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A Signed Distance Method for Solving Multi-Objective Transportation Problems in Fuzzy Environment

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A B S T R A C T

This paper aims to study the multi-objective transportation problem with fuzzy parameters. These fuzzy parameters represented as (α, β) interval-valued fuzzy numbers instead of the normal fuzzy numbers. Using the signed distance ranking, the problem converted into the corresponding crisp multi-objective transportation problem. Then, the solution method introduced by [8] for solving the problem is applied. This method provides the ideal and the set of all (α, β) fuzzy efficient solutions. The advantage of this method is more flexible than the standard multi-objective transportation problem, where it allows the decision maker to choose the (α, β) levels of fuzzy numbers he is willing. A numerical example to illustrate the utility, effectiveness, and applicability of the method is given.

Keywords: Multi-objective transportation problem, (α, β) fuzzy numbers, Signed distance function, Multi-objective decision making problem, Optimal transportation, Optimal flowing method.

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1. Introduction

Transportation Problem (TP) is a special type of Linear Programming (LP) problem, where the objective is to minimize the cost of distributing product from m sources or origins to n distributions and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_m$, respectively. In addition, there is a penalty c_{ij} associated with transportation a unit of product from source i to destination j; this penalty may be cost or delivery time of safety of delivery, etc. A variable x_{ij} represents the unknown quantity to be shipped from source i to destination j.

However, TP representing real-world situations involves a set of parameters whose values are assigned by Decision Makers (DMs). DMs required fixing exact values to the parameters in the conventional approach. In that case, DM does not precisely know the exact value of parameters,

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so the parameters of the problem are usually defined in an uncertain manner. In many scientific areas, such as systems analysis and operations research, a model has to be set up using data which is only approximately known. Fuzzy sets theory, introduced by Zadeh [24] make this possible. Fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers. Dubois and Prade [5] extended the use of algebraic operations on real numbers to fuzzy numbers by the use afuzzification principle. Bellmann and Zadeh [1] introduced the concept of fuzzy set theory into the decision-making problem involving uncertainty and imprecision. Zimmermann [25] was the first solved LP problem with several objectives through suitable membership functions. Tanaka and Asai [21] formulated a Fuzzy Linear Programming (FLP) problem to obtain a reasonable solution under consideration of the ambiguity of parameters. Oheigeartaigh [17] developed an algorithm for FTP. Chanas et al. [3] developed a parametric approach to solve single objective FTP. Leberling [12] solved vector maximum LP problem using a special type of non-linear membership functions. Thamaraiselvi and Santhi, [22] studied FTP with hexagonal fuzzy numbers. Sakawa and Yano [20] introduced the concept of fuzzy Multi-Objective Linear Programming (MOLP) problems. Kiruthiga and Loganathan [10] reduced the fuzzy MOLP problem to the corresponding ordinary one using the ranking function and hence solved it using the fuzzy programming technique. Hamadameen [7] proposed a technique for solving fuzzy MOLP problem in which the objective functions coefficients are triangular fuzzy numbers. Rommelfanger et al. [19] presented an interactive method for solving MOLP problem. Bit et al. [2] applied fuzzy programming approach for MOTP problem. Pandian and Anuradha [18] developed dripping method to solve bi-objective TP. Maity and Roy [14] solved multi-choice MOTP and MOTP with interval parameters. Nomani et al. [16] developed weighted approach based on goal programming to obtain compromise solution of MOTP. Yu et al. [23] proposed an approach for obtaining the solution of MOTP with interval parameters. Kaur et al. [9] proposed a simple approach to obtain the best compromise solution of MOTP problem. Kumar et al. [11] proposed an algorithm for solving TP, where they firstly extended an initial basic feasible solution, then used an existing optimality method to obtain the cost transportation. Mohmoudi and Nasseri [13] developed method to solve the fully fuzzy LP problem, the method demonstrates definitions introduced by Ezzati et al. [6]. Najafi et al. [15] proposed a method based on crisp nonlinear programming problem, which has a simple structure for solving fully fuzzy LP problems under nonnegative fuzzy variables restricted fuzzy coefficients.

In this paper, we attempt to solve the multi-objective transportation problem in fuzzy environment. A proposed algorithm introduced by [8] is applied to provide the ideal and the set of all efficient solution to the corresponding crisp MOTP problem.

The remainder of the paper is organized as follows: In Section 2, some preliminaries are presented. In Section 3, a General Fuzzy Multi-Objective Transportation Problem (GFMOTP) is formulated. Section 4 applies the method introduced by [8] to provide the ideal and the set of all fuzzy efficient solutions to the GFMOTP problem. In Section 5, a numerical example is given for illustration. Finally, some concluding remarks are reported in Section 6.

2. Preliminaries

In order to discuss our problem conveniently, the basic concepts and results related to the fuzzy numbers, and (α, β) interval valued fuzzy numbers are recalled.

Definition 1. (Fuzzy number). A fuzzy number $\widetilde{A}$ is a convex normalized fuzzy set on the real line $\mathbb{R}$ such that:

- (x) is piecewise continuous;
- $\exists\, x \in \mathbb{R}$, with $\mu_{\widetilde{A}}(x) = 1$.

Definition 2. (Level α of fuzzy number [4]). If the membership function of the fuzzy set $\widetilde{A}$ on $\mathbb{R}$ is

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{\alpha\,(x-r)}{(s-r)}, & r < x \leq s, \\ \dfrac{\alpha\,(t-x)}{(t-s)}, & s \leq x < t, \\ 0, \text{otherwise}, \end{cases}$$

where $0 < \alpha \leq 1$ then $\widetilde{A}$ is called a level α fuzzy number and it is denoted as $\widetilde{A} = (r, s, t;\ \alpha)$.

Definition 3. [4]. An interval-valued fuzzy set $\widetilde{A}$ on $\mathbb{R}$ is given by $\widetilde{A} \triangleq \{(x,\ [\mu_{A^-}(x), \mu_{A^+}(x)\ ]) \colon x \in \mathbb{R}\}$,

where $\mu_{A^-}(x), \mu_{A^+}(x) \in [0,\ 1]$ and $\mu_{A^-}(x) \leq\ \mu_{A^+}(x);$ for all $x \in \mathbb{R}$ is denoted as $\widetilde{A} = \left[\widetilde{A}^-, \widetilde{A}^+\right]$. Let

$$\mu_{\widetilde{A}^-}(x) = \begin{cases} \dfrac{\alpha\,(x-r)}{(s-r)}, & r < x \leq s, \\ \dfrac{\alpha\,(t-x)}{(t-s)}, & s \leq x < t, \\ 0, \text{otherwise}. \end{cases}$$

then, $\widetilde{A}^- = (r, s, t; \alpha)$.

Let

$$\mu_{\widetilde{A}^+}(x) = \begin{cases} \dfrac{\beta\,(x-a)}{(s-a)}, & a < x \leq s, \\ \dfrac{\beta\,(c-x)}{(c-s)}, & s \leq x < c, \\ 0, \text{otherwise}. \end{cases}$$

Then $\widetilde{A}^+ = (a, s, c;\ \beta)$.

It is clear that $0 < \alpha \leq \beta \leq 1$ and $a < r < s < t < c$.

Then the interval-valued fuzzy set is

$\widetilde{A} \triangleq \{(x, [\mu_{A^-}(x), \mu_{A^+}(x)\ ]) \colon x \in \mathbb{R}\}$ that is denoted as $\widetilde{A} = [(r, s, t; \alpha), (a, s, c; \beta)] = \left[\widetilde{A}^-, \widetilde{A}^+\right]$.

$\widetilde{A}$ is called a level (α, β) interval-valued fuzzy number.

Property 1. Let $F_{IVF}(\alpha,\beta) = \{[(r,s,t;\alpha),(a,s,c;\ \beta)]:\text{for all } a < r < s < t < c\},\quad 0 < \alpha \le \beta \le 1$ be the family of (α,β) interval-valued fuzzy numbers.

Let, $\widetilde{P} = [(r,s,t;\alpha),(a,s,c;\beta)] \in F_{IVF}(\alpha,\beta)$ and $\widetilde{Q} = [(r_1,s_1,t_1;\alpha),(a_1,s_1,c_1;\beta)] \in F_{IVF}(\alpha,\beta)$. Then

$$\widetilde{P}(+)\widetilde{Q} = [(r+r_1, s+s_1, t+t_1;\alpha),(a+a_1, s+s_1, c+c_1;\beta)],$$

$$k\tilde{P} = \begin{cases} [(kr,ks,kt;\alpha),(ka,ks,k\,c;\beta)], & k > 0, \\ [(kt,ks,kr;\alpha),(kc,ks,k\,a;\beta)], & k < 0, \\ [(0,0,0;\alpha),(0,0,0\,;\beta)], & k = 0. \end{cases}$$

Definition 4. Let $\widetilde{P} = [(r,s,t;\alpha),(a,s,c;\beta)] \in F_{IVF}(\alpha,\beta), 0 < \alpha \le \beta \le 1$. The signed distance of $\widetilde{P}$ from $\tilde{0}$ is given as:

$$d_0(\widetilde{P},\tilde{0}) = \frac{1}{8}\left[6s + r + t + 4a + 4c + 3(2s - a - c)\frac{\alpha}{\beta}\right].$$

Remark 1. $\widetilde{P} = [(a,a,a;\alpha),(a,a,a;\beta)]$, then $d_0(\widetilde{P},\tilde{0}) = 2a$.

Definition 5. Let $\widetilde{P},\widetilde{Q} \in F_{IVF}(\alpha,\beta)$, the ranking of level (α,β) interval-valued fuzzy numbers in $F_{IVF}(\alpha,\beta)$ using the distance function d_0 is defined as:

$$\tilde{Q} \prec \tilde{P} \Leftrightarrow d_0\,(\tilde{Q},\tilde{0}\,) < d_0(\widetilde{P},0)$$

$$\widetilde{Q} \approx \widetilde{P} \Leftrightarrow (\widetilde{Q},\tilde{0}\,) = d_0(\widetilde{P},0).$$

Property 2. Let $\widetilde{P} = [[(r,s,t;\alpha),(a,s,c;\beta)]]$ and $\widetilde{Q} = [(r_1,s_1,t_1;\alpha),(a_1,s_1,c_1;\beta)]$ be (α,β) interval-valued fuzzy numbers in $F_{IVF}(\alpha,\beta)$. Then

$$d_0(\tilde{P} \oplus \tilde{Q},\tilde{0}) = d_0(\tilde{P},\tilde{0}) + d_0(\tilde{Q},\tilde{0}),$$
$$d_0(k\,\widetilde{P},\tilde{0}) = k\,d_0(\widetilde{P},\tilde{0}), k > 0.$$

3. Problem Formulation and Solution Concepts

Consider the following (α,β) interval-valued fuzzy multi-objective transportation problem

$$(P_1) \qquad \min\ \tilde{Z}_k(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{C}_{ij}^{k}\ x_{ij}, k = 1,2,\dots,K,$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = \tilde{a}_i, i = 1,2,\dots,m,$$
$$\sum_{i=1}^{m} x_{ij} = \tilde{b}_j, j = 1,2,\dots,n,$$
$$x_{ij} \ge 0, i = 1,2,\dots,m; j = 1,2,\dots,n.$$

where, $\tilde{Z}_k = \{\tilde{Z}_1, \tilde{Z}_2, \dots, \tilde{Z}_K\}$ is (α, β) interval-valued vector of K objective functions, and $\tilde{C}_{ij}^k$, $\tilde{a}_I$, $\tilde{b}_j \in F_{IVF}(\alpha, \beta)$. Without loss of generality, it is assumed that $\tilde{a}_i > 0, \tilde{b}_j > 0$, $\tilde{C}_{ij}^k \geq 0$ for all (i, j) and $\sum_{j=1}^n \tilde{b}_j = \sum_{i=1}^n \tilde{a}_i$.

Definition 6. A point x satisfies the constraints in the $(P1)$ problem is said to be a feasible (α, β) interval-valued fuzzy point.

Definition 7. An (α, β) interval-valued fuzzy feasible point x° is called (α, β) interval-valued fuzzy efficient solution to $(P1)$ if and only if there does not exist another x such that $\tilde{Z}_k(x, \tilde{C}_{ij}^k) \leq \tilde{Z}_k(x^\circ, \tilde{C}_{ij}^k)$ and $\tilde{Z}_k(x, \tilde{C}_{ij}^k) \neq \tilde{Z}_k(x^\circ, \tilde{C}_{ij}^k)$.

According to the signed distance function in ***Definition 4***, the problem (P_1) is converted into the following crisp (P_2) problem as:

$$(P_2) \qquad \min \ Z_k(x) = \sum_{i=1}^m \sum_{j=1}^n C_{ij}^k \ x_{ij}, k = 1, 2, \dots, K$$

Subject to

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, \dots, m,$$
$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, \dots, n,$$
$$x_{ij} \geq 0, i = 1, 2, \dots, m; j = 1, 2, \dots, n.$$

4. Solution Procedure

A solution procedure based the method introduced by [8] to provide the ideal and the set of all (α, β) interval-valued fuzzy efficient solutions for the (P_1) problem is introduced as in the following steps:

Step 1. Formulate the problem (P_1).

Step 2. Convert the problem (P_1) into the corresponding crisp (P_2) problem using the signed distance function.

Step 3. Construct K single objective TP from the problem (P_2).

Step 4. Obtain the primal transportation for the problem $(P_u), u = 1, 2, \dots, K$ using existing method. Let the optimal transportation solution be $Y_u^*, u = 1, 2, \dots, K$ with the minimum cost Z_u^*, which is an ideal solution to the problem (P_2).

Step 5. Use the optimal solution of the problem (P_v) obtained in Step 4 in the problem

$P_1, P_2, \dots, P_{v-1}, P_{v+1}, \dots, P_K$.

Step 6. Repeat the Step 5 for all the problem (P_u) which provides all the efficient solution for the problem (P_2).

5. Numerical Example

Consider an (α,β) interval-valued fuzzy multi-objective transportation problem with the following characteristics:

Supplies:

$$\tilde{a}_1 = [(6,7,12;.6)(5,7,15;.9)], \quad \tilde{a}_2 = [(17,20,21;.6),(11,20,22;.9)],$$

$$\tilde{a}_3 = [(15,16,21;.6),(14,16,24;.9)] .$$

Demands:

$$\tilde{b}_1 = [(10,11,12;.6),(9,11,13;.9)], \quad \tilde{b}_2 = [(1.5,2,4.5,.6),(1,2,10;.9)],$$

$$\tilde{b}_3 = [(13,14,15;.6),(11,14,17;.9)], \tilde{b}_4 = [(14,15,20;.6),(13,15,23;.9)].$$

Penalties:

$$\tilde{C}^1 = \begin{bmatrix} \tilde{c}^1_{11} & \tilde{c}^1_{12} & \tilde{c}^1_{13} & \tilde{c}^1_{14} \\ \tilde{c}^1_{21} & \tilde{c}^1_{22} & \tilde{c}^1_{23} & \tilde{c}^1_{24} \\ \tilde{c}^1_{31} & \tilde{c}^1_{32} & \tilde{c}^1_{33} & \tilde{c}^1_{34} \end{bmatrix} \text{ and } \tilde{C}^2 = \begin{bmatrix} \tilde{c}^2_{11} & \tilde{c}^2_{12} & \tilde{c}^2_{13} & \tilde{c}^2_{14} \\ \tilde{c}^2_{21} & \tilde{c}^2_{22} & \tilde{c}^2_{23} & \tilde{c}^2_{24} \\ \tilde{c}^2_{31} & \tilde{c}^2_{32} & \tilde{c}^2_{33} & \tilde{c}^2_{34} \end{bmatrix},$$

where,

$$\tilde{c}^1_{11} = [(1,1,1;.6)\ (1,1,1;.9)], \quad \tilde{c}^1_{12} = [(0.5,1,1.5;.6),(0.25,1,1.75;.9)],$$

$$\tilde{c}^1_{13} = [(2,3,6;.6),(1,3,8;.9)], \quad \tilde{c}^1_{14} = [(3,3.5,4;.6),(2.5,3.5,4.5,;.9)],$$

$$\tilde{c}^1_{21} = [(0.2,0.25,1.3;.6),(0.125,.25,1.875;.9)], \tilde{c}^1_{22} = [(3,4,6;.6),(2.5,4,9;.9)]$$

$$, \quad \tilde{c}^1_{23} = [(1,1.5,2;.6),(0.5,1.5,2.5;.9)], \quad \tilde{c}^1_{24} = [(1,1.5,2;.6),(0.5,1.5,6.5;.9)],$$

$$\tilde{c}^1_{31} = [(2.5,3.5,6.5;.6),(1.5,3.5,8.5;.9)], \tilde{c}^1_{32} = [(4.25,4.5,4.75;.6),(3.5,4.5,5.5;.9)],$$

$$\tilde{c}^1_{33} = [(1,1.5,2;.6),(0.5,1.5,6.5;.9)], \quad \tilde{c}^1_{34} = [(2.5,3,3.5;.6),(1.5,3,4.5;.9)],$$

$$\tilde{c}^2_{11} = [(1,1.5,2;.6),(0.5,1.5,6.5;.9)], \quad \tilde{c}^2_{12} = [(1.5,2,2.5;.6),(1,2,3;.9)],$$

$$\tilde{c}^2_{13} = [(1.25,1.5,1.75;.6),(0.5,1.5,2.5;.9)], \tilde{c}^2_{14} = [(0.75,1,2.25;.6),(0.5,1,5;.9)],$$

$$\tilde{c}^2_{21} = [(1.5,2.5,3.5;.6),(1,2.5,4;.9)], \quad \tilde{c}^2_{22} = [(3,3.5,4;.6),(2.5,3.5,8.5;.9)],$$

$$\tilde{c}^2_{23} = [(3.5,5,6.5;.6),(0.5,5,6;.9)], \quad \tilde{c}^2_{24} = [(4.5,5,6.5;.6),(4,5,6;.9)],$$

$$\tilde{c}^2_{31} = [(2,2.5,4;.6),(1.5,2.5,7;.9)], \tilde{c}^2_{32} = [(.25,.5,2.7;.6),(0.125,.5,3.875;.9)],$$

$$\tilde{c}^2_{33} = [(1.5,2.5,3.5;.6),(1,2.5,4;.9)], \quad \tilde{c}^2_{34} = [(.25,.5,.75;.6),(0.125,.5,0.875;.9)].$$

According to the Definition 4, the above characteristics converted into the following crisp as

Supplies:

$$a_1 = 8, a_2 = 19, a_3 = 17 .$$

Demands:

$$b_1 = 11,\ \ b_2 = 3, b_3 = 14, b_4 = \ 16\ .$$

Penalties:

$$C^1 = \begin{bmatrix} 1 & 2 & 7 & 7 \\ 1 & 9 & 3 & 4 \\ 8 & 9 & 4 & 6 \end{bmatrix} \text{ and } C^2 = \begin{bmatrix} 4 & 4 & 3 & 4 \\ 5 & 8 & 9 & 10 \\ 6 & 2 & 5 & 1 \end{bmatrix}.$$

At the Step 3, the solution of each single objective is

$$X^1 = (5, 3, 0, 0, 6, 0\ ,0\ 13, 0,0\ 14, 3), Z_1(X^1) = 143,$$

$$X^2 = (0, 0, 8, 0, 11, 2\ ,6\ 0, 0,1\ 0, 16), Z_2(X^2) = 167.$$

At the Step 5, using the optimal transportation of Z_1in the Z_2, the efficient solution is $(Z_1, Z_2) = (143, 265)$ and hence the fuzzy efficient solution become

$$\tilde{Z}_1 = [(42.2, 59, 81.8; .6), (24.5, 59, 205.25; .9)],$$

$$\tilde{Z}_2 = [(113.5, 130, 174.25; .6), (77.875, 130, 202.125; .9)].$$

At the Step 6, using the optimal transportation of Z_2in the Z_1, the efficient solution is $(Z_1, Z_2) = (167, 208)$ and hence the fuzzy efficient solution become

$$\tilde{Z}_1 = [(74.25, 96.25, 147.05; .6), (47.875, 96.25, 195.125; .9)],$$

$$\tilde{Z}_2 = [(57.75, 85, 114.2; .6), (25.125,85, 134.875; .9)].$$

Therefore, the ideal fuzzy solution of the problem $(\tilde{Z}_1^{\ *}, \tilde{Z}_2^{\ *})$ is equal to

$$([42.2, 59, 81.8; .6, 24.5, 59, 205.25; .9], [57.75, 85, 114.2; .6, 25.125, 85, 134.875; .9]).$$

Also, the set of all fuzzy efficient solutions $(\tilde{Z}_1^{\ \circ}, \tilde{Z}_2^{\ \circ})$ are

$$([42.2, 59, 81.8; .6, 24.5, 59, 205.25; .9], [74.25, 96.25, 147.05; .6,47.875, 96.25, 195.125; .9]),$$

$$([57.75, 85, 114.2; .6, 25.125, 85, 134.875; .9], [113.5, 130, 174.25; .6, 77.875, 130, 202.125; .9]).$$

6. Conclusion

In this paper, a multi-objective transportation problem with(α, β) interval-valued fuzzy numbers is studied. The problem has converted into the corresponding crisp multi-objective transportation problem using the signed distance ranking of the (α, β) interval-valued fuzzy numbers. The solution method introduced by Jayalakshmi and Sujatha [8] was applied to obtain ideal and the set of all (α, β) fuzzy efficient solutions for the problem. The advantage of this method is more flexible than the standard multi-objective transportation problem, where it allows the decision maker to choose the (α, β) levels of fuzzy numbers he is willing.

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