin{pmatrix} 0.550 & 0.450 \\ 0.150 & 0.850 \end{pmatrix}$.

5.5.2 | Consumption cost matrices

Residential area 1 Residential area 2

$C^1(1) = \begin{pmatrix} 6.4 & 1.28 \\ 23.2 & 3.2 \end{pmatrix}$, $C^1(2) = \begin{pmatrix} 9.28 & 3.2 \\ 3.2 & 23.2 \end{pmatrix}$.

$C^0(1) = \begin{pmatrix} 6.94 & 3.2 \\ 3.2 & 3.2 \end{pmatrix}$, $C^0(2) = \begin{pmatrix} 1.60 & 12.8 \\ 69.6 & 6.4 \end{pmatrix}$.

5.5.3 | Expected consumption costs

Residential area 1

$e_F^1(1) = (0.700)(6.4) + (0.300)(1.28) = 8.32,$

$e_F^0(1) = (0.800)(6.94) + (0.200)(3.2) = 6.172,$

$e_U^1(1) = (0.400)(23.2) + (0.600)(3.2) = 11.200,$

$e_U^0(1) = (0.250)(3.2) + (0750)(3.2) = 3.20.$

Residential area 2

$e_F^1(2) = (0.900)(9.28) + (0.100)(3.2) = 8.672,$

$e_F^0(2) = (0.55)(1.6) + (0.45)(8.32) = 6.640,$

$e_U^1(2) = (035)(3.2) + (0.65)(23.2) = 16.200,$

$e_U^0(2) = (0.15)(69{,}6) + (0.850\backslash)(6.4) = 15.880.$

5.5.4 | Accumulated consumption costs

Residential area 1

$a_F^1(1) = 13.60 + (0.700)(6.172) + (0.300)(3.200) = 13.600,$

$a_F^0(1) = 6.172 + (0.800)(6.172) + (0.200)(0.540) = 11.750,$

$a_U^1(1) = 11.200 + (0.400)(6.172) + (0.600)(3.20) = 15.589,$

$a_U^0(1) = 3.200 + (0.250)(6.172) + (0750)(3.2) = 7.143.$

Residential area 2

$a_F^1(2) = 8.672 + (0.900)8.672) + (0.100)15.880) = 18.065,$

$a_F^0(2) = 6.64 + (0.55)(8.672) + (0.45)(15.880) = 18.556,$

$a_U^1(2) = 16.20 + (0.35)(8.672) + (0.65)(15.880) = 29.557,$

$a_U^0(2) = 15.880 + (0.150)(8.672) + (0850)(15.880) = 30.679.$

Table 7. Expected and accumulated consumption costs (in USD) for residential areas.

Residential Area (r)	State of Demand (i)	Expected Consumption Costs $e^S(r)$		Accumulated Consumption Costs $a^S(r)$	
		Load Additional Units (S=1)	Do not Load Units (S=0)	Load Additional Units (S=1)	Do not Load Unit (S=0)
1	F	8.320	6.170	13.360	11.750
	U	11.200	3.20	15.589	7.143
2	F	8.672	9.200	18.065	18.556
	U	16.200	15.880	29.557	30.679

5.6 | Analysis of Results

Week 1 (Residential area 1)

Considering residential area 1, when demand was favourable and noting that 6.17 < 8.32, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 6.17 USD for the case of favourable demand. Since 3.20<11.20, S=0 was chosen as an optimal electricity loading decision for week 1 with associated expected consumption costs of 3.20 USD for the case of unfavourable demand.

Week 1 (Residential area 2)

When demand was favourable (state F) and noting that 8.672<9.200, S=1 was chosen as an optimal electricity loading decision with associated expected consumption costs of 8.672 USD. When demand was unfavourable (state U), S=0 was chosen as an optimal electricity loading decision with associated expected consumption costs of 8.67 USD for the case of unfavourable demand (state U).

Week 2 (Residential area 1)

When demand was favourable (state F) and noting that 11.750<13.60, S=0 was the optimal electricity loading decision with associated accumulated consumption costs of 11.750 USD. Similarly, since 7.143<15.589, S=0 was the optimal electricity loading decision with accumulated consumption costs of 7.143 USD for the case of unfavourable demand (state U).

Week 2 (Residential area 2)

When demand was favourable (state F) and noting that 18.065<21.116, S=1 was the optimal electricity loading decision with associated accumulated consumption costs of 18.065 USD for the case of favourable demand (state F). Since 29.557<30.679, then S=1 was the optimal electricity loading decision with associated accumulated consumption costs of 29.5557 USD for the case of unfavourable demand (state U).

5.5 | Discussion of Results

Considering the case study of UEDC presented, the optimality of electricity loading decisions and consumption costs over a finite period planning horizon yielded important results for discussion. Results indicated optimal state-dependent electricity loading decisions and consumption costs were dependent and consistent at every stage of the decision problem. This was attributed to the stationary demand transition probabilities considered at the decision epochs.

When demand was initially favourable (state F), additional electricity units were needed for weeks 1 and 2 of residential area 1. However, when demand was initially unfavourable (state U), additional electricity units were not required for weeks 1 and 2 of residential area 2.

6 | Conclusions

A Markov decision process model that optimized electricity loading decisions and consumption costs with stochastic demand was presented in this paper. An optimal electricity loading decision was determined for residential areas over a multi-period planning horizon using dynamic programming. Therefore, as an optimization strategy for electricity loading decisions and consumption costs in residential areas, computational efforts using the Markov decision process model showed promising results.

6.1 | Model Implications on the Electricity Industry

The proposed model has interesting implications in practical terms as a decision-making tool for sustaining electricity regulation strategies in industry. Considering the case study results, demand uncertainty affected electricity regulatory policies, which was a driving force for the comparative analysis of electricity consumption. Although Markov decision processes for optimizing electricity loading options were fundamental for practical purposes, stationarity of demand transition probabilities raised a number of salient issues to consider. For example, changing demand patterns for electricity consumption among users, unpredictable power outages during the demand cycle, price fluctuations of electricity supply etc, left a lot to be examined; especially in the Ugandan context. It was also noted that the study used a smaller number of residential areas, considering the two areas captured in the case study and the model validation section. Future studies must aim to increase the number of residential areas to establish a realistic representative sample. This can also improve the model's predictive ability as a decision-making tool. In effect, the electricity regulatory authorities for energy distribution can be in a position to gain a competitive advantage over energy providers for domestic consumption.

Statements and Declarations

The corresponding author declares that this manuscript is original, has not been published before and is not currently being considered for publication elsewhere.

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Data Availability

The data used during this study is not publicly available due to company policy but can be obtained from the corresponding author on reasonable request.

Conflicts of Interest

No financial/non-financial personal interests could be considered inappropriate to influence the presentation or interpretation of the research findings.

Competing Interests

There are no competing interests whatsoever for the submitted manuscript to the Journal of Industrial Engineering Research.

Financial Interests

The authors have no financial interests whatsoever in submitting this work.

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Toward Characterizing Solutions to Complex Programming Problems Involving Fuzzy Parameters in Constraints

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Abstract

The current study investigates to characterize the Complex Programming Problem (CPP) solution in a fuzzy environment. The paper is divided into two parts: 1) the first presents a Fuzzy Complex Programming Problem (F-CPP) with fuzzy complex constraints, and 2) the second presents the optimality criteria using the fuzzy complex cone. The CPP is suggested by involving fuzzy numbers in the constraints in parts. Using the $\alpha-$cut set concepts, the problem is converted into the $\alpha-$complex programming. A number of basic theorems with proofs are established concerning the basic results for the fuzzy complex set of solutions for the F-CPP, and the optimality criteria of the saddle point for F-CPP with fuzzy cones is derived.

Keywords: Complex programming, Fuzzy numbers, α-cut set, Kuhn-Tucker's optimality conditions, Saddle point, Cone, Duality.

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1 | Introduction

Complex Programming Problem (CPP) applications are found in [1]–[3]. In earlier works in the field of CPP, the majority of the authors considered only the real part of the objective function of the problem as the objective function of the problem, neglecting the imaginary part, and the corresponding constraints have been assumed as a cone in the complex space $\mathbb{C}^n$. However, in several applications of the real-world problem, the imaginary part involved in objective function especially plays a crucial role.

Mathematical programming in complex space has been originated by [1], where Farkas's theorem has generalized to the complex space. Hanson and Mond [4] have generalized Wolfe's duality optimization from real lines to complex numbers. Duca [5] formulated the vectorial optimization model involving complex numbers and derived the necessary and sufficient conditions at the point to be the efficient solution to the problem. Ferrero [6] considered the finite dimensional spaces using the separation arguments. In addition, the optimality conditions have been established in the real and imaginary parts of the objective function. Abrams [7] established sufficient conditions for optimal points of the objective function's real part, neglecting the imaginary part. Ben-Israel [8] introduced two theorems with proofs for equalities. Smart and Mond [9] have shown that the necessary conditions for optimality in polyhedral-cone-constrained nonlinear programming problems are sufficient with the special type of invexity hypothesis. In addition, they extended the duality results for a Wolfe-type dual. Youness and Elborolosy [10] formulated the optimization problem involving complex numbers. Malakooti [11] developed a complex method with interior search directions to solve linear and nonlinear programming problems.

One of the difficulties that emerged with the application of MP is that the parameters are not constants but uncertain. In fuzzy sets, as Zadeh [12] proposed, fuzzy numerical data is used as the fuzzy subsets of real lines, referred to as fuzzy numbers. Later, Dubois and Prade [13] studied the applications of mathematical operations on real lines to fuzzy numbers with the help of the fuzzification principle. The fuzzy nature of a goal programming problem was discussed by [14] and later followed by [15], [16], and many other authors working in that field. Tanaka et al. [17] introduced the concepts of fuzzy mathematical programming problems, following Bellman and Zadeh [18] and then by Tanaka and Asai [19].

Buckley [20] proposed the definition of fuzzy complex numbers following the concept of $\alpha-\mathrm{cut}$ and investigated two special types depending on the forms $z = x + iy$ and $z = re^{i\gamma}$. Zhang and Xia [21] proposed two algorithms for solving complex quadratic mathematical programming problems with linear equality constraints and both an l_1 -norm and linear equality constraints. Zhang and Xia [3] proposed two complex-valued optimization solutions to constrained nonlinear programming problems of real functions in complex variables. Khalifa et al. [22] characterized the solution of complex nonlinear programming with interval-valued neutrosophic trapezoidal fuzzy parameters. Many researchers have developed the optimization model in complex spaces (for instance, Mond [23], Huang [24], and Elbrolosy [25]). Usha and Kumar [26] studied the queuing model using the fuzzy ranking method in a fuzzy environment. Khalifa et al. [22] characterized the solution of complex nonlinear programming with interval-valued neutrosophic trapezoidal fuzzy parameters. Qiu et al. [27] developed a fuzzy relation bi-level optimization model. They presented an application in the wireless communication station system. Khalifa and Kumar [28] recently applied fuzzy goal programming for nonlinear complex programming in a neutrosophic environment.

This paper is divided into two parts: The first part considers an Fuzzy Complex Programming Problem (F-CPP) with fuzzy complex constraints, and the second part presents the optimality criteria using the fuzzy complex cone. The CPP is considered in all the two parts by involving fuzzy numbers in the constraints. Using the $\alpha-$cut set concepts, the problem is converted into the corresponding $\alpha-$complex programming. Some basic theorems with proofs are established concerning the basic results for the fuzzy complex set of solutions for the F-CPPs, and the optimality criteria of the saddle point for F-CPP with fuzzy cones is derived.

The remaining structure of this study is outlined below:

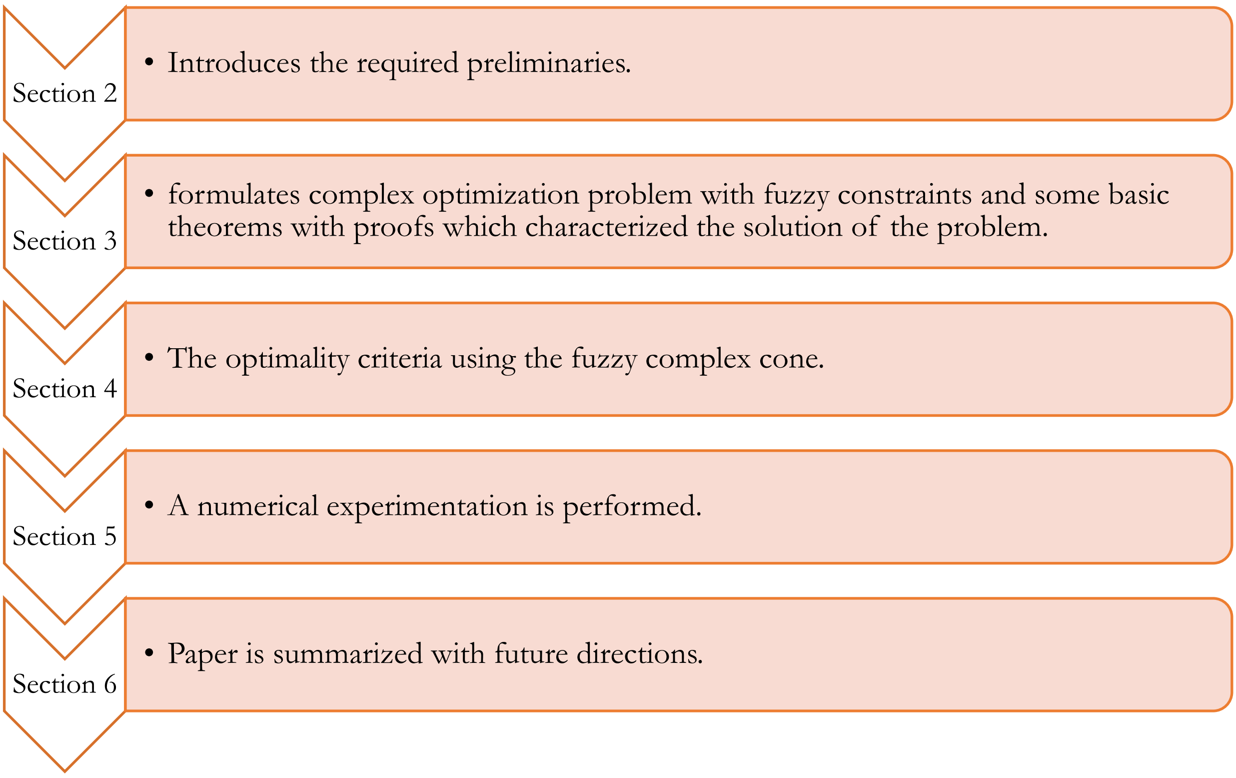

Fig. 1. Research method.

2 | Preliminaries

Represent $\Re = (-\infty, \infty)$ as real line, and $z = \{x + iy: x, y \in \Re,\ i = \sqrt{-1}\}$ as a complex number, $\mathbb{C}$ = field of complex numbers.

Definition 1 ([20]). A map $\tilde{Z}\colon\ \mathbb{C} \to [0,1]$ is referred to as a fuzzy complex set, $\mu_{\tilde{Z}}(z)$ is said to be a membership function of $\tilde{Z}$ for z, $F(\mathbb{C}) = \{\tilde{Z}: \tilde{Z}\colon\ \mathbb{C} \to [0,1]\}$ represents all fuzzy complex sets on $\mathbb{C}$.

Definition 2. The α −cut set of a complex set $\tilde{Z}$, designated by $\tilde{Z}_\alpha$, can be written as follows:

$\tilde{Z}_\alpha = \{z = x + i\, y \in \mathbb{C}: \mu_{\tilde{Z}}(z) \geq \alpha\}$.

Definition 3 ([20]). Supp $\tilde{Z} = \{z = x + i\, y \in \mathbb{C}: \mu_{\tilde{Z}}(z) \geq 0\}$ is referred to as the support of $\tilde{Z}$.

Definition 4 ([20]).

I. $\tilde{Z} \in F(\mathbb{C})$ is convex fuzzy complex set on $\mathbb{C}$, iff for all $\alpha \in [0,1]$, $\tilde{Z}_\alpha$ is a convex complex set.

II. $\tilde{Z} \in F(\mathbb{C})$ is a closed fuzzy complex set on $\mathbb{C}$, iff for $\alpha \in [0,1]$, $\tilde{Z}_\alpha$ is a closed complex set.

III. If $\tilde{Z} \in F(\mathbb{C})$, Supp $\tilde{Z}$ is bounded set on $\mathbb{C}$, iff for $\alpha \in [0,1]$, $\tilde{Z}_\alpha$ is bounded on $\mathbb{C}$.

IV. $\tilde{Z} \in F(\mathbb{C})$ is normal fuzzy complex set on $\mathbb{C}$, iff for $\alpha \in [0,1]$, $\{z \in \mathbb{C}: \mu_{\tilde{Z}}(z) = 1\} \neq \emptyset$.

Definition 5. A normal convex fuzzy complex set on $\mathbb{C}$ is referred to as a fuzzy complex number.

Definition 6 ([5], [29]). Let us consider that $\phi \neq H \subset \mathbb{C}^n$.

I. H is a cone, provided that for each $z \in H$ and each $\alpha \in\ ]0, \infty[$, we have $\alpha\, z \in H$.

II. H is a convex cone, when it is both – convex as well as a cone.

III. Intersection of all convex cones in $\mathbb{C}^n$ containing the set H is a convex cone spanned by H, and it is designated by con (H).

Definition 7 ([5]). A polyhedral cone S in $\mathbb{C}^n$ is a convex cone generated by finitely many vectors, that is, a set of the form $S\,\Re_+^k = \{Ax: x \in \Re_+^k\}$, for some positive integer k and $A \in \mathbb{C}^{n\times k}$.

3 | Problem Definition (Part I)

Let us recall a problem in complex space as follows:

$$\begin{aligned} &\min \operatorname{Re} f(z),\\ &\text{s.t.}\\ &z \in \widetilde{M}, \end{aligned} \tag{1}$$

where $\widetilde{M}$ referrers to a fuzzy complex non-empty subset of $\mathbb{C}^n$ (i.e., $\emptyset \neq \widetilde{M} \subset \mathbb{C}^n$), and $f: \widetilde{M} \to \mathbb{C}$ is a function of complex variable z.

Problem (1) is transformed to its crisp counterpart as below:

$$\begin{aligned} &\min \operatorname{Re} f(z),\\ &\text{s.t.}\\ &z \in \mu_{\widetilde{M}}(z), \end{aligned} \tag{2}$$

where $\mu_{\widetilde{M}}(z) = \mu_{\widetilde{M}}(x, y) = \min\big(\mu_{\widetilde{M}}(x), \mu_{\widetilde{M}}(y)\big)$, $\mu_{\widetilde{M}}(x): \Re^n \to [0,1], x \in \Re^n$; $\mu_{\widetilde{M}}(y): \Re^n \to [0,1], y \in \Re^n$.

Definition 8. The fuzzy feasible point $z^\circ \in \widetilde{M}$ with the membership function $\mu_{\widetilde{M}}(z^\circ) = \mu_{\widetilde{M}}(x^\circ, y^\circ) = \min\Big(\mu_{\widetilde{M}}(x^\circ), \mu_{\widetilde{M}}(y^\circ)\Big)$, $z^\circ = x^\circ + i\,y^\circ$ is a fuzzy optimal solution to the *Problem (2)* when $\operatorname{Re} f(z^\circ) = \min\big(\operatorname{Re} f(z): z \in \widetilde{M} \text{ with membership } \mu_{\widetilde{M}}(z)\big)$.

Theorem 1. Let $\widetilde{M}$ be a non-empty fuzzy complex subset of $\mathbb{C}^n$ and $f: \widetilde{M} \to \mathbb{C}$ be a function of quasi-convex real part on $\widetilde{M}$ relative to $\Re_+ = [0, \infty[$. Then, $\widetilde{M}$ is convex.

Proof: Let z_1 and z_2 be two solutions of *Problem (2)*, then $z_1, z_2 \in \widetilde{M}$ with $\mu_{\widetilde{M}}(z_1), \mu_{\widetilde{M}}(z_2)$, and $\operatorname{Re} f(z_1) = \operatorname{Re} f(z_2) = \min\big(\operatorname{Re} f(z): z \in \widetilde{M}\big)$ and therefore

$$0 = \operatorname{Re} f(z_1) - \operatorname{Re} f(z_2) \in \Re_+. \tag{3}$$

Since $\widetilde{M}$ is a fuzzy complex set, then for $0 \le \zeta \le 1 \Rightarrow z_\zeta = \zeta z_1 + (1-\zeta) z_2 \in \widetilde{M}$ with $\mu_{\widetilde{M}}(\zeta z_1 + (1-\zeta) z_2) \ge \min(\mu_{\widetilde{M}}(z_1), \mu_{\widetilde{M}}(z_2)\,)$. From the assumption of the *Theorem 1* and from *Eq. (3)*, we have

$$\operatorname{Re} f(z_1) - \operatorname{Re} f(z_2) \in \Re_+. \tag{4}$$

Or equivalently, $\operatorname{Re} f(z_1) \le \operatorname{Re} f(z_2)$. Thus, z_ζ is a convex set.

Theorem 2. Let us consider that $\phi \neq \widetilde{M} \subset \mathbb{C}^n$, and $f: \widetilde{M} \to \mathbb{C}$ be a function and z_1 be a solution to *Problem (1)*. If f has a strictly convex real part of z_1 following $\Re_+$, then z_1 is the unique solution to the *Problem (1)*.

Proof: Let z_2 be another solution to the *Problem (1)*, $z_1 \neq z_2$, then $z_1, z_2 \in \widetilde{M}$ with membership function $\mu_{\widetilde{M}}(z_1) = \mu_{\widetilde{M}}(x_1, y_1) = \min(\mu_{\widetilde{M}}(x_1), \mu_{\widetilde{M}}(y_1))$, $\mu_{\widetilde{M}}(z_2) = \mu_{\widetilde{M}}(x_2, y_2) = \min(\mu_{\widetilde{M}}(x_2), \mu_{\widetilde{M}}(y_2))$, with $\mu_{\widetilde{M}}(x_1): \Re^n \to [0,1]$. Since $z_1 = x_1 + i\,y_1, z_1 \in \widetilde{M} \subset \mathbb{C}^n, \operatorname{Re}(z_1) = \operatorname{Re}(z_2) \min \operatorname{Re}\{f(z): z \in \widetilde{M}\}$, this leads to

$$\operatorname{Re} f(z_1) - \operatorname{Re}(z_2) = 0. \tag{5}$$

Let $0 \le \zeta \le 1$. Since, $\widetilde{M}$ is convex in $\mathbb{C}^n$, then $z_\zeta = (1-\zeta) z_1 + \zeta z_2 \in \widetilde{M}$ with membership $\mu_{\widetilde{M}}\big((1-\zeta) z_1 + \zeta z_2\big) \ge \min(\mu_{\widetilde{M}}(z_1), \mu_{\widetilde{M}}(z_1))$. Moreover, since f has a strictly convex real part at z_1 with respect to $\Re_+$, it follows that

$$\operatorname{Re}\Big((1-\zeta) f(z_1) + \zeta f(z_2) - f\big(z_\zeta\big)\Big) > 0, \tag{6}$$

Therefore,

$$\operatorname{Re} f(z_\zeta) < \operatorname{Re}\left((1-\zeta)f(z_1)+\zeta f(z_2)\right) < \operatorname{Re} f(z_1) - \zeta \operatorname{Re} f(z_1) + \zeta \operatorname{Re} f(z_2) = \operatorname{Re} f(z_1). \quad \textbf{(7)}$$

This contradicts that z_1 is a solution to *Problem (1)*.

Definition 9. The point $z^\circ \in \widetilde{M}$ with membership $\mu_{\widetilde{M}}(z^\circ) = \mu_{\widetilde{M}}(x^\circ, y^\circ) = \min(\mu_{\widetilde{M}}(x^\circ), \mu_{\widetilde{M}}(y^\circ)$ is a local solution to *Problem (1)* if there is a neighbourhood $\tilde{u}$ of the point z° so that $\operatorname{Re} f(z^\circ) = \min\{\operatorname{Re} f(z): z \in \widetilde{M} \cap \tilde{u}\}$, with membership function $\mu_{\widetilde{M}\cap\tilde{u}} = \min(\mu_{\widetilde{M}}(z)), \mu_{\tilde{u}}(z))$.

Theorem 3. Let us consider that $\phi \neq \widetilde{M} \subset \mathbb{C}^n$, and $f: \widetilde{M} \to \mathbb{C}$ be a function with concave real part on $\widetilde{M}$ relative to $\mathfrak{R}_+$, under the assumption that its real part is not a constant, if z° is a solution to *Problem (1)*, then z° is a member of the set representing the boundary of $\widetilde{M}$.

Proof: If $\operatorname{int}(\widetilde{M}) = \emptyset$, then z° belongs to the boundary of $\widetilde{M}$. Assume the case when $\operatorname{int}(\widetilde{M}) \neq \emptyset$. Since $\operatorname{Re} f(z)$ is not constant, it follows that there exists $z_1 \in \widetilde{M}$ with the membership function $\mu_{\widetilde{M}}(z_1)$ such that

$$\operatorname{Re} f(z^\circ) < \operatorname{Re} f(z_1). \quad \textbf{(8)}$$

Now, let $z \in \operatorname{int}(\widetilde{M})$, then there exists a real number $\varepsilon > 0$ so that $\widetilde{B}(z,\varepsilon) \subseteq \widetilde{M}$ with $\mu_{\widetilde{B}(z,\varepsilon)} < \mu_{\widetilde{M}}(z)$. Let us denote

$$\zeta = \frac{\|z - z_1\| + 1}{\|z - z_1\| + 1 + \varepsilon}, 0 < \zeta < 1.$$

Let $w = \frac{1}{\zeta} z + \left(1 - \frac{1}{\zeta}\right) z_1$. Since

$$\|z - w\| = \left\|z - \frac{1}{\zeta} z - z_1 + \frac{1}{\zeta} z_1\right\|$$
$$= \left\|z\left(1 - \frac{1}{\zeta}\right) - \left(1 - \frac{1}{\zeta}\right) z_1\right\|$$
$$= \left\|\left(1 - \frac{1}{\zeta}\right)(z - z_1)\right\|$$
$$= \left|1 - \frac{1}{\zeta}\right| \|z - z_1\|$$
$$= \left|1 - \frac{\|z - z_1\| + 1 + \varepsilon}{\|z - z_1\| + 1}\right| . \|z - z_1\|$$
$$= \left|1 - \frac{\|z - z_1\| + 1}{\|z - z_1\| + 1} - \frac{\varepsilon}{\|z - z_1\| + 1}\right| . \|z - z_1\|$$
$$= \left|\frac{-\varepsilon}{\|z - z_1\| + 1}\right| . \|z - z_1\| < \varepsilon$$
$$= \frac{\varepsilon}{\|z - z_1\| + 1} . \|z - z_1\| < \varepsilon.$$

It follows that, $w \in \widetilde{B}(z,\varepsilon)$, and $w \in \widetilde{M}$ wit memberships $\mu_{\widetilde{B}(z,\varepsilon)}(w) \leq \mu_{\widetilde{M}}(w)$. Then, we deduce that $\operatorname{Re} f(z^\circ) \leq \operatorname{Re}(w)$, referring to the equation

$$w = \frac{1}{\zeta} z + \left(1 - \frac{1}{\zeta}\right) z_1 \Rightarrow \frac{1}{\zeta} z = w - z_1 + \frac{1}{\zeta} z_1 \Rightarrow \frac{1}{\zeta} z = w + \left(\frac{1}{\zeta} - 1\right) z_1.$$

Hence, we obtain $z = \zeta w + z_1 - \zeta z_1$, i.e., $z = \zeta w + (1 - \zeta) z_1$, since $z_1 \in \widetilde{M}, w \in \widetilde{B}(z,\varepsilon) \subseteq \widetilde{M}$, $0 < \zeta < 1$, then $z \in \widetilde{M}$ with the following membership function

$$\mu_{\widetilde{M}} \geq \min(\mu_{\widetilde{M}}(w), \mu_{\widetilde{M}}(z_1)).$$

Since $f(z)$ has a concave real part on $\widetilde{M}$ relative to $\mathfrak{R}_+$, we have

$$\text{Re } f(z) = \text{Re } f\big((1-\zeta)z_1 + \zeta w\big)$$
$$\geq (1-\zeta)\text{Re } f(z_1) + \zeta \text{ Re } f(w)$$
$$> (1-\zeta)\text{Re } f(z^{\circ}) + \zeta \text{ Re } f(z^{\circ}) = \text{Re } f(z^{\circ}).$$

If $z \in \text{int}\,(\widetilde{M})$. Then

$$\text{Re } f(z^{\circ}) < \text{Re } f(z). \tag{9}$$

Thus, Re f cannot attain its minimum at an interior point of $\widetilde{M}$.

Remark 1: Let z° be a solution of *Problem (1)*. Then, z° is a local solution to *Problem (1)*. But the converse does not generally hold.

Theorem 4. Let z° be a non-empty fuzzy subset of $\mathbb{C}^n$ with membership function $\mu_{\widetilde{M}}(z) = \mu_{\widetilde{M}}(x,y) = \min(\mu_{\widetilde{M}}(x), \mu_{\widetilde{M}}(y))$, where $\mu_{\widetilde{M}}(x): \Re^n \to [0,1]$, $\mu_{\widetilde{M}}(y): \Re^n \to [0,1], x, y \in \Re^n$, and $f: \widetilde{M} \to \mathbb{C}$ be a function with a convex real part on $\widetilde{M}$ relative to $\mathbb{R}_+$. If z° is a local solution of *Problem (1)*, then z° is a solution of *Problem (1)* with membership $\mu_{\widetilde{M}}(z^{\circ}) = \mu_{\widetilde{M}}(x^{\circ}, y^{\circ}) = \min(\mu_{\widetilde{M}}(x^{\circ}), \mu_{\widetilde{M}}(y^{\circ}))$, $\mu_{\widetilde{M}}(x^{\circ}): \Re^n \to [0,1]$, $\mu_{\widetilde{M}}(y^{\circ}): \Re^n \to [0,1], x^{\circ}, y^{\circ} \in \Re^n$.

Proof: Let z° be a local solution to *Problem (1)*. Then there is a $\varepsilon \in \Re$, $\varepsilon > 0$, so that

$$\text{Re } f(z^{\circ}) < \text{Re } f\,(z), \quad \text{for all } \; z \in \widetilde{M} \cap \mu_{\widetilde{B}(z^{\circ},\varepsilon)}, \tag{10}$$

with membership function

$$\mu_{\widetilde{M}}(z) = \min(\mu_{\widetilde{M}}(x^{\circ}), \mu_{\widetilde{M}}(y^{\circ})), \mu_{\widetilde{B}(z^{\circ},\varepsilon)}(z) = \min\big(\mu_{\widetilde{M}}(z), \mu_{\widetilde{B}(z^{\circ},\varepsilon)}\big).$$

Let us assume that there exists a point z_1 with the property that

$$\text{Re } f(z_1) < \text{Re } f(z^{\circ}). \tag{11}$$

It can be observed that $z_1 = z^{\circ}$. Since, $\widetilde{M}$ is a fuzzy convex set, it follows that $z_{\zeta} = (1-\zeta)z^{\circ} + \zeta z^{\circ} \in \widetilde{M}$; for all $0 \leq \zeta \leq 1$, and membership $\mu_{\widetilde{M}}\big((1-\zeta)z^{\circ} + \zeta z^{\circ}\big) \geq \min(\mu_{\widetilde{M}}(z^{\circ}), \mu_{\widetilde{M}}(z^1))$, let us choose $0 \leq \zeta \leq 1$ such that $0 < \zeta < \frac{\varepsilon}{\|z^{\circ} - z_1\|}$.

Then, $\|z_{\zeta} - z^{\circ}\| = \|(1-\zeta)z^{\circ} + \zeta z_1 - z^{\circ}\| = \zeta\,\|z^{\circ} - z_1\| \leq \varepsilon$, and thus $z_{\zeta} \in \widetilde{M} \cap \widetilde{B}(z^{\circ}, \varepsilon)$, with membership $\mu_{\widetilde{M} \cap \widetilde{B}(z^{\circ},\varepsilon)} = \min\left(\mu_{\widetilde{M}(z_{\zeta})}, \mu_{\widetilde{B}(z^{\circ},\varepsilon)}(z_{\zeta})\right)$. Hence, we obtain

$$\text{Re } f(z^{\circ}) < \text{Re } f(z_{\zeta}).$$

Moreover, since the function f has a convex real part on $\widetilde{M}$ relative to $\mathbb{R}_+$, it follows that

$$\begin{aligned} &\text{Re } f(z_1) < \text{Re } f(z^{\circ}), \\ &\text{Re } f(z_{\zeta}) < (1-\zeta)\text{Re } f(z^{\circ}) + \zeta \text{ Re } f(z_1) < (1-\zeta)\text{Re } f(z^{\circ}) + \zeta \text{ Re } f(z^{\circ}) \Rightarrow \text{Re } f(z_1) < \\ &\text{Re } f(z^{\circ}). \end{aligned} \tag{12}$$

Which contradicts *Eq. (11)* and so there is no other solution for the *Problem (1)*, and so the local solution z° is a global solution to *Problem (1)*.

4 | Problem Statement (Part II)

Let us recall an optimization problem as below:

$$\begin{aligned} &\min \text{Re } f(z), \\ &\text{s.t.} \\ &z \in \widetilde{X}, \\ &g(z) \in \widetilde{U}, \end{aligned} \tag{13}$$

where $\emptyset \neq \tilde{X} \subset \mathbb{C}^n, \emptyset \neq \tilde{U}$, with memberships $\mu_{\tilde{X}}(z) = \mu_{\tilde{X}}(x,y) = \min(\mu_{\tilde{X}}(x), \mu_{\tilde{X}}(y))$, $\mu_{\tilde{X}}(x): \mathfrak{R}^n \to [0,1], x \in \mathfrak{R}^n, \mu_{\tilde{X}}(y): \mathfrak{R}^n \to [0,1], y \in \mathfrak{R}^n, \mu_{\tilde{U}}(g(z)) = \mu_{\tilde{U}}(u(x,y), v(x,y)) = \min(\mu_{\tilde{U}}(u), \mu_{\tilde{U}}(v)), z = x + i\,y, g(x,y) = u(x,y) + i\,v(x,y)$, $\mu_{\tilde{U}}(u): \mathfrak{R}^m \to [0,1], u \in \mathfrak{R}^m, \mu_{\tilde{U}}(v): \mathfrak{R}^m \to [0,1], v \in \mathbb{R}^m$, and $f: \tilde{X} \to \mathbb{C}, g: \tilde{X} \to \mathbb{C}^m$ are two functions. For $r \in \mathbb{R}$, and *Problem (13)*, we define the function as follows:

$$\varphi_r(z,v) = r\,f(z) - \langle g(z), v)\rangle, \text{for all } (z,v) \in \tilde{X} \times \tilde{U}_*, \mu_{\tilde{X}\times\tilde{U}_*}(z,v) = \mu_{\tilde{X}}(z) \times \mu_{\tilde{U}_*}(v). \quad \textbf{(14)}$$

Definition 10. $Y \subset \mathbb{C}^n$ referrers to fuzzy cone with membership function $U: Y \to [0,1]$, if:

I. $U(0) = 1$.

II. $U(\zeta z) \leq U(z)$; for all $z \in Y, \zeta \geq 0, z = x + i\,y, U(z) = U(x,y) = \min(U(x), U(y)), U(x): \mathbb{R}^n \to [0,1], U(y): \mathbb{R}^n \to [0,1]$.

Remark 2: In the case of the complex cone U generalized by infinite vectors, a fuzzy complex cone is termed a fuzzy polyhedral cone.

Example 1. Let $Y \subset \mathbb{C}^n$ be the space of all complex numbers, the function U, defined by

$$U(z) = U(x,y) = \begin{cases} 0, & \text{if } x < 0 \vee y < 0, \\ \dfrac{y}{x}, & \text{if } x > 0, y > 0 \wedge y < x, \\ 1, & \text{if } x \geq 0, y \geq 0 \wedge y \geq x, \end{cases}$$

is an example of a fuzzy complex cone in $\mathbb{C}$.

Theorem 5. Suppose $\emptyset \neq \tilde{X} \subset \mathbb{C}^n$, $\tilde{U}$ is a fuzzy polyhedral cone in $\mathbb{C}^m$ with non-empty interior $f: \tilde{X} \to \mathbb{C}$ representing a function with a convex real part on $\tilde{X}$ relative to $\mathfrak{R}_+$, and $g: \tilde{X} \to \mathbb{C}^m$ be a concave function on $\tilde{X}$ with respect to $\tilde{U}$. If z° is a solution of *Problem (13)*, then there is a $r_\circ \in \mathfrak{R}, v_\circ \in \tilde{U}_*$ with membership

$$\mu_{\tilde{U}_*}(v_\circ) = \mu_{\tilde{U}_*}(x_{v_\circ}, y_{v_\circ}) = \min(\mu_{\tilde{U}_*}(x_{v_\circ}), \mu_{\tilde{U}_*}(y_{v_\circ})). \quad \textbf{(15)}$$

$$\operatorname{Re}\langle g(z^\circ), v_\circ\rangle = 0. \quad \textbf{(16)}$$

$$\operatorname{Re}\varphi_{r_\circ}(z^\circ, v) < \operatorname{Re}\varphi_{r_\circ}(z^\circ, v_\circ) \leq \operatorname{Re}\varphi_{r_\circ}(z, v_\circ), \quad \text{for all } (z^\circ, v_\circ) \in \tilde{X} \times \tilde{U}, \quad \textbf{(17)}$$

where $\varphi_{r_\circ}(z,v) = r\,f(z) - \langle g(z), v)\rangle$; for all $(z,v) \in \tilde{X} \times \tilde{U}_*$.

Proof: Since z° is a solution to *Problem (13)*, therefore the following system:

$$\begin{aligned} &\operatorname{Re}\left(f(z) - f(z^\circ)\right) < 0, \\ &z \in \tilde{X}, \\ &g(z) \in \tilde{U}, \end{aligned} \quad \textbf{(18)}$$

is inconsistent.

Denote $\widetilde{\mathbb{C}\,\mathfrak{R}_+}$ be the closed fuzzy convex cone, which is defined by $\mu_{\widetilde{\mathbb{C}\mathfrak{R}_+}}(x,y) = \min\left(\mu_{\widetilde{\mathbb{C}\mathfrak{R}_+}}(x), \mu_{\widetilde{\mathbb{C}\mathfrak{R}_+}}(y)\right)$, $\mu_{\widetilde{\mathbb{C}\mathfrak{R}_+}}(x): \mathfrak{R} \to [0,1], \mu_{\widetilde{\mathbb{C}\mathfrak{R}_+}}(y): \mathfrak{R} \to [0,1]$, $v = x + i\,y$, for all $x, y \in \mathfrak{R}$.

Let us rewrite the *System (18)* as expressed in the following form:

$$\begin{aligned} &f(z^\circ) - f(z) \in \operatorname{int}\left(\widetilde{\mathbb{C}\,\mathfrak{R}_+}\right), \\ &z \in \tilde{X}, \\ &g(z) \in \tilde{U}, \end{aligned} \quad \textbf{(19)}$$

Since the *System (19)* is consistent, it follows that there exists $r_\circ \in \mathfrak{R}, v_\circ \in \mathbb{C}^m$ such that

$$r_\circ \in (\widetilde{\mathbb{C}\,\mathfrak{R}_+})_* = \mathfrak{R}_+, v_\circ \in \mathbb{C}^m, (r_\circ, v_\circ) \neq (0,0). \quad \textbf{(20)}$$

$$\operatorname{Re}\left(\langle f(z^\circ) - f(z), r_\circ\rangle + \langle g(z), v_\circ\rangle\right) \leq 0, \quad \text{for all } z \in \tilde{X}. \quad \textbf{(21)}$$

Let us choose $z = z^\circ \in X$, we have

$$\mathrm{Re}\,\langle g(z^{\circ}), v_{\circ}\rangle \leq 0. \tag{22}$$

Because $v_{\circ} \in U_{*}$ and $g(z^{\circ}) \in \tilde{U}$, we get $\mathrm{Re}\,\langle g(z^{\circ}), v_{\circ}\rangle \geq 0$, and therefore, $\mathrm{Re}\,\langle g(z^{\circ}), v_{\circ}\rangle = 0$.

From *Eq. (16)* and *Inequality (21)* and the fact that z° is a solution of *Problem (13)*, it follows that

$$\mathrm{Re}\big(r_{\circ}f(z^{\circ}) - \langle g(z^{\circ}), v_{\circ}\rangle\big) \leq \mathrm{Re}\,(r_{\circ}f(z) - \langle g(z), v_{\circ}\rangle), \quad \text{for all } z \in \tilde{X}, \tag{23}$$

which is the same as in *Inequality (18)*.

In order to choose that the *Inequality (17)* is satisfied, from $g(z^{\circ}) \in \tilde{U}$ with membership

$$\mu_{\tilde{X}}\left(g(z^{\circ})\right) = \min\big(\mu_{\tilde{X}}u(x^{\circ}, y^{\circ}), \mu_{\tilde{X}}v(x^{\circ}, y^{\circ})\big).$$

We get $\mathrm{Re}\,\langle g(z^{\circ}), v\rangle \geq 0$; for all $v \in \tilde{U}_{*}$, and therefore $\mathrm{Re}\,(r_{\circ}f(z^{\circ}) - \langle g(z^{\circ}), v_{\circ}\rangle) \leq \mathrm{Re}\,(r_{\circ}f(z^{\circ}) - \langle g(z^{\circ}), v_{\circ}\rangle)$; for all $v \in \tilde{U}_{*}$, because of $\mathrm{Re}\langle g(z^{\circ}), v_{\circ}\rangle = 0$.

5 | Concluding Remarks

In the current study, we introduced some results for optimization in complex space with fuzzy complex set in the constraints, and also, the F-CPP with fuzzy complex cone in the constraint has been introduced. Some basic theorems that characterized the problem's solution have been stated with proof. There are many problems and research points to be investigated in the field of fuzzy complex MP problems; some of these points are as follows:

I. Study of fuzzy complex linear programming problem considering the real and imaginary parts.

II. Study of fuzzy complex fractional programming problems in both single-objective and multi-objective functions.

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The Joint Policy of Production, Maintenance, and Product Quality in a Multi-Machine Production System by Reinforcement Learning and Agent-based Modeling

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Abstract

Adopting an integrated production, maintenance, and quality policy in production systems is of great importance due to their interconnected influence. Consequently, investigating these aspects in isolation may yield an infeasible solution. This paper aims to address the joint optimal policy of production, maintenance, and quality in a two-machine-single-product production system with an intermediate buffer and final product storage. The production machines have degradation levels from as-good-as-new to the breakdown state. The failures increase the production machine's degradation level, and maintenance activities change the status to the initial state. Also, the quality of the final product depends on the level of degradation of the machines and the correlation between the degradation level of the production machines and the product's quality in the case that high degradation of the previous production machines leads to a high probability to produce wastage by the following machines is considered. The production system studied in this research has been modeled using the agent-based simulation, and the Reinforcement Learning (RL) algorithm has obtained the optimal integrated policy. The goal is to find an integrated optimal policy that minimizes production costs, maintenance costs, inventory costs, lost orders, breakdown of production machines, and low-quality production. The meta-heuristic technique evaluates the joint policy obtained by the decision-maker agent. The results show that the acquired joint policy by the RL algorithm offers acceptable performance and can be applied to the autonomous real-time decision-making process in manufacturing systems.

Keywords: Agent-based modeling, Reinforcement learning, Simulation-optimization, Production planning, Maintenance, Quality control.

1 | Introduction

Production planning, maintenance, and quality control are the most critical challenges of a production system that directly affect the system's performance. Production planning aims to decide how to divide and schedule the tasks to achieve specific goals, such as minimizing tardiness or maximizing production. The maintenance

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goal is to maximize machine availability time while minimizing associated costs. According to the definition of European standard EN13306 version 2010 [1], maintenance and repair is "the combination of all technical, administrative and managerial activities during the life of a device in order to maintain or return it to the required function."

Since performing any maintenance and repair activities will make the production machines inaccessible, optimizing maintenance and repair activities without considering the production planning limitations will lead to an infeasible solution. On the other hand, production planning is entirely affected by the time of availability of production machines. If the production machines are not available at the right time, achieving the desired goals in production planning will be almost impossible. In addition, if the maintenance and repair duration is allocated to the production, there will be a possibility of failure and breakdown of the production machines. Therefore, maintenance and production are two activities that conflict with each other [2].

The quality of the final products is also affected by the degradation level of the production machines. The high level of degradation increases the probability of producing low-quality products. Therefore, production planning, maintenance, and quality control must be considered as an integrated problem to obtain a feasible solution. However, very little research has been done on the integrated optimization of these aspects [3].

There has been significant development in artificial intelligence and machine learning in recent years, and their applications have been extended to many fields, including manufacturing systems. Machine learning is a subset of artificial intelligence in which computers can explore the patterns in the data and learn the policies. Machine learning is divided into three categories: supervised learning, unsupervised learning, and Reinforcement Learning (RL), which are shown in *Fig. 1* [4].

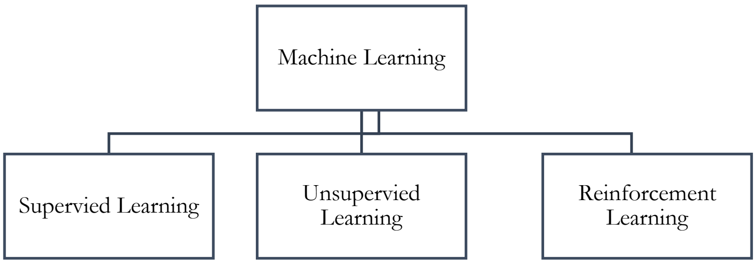

Fig. 1. The machine learning categories.

The required data for supervised learning has been labeled or categorized, e.g., data classification and regression for future prediction. In unsupervised learning, the data is unlabeled. This method is used to explore hidden patterns in the data, such as clustering methods. However, in RL, the learning process is done by trial and error in a dynamic environment. The agent takes action and receives associated rewards to maximize the expected rewards. This method is inspired by behavioral psychology. The agent's learning process can occur in a real-world environment or a simulation model. The application of RL in optimizing the production system is almost nascent. This research aims to apply RL to achieve the optimal joint policy of production planning, maintenance, and quality in a multi-machine production system. In order to evaluate the obtained policy, the Simulation-Optimization (SO) approach has been used. The main contributions of this paper are:

- *Applying RL and Agent-Based Modeling (ABM) to obtain a joint optimal policy of production, maintenance, and quality in a multi-machine single-product manufacturing system.*
- *Investigating the correlation between the degradation level of the production machines and the product's quality in the case that high degradation of the previous production machines leads to a high probability of produce wastage by the following machines.*
- *Comparing the acquired joint policy by the decision-maker agent with the meta-heuristic method and evaluating the performance of the RL-based policy.*

The article's structure will be as follows. In Section 2, the literature review will be presented. In Section 3, the production system, the agent-based model, and the RL algorithm are described. In Section 4, the obtained policy has been evaluated by SO techniques. Finally, the conclusion and suggestions are provided in Section 5.

2 | Literature Review

As mentioned, RL is an emerging approach to optimize production systems. Zheng et al. [5] consider a production system consisting of two machines and one intermediate buffer [5]. The production system has been modeled by Discrete Event Simulation (DES), and RL has been used to obtain the optimal joint policy of production and maintenance based on the inventory level and the state of the production machines. Kuhnle et al. [6] optimized a parallel multi-machine production system using RL to achieve the best schedule for maintenance activities, increase production rates, and reduce maintenance costs [6]. ABM has been applied to model the production system, and each machine is considered an agent. In addition, each agent independently satisfies the demand. Xanthopoulos et al. [7] investigated a joint production and maintenance problem in a single machine–single product system. The system has a downstream buffer to store the final products and satisfy the demand. The optimal joint policy to minimize the inventory level and demand backorders has been acquired by RL. The extension of the previous research has been done by Paraschos et al. [8]. They consider the quality of products so that the production machine has a degradation level that affects the quality of the final products. The maintenance and repair activities can be performed to improve the degradation level. The optimal joint production, maintenance, and quality have been yielded by RL.

Yang et al. [9] have considered the joint optimal Preventive Maintenance (PM) and production scheduling policy in a similar production system by RL approach [9]. Deep Reinforcement Learning (DRL) has acquired the PM policy of a multi-machine, single-product system in [10]. Su et al. [11] investigate the challenge of designing PM policies for large-scale manufacturing systems and propose a novel approach using multi-agent RL to address the complexity of such systems [11]. They discuss that Designing efficient PM policies for large-scale manufacturing systems is difficult due to non-linearity and stochasticity in these complex systems. Zhao and Smidts [12] address challenges in maintenance policy optimization, in which decision-maker agents encounter an imperfect understanding of system degradation models and have a limited ability to observe system degradation states [12]. They proposed RL to tackle these challenges, specifically for maintenance problems with Markov degradation processes. Ye et al. [13] investigate the joint optimization of manufacturing systems, specifically focusing on large-scale dynamic systems, such as manufacturing networks, which have complex structures [13]. The paper proposes a novel approach using RL, specifically the Deep Deterministic Policy Gradient (DDPG) algorithm, to achieve joint optimization of PM and work-in-process quality inspection in manufacturing networks with reliability-quality interactions.

SO is another method to obtain optimal policies in production systems. In a study conducted by Lavoie et al. [14] they proved the effectiveness of SO algorithms in optimizing production systems. These algorithms have also been used in joint production planning, maintenance, and quality control optimization. Bouslah et al. [15] investigated a production system with two machines. In this research, hybrid optimization was performed using SO techniques to minimize the total costs. The problem of production planning and quality control is presented in [16]. They use SO to optimize a single machine-single product system jointly. The extension of the previous research has been done in [3]. They optimized combined production planning, maintenance, and quality control of a continuous single-machine production machine. Tambe and Kulkarni [17] introduce an integrated planning approach for optimizing the three core functions of shop floor management: maintenance, production scheduling, and quality. The methodology focuses on the conditional reliability of components and their impact on the overall system operation. The primary objective is to minimize the system operation cost through combined decision-making and investigate the integrated policy's cost-effectiveness compared to non-integrated planning approaches. The mathematical model is used to build a system model, and the meta-heuristics methods, such as simulated annealing and genetic algorithm, are applied to solve the optimization problem.

A summary of the presented papers is given in *Table 1*.

Table 1. The summary of the papers.

No	Rf.	Method	Production System	Production Planning	Maintenance	Quality
1	[5]	RL	Two machines–Single product	✓	✓	
2	[6]	RL	Multi parallel machines–Single product	✓	✓	
3	[7]	RL	Single machine–Single product	✓	✓	
4	[8]	RL	Single machine–Single product	✓	✓	✓
5	[9]	RL	Single machine–Single product	✓	✓	
6	[10]	RL	Multi Machine–Single Product	✓	✓	
7	[15]	SO	Two machines–Single product	✓	✓	✓
8	[16]	SO	Single machine–Single product	✓	✓	
9	[3]	SO	Single machine–Single product	✓	✓	✓
10	[17]	SO	Multi Machine–Single Product	✓	✓	✓
11	[11]	RL	Multi Machine–Single Product		✓	
12	[12]	RL	Single machine–Single product		✓	
13	[13]	RL	Multi Machine–Single Product		✓	✓

The main contribution of this paper is to propose an RL-based method to jointly optimize the production, maintenance, and quality problem in a multi-machine single-product system by considering the correlation between the degradation level of the production machines and the product's quality in the case that high degradation of the previous production machines leads to a high probability to produce wastage by the following machines. To the author's knowledge, the proposed method has not been discussed in other research.

To evaluate the obtained RL-based policy, the SO technique has been implemented to compare the results. For this purpose, an agent-based model of the production system is developed, and a commercial SO package is used to obtain the optimal joint policy. The SO package combines heuristic and meta-heuristic methods such as tabu search, neural network, and scattered search to optimize the objective function.

3 | Problem Description

In this research, a production system consisting of two production machines M_1 and M_2, and one intermediate buffer with a maximum capacity of B_{max} and a final product warehouse with a capacity of I_{max} has been investigated (*Fig. 2*).

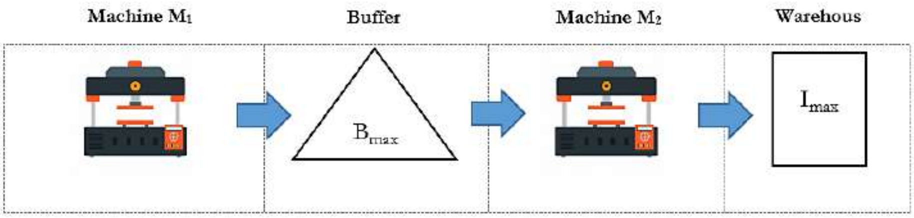

Fig. 2. The production system.

Manufacturing machines have depreciation levels that vary from the as-good-as-new d_0 to the breakdown d_{max}. Depreciation level increases due to failures at the rate of λ_f during the production process. The probability of breakdown at each level of degradation levels has a probability of p_{b,d_i}. The higher degradation level increases the breakdown probability. Maintenance and repair activities return the degradation level of the production machine to the as-good-as-new state d_0.

The semi-final parts are produced by machine M_1. The production time follows the exponential distribution with the parameter λ_{p_1}. After that, the semi-final parts are stored in the intermediate buffer to be processed by the machine M_2. The processing time to produce the final parts follows the exponential distribution with parameter λ_{p_2}. The quality of the semi-final and final parts varies depending on the degradation level of the machine M_1 and M_2. The probability of producing low-quality semi-final parts in each degradation level of the machine M_1 is p_{d_i}. The quality of the final parts is related to the quality of the semi-final part and the degradation level of the machine M_2. The probability of producing high-quality parts by machine M_2 is $1\text{-}p_{d_{i,q_h}}$when the semi-final product's quality is high, or $1\text{-}p_{d_{i,q_l}}$ when the quality is low. On the other hand, the wastage is produced with the portability $p_{d_{i,q_h}}$ and $p_{d_{i,q_l}}$ when the quality of semi-final parts is high or low, respectively. The probability of producing high-quality parts from the low-quality semi-final parts increases when the degradation level of the machine M_2 is close to d_0. Therefore, at a certain degradation level, $p_{d_{i,q_h}}<p_{d_{i,q_l}}$.

At the end of each day, the demand that follows the Poisson distribution with parameter λ_d arrives, and satisfies when the inventory of the final parts is sufficient. Otherwise, the demand is backordered until the inventory is available. The maximum number of allowed backorders is S_{max} and the number of missed orders is calculated by *Eq. (1)*.

$$\text{Missed orders}=|I\text{-}D+S_{max}|. \tag{1}$$

4 | Methodology

4.1 | The Agent-Based Modeling

There are three major paradigms in simulation modeling: DES, System Dynamics (SD), and ABM. Despite the traditional approaches such as DES and SD, ABM is relatively new and is more general. ABM enables the modelers to capture more complexities in dynamic systems [18]. So, in dynamic systems where the events are time-related, ABM can be used to model the system. There is no universal agreement for the definition of agents; e.g., the agent is defined as an entity with autonomous behavior [19] or an independent component [20]. In much literature, self-contained, autonomous, self-directed, and the ability to interact are mentioned as the essential characteristics of the agent [21].

The ABM consists of the following elements [21]:

- *The agents, characteristics, and behaviors.*
- *The way the agents interact.*
- *The environment.*

This paper uses ABM to simulate the production system due to the flexibility of the approach to capture all details and ease communication with the decision-maker agent. In the proposed production system, the agents are:

- *The Production machine* M_1.
- *The production machine* M_2.
- *The decision-maker agent.*

And the environment consists of the storage of the final products and the place where the agents interact.

4.1.1 | The agent's behavior

In *Fig. 3*, The behavior of the production agents is shown. The production agents can be in one of the following states:

- *ReadyForMessage: In this state, the production agent is waiting to receive a message from the decision-maker agent*
- *ReadyForProduction: In this state, the production agent investigates the buffer and storage volume and the remaining time till the end of the shift, and if all the required conditions are met, the production process will be started.*
- *Produce: The production agent starts the production process at the rate of* λ_p .
- *Check condition: When the production process is going to be completed, depending on the degradation level of the production agent* M_1, *The production has high quality with* $1\text{-}p_{d_i}$ *and low-quality with probability* p_{d_i}. *In the case of the production agent* M_2, *depending on the quality of the semi-final part and the degradation level of the agent, The production has high quality with the probability* $1\text{-}p_{d_{i,q}}$ *or the production becomes wastage with* $p_{d_{i,q}}$. *Also, the agent's state transits to the Breakdown state with probability* p_{b,d_i} *or to the ReadyForProduction state with the probability* $1\text{-}p_{b,d_i}$.
- *Breakdown: The operating agent will fail, and maintenance and repair activities will be necessary.*
- *Maintain: The maintenance and repair activities are performed to return the degradation level of the agent to as-good-as-new* d_0.
- *Idle: the production agent does not perform any activity during the shift.*

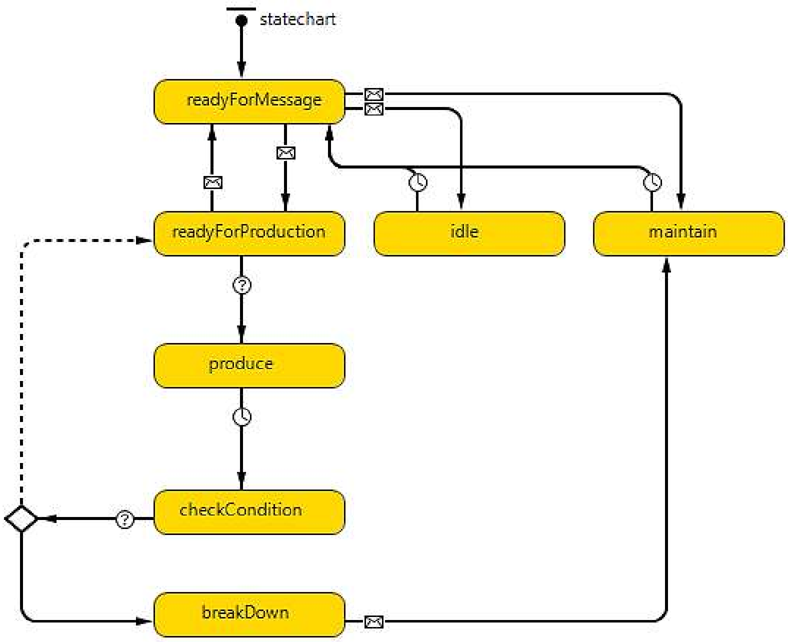

Fig. 3. The production agent's behavior.

4.1.2 | The decision-maker agent's behavior

In this paper, the decision-making process of the decision-maker agent is implemented based on the Markov-decision process, which consists of the following elements: state-space, actions, transition probability, and reward [4].

First, The decision-maker agent observes the state of the system and authorizes an action. The action is sent to the production agents as a message. Next, the agent considers the new state of the system and the corresponding reward of the authorized action. Then, this process continues until the optimal policy is obtained (*Fig. 4*).

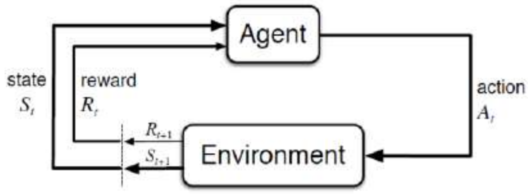

Fig. 3. The Markov-decision process of the decision-maker agent.

The decision-maker agent authorizes the actions that maximize the discounted future reward. Therefore, the agent authorizes an action in such a way that the following equation is maximized:

$$G_t=R_{t+1}+\gamma R_{t+1}+\gamma^2 R_{t+1}+\ldots= \sum_{k=0}^{\infty}\gamma^k R_{t+k+1}. \tag{2}$$

4.1.3 | The State of the system

In every decision epoch, the decision-maker agent observes the state of the system as the following vector

$$S(t)=\left(S_{1,t},S_{2,t},S_{3,t},S_{4,t},S_{5,t}\right)=(q_{lq},q_{hq},I,d_1,d_2). \tag{3}$$

Where q_{lq} and q_{hq} are the number of low-quality and high-quality semi-final parts in the intermediate buffer, respectively. I is the inventory level of final parts, and d_1 and d_2 are the degradation level of machine M_1 and M_2. The system state-space is

$$\begin{aligned}
&S_1: 0, \ldots, B_{max},\\
&S_2: 0, \ldots, B_{max},\\
&S_3: -B_{max}, \ldots, I_{max},\\
&S_4: 0, \ldots, d_{max}, \text{ for the machine } M_1,\\
&S_5: 0, \ldots, d_{max}, \text{ for the machine } M_2.
\end{aligned} \tag{4}$$

Consequently, the number of states that the decision-maker agent can find itself in is:

$$N(s)=(B_{max}+1)\times(B_{max}+1)\times(I_{max}+S_{max}+1)\times(d_{max\,M1}+1)\times(d_{max\,M2}+1). \tag{5}$$

4.1.4 | Actions

In every decision epoch, the decision-maker agent authorizes one of the following actions based on the system's state: produce, maintain, and idle. The feasible action set in every state is:

- *If the number of final parts reaches the maximum storage capacity I_{max}, the "maintain" or "idle" action can only be performed for machine M_2.*
- *The maintenance and repair activities are the only options if the production agents are in "breakdown" state.*
- *All the actions are feasible if the inventory of the final parts is less than the maximum allowed capacity and the production agent is in "readyformessage" state.*

4.1.5 | Reward

Let $A = \{A_1, A_2, A_3, \ldots\}$ denotes the actions that the decision-maker agent takes in each decision epoch. The agent receives a numerical reward in each decision epoch regarding the previously performed action by

$$R = C_{h_b} + C_{h_I} + C_s + C_p + C_m + C_w + C_b + C_l - S_p. \quad (6)$$

Where

C_{h_b}: The holding cost of the semi-final parts stored in the buffer.

C_{h_I}: The holding cost of the final parts stored in the storage.

C_s: The cost of backordered orders.

C_p: The cost of production.

C_m: The cost of maintenance and repair activities.

C_w: The cost of producing wastages.

C_b: The cost of production machine breakdown.

C_l: The cost of missed orders.

S_p: The sales profit of the final parts.

The main goal of the decision-maker agent is to obtain a policy that maximizes the total acquired rewards (minimizing *Eq. (6)*).

4.2 | Reinforcement Learning

In this research, the optimal joint policy of the decision-maker agent is achieved by a RL algorithm called R-learning [22]. The R-learning algorithm has been applied in recent research [7], [8].

Let $t_d = \{t_{d,1}, t_{d,2}, t_{d,3}, \ldots\}$ denotes the decision epochs that the decision-maker agent takes action and $R_{t_{d,i}}$ is the obtained reward in decision epoch i. The R-learning seeks to maximize the following equation:

$$\rho = \lim_{n\to\infty} \frac{1}{n} E\left\{\sum_{i=1}^{n} R_{t_{d,i}}\right\}. \quad (7)$$

In the R-learning, the value of the performed action in the state (state-action) is calculated by

$$Q(S_t,A_t) \leftarrow Q(S_t,A_t) + \alpha \left[R_{t+1} - \varrho + \max_a Q(S_{t+1},a) - Q(S_t,A_t)\right]. \quad (8)$$

Where

$Q(S_t,A_t)$: The value of action A in state S in time t.

α: Learning rate.

R_{t+1}: The obtained reward regarding the performed action A in state S in time t.

$\max_a Q(S_{t+1},a)$: The action that yields the maximum value in state S in time t+1.

Also, the following equation updates the average reward ρ:

$$Q(S_t,A_t) \leftarrow Q(S_t,A_t) + \alpha \left[R_{t+1} - \varrho + \max_a Q(S_{t+1},a) - Q(S_t,A_t)\right]. \quad (9)$$

Where β is a real-valued parameter between 0 and 1. The decision-maker agent applied the ε-greedy "policy" to take action. In this policy, the agent chooses the action that yields the maximum value by the probability $1 - \varepsilon$, and selects the action randomly with the probability ε.

It is worth mentioning that in methods such as dynamic programming, the state transition probability matrix is needed to solve the MDP. However, in many real-world problems, it is impossible to calculate the state transition probabilities matrix of the system. In this case, the RL algorithms are helpful because there is no need to calculate the transition probabilities. The algorithm uses the simulation model to act and observe the next state of the system and reward.

4.3 | Simulation-Optimization

The term "SO" refers to techniques used to optimize stochastic problems in parametric optimization [23]. Specifically, it involves searching for the optimal values of input parameters in a simulation model to achieve a specific objective.

The integrated production, maintenance, and quality control in this research can be represented as a discrete parametric optimization problem. The input parameters in this context refer to the set of feasible actions available in each state of the system. SO techniques can be employed to find the best action in each state, maximizing the total reward or objective function. In this research, the SO package is utilized to find the optimal or near-optimal values for the combined optimization problem. The SO package integrates various metaheuristic approaches, including scatter search, tabu search, and neural networks, into a single optimization procedure, enabling efficient and effective optimization. The SO process is shown in *Fig. 5*.

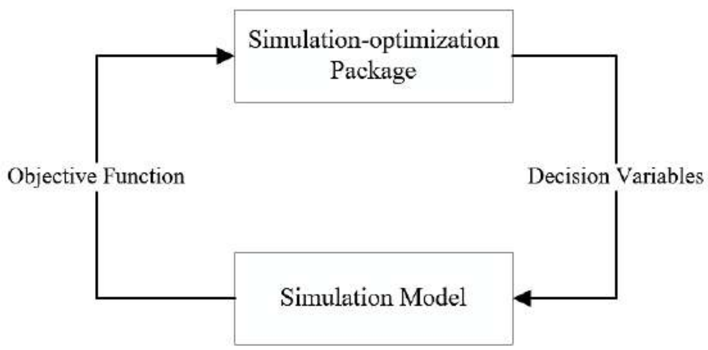

Fig. 4. The process of SO.

5 | Numerical Results

Four scenarios evaluate the efficiency of the proposed method. The first scenario is the base case, as illustrated in *Table 2*. In the second scenario, the efficiency of the policies to decrease missed orders is examined. In the third scenario, the effect of increasing the production rate is studied. The efficiency of the policies in reducing missed orders and wastages is evaluated in Scenario 4.

In the "policy learning" phase, the RL-based decision-maker agent acquires the optimal joint production, maintenance, and quality policy using the agent-based simulation of the production system of each scenario. Next, in the "policy evaluation" phase, the decision-maker agent selects the action based on the obtained policy in the simulation model of the scenarios. By the Monte-Carlo, the policy will be evaluated, the simulation model is iterated for a certain number of runs, and a unique random seed performs each iteration to capture all the events.

Two alternatives evaluate the RL-based policy: the random decision-maker agent and the SO method.

As discussed in Section 5, the decision-maker agent uses the ε-greedy policy to select an action in every decision epoch. If the value of ε is set to be 1, the agent chooses the action randomly in all decision epochs. This case is considered as an alternative to evaluating the policies.

The SO method is also studied to evaluate the policies. The SO package observes the simulation model of each scenario as a black box, and the system's states are defined as the input parameters of the simulation model. In each iteration, the state-action pairs are set as decision variables to minimize the obtained cost.

Table 2. The parameter's value of the production system.

Environment	I_{max}				10
	B_{max}				10
	S_{max}				10
	C_{h_I}				1
	C_{h_b}				3
	C_s				5
	C_w				15
	C_l				50
	S_p				10
Machine M_1	d	(0,1,2,3)	**Machine** M_2	d	(0,1,2,3)
	λ_f	$20\times\lambda_p$		λ_f	$20\times\lambda_p$
	λ_p	1.5		λ_p	2
	p_{b,d_i}	(0,0.05,0.2,0.8)		p_{b,d_i}	(0,0.06,0.25,0.8)
	p_{d_i}	(0,0.1,0.5,1)		p_{d_i}	-
	p_{d_i,q_l}	-		p_{d_i,q_l}	(0,0.2,0.5,0.9)
	p_{d_i,q_h}	-		p_{d_i,q_h}	(0,0.1,0.25,0.6)
	C_p	0.5		C_p	0.5
	C_m	100		C_m	120
	C_b	150		C_b	200
The Agent-Based Model	Shift duration				12 Hours
	Model execution time				2160 Hours

Due to the stochasticity of the simulation model, the model is replicated for a certain number of runs, and the average obtained costs are considered as the objective function value. The best state-action pairs that yield the minimum total cost are the acquired optimal joint policy by the SO package. In *Table 3*, the input parameters of the RL algorithm, the Monte Carlo method, and the SO package are illustrated.

Table 3. The input parameter's value.

RL	Number of episodes	25000
	Number of steps	2160 Hours
Monte-Carlo	Number of Iterations	5000
	The simulation model execution time	2160 Hours
SO Package	Number of iterations	25000
	Minimum number of replications	30
	Maximum number of replications	100
	The simulation model execution time	2160 Hours

5.1 | Scenario 1-Base Case

In Scenario 1, the input parameters of the agent-based simulation are set by the values of *Table 2*. The RL-based decision-maker agent observes the state of the system and authorizes the action. In the next step, the new state of the system and the corresponding reward (*Eq. (6)*) is returned. This process continues until the optimal policy is obtained. The acquired reward by the agent in each episode is shown in *Fig. 6*.

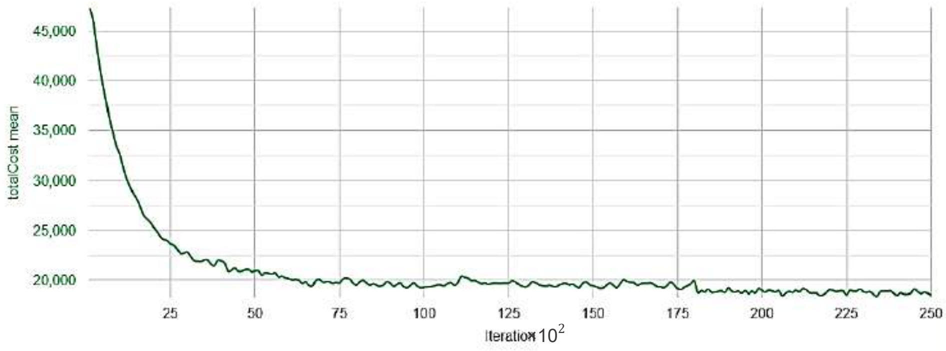

Fig. 5. The obtained reward by the decision-maker agent in scenario 1.

It is observed that the decision-maker agent initially receives a reward of about 47000 due to the random action selection. However, the policy gradually became goal-oriented and finally converged to the reward of about 19000. To evaluate the policy, the agent's decision-making process in the simulation model is implemented according to the acquired policy, and the Monte Carlo method is used for evaluation. The results are presented in *Table 4*.

Table 2. The results obtained by the RL-based policy in scenario 1.

Title	Average	Std. Dev
Total cost	18,884.25	1,900.18
Number of performed maintenance activities on machine M_1	44.699	1.168
Number of performed maintenance activities on machine M_2	44.609	1.14
Number of breakdowns of machine M_1	8.134	2.556
Number of breakdowns of machine M_2	8.428	2.607
Number of missed orders	165.097	31.549
Number of wastages	15.135	4.039

In order to evaluate the policy obtained by the decision-maker agent, the random decision-making policy is implemented in the agent-based simulation model. A comparison is given in *Table 5*.

Table 3. The comparison of the RL-based and random policy in scenario 1.

Title	RL-Based Decision-Maker Agent	Random Decision-Maker Agent
Number of performed maintenance activities on machine M_1	44.699	58.452
Number of performed maintenance activities on machine M_2	44.609	58.801
Number of breakdowns of machine M_1	8.134	4.277
Number of breakdowns of machine M_2	8.428	4.685
Number of missed orders	165.097	603.038
Number of wastages	15.135	8.092

The random decision-maker agent performed more maintenance and repair activities than the RL-based decision-maker agent, resulting in reduced produced wastage. However, the number of missed orders by the random policy is significantly higher than the RL-based policy. The RL-based policy performed maintenance and repair activities on time and succeeded in reducing 438 units of missed orders and not drastically increasing the wastage.

5.2 | Scenario 2- Decreasing the Missed Orders

In this scenario, the cost associated with missed orders is denoted as C_l is ten times higher than the base case. This intentional adjustment aims to incentivize the RL-based algorithm to minimize missed orders effectively. The achieved rewards by the agent in each episode are depicted in *Fig. 7*.

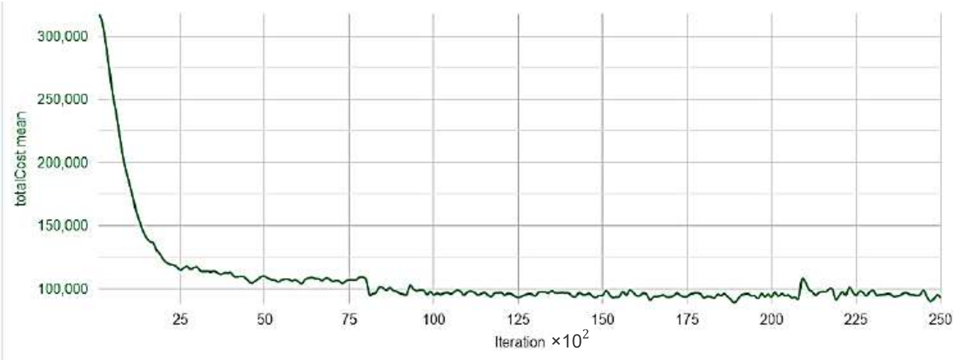

Fig. 6. The obtained reward by the decision-maker agent in scenario 2.

The agent receives a reward of about 320000 at the beginning of episodes, But with more learning, the reward converged to 93000. The results of the policy evaluation by the Monte-Carlo method are presented in *Table 6*.

Table 4. The results obtained by the RL-based policy in scenario 2.

Title	Average	Std. Dev
Total cost	93,775.34	16,349.80
Number of performed maintenance activities on machine M_1	42.927	1.948
Number of performed maintenance activities on machine M_2	44.724	1.168
Number of breakdowns of machine M_1	16.281	3.29
Number of breakdowns of machine M_2	8.584	2.561
Number of missed orders	155.145	32.159
Number of wastages	15.955	4.138

The RL-based policy decreases missed orders compared to the base case. But the number of breakdowns of the machine M_1 has increased. It means that the machine M_1 must produce even at a high level of degradation to satisfy the demand.

The comparison between the RL-based policy and the random policy is presented in *Table 7*.

Table 5. The comparison of the RL-based and random policy in scenario 2.

Title	RL-Based Decision-Maker Agent	Random Decision-Maker Agent
Number of performed maintenance activities on machine M_1	42.927	58.529
Number of performed maintenance activities on machine M_2	44.724	58.866
Number of breakdowns of machine M_1	16.281	4.269
Number of breakdowns of machine M_2	8.584	4.631
Number of missed orders	155.145	603.481
Number of wastages	15.955	8.048

Although the RL-based decision-maker agent decreases the missed orders, it is not significant. The main reason is that the production rate of machine M_1 and M_2, by considering the required maintenance and repair activities, can not meet the demand. In the following scenario, the effect of increasing the production rate is examined.

5.3 | Scenario 3-Increasing the Production Rate

In order to increase the possibility of satisfying the demand, the production rate is tripled. The cost of missed orders is the same as in the previous scenario. In this case, the acquired reward by the agent in each episode is shown in *Fig. 8*.

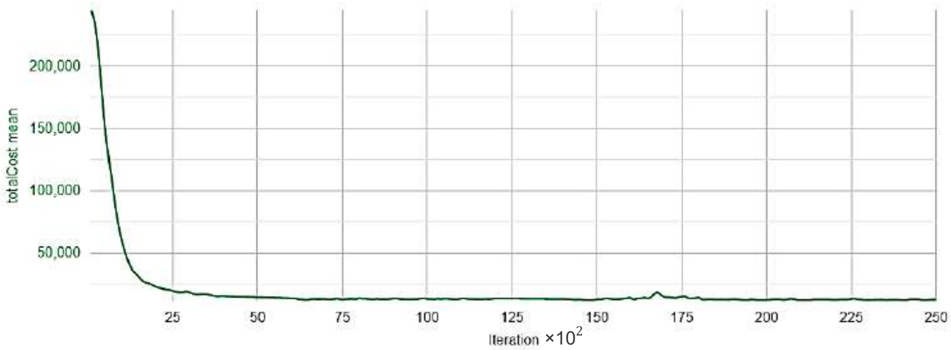

Fig. 7. The obtained reward by the decision-maker agent in scenario 3.

In scenario 3, the agent initially receives a reward of about 250000, but over time, the reward converges to 14000. The results of the policy evaluation by the Monte-Carlo method are presented in *Table 8*.

Table 6. The results obtained by the RL-based policy in scenario 3.

Title	Average	Std. Dev
Total cost	14,256.19	4,274.82
Number of performed maintenance activities on machine M_1	56.095	2.24
Number of performed maintenance activities on machine M_2	56.083	1.936
Number of breakdowns of machine M_1	18.613	3.762
Number of breakdowns of machine M_2	11.937	3.299
Number of missed orders	5.727	7.513
Number of wastages	21.553	5.65

The RL-based policy succeeded in decreasing missed orders by almost 96% compared to scenario 2. The number of maintenance, breakdowns, and wastages has also increased due to the increased production.

The comparison between the RL-based policy and the random policy is provided in *Table 9*.

Table 7. The comparison of the RL-based and random policy in scenario 3.

Title	RL-Based Decision-Maker Agent	Random Decision-Maker Agent
Number of performed maintenance activities on machine M_1	56.095	62.92
Number of performed maintenance activities on machine M_2	56.083	62.842
Number of breakdowns of machine M_1	18.613	10.889
Number of breakdowns of machine M_2	11.937	10.871
Number of missed orders	5.727	470.454
Number of wastages	21.553	18.571

As it is observed, the performance of the RL-based decision-maker agent is impressive in the current scenario. The increased production gives more flexibility to the agent in selecting the time of production, maintenance, and repair activities or being idle so that the total cost is minimized. *Table 9* shows that although there is no significant difference in the number of maintenance and repair activities performed between the two agents, the RL-based decision-making agent's missed orders are much lower. However, the increase in production rate has not led to a significant increase in the number of wastages, which has been due to the timely authorized maintenance and repair activities by the RL-based decision-maker agent.

5.4 | Scenario 4-Decreasing the Wastages

In this scenario, the algorithm's efficiency in maintaining the quality of the final parts and reducing the number of wastages is examined. So, the probability of producing low-quality semi-finished parts in machine M_1 at different degradation levels are increased. In addition, the probability of making waste from high-quality and low-quality semi-final parts at different degradation levels of machine M_2 has also increased. The new values are presented in *Table 10*.

Table 10. The quality-related probabilities in scenario 4.

Quality-Related Probabilities	The Production Machine	New Values
p_{d_i}	Machine M_1	(0, 0.3, 0.7, 1)
$p_{d_{i,q_l}}$	Machine M_2	(0, 0.3, 0.8, 0.9)
$p_{d_{i,q_h}}$	Machine M_2	(0, 0.2, 0.6, 0.8)

Therefore, a large percentage of the final parts will be wasted in the case of insufficient maintenance and repair activities. Also, to jointly optimize the production, maintenance, and quality at the same time, the cost of missed order is the same as the third scenario, and the cost of wastage C_w is examined in two cases: 15 and 500. The yielded reward by the agent in each episode is presented in *Fig. 9* (C_w=15) and *Fig. 10* (C_w=500).

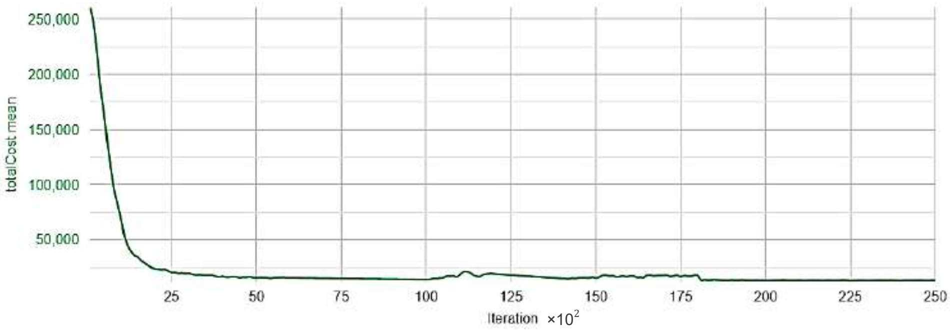

Fig. 8. The obtained reward by the decision-maker agent in scenario 4 (C_w=15).

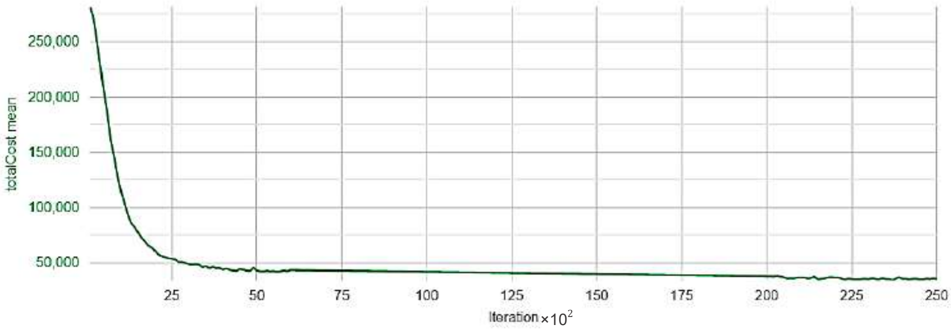

Fig. 9. The obtained reward by the decision-maker agent in scenario 4 (C_w=500).

Similar to the previous scenarios, the performance of the RL-based decision-maker agent and the random decision-maker agent are compared in *Table 11*.

Table 8. The comparison of the RL-based and random policy in scenario 4.

Title	Scenario 4-1 (C_w=15)		Scenario 4-2 (C_w=500)	
	RL-Based Decision-Maker Agent	Random Decision-Maker Agent	RL-Based Decision-Maker Agent	Random Decision-Maker Agent
Number of performed maintenance activities on machine M_1	57.845	62.926	58.049	62.992
Number of performed maintenance activities on machine M_2	58.13	63.067	57.909	63.065
Number of breakdowns of machine M_1	18.75	11.056	18.846	10.977
Number of breakdowns of machine M_2	12.415	11.052	11.721	11.028
Number of missed orders	2.199	488.505	2.852	489.384
Number of wastages	47.048	39.222	44.876	38.969

It can be seen that although the increased probability of producing wastages leads to a decrease in final products, the RL-based agent has succeeded in balancing the number of missed orders and wastages in the current scenario.

5.5 | The Policy Evaluation by SO

Meta-heuristic methods have been widely used to obtain optimal or near-optimal policies for different mixes of production, maintenance, and quality problems [3], [16], [17], [20]. In this research, in addition to the Random Agent (RA), the SO method is also used as an alternative to evaluating the obtained policy by the RL-based decision-maker agent. The SO package is initialized by related parameters (*Table 3),* and the connection of the package and simulation model of each scenario is established, as shown in *Fig. 5*. Similar to the evaluation process of RL-based policy, the acquired joint policy by SO package is evaluated by the Monte Carlo method. The results are shown in *Fig. 11*.

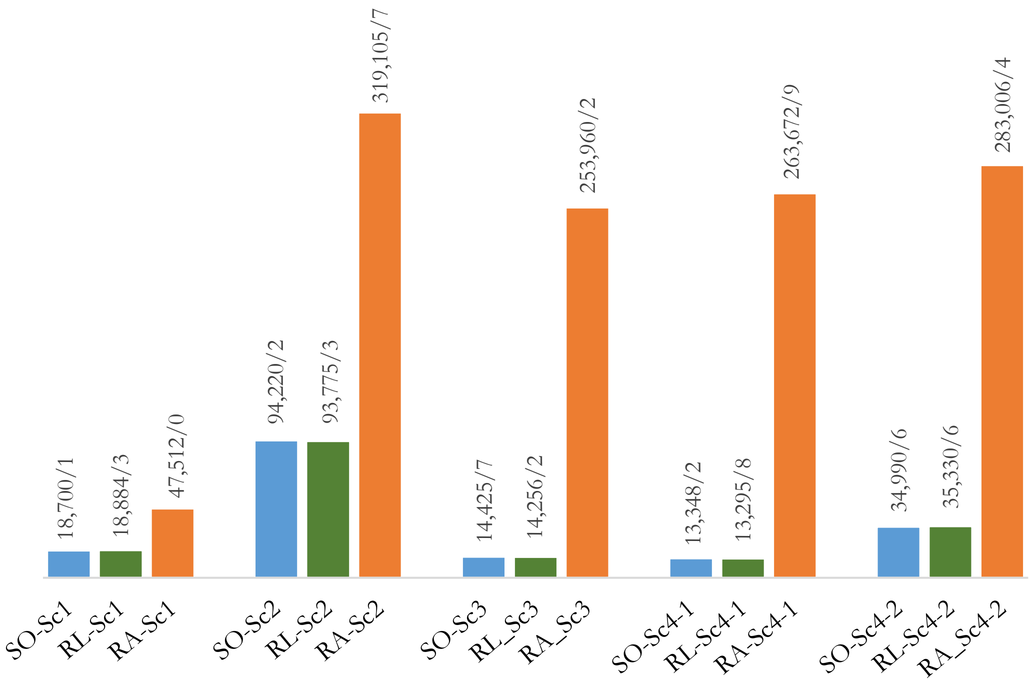

Fig. 10. The comparison between SO, RL, and RA policies.

The results indicate that the decision-maker agent, through RL, has successfully achieved an optimal or near-optimal policy. Its performance is very close to, and in some cases, even better than, the SO approach, and both methods have superior performance in minimizing the cumulative reward of *Eq. (6)*. SO methods, due to the use of metaheuristic algorithms, provide near-optimal solutions, which serve as a suitable benchmark for evaluating the performance of other proposed methods. Therefore, it can be concluded that the derived joint policy through the RL algorithm can be an effective policy for the real-time decision-making process in manufacturing systems. RL can provide the best action regarding production, maintenance, repair, and quality in various states of the production system.

6 | Conclusion

Production planning, maintenance, and quality control are always the most critical challenges of production systems. Regarding mutual interactions, it is necessary to investigate these issues jointly to achieve the optimal integrated policy of the system.

This paper examined the joint optimization of production, maintenance, and quality of the multi-machine single-product system with an intermediate buffer and final product. The ABM approach was applied to simulate the production system, and an RL-based agent was designed to interact with the simulation model to obtain the combined policy. The random policy and the meta-heuristic methods in the form of the SO approach were used to evaluate the acquired RL-based optimal policy.

Four scenarios were considered to cover all aspects of the production system. In each scenario, the performance of the policies in authorizing maintenance and repair activities, reducing missed orders, and reducing wastage were examined. The results showed that RL-based policy has superior performance in minimizing production costs, maintenance costs, inventory costs, lost orders, breakdown of production machines, and low-quality production. By the RL-based achieved joint policy, the decision-maker agent authorizes the most proper action in each system's state. Thus, it can be used for an autonomous real-time decision-making process necessary for industry 4.0.

The results also showed that the RL algorithm has a high potential to solve problems defined in the Markov Decision Process (MDP). Dynamic programming is another approach to solving the MDPs, but the transition probabilities matrix is required, which is very hard or impossible to define in many real-world problems. In addition, the Curse-of-dimensionality is another challenge, specifically when the system's state space is numerous.

SO methods prove their efficiency in solving the complex optimization problem. However, it is computationally expensive to obtain a joint policy in large states and action spaces. It is necessary to assign an action to every state of the system as a decision variable in each iteration. However, with the advent of DRL, agents can now bypass exhaustive state exploration by integrating RL algorithms with neural networks. By harnessing the learning capabilities of neural networks, RL agents can adapt and make decisions even in unencountered states. This advantage positions RL as a promising approach to finding the joint optimal policy. However, there are also some limitations, such as reward design, stability, and convergence issues when applying RL. So, acquiring optimal policies for diverse problem domains relies crucially on selecting the most fitting methodology.

Finally, this research can be extended by considering the multi-machine multi-product system. In this case, DRL or Multi-agent RL is proposed to find the joint optimal or near-optimal policy because the state of the system will be dramatically increased. Dealing with multiple machines and products simultaneously introduces a more complex and extensive state space, making traditional RL methods less effective. By leveraging DRL techniques, such as Deep Q Networks (DQNs) or Policy Gradient methods, the agent can handle high-dimensional state representations and learn more sophisticated strategies. Alternatively, Multi-agent RL can be employed to model interactions and dependencies among multiple machines and products, allowing the agents to coordinate and collectively optimize the system performance.

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Leveraging Deep Feature Learning for Handwriting Biometric Authentication

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Abstract

The authentication of writers through handwritten text stands as a biometric technique with considerable practical importance in the field of document forensics and literary history. The verification process involves a meticulous examination of the questioned handwriting in comparison to the genuine handwriting of a known writer, aiming to determine whether a shared authorship exists. In real-world scenarios, writer verification based on the handwritten text presents more challenges compared to signatures. Signatures typically consist of fixed designs chosen by signers, whereas textual content can vary and encompass a diverse set of letters, numbers, and punctuation marks. Moreover, verifying a writer based on limited handwritten texts, such as a single word, is recognized as one of authentication's open and challenging aspects. In this paper, we propose a Customized Siamese Convolutional Neural Network (CSCNN) for offline writer verification based on handwritten words. Additionally, a combined loss function is employed to achieve more accurate discrimination between the handwriting styles of different writers. The designed model is trained with pairs of images, each comprising one authentic and one questioned handwritten word. The effectiveness of the proposed model is substantiated through experimental results obtained from two well-known datasets in both English and Arabic, IAM and IFN/ENIT. These results underscore the efficiency and performance of our model across diverse linguistic contexts.

Keywords: Writer verification, Siamese neural network, Feature learning, Combined loss function.

1 | Introduction

Handwritten texts have held a special significance in human relations, and the writer verification of a written text has always been considered a biometric authentication method [1]. Due to the fact that each person's handwriting has unique and measurable traits that distinguish the writer, the verification process can be done

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[2]. The writer verification involves the task of determining whether two handwritten texts belong to the same writer [3]. Based on the methods of handwritten data collection, two approaches are proposed in the writer verification systems: online and offline. A digital device collects the handwriting samples in the online approach. Therefore, various dynamic features such as the writing speed and the pen pressure level can be accessible. However, in the offline approach, no temporal and dynamic data about the handwriting is available, and only the scanned image of the written text can be used [3]. However, because it is impossible to collect dynamic information in the analysis of historical documents or some forensic documents, the offline approach is vastly applicable [4]. Handwriting-based authentication systems can also be divided into text-dependent and text-independent categories. In the text-dependent group, the text of the training data and the test data must be identical, but there is no limit on textual content in the text-independent group [5]. The writer verification is a significant topic in computer vision, and many verification methods have been proposed; nevertheless, there are still some open challenges. For instance, most proposed verification methods are ineffective for writer verification based on a single handwritten word rather than multiple handwriting lines.

In this paper, a new offline text-independent framework is proposed to verify the writer based on handwriting. In the proposed approach using the Siamese network, a pair of images is entered into two identical subnetworks, and the obtained feature vectors are used to detect the similarity between the original and the questioned handwriting images.

The significant contributions of this study can be summarized as follows:

I. An efficient writer verification framework is proposed to extract discriminative characteristics based on handwritten word images.

II. As the backbone of the proposed Siamese network, a customized Convolutional Neural Network (CNN) model is designed for the subnetworks.

III. A combined loss function that incorporates both contrastive loss and cross-entropy loss is utilized to enhance the accuracy in discriminating between the handwriting styles of different writers. This approach allows us to simultaneously leverage the strengths of these two loss functions for more effective discrimination.

IV. The proposed approach is tested on two well-known English and Arabic handwriting datasets, IAM and IFN/ENIT, to demonstrate its generalizability to different languages and handwriting styles.

The rest of this paper is organized as follows. Section 2 includes a brief overview of the related research. The description of the suggested method is provided in the Section 3. Section 4 is dedicated to describing the datasets used to evaluate the performance of the proposed verification model, as well as the results and discussion. Finally, Section 5 concludes the paper.

2|Literature Review

In this section, some state-of-the-art writer verification approaches are reviewed. Various methods have been proposed to verify the writer using machine learning and deep learning techniques, but most of these studies are focused on signatures, not handwriting.

Yilmaz et al. [6] applied several descriptors and a concatenation of classifiers to use the local signature features and the overall shape of the signature. In [7], a writer verification approach is proposed by examining a unique fragment of the handwritten word and using the Levenshtein edit distance. The proposed approach was tested on a part of the IAM dataset including 100 authors, and 87% accuracy was achieved.

In [8], a CNN was applied to extract the features of the signature samples. The proposed model was writer-dependent, meaning that a separate training set was assigned to each writer. Dey et al. [9] proposed a structure named SigNet, which was based on the Siamese network and received pairs of signature images to verify the signature.

Jang et al. [10] presented an approach based on the modified Hausdorff distance and the geometric features to verify the writer from Korean texts. In this method, the RGB images were converted into grey images, and the images were separated from the background with a binarization threshold. Then, the noises were detected and removed. Next, the images were created using the narrowing algorithm. The corners, endings, and intersection points were detected, and the segments between these points were removed. Finally, the distance criterion was determined without normalization and with normalization. The error rate was 37% without normalization and 36% with normalization.

Adak et al. [11] proposed an offline writer verification approach for Bengali scripts using hybrid features. In this method, the probability distribution function was first used to extract the handcrafted features. Then, these features were combined with the features automatically obtained from the CNNs. Finally, the combined features were applied as the input of the Siamese network. The proposed approach was evaluated using a dataset of 100 authors with 300 pages of Bengali manuscripts. Another hybrid deep method is proposed by Shaikh et al. for the handwriting verification of Shaikh et al. [1]. In this research, the engineered features were extracted using SIFT and GSC methods and combined with extracted features using CNN. In order to evaluate the proposed model, only the most frequent word "and" extracted from the CEDAR dataset, which contains 15518 words from 1567 authors, was used. Although the feature extractions using SIFT and GSC complement the deep networks, according to the results, the manual feature extraction process is very time-consuming, and 70% of the learning time is spent on this task. Using a combination of SIFT features and the deep Siamese network, the training accuracy of 99% and the test accuracy of 63% were achieved, indicating overfitting in the system.

Calik et al. [3] provided a new structure based on CNN for signature recognition on large-scale datasets. In the proposed structure, the nearest neighbor algorithm was also used to classify the extracted feature vectors of the dense layers. The evaluations were performed on GPDS, MCYT, and CEDAR dataset samples.

Maergner et al. [12] proposed a combination of two structural and statistical classifiers for signature verification. A triplet Siamese-based architecture, including three subnetworks sharing the same weights, was used as a statistical model. In the training phase, a triplet of signatures (the anchor image, the positive image, and the negative image) was fed to the model. Also, the graph edit distance was employed in the structural classifier. Parcham et al. [4] provided a new model with high performance for signature verification. In this model, a hybrid architecture named CBCapsNet, including the customized CNN and capsule neural network models, was presented to improve the model's capability in feature extraction and increase accuracy. The researchers tried to use the benefits of the convolutional networks to identify and extract the features. They also attempted to lessen the weakness of the CNNs in distinguishing the spatial changes and variations in image properties by utilizing the capsule neural networks.

Gosh [13] proposed a deep model using recurrent neural networks for signature verification and recognition. In this research, several features of the signature images were extracted, and the obtained feature vectors were classified. The method was evaluated on six public databases. Furthermore, a comparison was made with the CNNs, and a better result was achieved than with these networks. In [14], researchers proposed a new method of using generative adversarial networks as a data augmentation approach in training sets to solve the problem of limited data in the signature verification problem. The researchers evaluated their proposed method using two popular datasets GPDS and MCYT and four pre-trained CNN models, and acceptable results were obtained.

Aubin et al. [15] presented a new method using small segments of the graphemes. Two texture descriptors were used on five ordinary graphemes of a collected dataset of 3000 sample images written by 50 people. Using the support vector machine as a classifier, this research reported an average verification accuracy of 97%. Khan et al. [16] provided an approach to verify the writer based on partly damaged Arabic documents. The authors attempted to improve the verification performance by omitting the ineffective characters and focusing on the character shapes. By evaluating a collected Arabic dataset using a CNN, an accuracy of 95% was obtained.

The review of related works reveals a scarcity of studies in the realm of writer verification centered on handwriting, with the majority concentrating on signature-based approaches. Signatures, typically fixed designs chosen by the writer, offer a limited scope. However, the realm of writer verification through handwriting introduces a greater complexity, given the variability in textual content, encompassing a diverse array of letters, numbers, and various symbols. This intricacy makes writing verification based on handwriting more challenging than signature-based methods. Unlike signature-based approaches, which focus on a fixed design, our study delves into the dynamic nature of handwriting, specifically concentrating on single handwritten words.

Furthermore, our approach stands out by removing the dependence on language-specific graphemes. Our method avoids this limitation, unlike many existing methodologies in previous research, which focus on extracting features tied to particular graphemes. This departure enhances the generalizability of our approach, making it more versatile and applicable across diverse languages.

In light of these observations, our current study assumes a pivotal role in bridging this research gap. By presenting an innovative approach to writer verification based on a single handwritten word independent of language-specific graphemes, we aim to contribute to a more applicable and robust methodology in the realm of handwriting-based writer verification.

3 | The Proposed Customized Siamese Convolutional Neural Network Methodology

In this section, we present the architecture of the proposed methodology for writer verification. This architecture consists of a customized convolutional Siamese network (cSCNN), including two convolutional subnetworks with a shared architecture and equal weights.

CNNs are a special class of deep neural networks widely used in various machine vision tasks [17]. CNNs include several important layers, such as convolutional, pooling, and Fully Connected (FC) layers [18]. As the core block of the network, the convolutional layers can have different kernel sizes. The lower convolutional layers extract low-level features like color and edge, while high-level features like lines and objects are extracted in the upper layers. After the convolutional layer, the pooling layer performs the down-sampling of the outputs. Based on the extracted features, the prediction can be done in the FC layer [19].

The Siamese neural network includes two subnetworks with identical configurations as a specific neural network architecture. The parameters update is reflected in both branches and is connected through a loss function. This function calculates a similarity measure between the feature vectors obtained from two subnetworks [20]. This network has achieved acceptable results in various challenging issues of machine vision, such as face verification and signature verification [9].

The detailed information about the architecture of the proposed cSCNN is illustrated in *Fig. 1*. The subnetworks of our cSCNN utilize a proposed deep architecture consisting of five convolutional blocks (Conv-Block). Two convolutional layers are embedded in each Conv-Block. A Batch Normalization (BN) layer is applied after each convolutional layer to extract the most salient features, discard less significant details, improve the generalization, and aid in faster and more effective training. Applying dropout after BN in the first three Conv-Blocks is attempted to enhance the generalizability and decrease the risk of overfitting. Each conv-Block is ended with a max pooling layer with the size of 2×2 and stride 2 to capture the most important handwriting information while discarding the redundant details. The pair of handwriting images (size: 80×180) is fed as the input to the customized subnetworks. In all convolutional layers, the convolution operation is performed by sliding the filters of size 3×3 over the input handwriting images. The stride and the padding are also set to 1. To enhance the non-linear transformations in the feature learning process [21], we use the interval type 2 fuzzy unit proposed in [22] as the activation function. The high-level handwriting features can be captured by increasing the number of filters from 32 to 512 while going deeper into the Conv-Blocks and combining the low-level features. Following the convolutional blocks, Global Average

Aggregation (GAP), aiming to reduce the number of parameters and the complexity of the model, and the flattened layer, converting the multi-dimensional dimension into a one-dimensional vector, finalize the architecture of the customized Siamese subnetworks. The architecture configuration details of each subnetwork are also presented in *Table 1*. Moreover, *Fig. 2* illustrates the pseudocode of the proposed writer verification framework.

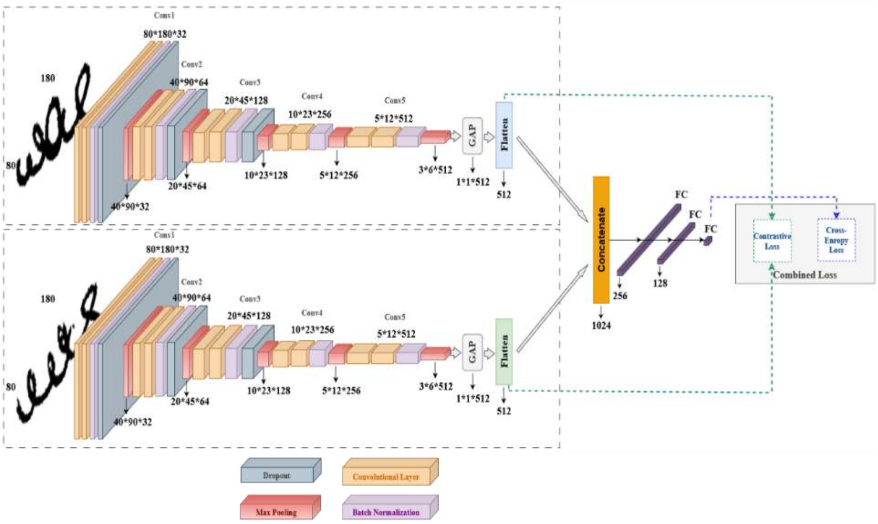


Fig. 1. The custom architecture used in cSCNN.

A concatenation layer is then applied to the two output vectors of the deep subnetworks. Afterward, the concatenated vector is passed through the three FC layers. The output of the last FC layer determines whether the verification result is Genuine or Forged, meaning that both images of the pair are written by the same writer (Similar) or different writers (Dissimilar), respectively.

Table 1. The architecture configuration details of the identical subnetworks.

Layers	Number of Kernels	Kernel Size	Output Size
2 × Convolution2D+BN	32	3×3	80×180×32
Max pooling2D			40×90×32
2 × Convolution2D+BN	64	3×3	40×90×64
Max pooling2D			20×45×64
2 × Convolution2D+BN	128	3×3	20×45×128
Max pooling2D			10×22×128
2 × Convolution2D+BN	256	3×3	10×22×256
Max pooling2D			5×11×256
2 × Convolution2D+BN	512	3×3	5×11×512
Max pooling2D			3×5×512
Global Average Pooling			1×1×512
Flatten			512

```
Input:
  Handwriting Dataset: Handwriting images;
  Hyperparameters: num_epochs: Number of Epochs; batch_size: Batch size; learning_rate: learning rate
Output:
   Evaluated performance of the model

Preprocessing
  Convert handwriting_samples to grayscale images
  Crop images into fixed-size patches
  Normalize pixel values
  Create pairs of images and corresponding labels (1 for positive/similar pairs, 0 for negative/dissimlar pairs)
  Implement Data Augmentation(Brightness Adjustment, Zooming, Random Rotation, Random Noise)

Training the Proposed Verification Model
 Initialize the customized Siamese Convolutional Neural Network with two identical Subnetworks

Training Loop:
while epoch ≤ num_epochs do
   Divide the Data into batches of batch_size
   for each batch of handwriting paired_images do
     embedding1 ← Extract deep features from Subnetwork1
     embedding2 ← Extract deep features from Subnetwork2
     concatenated_features ← Concatenate the extracted features
     output ← Pass concatenated_features through two fully connected layers
     prediction ← Pass output through dense layer for writer classification
     Contrastive_Loss ← Compute the loss (embedding1, embedding2 )
     Binary_Cross_Entropy_loss ← Compute the loss (prediction, true label)
     Combined_Loss ← a.Contrastive_Loss + b.Binary_Cross_Entropy_loss
     update the model parameters by computing gradients through backpropagation and utilizing Adam optimizer with the learning_rate
   end for
end while

Evaluating the Proposed Verification Model
Accuracy; FAR; FRR ← Evaluate the model performance based on the test data
```

Fig. 2. The pseudocode of the proposed writer verification framework.

3.1 | Training and Evaluating of the Proposed Model for Writer Verification

The training process of the proposed model for writer verification requires the creation of image pairs after preprocessing. Thus, for a pair of images, if two handwritten images belong to the same writer, it is a positive pair; otherwise, it is considered a negative pair. The proposed deep Siamese network is trained on these positive and negative pairs. During the training phase, the proposed model extracts embeddings from each image in the input pair. Then, these embeddings are used to calculate the loss. By iteratively adjusting the parameters based on the calculated loss, the model learns to produce embeddings that effectively discriminate between genuine and questioned handwriting. A pair of two handwriting images is presented as an input element to test the model. One image belongs to a specific writer, and the other is the questioned handwriting that needs to be verified. An overview of the training and testing of the proposed model for writer verification is shown in *Fig. 3*.

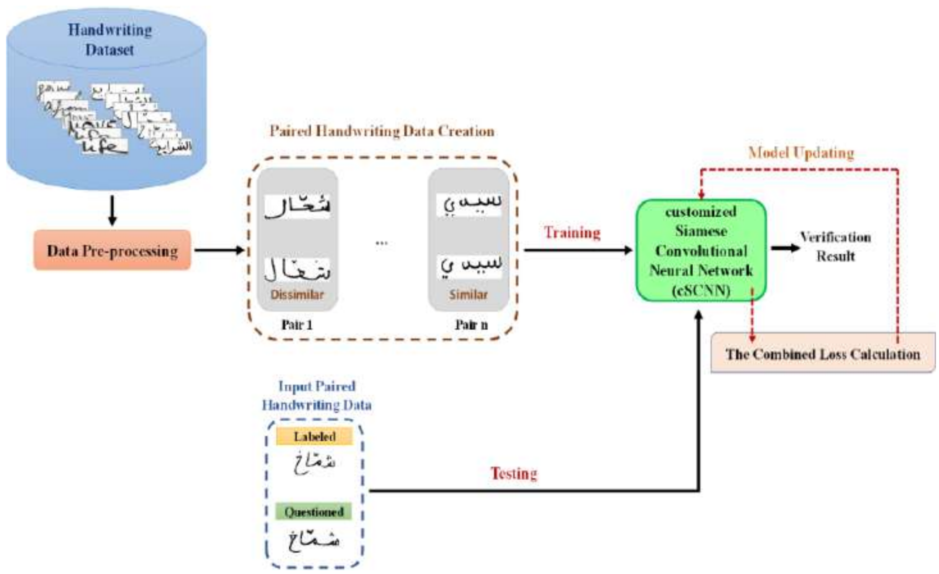


Fig. 3. The flow diagram of cSCNN for writer verification.

3.1.1 | The proposed combined loss

Loss function is essential for training neural networks or machine learning models [23]. This function provides evidence of estimation errors in the training phase and directly affects the learning performance [24]. Each loss function can include different aspects of learning objectives.

In this study, a combined loss function is provided to enhance the training process and model performance. During the training of the proposed model, two vectors obtained from the Flatten layers in two Siamese subnetworks are used to apply the contrastive loss function [25], and the cross-entropy loss function [26] is also applied to the output of the last FC layer. As the components of the proposed combined function, the cross-entropy loss function focuses on Binary classification accuracy, and the Contrastive loss function emphasizes the accurate separation and discrimination between similar and dissimilar handwriting samples.

Contrastive loss

The contrastive loss function is widely employed for comparing two samples. The cSCNN leverages this loss function to highlight significant features within the feature space, bringing similar samples (written by the same writer) closer together while pushing dissimilar samples (written by different writers) farther apart. This loss function is applied to the feature vectors obtained from two siamese subnetworks.

The Contrastive loss is obtained as follows:

$$L_C = \frac{1}{2N}\sum_{i=1}^{N}\left(y_i D_i^2 + (1 - y_i)\max(0, \text{margin} - D_i)^2\right), \quad (1)$$

where D_i denotes the Euclidean distance between two feature representations of the pair of handwritten images. A pair of input images is positive/similar when the same writer writes yi=1 and the two handwriting images; otherwise, it is considered a negative/dissimilar pair for y_i=0. The margin determines the desired separation threshold between similar and dissimilar samples.

Cross-entropy loss

The cross-entropy loss function is one of the most beneficial and widely used loss functions in neural networks. The measured prediction is a number between zero and one. The main goal is to achieve a model with a log loss around zero [24].

The cross-entropy loss is formulated as follows:

$$L_{CE} = -\frac{1}{N}\sum_{n=1}^{N}(\text{similarity\_label} * \log(\text{predicted\_similarity}) + (1 - \text{similarity\_label}) * \log(1 - \text{predicted\_similarity}))], \quad (2)$$

where the similarity_label is either 0 or 1, indicating whether the pair is dissimilar (0) or similar (1). The predicted_similarity is the network output that represents the predicted similarity between the pair of handwriting samples, and it is a value between 0 and 1. The 1/N factor is applied to average the loss of overall data pairs. This averaging helps ensure that each pair of handwriting data equally influences the overall loss calculation.

In the suggested approach, cross-entropy loss is applied to the outputs of the last FC layer and allows the model to be optimized for accurate classification.

The combined loss formula

The combined loss is formulated as the linear combination of the employed loss functions *Eq. (3)* and applied to train the proposed network structure by optimizing all the loss functions with back-propagation at the same time.

$$\text{Combined Loss} = \alpha L_C + \beta L_{CE}, \quad (3)$$

where α and β are the weights to balance the contributions of each loss function based on the desired trade-off.

4 | Experiments and Results

This section details the datasets and experiments conducted to evaluate the proposed method.

4.1 | Data

The well-known datasets of IAM [27] and IFN/ENIT [28] are employed to train and evaluate the proposed model.

4.1.1 | The IAM dataset

The IAM dataset, renowned for its collection of English handwriting samples, encompasses 1539 documents originating from 657 unique writers. This dataset is meticulously organized, featuring handwriting images across various document elements, including pages, lines, and words. Within this rich dataset, a total of 1539 pages, 5685 sentences, 13353 text lines, and 115320 labeled words are available [27]. For our study, we utilized the handwriting images of 130 writers from this dataset to generate both similar and dissimilar handwriting samples. Refer to *Fig. 4* for visual representations of sample images from this dataset.

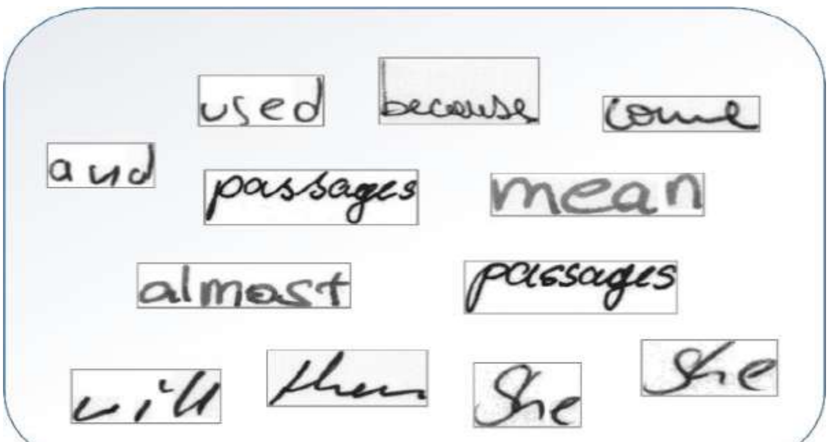


Fig. 4. Some instances of handwritten words from the IAM dataset.

4.1.2 | The IFN/ENIT dataset

The IFN/ENIT dataset, a comprehensive collection of Arabic handwriting, encompasses 26,459 handwritten names representing Tunisian cities or villages, contributed by 411 participants [28]. To construct a diverse set of handwriting samples, we specifically leveraged the handwriting images of 120 writers from this extensive dataset. For a visual representation, please refer to *Fig. 5*, which showcases a selection of examples featuring handwritten words extracted from the IFN/ENIT dataset. This dataset provides a rich source of Arabic handwriting and facilitates the creation of varied samples for our study.

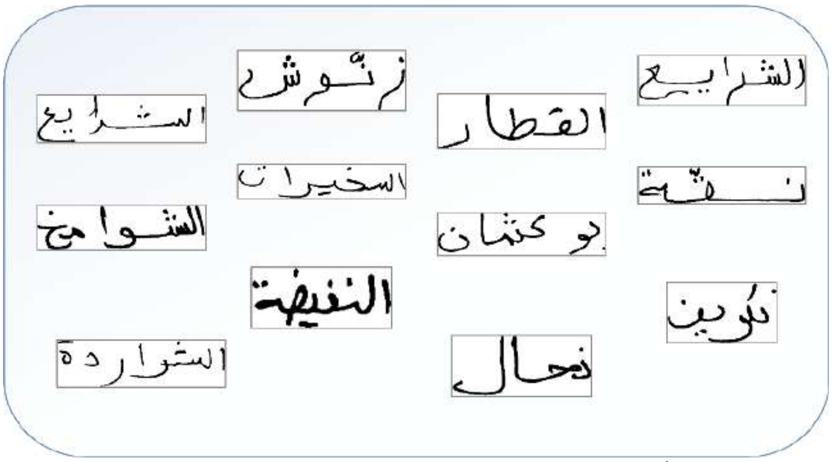


Fig. 5. Some exemplars of handwritten words from the IFN/ENIT dataset.

4.1.3 | Preprocessing and preparing the pair of input images

Preprocessing is crucial for effective feature extraction and precise analysis in the field of writer verification using handwriting images. Thus, as the initial step, we converted the collected samples into grayscale images. The handwritten word images within the datasets vary in size, requiring standardization for consistent model input. To achieve this, we meticulously cropped the images into uniform 80×180 patches to ensure that the model receives standardized input across all samples. Moreover, we normalized pixel values to foster an environment conducive to effective model learning and performance optimization.

In the realm of data augmentation, we employed a comprehensive set of transformations to enhance dataset variability. Rotation, with a variability of 0.8, was introduced to simulate diverse perspectives. Zooming, within a range of 0.1, and brightness adjustments spanning from 0.4 to 0.7 were applied to augment the dataset further, providing the model with a more diverse and robust set of samples for training.

Addressing the creation of pairs for training, positive (similar) pairs were meticulously curated by selecting two samples from the handwriting of a single writer. Similarly, negative (dissimilar) pairs were formed by pairing two samples from different writers. Importantly, to counteract potential biases and ensure a balanced dataset, an equal number of positive and negative samples were generated for each writer.

Then, turning to the data-splitting strategy, 70% of the word samples were allocated for training, providing the model with a substantial foundation for learning. A reserved 10% was set aside for validation and fine-tuning during the training process, aiding in parameter adjustments and enhancing model generalization. The final 20% was dedicated to evaluating the model's performance on unseen data during the testing phase. The distribution of training and testing data for the two datasets is detailed in *Table 2*.

Table 2. The data splitting on IAM and IFN/ENIT datasets.

Dataset	# Pair Images	# Training Data (70%)		# Validation Data (10%)		# Testing Data (20%)	
		Similar	Dissimilar	Similar	Dissimilar	Similar	Dissimilar
IAM	2600	910	910	130	130	260	260
IFN/ENIT	2040	714	714	102	102	204	204

4.2|Hyperparameters Setting

Fine-tuning the hyperparameters, as detailed in *Table 3*, included setting the learning rate to 0.0001 and the batch size to 64. A dynamic adjustment strategy was applied to address potential learning stagnation, reducing the learning rate by 0.2 whenever stagnation persisted for five consecutive epochs.

The model was trained for 70 epochs, optimizing parameters using the Adam optimizer and the Binary cross-entropy loss function. The experiments were conducted on an Nvidia GEFORCE GTX 1070 with 8 GB RAM, ensuring computational efficiency and robust model training.

Table 3. List of training hyperparameters.

Hyperparameters	Value
Initial learning rate	0.0001
Batch size	64
Dropout	0.3
Optimizer	Adam
Loss function	Binary cross-entropy
Epoch	70

4.3 | Evaluation Metrics

In our study, the chosen criteria to evaluate the verification performance are as follows:

I. Accuracy (Acc): the standard metric of accuracy is used to evaluate how well the proposed approach can verify the writer of the handwritten text and identify the genuine/similar and forged/dissimilar handwriting.

$$\mathrm{Acc} = (\mathrm{TP} + \mathrm{TN}) / (\mathrm{TP} + \mathrm{TN} + \mathrm{FP} + \mathrm{FN}), \quad (4)$$

where True Positives (TP) is the number of samples correctly predicted as genuine/similar handwriting. True Negatives (TN) is the number of samples correctly predicted as forged/dissimilar handwriting. False Positives (FP) is the number of samples incorrectly predicted as genuine/similar handwriting. False Negatives (FN) is the number of samples incorrectly predicted as negative forged/dissimilar handwriting.

II. False Acceptance Rate (FAR): This metric is commonly used in verification systems and refers to the percentage of forged/dissimilar handwriting pairs classified as genuine/similar. A lower FAR demonstrates that the biometric system is more reliable and mistakenly accepts fewer forged inputs.

$$\mathrm{FAR} = \mathrm{FP} / (\mathrm{TN} + \mathrm{FP}). \quad (5)$$

III. False Rejection Rate (FRR): this is another common biometric performance metric used in verification systems and refers to the percentage of genuine/similar handwriting pairs classified as forged/dissimilar. A lower FRR indicates that the biometric system mistakenly rejects fewer genuine handwritings.

$$\mathrm{FRR} = \mathrm{FN} / (\mathrm{TP} + \mathrm{FN}). \quad (6)$$

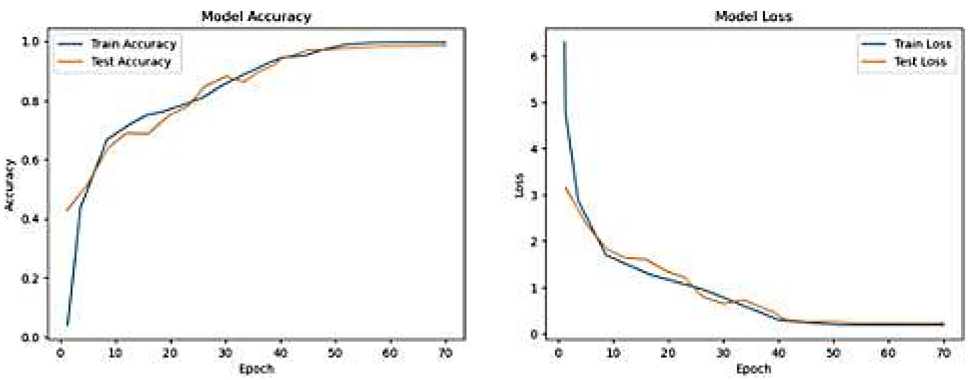
4.4|Evaluation Results of the Proposed Writer Verification Architecture

The effectiveness of the cSCNN architecture is assessed using IAM and IFN/ENIT datasets. The evaluation outcomes, measured in terms of accuracy, FAR, and FRR, are presented in *Table 4*.

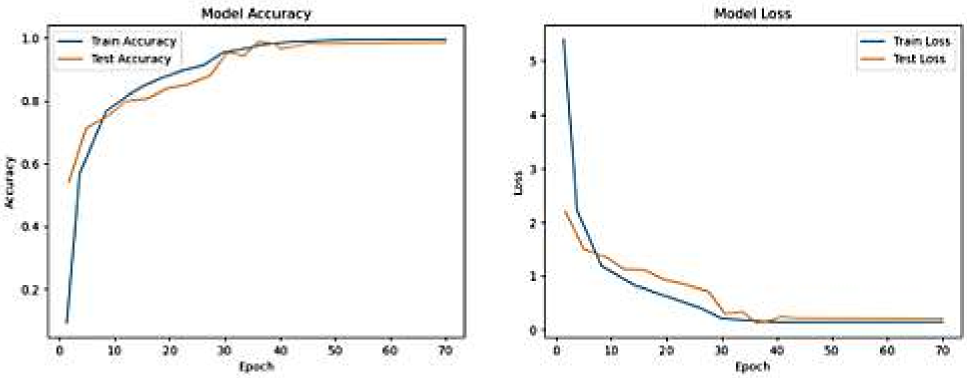
Table 4. The performance results of the cSCNN model on IAM and IFN/ENIT datasets.

Dataset	Language	Train accuracy (%)	Validation accuracy (%)	Test accuracy (%)	FAR (%)	FRR (%)
IAM	English	99.61	98.65	98.46	1.92	1.15
IFN/ENIT	Arabic	99.50	98.77	98.52	1.47	1.47

Moreover, *Fig. 6* illustrates the performance of the proposed model during both the training and test phases.

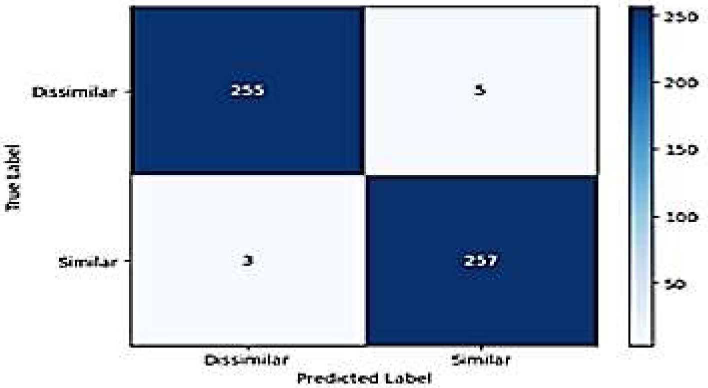


a.

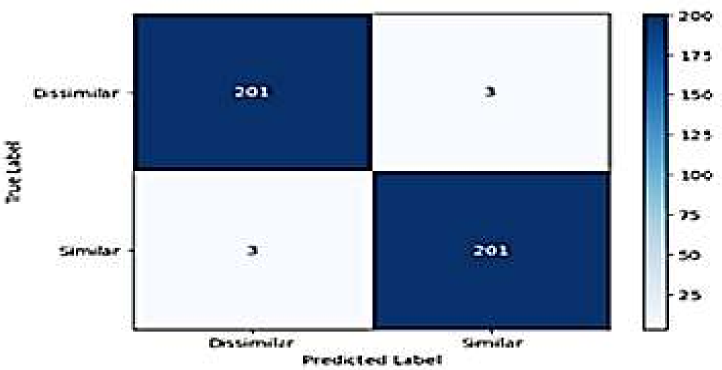


b.

Fig. 6. Accuracy and loss plots of the proposed cSCNN model on; a. IAM and b. IFN/ENIT.



a.



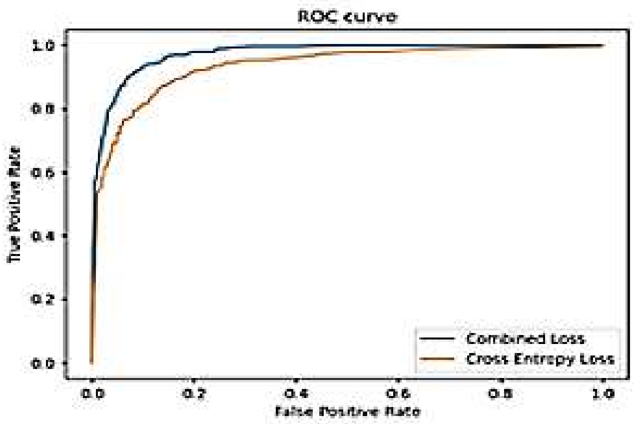
b.

Fig. 7. The confusion matrices of cSCNN on; a. IAM and b. IFN/ENIT.

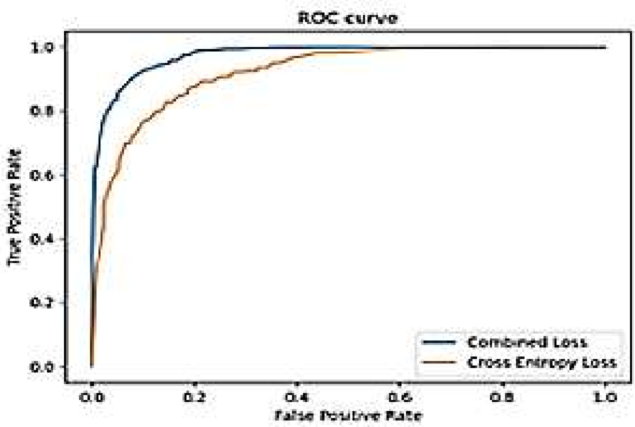
In addition, the confusion matrices of the cSCNN model on two datasets are displayed in *Fig. 7*.

4.5 | The Impact of Different Loss Functions

The next experiment evaluated the influence of using the proposed combined loss function compared to a single cross-entropy loss function. ROC curves were used to illustrate the comparison results. As shown in *Fig. 8*, applying the combined loss function to the proposed model, a higher area under the curve was obtained on both IAM and IFN/ENIT.



a.



b.

Fig. 8. ROC curve of the verification model with proposed combined loss function and cross-entropy loss function on; a. IAM and b. IFN/ENIT.

4.6 | The Impact of Different Numbers of Training Pairs

In another experiment, the effect of the number of training pairs per writer on the performance of the designed model is assessed. We assessed the verification performance by utilizing varying similar and dissimilar pairs (3 to 7) for each writer in the IAM dataset. As seen in *Table 5*, the verification accuracy is impacted by the increase in the number of training pairs. Also, a substantial improvement is observed when 5 training pairs are utilized for each author.

Table 5. The effect of different numbers of the training pairs on the verification performance.

The number of training pairs (similar, dissimilar)	Train accuracy (%)	Test accuracy (%)
(3,3)	92.50	89.53
(4,4)	95.82	93.74
(5,5)	99.32	98.15
(6,6)	99.47	98.41
(7,7)	99.60	98.44

4.7 | The Comparison with the Related Writer Verification Methods and Discussion

Table 6 compares the accuracy of the proposed method with other writer verification approaches. In comparison to the study by Bensefia et al. [7], which sought to verify writers by analyzing the constituent graphemes of a handwriting word sample, our research demonstrates a notable improvement in accuracy. Specifically, our proposed method achieved a significant 11.46% enhancement over [7] when evaluated on the IAM dataset. This improvement underscores the efficacy and superiority of our cSCNN model in comparison to the approach presented by Bensefia et al.

Aubin et al.'s method [15], while achieving 97% accuracy on a specific dataset, is confined by its reliance on only five typical graphemes. This limitation raises concerns about the model's generalizability. In contrast, our proposed cSCNN is designed to offer greater versatility by not being restricted to a small set of graphemes. Our comprehensive evaluation of distinct datasets emphasizes the importance of considering model performance across diverse datasets to ensure reliability and robustness.

Similarly, the approach outlined by Khan et al. [16], which focuses on specific shapes of Arabic alphabets, presents limitations in its applicability to other languages and writing systems. Its reliance on predefined shapes hinders adaptability to different writing styles and scripts. In contrast, our method prioritizes feature extraction and generalization, offering a more flexible and adaptable solution for writer verification. By not solely depending on predefined shapes, our approach becomes applicable to a broader range of languages and handwriting styles.

Our novel writer verification method focuses specifically on handwritten words, addressing the observed limitations in previous approaches. Through extensive experimentation and evaluation of datasets representing both English and Arabic languages, our cSCNN showcased its ability to extract distinctive features effectively, thereby enabling reliable writer verification. The promising results obtained on datasets with different languages highlight the potential for our approach to be widely applicable and adaptable in real-world scenarios.

Table 6. The performance comparison with the related writer verification methods.

Study	Method	Dataset	Accuracy (%)
Bensefia et al. [7]	Levenshtein edit distance	IAM (English)	87.00
Aubin et al. [15]	Two texture descriptors and support vector machine	Collected English dataset	97.00
Khan et al. [16]	CNN	Collected Arabic dataset	95.00
Present study	cSCNN	IAM (English)	98.46
		IFN/ENIT (Arabic)	98.52

5 | Conclusion

Various challenges are involved in analyzing handwritten text for writer verification, such as extracting more informative and differentiating features, achieving higher performance in real-world applications, and enabling writer authentication based on a small amount of handwritten text. In response to these challenges, this paper introduced an effective method for improved writer verification from single handwritten words.

Our proposed deep architecture is based on a Siamese network with a custom CNN model as the backbone of the sub-networks. This design aims to enhance handwriting verification performance by incorporating specially designed convolution blocks in each Siamese branch. The model is trained using a composite loss function, including contrastive and cross-entropy losses. This integration of multiple loss functions facilitates efficient extraction of handwriting characteristics, enhancing the model's ability to capture fundamental patterns and, consequently, improving overall performance compared to using a single loss function.

Notably, our model underwent evaluation on two diverse datasets representing different languages, English and Arabic, with the added distinction that one dataset is written right-to-left and the other left-to-right. The experimental results underscored the versatility and efficacy of our proposed model in handling various handwriting styles and languages, including the directional nature of the script.

In addition, our innovative approach diverges from the reliance on language-specific graphemes, setting it apart from numerous existing methodologies in prior research that concentrate on extracting features associated with specific graphemes. This deviation eliminates a potential limitation and significantly bolsters the generalizability of our method, rendering it more adaptable and applicable across a wide spectrum of languages.

Looking ahead, the potential of our proposed framework for effectively detecting forged signatures and identifying writers can be explored in future works. Additionally, we plan to enhance the model's capabilities by training it on a dataset of cursive English handwritten samples, presenting unique challenges due to their intricate connections.

In summary, our study contributes a novel and robust approach to writer verification from single handwritten words, demonstrating versatility across languages and superior performance compared to existing methods. The consideration of different directional datasets and the departure from language-specific graphemes add additional layers of complexity and relevance to the evaluation, highlighting the adaptability of our proposed model to diverse handwriting styles, scripts, and languages. The potential applications for detecting forged signatures and handling cursive English handwriting present exciting avenues for future exploration and expansion of our proposed framework.

Data Availability

The used datasets can be found at (https://fki.tic.heia-fr.ch/databases/download-the-iam-handwriting-database) and (http://www.ifnenit.com/download.htm).

Conflicts of Interest

The authors declare no conflict of interest.

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