



## Influence of Wall Properties on the Peristaltic Flow of a Jeffrey Fluid in a Uniform Porous Channel under Heat Transfer

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### ABSTRACT

Incompressible Jeffrey fluid under peristalsis is considered into permeable conduit. Magnetic effect and slip effect are studied for this channel in the existence of wall slip and heat transfer. Time average velocity, heat transfer coefficient and temperature are obtained analytically underneath the presumption of large wavelength approximation and also small Reynolds number. Effects of magnetic number, slip parameter, elasticity parameters and Brinkman number on coefficient of heat transfer and temperature field are graphically discussed. It is observed that in the case of temperature distribution the flow intensity enhances with rise in the Darcy number, while it reduces with enhancement in the Brinkman number and slip parameter.

**Keywords:** Peristaltic flow, darcy number, jeffrey fluid, compliant walls, slip parameter, heat transfer.

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## 1. Introduction

Peristalsis is a transport process for passing of fluids inside a conduit by means of surface deformations moving on the conduit. For example chyme movement within the region of the gastrointestinal tract, urine flow from kidney to bladder through ureter and blood flow through tiny blood vessels. Commercial peristaltic pumps like Roller and Finger pumps make use of peristalsis, where it is required to split the fluid and move forward from the pumping device, generally used to move forward a bolus of fluid. The outcomes of Shapiro et al. [1] under peristalsis of Newtonian fluids at large wavelength and small Reynolds numbers are related to arbitrary significance of the amplitude ratio.

It is found that various physiological fluids are non-Newtonian and their rheological properties (extensional viscoelasticities and shear) play a significant role in the peristaltic motion

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characteristics. The majority of the physiological fluids as well as blood act as a non-Newtonian fluids. Thus, non-Newtonian fluids study under peristalsis can be useful in understanding the biological structure.

Numerous scientific researchers examined peristaltic motion of non-Newtonian fluids. Popel [2] investigated a continuum model of blood flow. Some researchers later extended their research on this area by taking into account the different types of fluid forms; slip condition, porous medium, compliant walls along with presumptions of small Reynolds number, small wave number, long wavelength and small amplitude ratio. Effect of slip appears in molten polymers and concentrated polymer solutions and non-Newtonian fluids. EL-Shehaway et al. [3] observed the peristaltic transport of Maxwell fluid accompanied by slip effects. Ellahi [4] reports that effect of slip is essential in non-Newtonian system in a channel. A porous medium contains a number of tiny holes spread around the substance. For understanding complexity of numerous diseases like bladder and bacterial stones, cystitis and bacterial affection of kidneys, reflux conditions of peristaltic motion has been studied through porous medium. Beaver and Joseph [5] first initiated the boundary condition used in porous wall justifying statistically and verify using theoretical calculation. Saffman [6] further improved this and recommended an appropriate related condition for tiny permeability. Several researchers [7-11] studied wall effects under peristalsis within a channel.

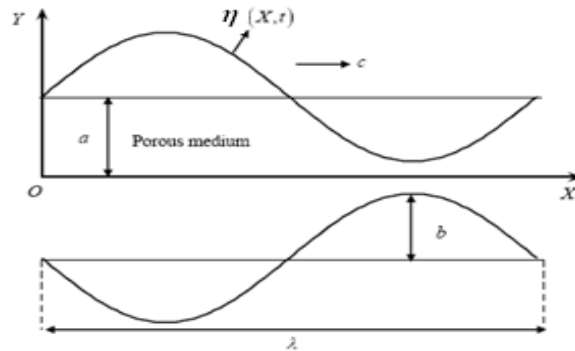
In oxygenation processes peristalsis accompanied by heat transfer is useful. The further applications of this concept are transport of corrosive materials and sanitary fluid transport. Heat transfer applications can also be seen in medical field as in hyper and hypothermic ranges. Many more applications are skin burns, heating of body tissues and organs, heat and mass transfer in the human respiratory system, electro surgery, preservation of tissues by freezing, application of cryosurgery, heat transfer in teeth, temperature measurement and thermograph. Several researchers have worked on peristaltic transport taking into account the heat transfer and slip effects [12-16]. Kothandapani and Srinivas [17] gave importance on magnetic effect on peristaltic motion of a couple stress fluid under heat transfer and studied the wall property effect. Eldabe et al. [18] examined wall effect properties of couple stress fluid in a permeable conduit influenced by peristaltic flow under mass and heat transfer property. Lakshminarayana et al. [19] analyzed the effect of slip, wall properties on the peristaltic motion of a conducting Bingham fluid with heat transfer. Sankad and Nagathan [20] examined the properties of magnetohydrodynamic (MHD) couple stress fluid within peristaltic transport in porous medium channel along with the compliant wall properties, slip effects and heat transfer.

Jeffrey model is a simpler model compared to other non-Newtonian models. In this model time derivatives are used in place of convective derivatives. Study of peristaltic flow of a Jeffrey fluid is quite useful in physiology and industry because of its large number of applications and in mathematics due to its complicated geometries and solutions of nonlinear equations. Saravana et al. [21] examined the effects on wall, slip and heat transfer distribution considering a non-

uniform, permeable peristaltic conduit of a magnetohydrodynamic Jeffrey fluid underneath the presumptions of small Reynolds number and large wavelength. Considering the movement of food bolus moving inside the esophagus in a channel with Jeffrey fluid, Arun kumar et al. [22] observed the heat transfer and wall effects. Dheia and Al-Khafajy [23] are discussed properties of wall effect and heat transfer distribution under the peristalsis in a porous medium channel containing Jeffrey fluid. They considered Jeffrey fluid as a food bolus and peristaltic wave through porous medium as the wall surface of esophagus. Vajravelu et al. [24] studied the result of heat transfer on the nonlinear peristaltic flow of a Jeffrey fluid through a finite vertical porous channel. Effects of hall and ion slip on MHD peristaltic transport of Jeffrey fluid in a non-uniform rectangular duct is examined by Ellahi et al. [25].

To analyze the importance of magnetic effect on peristaltic motion of Jeffrey fluid inside a uniform porous medium channel is the main objective of this concept. Compliant wall slip effects and heat transfer are also taken into consideration. Due to oxygenation process and hemodialysis heat transfer is important.

**2. Mathematical formulation**



**Figure1.** Physical model.

A uniform permeable channel of thickness 2d is considered to be held fixed wherein the peristaltic motion of a non-Newtonian magnetohydrodynamic Jeffrey fluid is moving under slip conditions. The flow is unsteady and symmetric. The motion regarded as two dimensional incompressible, peristaltic motions in an elastic conduit with speed c along the conduit walls described in figure 1. Here dimensions of the uniform flow are set as wave amplitude and wave length λ. The axial coordinates are x and y are normal to it. We consider the uniform flow structure in the wave frame (x, y), where (u, v) are velocities in the direction of (x, y) respectively. Here η, the wall deformation is the tangential coordinate of the channel wall.

$$\eta(x,t) = y = d + a \sin 2\pi / \lambda (x - ct). \tag{1}$$

Here, t denotes the time.

The elastic wall is governed by the equation:

$$L(\eta) = p - p_0 \tag{2}$$

Due to the tension in the muscles, pressure is exerted on the outer surface of the wall and is denoted by  $P_0$ . Here we assume  $p_0=0$ , where  $p$  is the pressure.  $L$  in the operator form signifies the motion of an elastic wall having viscosity damping force and is given by:

$$L = -\tau \frac{\partial^2}{\partial x^2} + m \frac{\partial^2}{\partial t^2} + C \frac{\partial}{\partial t} \tag{3}$$

Here,  $\tau$ ,  $m$  and  $C$  represents elastic tension within the wall, mass of a substance applied to a given area and viscous damping force respectively.

The fundamental equations are:

The mass conservation equation (continuity equation) is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

The momentum conservation equations are:

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\mu}{1 + \lambda_1} \nabla^2 u - \mu \frac{u}{\mathcal{K}} - \sigma B_0^2 u - \frac{\partial p}{\partial x} \tag{5}$$

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \frac{\mu}{1 + \lambda_1} \nabla^2 v - \mu \frac{v}{\mathcal{K}} - \frac{\partial p}{\partial y} \tag{6}$$

Equation of motion for temperature is:

$$G \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{K}{\rho} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \vartheta \left\{ \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 + 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] \right\} \tag{7}$$

Due to symmetrical plane the normal velocity is zero. Experimentally it is proved in several physiological situations that flow is accompanied with very small Reynolds number. Hence infinite wavelength is assumed. The ratio of relaxation to retardation is  $\lambda_1$ , porous medium permeability is  $\mathcal{K}$ , fluid density is  $\rho$ , fluid viscosity coefficient is  $\mu$  and  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ ,  $\nabla^4 = \nabla^2 \nabla^2$ .

The corresponding boundary conditions in the wave frame are given by:

$$\frac{\partial u}{\partial y} = 0, \text{ at } y = 0 \text{ (the regularity condition),} \tag{8}$$

$$u = -d \frac{\sqrt{Da}}{\beta} \frac{\partial u}{\partial y}, \text{ at } y = \pm \eta(x, t) \text{ (the slip condition).} \tag{9}$$

Here,  $Da$  and  $\beta$  denote Darcy number and slip parameter respectively.

The dynamic boundary condition referred by Mittra and Prasad [26] is:

$$\frac{\partial}{\partial x} L(\eta) = -\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{\mu}{1 + \lambda_1} \nabla^2 u - \mu \frac{u}{\mathcal{K}} - \sigma B_0^2 u, \quad y = \eta(x, t), \tag{10}$$

Where, (11)

$$\frac{\partial}{\partial x} L(\eta) = \frac{\partial p}{\partial x} = -\tau \frac{\partial^3 \eta}{\partial x^3} + m \frac{\partial^3 \eta}{\partial x \partial t^2} + C \frac{\partial^2 \eta}{\partial x \partial t}.$$

In order to get dimensionless forms of Eqs. (4 -10), let us define the dimensionless quantities as follows:

$$\begin{aligned} x' &= \frac{x}{\lambda}, \quad y' = \frac{y}{d}, \quad u' = \frac{u}{c}, \quad v' = \frac{\lambda v}{cd}, \quad \psi' = \frac{\psi}{cd}, \quad \eta' = \frac{\eta}{d}, \quad t' = \frac{ct}{\lambda}, \quad \mathcal{P}' = \frac{d^2 \mathcal{P}}{\mu \lambda c}, \quad \mathcal{K}' = \frac{\mathcal{K}}{d^2}, \quad \theta \tag{10} \\ &= \frac{T - T_0}{T_1 - T_0}. \end{aligned}$$

Here,  $\theta$  is the non dimensional parameter.

Introducing dimensionless variables in (4-7),

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{13}$$

$$Re_e \delta \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \tag{14}$$

$$= \frac{1}{1 + \lambda_1} \left( \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{u}{\mathcal{K}} - M^2 u - \frac{\partial p}{\partial x},$$

$$Re_e \delta^3 \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = \frac{\delta^2}{1 + \lambda_1} \left( \delta^2 \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \delta^2 \frac{v}{\mathcal{K}} - \frac{\partial p}{\partial y}, \tag{15}$$

$$Re_e \delta \left( \frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) \tag{16}$$

$$= \frac{1}{Pr} \left( \delta^2 \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + Ec \left[ 2 \left\{ \delta^2 \left( \frac{\partial u}{\partial x} \right)^2 + \delta^2 \left( \frac{\partial v}{\partial y} \right)^2 \right\} + \left( \delta^2 \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right].$$

Equations (8 -10) becomes:

$$\frac{\partial u}{\partial y} = 0, \quad \text{at } y = 0, \tag{17}$$

$$u = -\frac{\sqrt{Da}}{\beta} \frac{\partial u}{\partial y}, \quad \text{at } y = \pm \eta(x, t) = \pm (1 + \epsilon \sin 2\pi(x - t)). \tag{18}$$

And,

$$\begin{aligned} -Re_e \delta \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \frac{1}{1 + \lambda_1} \left( \delta^2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{u}{Da} - M^2 u \tag{19} \\ = \mathbb{E}_1 \frac{\partial^3 \eta}{\partial x^3} + \mathbb{E}_2 \frac{\partial^3 \eta}{\partial x \partial t^2} + \mathbb{E}_3 \frac{\partial^2 \eta}{\partial x \partial t} \quad \text{at } y = \pm \eta(x, t). \end{aligned}$$

Here,  $\delta = \left(\frac{d}{\lambda}\right)$  is wall slope parameter and  $\varepsilon = \left(\frac{a}{d}\right)$  is the amplitude ratio are the geometric parameters,  $Re = \left(\frac{\rho cd}{\mu}\right)$  is the Reynolds number,  $Da = \frac{\mathcal{K}}{d^2}$  is Darcy number,  $E_c = \frac{c^2}{G(T_1 - T_0)}$  is the Eckert number,  $M = \sqrt{\frac{\sigma}{\mu}} B_0 d$  is magnetic effect parameter,  $P_r = \frac{\rho \vartheta G}{K}$  is the Prandtl number. the stretchy parameters are defined as  $\mathbb{E}_1 = \left(\frac{-\tau d^3}{c\mu\lambda^3}\right)$ ,  $\mathbb{E}_2 = \left(\frac{mcd^3}{\mu\lambda^3}\right)$ ,  $\mathbb{E}_3 = \left(\frac{cd^3}{\mu\lambda^2}\right)$ . The parameter  $\mathbb{E}_1$  represents the inflexibility (rigidity),  $\mathbb{E}_2$  stiffness (mass characterizing parameter) and  $\mathbb{E}_3$  viscous damping force (damping nature of the membrane).

### 3. Method of solution

Usually the solution of the governing equations is not possible in general, hence we assume long wavelength approximation to solve Eqs. (13-19).

Equations (13-16) yield the compatibility equation as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{20}$$

$$0 = -\frac{\partial \mathcal{P}}{\partial x} + \frac{1}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{Da}\right)u, \tag{21}$$

$$0 = -\frac{\partial \mathcal{P}}{\partial y}, \tag{22}$$

$$E_c \left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} = 0. \tag{23}$$

The dimensionless boundary conditions (17-19) become:

$$\frac{\partial u}{\partial y} = 0, \text{ at } y = 0, \tag{24}$$

$$u = -\frac{\sqrt{Da}}{\beta} \frac{\partial u}{\partial y}, \text{ at } y = \pm\eta(x, t) = \pm(1 + \varepsilon \sin 2\pi(x - t)), \tag{25}$$

$$\frac{1}{1 + \lambda_1} \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{Da}\right)u = \mathbb{E}_1 \frac{\partial^3 \eta}{\partial x^3} + \mathbb{E}_2 \frac{\partial^3 \eta}{\partial x \partial t^2} + \mathbb{E}_3 \frac{\partial^2 \eta}{\partial x \partial t}, \text{ at } y = \pm\eta(x, t). \tag{26}$$

Further,

$$\theta = 0 \text{ on } y = -\eta(x, t), \tag{27}$$

$$\theta = 1 \text{ on } y = \eta(x, t). \tag{28}$$

Solving Eqs. (20) and (22) with boundary conditions (24-26), we get:

$$u = \frac{E}{N^2} \left[ -1 - \frac{\cosh(Ny)}{T_1} \right]. \tag{29}$$

The stream function  $u = \frac{\partial \psi}{\partial y}$  can be found by Eq. (29) and using the condition  $\psi = 0$  at  $y = 0$  is:

$$\psi = \frac{E}{N^2} \left[ -y - \frac{\sinh Ny}{NT_1} \right]. \tag{30}$$

Solving Eq. (23), with boundary conditions (27) and (28), we get:

$$\theta = B \left( \frac{\cosh(2Ny)}{4N^2} - \frac{y^2}{2} \right) + \frac{y}{2\eta} + \frac{1}{2} - \frac{B}{2} \left( \frac{\cosh(2N\eta)}{2N^2} - \eta^2 \right). \tag{31}$$

Here,

$$E = -\varepsilon[(2\pi)^3 \cos 2\pi(x - t) (\mathbb{E}_1 + \mathbb{E}_2) - \mathbb{E}_3(2\pi)^2 \sin 2\pi(x - t)], \quad D = -\frac{\sqrt{Da}}{\beta},$$

$$T_1 = DNSinhN\eta - SinhN\eta, \quad Br = Ecpr, \quad N = \sqrt{\left(M^2 + \frac{1}{Da}\right)(1 + \lambda_1)}, \quad B = -\frac{BrE^2}{2(1 + \lambda_1)T_1^2}.$$

Thus, it is obvious that  $Br$  signifies the relative importance between dissipation effects and the fluid conduction.

The time average velocity  $\bar{u}$  is:

$$\bar{u} = \int_0^1 u \, dt. \tag{32}$$

Heat transfer coefficient ( $H$ ) at the membrane is obtained as:

$$H = [\eta_x] [\theta_y(\eta)], \tag{33}$$

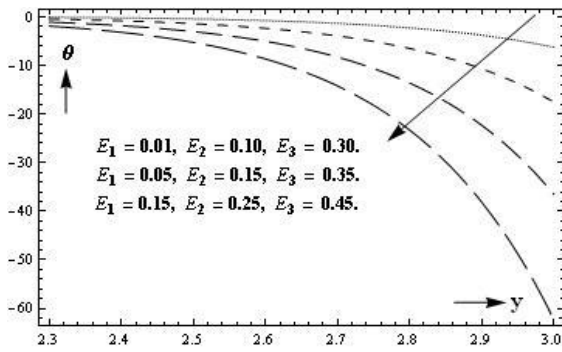
$$H = [2\pi\varepsilon \cos 2\pi(x - t)] \left[ \left\{ B \left( \frac{\sinh 2N\eta}{2N} - \eta \right) \right\} + \frac{1}{2\eta} \right]. \tag{34}$$

#### 4. Discussion of Results

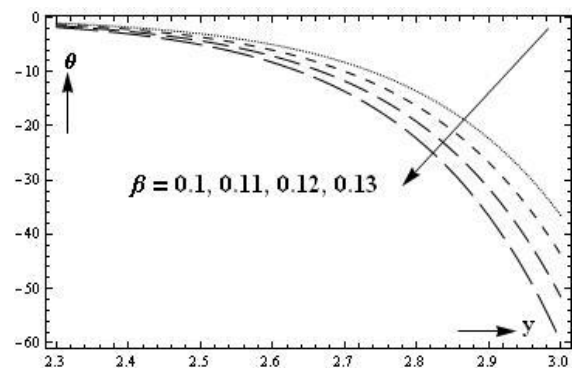
In this section, the uniform flow of a symmetric, incompressible, Jeffrey fluid in a porous medium channel having slip on its surface is considered. Solution of the problem is obtained using MATHEMATICA. This investigation has been carried out to study wall effect and slip factors which influence the flow in porous uniform channel. The non-Newtonian fluid of Jeffrey model has been considered in the analysis. Heat transfer coefficient ( $H$ ), temperature distribution ( $\theta$ ), velocity profile ( $u$ ) and stream function ( $\psi$ ) have been analytically derived using momentum and energy equations. Results are analyzed through graphical plots to study the behavior of various parameters of temperature distribution, heat transfer coefficient, velocity profile and stream function. The values considered for the parameters are:

$$\varepsilon = 0.2; M = 1, Br = 3; \mathbb{E}_1 = 0.1; \mathbb{E}_2 = 0.2; \mathbb{E}_3 = 0.4; Da = 0.5; \beta = 0.1; \lambda_1 = 1; x = 0.5.$$

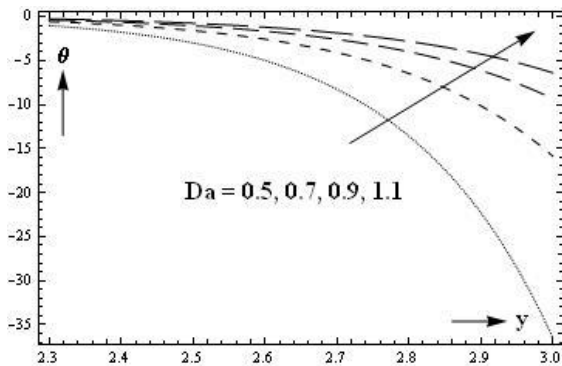
The analytical solutions are analyzed for various values of magnetic number ( $M$ ), Darcy number ( $Da$ ), Brinkman number ( $Br$ ), slip parameter ( $\beta$ ), Jeffrey parameter ( $\lambda_1$ ), the elastic parameters ( $E_1, E_2, E_3$ ) to study their behavior. Figure 2 illustrate the temperature distribution for changing  $E_1, E_2$  and  $E_3$  with constant values of other parameters. It can be observed from the graph that temperature distribution reduces with the rise of elastic parameters. Figure 3 depicts the effects of temperature distribution on the peristaltic flow used for different slip parameter values. It demonstrates that temperature distribution diminishes with increasing slip parameter. Temperature distribution enhances with increasing Darcy number as in Figure 4. Figure 5 depicts that the effect of Jaffrey parameter on temperature distribution. One can see that the temperature distribution decreases with rise in the Jaffrey parameter. The consequence of non-identical values of magnetic parameter upon temperature distribution is drawn in Figure 6. Interestingly it can be noted that the impact of enhancing magnetic parameter leads to a drop in the temperature in the conduit. A related result can be showed in Figure 7, if magnetic parameter replaced by Brinkmen number.



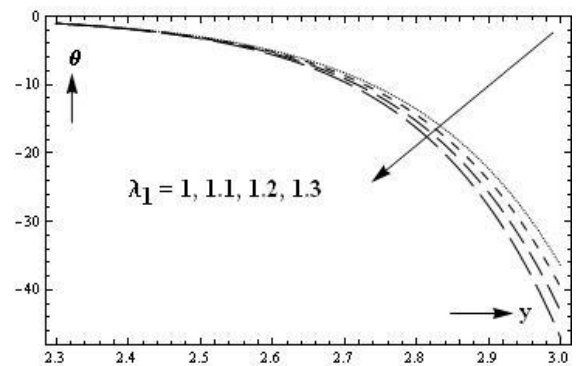
**Figure 2.** Variation of dimensionless temperature distribution for various values of elastic parameters.



**Figure 3.** Variation of dimensionless temperature distribution for various values of slip parameter.

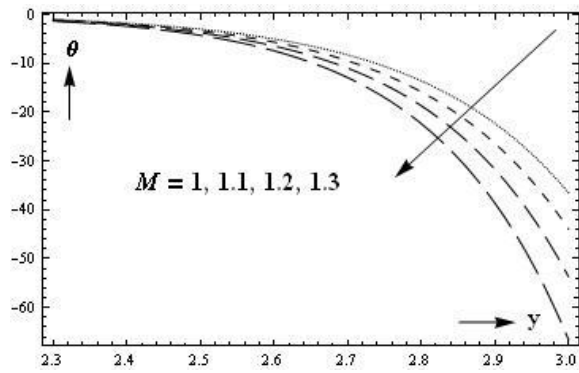


**Figure 4.** Variation of dimensionless temperature distribution for various values of Darcy number.

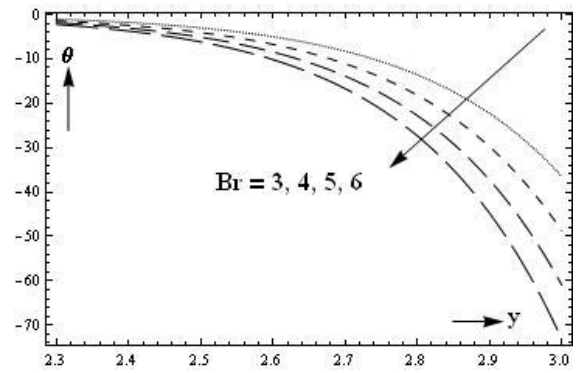


**Figure 5.** Variation of dimensionless temperature distribution for various values of Jeffrey parameter.



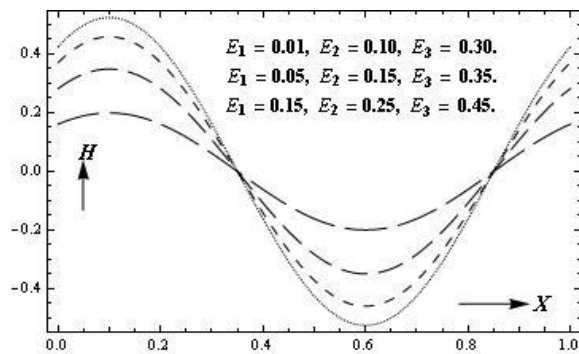


**Figure 6.** Variation of dimensionless temperature distribution for various values of magnetic number .

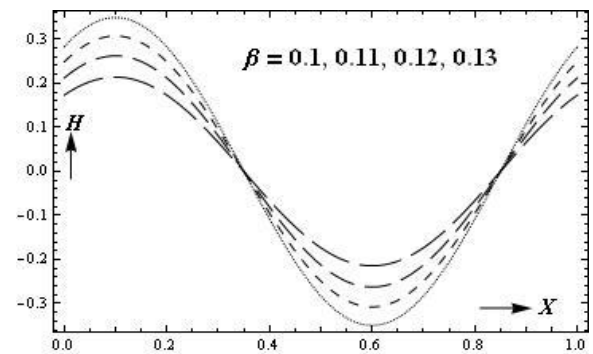


**Figure 7.** Variation of dimensionless temperature distribution for various values of Brinkman number.

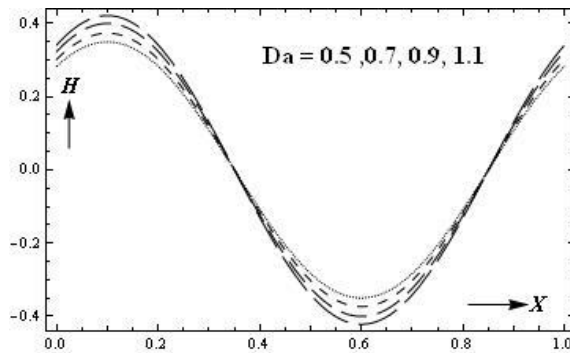
The effect of  $M, Da, \beta, Br, \lambda_1$  and elastic parameters ( $E_1, E_2, E_3$ ) on heat transform coefficient ( $H$ ) in the symmetric porous channel are plotted in Figures (8-13) using Eq. (33). The significant characteristic of uniform peristaltic motion of Jeffrey fluid through the channel having porosity under heat transfer and magnetic field effects is explored through these figures. It is noticed that, due to peristalsis in conduit, the heat transfer coefficient is having a periodically oscillatory behavior. The heat transfer coefficient drops as the elastic parameters, slip parameter and Brinkman number increase. One can examine that the heat transfer coefficient enhances with enhancing Darcy number, magnetic number and Jeffrey parameter.



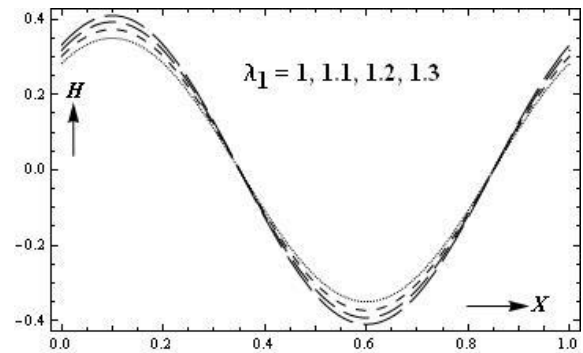
**Figure 8.** Heat transfer coefficient for various values of elastic parameters.



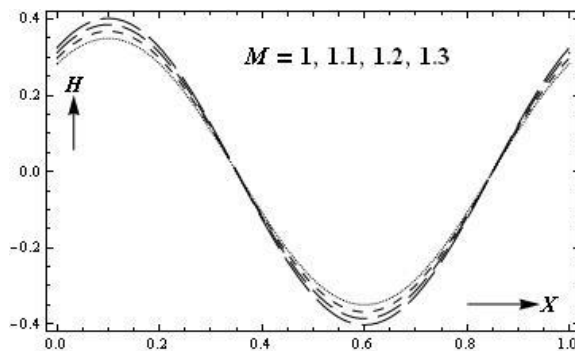
**Figure 9.** Heat transfer coefficient for various values of slip parameter.



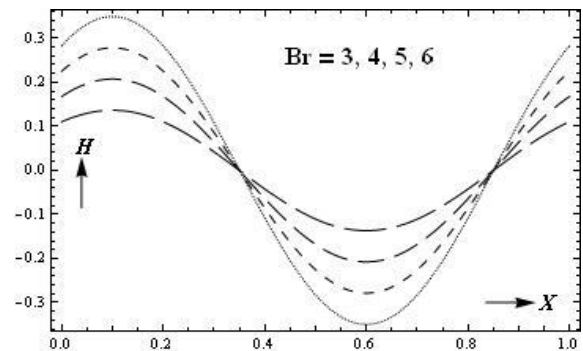
**Figure 10.** Heat transfer coefficient for various values of Darcy number.



**Figure 11.** Heat transfer coefficient for various values of Jeffrey parameter.



**Figure 12.** Heat transfer coefficient for various values of magnetic number.



**Figure 13.** Heat transfer coefficient for various values of Brinkman number.

The behavior of velocity profile for different values of  $M$ ,  $Da$ ,  $\beta$ ,  $\lambda_1$  and elastic parameters ( $\mathbb{E}_1$ ,  $\mathbb{E}_2$ ,  $\mathbb{E}_3$ ) are depicted through graphs as shown in Figures (14-18). The velocity profile depicts the variation of dimensionless velocity ( $u$ ) with dimensionless distance ( $y$ ) along the uniform channel.

Figure 14 depicts the variation of velocity for different values of elastic parameters. It is observed that velocity decreases with an increase in different values of elastic parameters. The velocity profile has been shown in Figure 15 for different slip parameters to study the flow behavior. It is examined that the velocity profile enhances with rise in slip parameter. The velocity profile decreases for increasing values of Darcy number and is demonstrated in Figure 16. It is seen that velocity is an increasing function of Jeffrey fluid parameter as shown in Figure 17. It is observed from Figure 18 that the behavior of velocity in case of magnetic number is quite opposite as compared with Darcy number.

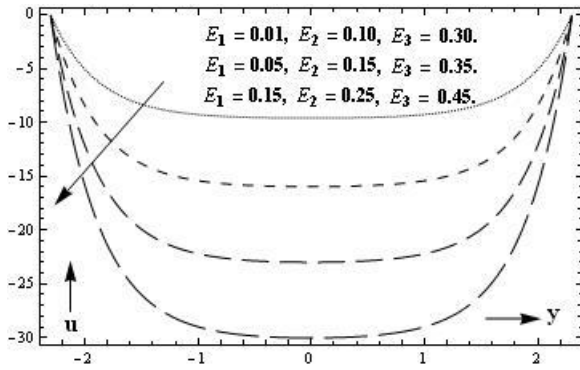


Figure 14. Velocity profile for various values of elastic parameters.

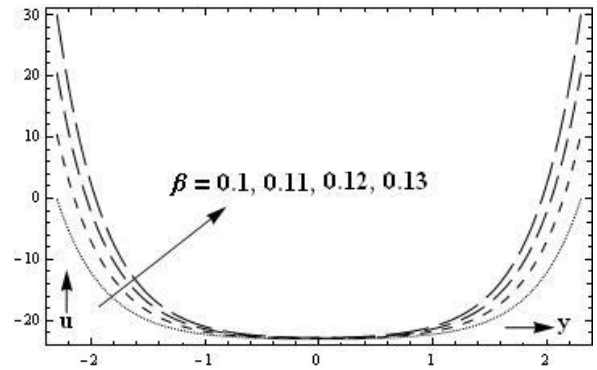


Figure 15. Velocity profile for various values of slip parameters.

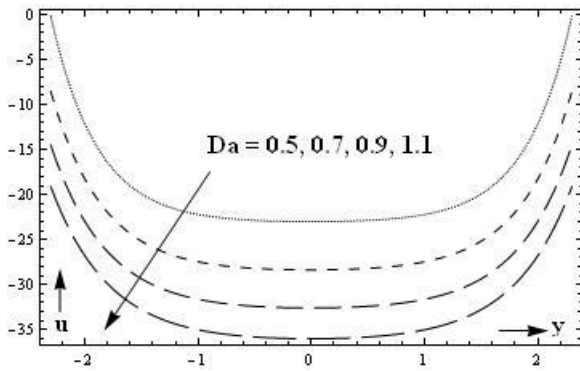


Figure 16. Velocity profile for various values of Darcy number.

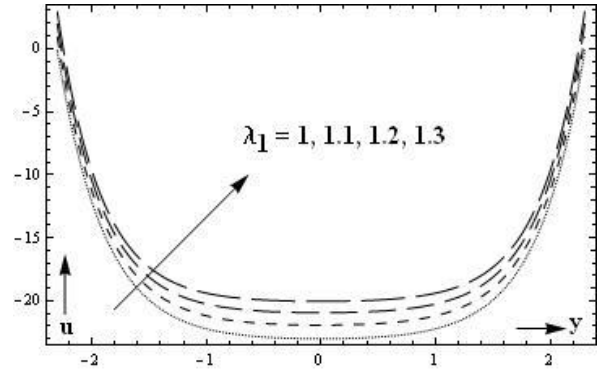


Figure 17. Velocity profile for various values of Jeffrey parameters.

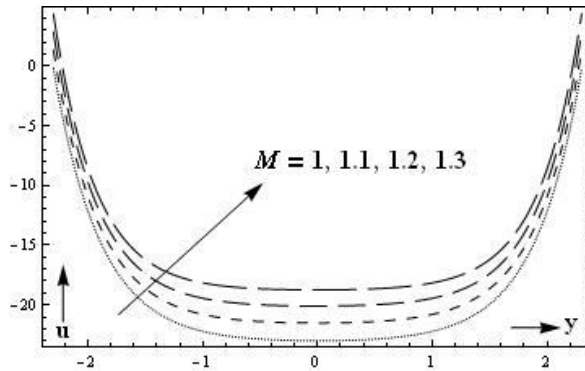
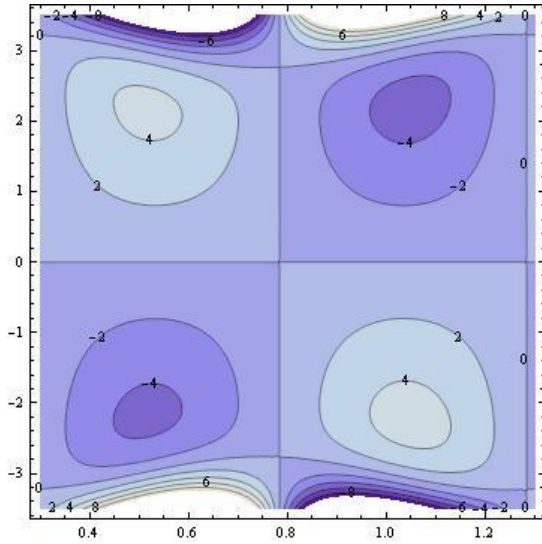


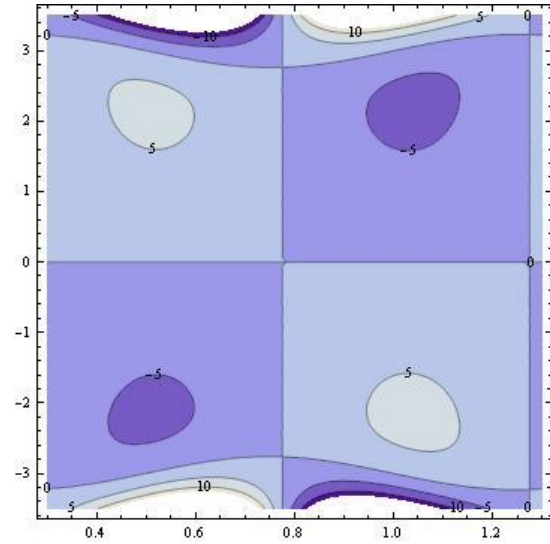
Figure 18. Velocity profile for various values of magnetic number.

The stream lines in general in the wave frame have a contour similar to the walls as the walls are immobile. Figures (19-24) show the stream lines for different values of  $M, Da, \beta, \lambda_1$  and elastic parameters ( $E_1, E_2, E_3$ ). From Figures 19(a) and 19(b), and Figures 20(a) and 20(b) it is observed that the size and number of the trapping bolus increases with an increase in  $E_1$  and  $E_3$ . Figures (21-23) show the stream lines for different values of  $\beta, M,$  and  $\lambda_1$ . It is observed from

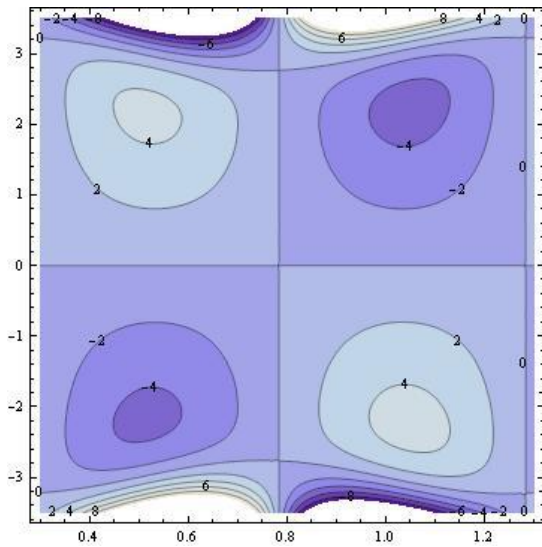
Figures 21(a) and 21(b) to Figures 23(a) and 23(b) that the size of the trapping bolus decreases with an increase in  $\beta$ ,  $M$ , and  $\lambda_1$  respectively. Stream lines for different values of  $Da$  as shown in Figures 24(a) and 24(b). It is depicted that the number of the trapping bolus rises with an increase in  $Da$ .



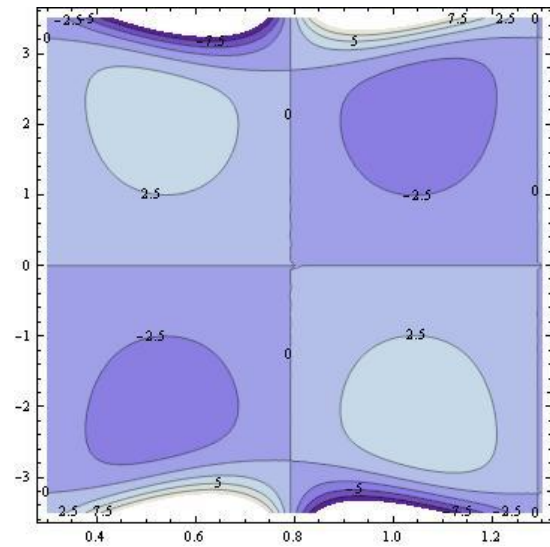
**Figure 19(a).** Stream lines for  $E_1 = 0.1$ .



**Figure 19(b).** Stream lines for  $E_1 = 0.2$ .



**Figure 20(a).** Stream lines for  $E_3 = 0.4$ .



**Figure 20(b).** Stream lines for  $E_3 = 0.5$ .

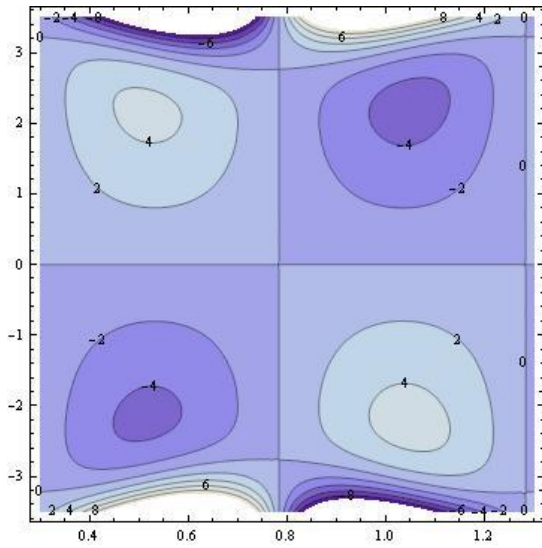


Figure 21(a). Stream lines for  $\beta = 0.1$ .

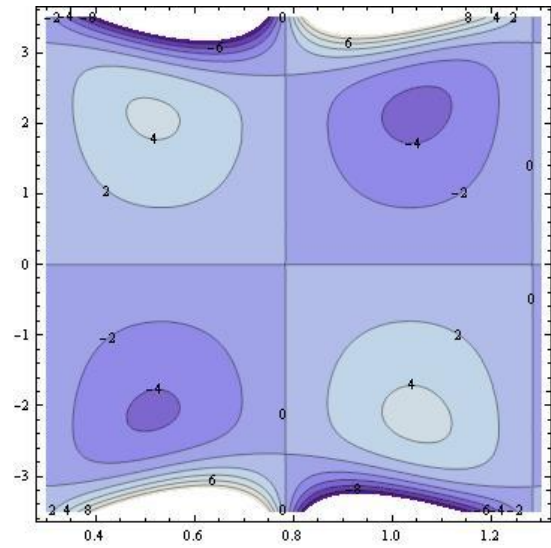


Figure 21(b). Stream lines for  $\beta = 0.12$ .

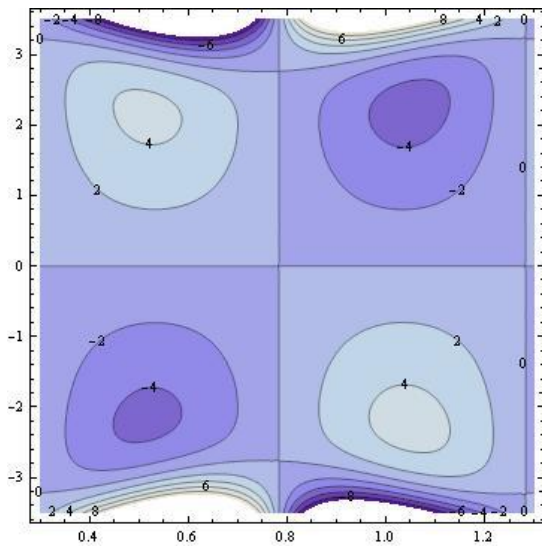


Figure 22(a). Stream lines for  $M = 1$ .

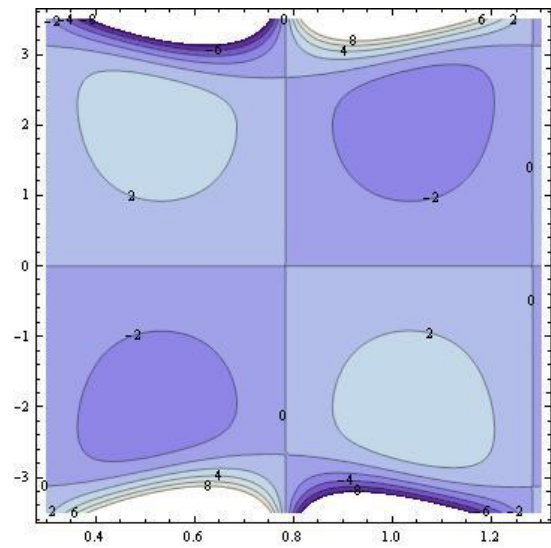


Figure 22(b): Stream lines for  $M = 1.2$ .

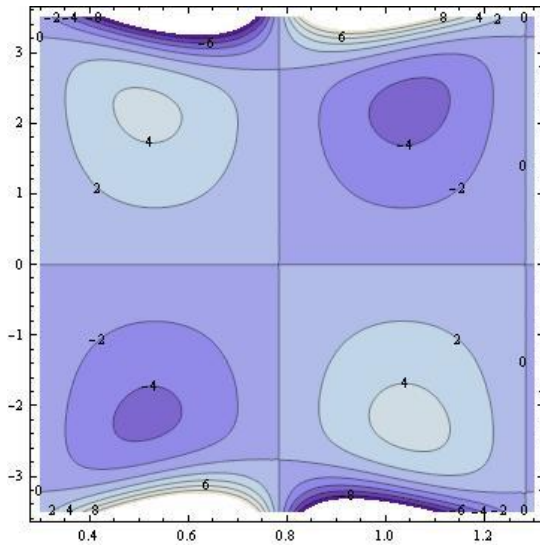


Figure 23(a). Stream lines for  $\lambda_1 = 1$ .

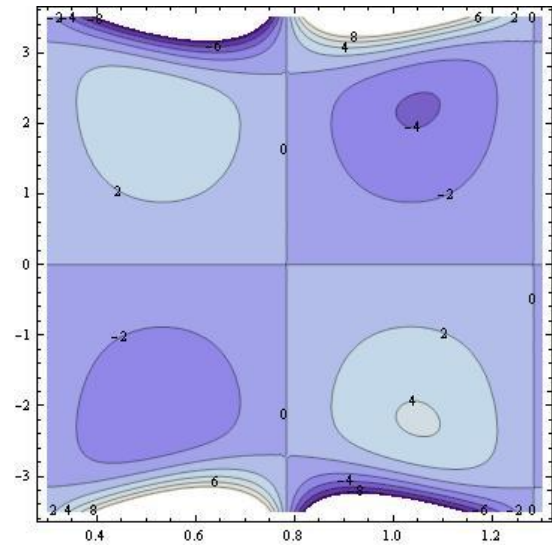


Figure 23(b). Stream lines for  $\lambda_1 = 1.2$ .

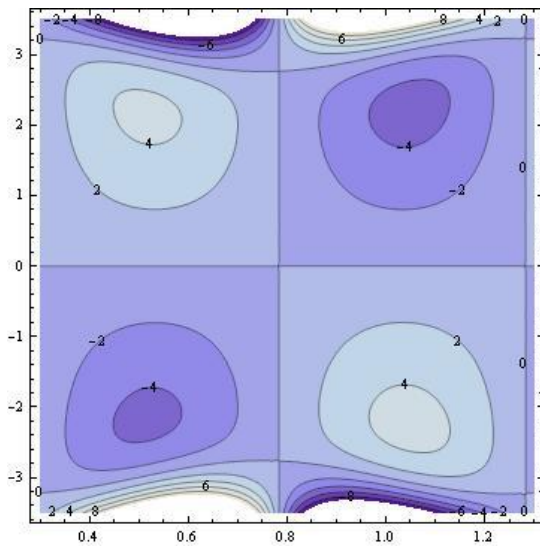


Figure 24(a). Stream lines for  $Da = 0.5$ .

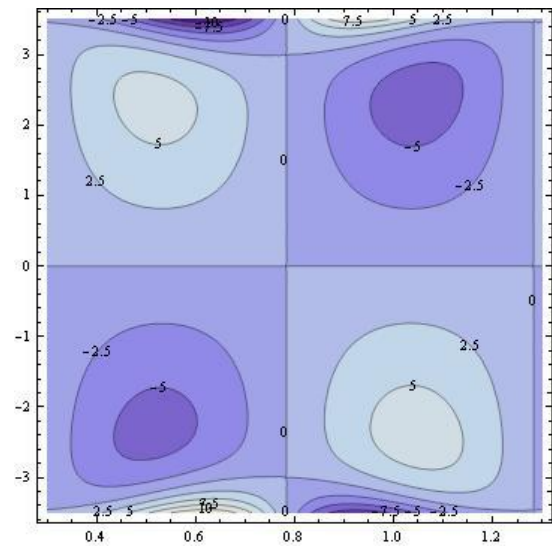


Figure 24(b). Stream lines for  $Da = 0.7$ .

## 5. Conclusion

The results of various parameters of a Jeffrey fluid model in a symmetric conduit on the heat transfer are observed graphically for different applicable parameters. The analytical solutions are obtained for velocity, stream function, temperature and heat transfer coefficient. Wall effects are analyzed since the parameters like elastic tension and damping have vast significance in realistic situations. The major findings can be reviewed as:

- I. The temperature distribution enhances with gain in Darcy number and Jeffrey parameter. The result of Darcy number is in agreement with that of Saravana et al. [21] and the Jeffrey parameter result is in agreement with Dheia et al. [23].
- II. The temperature reduces as the elastic parameters, Brinkman number, slip parameter, and magnetic number increase.
- III. The heat transfer coefficient enhances absolutely with increasing Jeffrey parameter and magnetic number but reduces with elastic parameters and Brinkman number.
- IV. The heat transfer coefficient enhances absolutely with increasing Darcy number but reduces with slip parameter. This result is in agreement with that of Saravana et al. [21].
- V. The velocity enhances with gain in Jeffrey and slip parameters and diminish with elastic parameters and Darcy number.
- VI. The size of trapped bolus is smaller in Jeffrey fluid when compared with that of Newtonian fluid ( $\lambda_1 = 0$ ).
- VII. As the Jeffrey parameter  $\lambda_1 \rightarrow 0$ , the results deduced are found to be in agreement with Srinivas et al. [12].

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