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Deriving Priorities the Alternatives in an Analytic Hierarchy Process

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ABSTRACT

The data envelopment analysis (DEA) is a mathematical programming technique, which is used for evaluating relative efficiency of decision making units (DMUs). However, the DEA does not provide more information about the efficient DMUs. Recently, some researchers have been carried out in the background of using DEA models to generate local weights of alternatives from pairwise comparison matrices used in the analytic hierarchy process (AHP). In this paper, an application of a common set of weights is used for determining priorities in the AHP. First, we determine DEA efficient alternatives as DMUs. Then, these alternatives are ranked according to the efficiency score weighted by the common set of weights in the AHP. This application is applied successfully and the result is valid and assured. A numerical example is utilized to illustrate the capability of this procedure.

1. Introduction

Since the introduction of the data envelopment analysis (DEA) model for relative efficiency analysis by Charnes et al. [1], the model has attracted a great amount of interest from both academic and industrial communities. It is particularly useful where no a priori information on the tradeoffs or relations among various performance measures is available. The decision making units (DMUs) usually use a set of resources, referred to as input indices, and transform them into a set of outcomes, referred to as output indices. Nowadays, the DEA becomes the important analysis tool and research method in management science, operational research, system engineering, decision analysis, and so on. The analytic hierarchy process (AHP) [2] has emerged as a useful decision making technique for solving and analyzing the complex problems. Indeed, the AHP converts a

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complex problem to several simple problems and solve them. The AHP technique is widely used in individual and group decision-making environments. In general models, defined using the AHP can be used for two purposes: (i) to maintain ranking of alternatives as part of a specific individual or group decision making activity or (ii) to model subjective performances of an individual or a group of decision makers.

The DEA divides DMUs into two categories; efficient DMUs and inefficient DMUs. Most of ranking methods are given for inefficient DMUs. However, efficient DMUs cannot be ranked. Some of these methods, which are proposed for ranking efficient DMUs, are mentioned here. Some samples of these approaches are: supper efficiency [3], cross efficiency [4], imposing restrictions on the weights and/or using common set of weights [5-12]. Cook et al. [13] developed prioritization models to rank only the efficient units in DEA. Mehrabian et al. [14] presented of the popular of these methods. These methods would have some weaknesses if data have certain structures. There are some methods based on norms. Jahanshahloo et al. [15] introduced an l1-norm approach that removes some deficiencies arising from AP and MAJ, but that cannot rank non-extreme DMUs. Liu and Peng [16] searched a common set of weights to create the best efficiency score of one group composed of efficient DMUs to rank them. Jahanshahloo et al. [17] proposed two ranking methods based on an ideal line and a special line respectively.

This paper contributes to the AHP-DEA approach by searching a common set of weights to create the best efficiency score of one group composed of efficient alternatives in AHP. The result is consisting a suitable ranking which can achieve decision makers' justifications and it can show that DEA models can be also useful in AHP. The remainder of this paper is organized as follows. Section 2 provides a brief review literature on the concept of AHP framework and two ranking methodologies proposed by Jahanshahloo et al. [17] and Liu and Peng [18] for ranking of DMUs. A numerical example is employed in section 3 to illustrate the application of mentioned ranking method in determining the priorities of alternatives in AHP. Finally, conclusions and remarks are provided in section 4.

2. Literature Review

2.1. AHP

The analytic hierarchy process (AHP) is a structured technique for organizing and analyzing complex decisions. Based on mathematics, it was developed by Saaty in the 1970s [2] and has been extensively studied and refined since then. It has particular application in group decision making, and is used around the world in a wide variety of decision situations, in fields such as government, business, industry, healthcare, and education. Users of the AHP first decompose their decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. Rather than prescribing a "correct" decision, the AHP helps decision makers find one that best suits their goal and their understanding of the problem. It provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. Both qualitative and quantitative criteria can be compared using informed judgments to derive weights and priorities. In the final step of the process, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal, so they allow a straightforward consideration of the various courses of action. The information is then synthesized to determine relative rankings of alternatives. The procedure for using the AHP can be summarized as:

1. Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives.
2. Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements. For example, when comparing potential real-estate purchases, the investors might say they prefer location to price and price over timing.
3. Synthesize these judgments to yield a set of overall priorities for the hierarchy. This would combine the investors' judgments about location, price and timing for properties A, B, C, and D into overall priorities for each property.
4. Check the consistency of the judgments.
5. Come to a final decision based on the results of this process.

Most of the papers focus on determining the priorities for alternatives [19-22]. This paper employs common set of weights (CSW) to focus on determining the priorities for alternatives in AHP, in order to rank them [17].

2.2. Ranking

One of the important issues in DEA is ranking. DEA methods commonly do not rank the efficient DMUs. Some papers are presented various methods and techniques to rank the efficient DMUs [3-13]. Most existing models are only able to rank extreme efficient DMUs, but not non-extreme efficient ones. Therefore, no single model can be specified as the best ranking model whose results can be relied on in all cases. This paper researched a common set of weights based on one of the resent ranking method that was the most favorable for determining the absolute efficiency for all of alternatives as DMUs [17]. The mentioned procedure is suitable for consistent pairwise comparison matrices and inconsistent pairwise comparison matrices. It produces true weights for perfectly consistent pairwise comparison matrices and the best local priorities that are logical and consistent with decision makers (DMs)' individual judgments for inconsistent pairwise comparison matrices.

Jahanshahloo and et al. [17] formulated two ranking methods. We obtain research idea from their models. We use model (1) to obtain efficient alternatives. Then model (2) is used to determine priorities alternatives in AHP to get a suitable decision.

$$\Lambda^* = \min \quad \sum_{j=1}^{n} \Lambda_j$$

$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j = 1,...,n \tag{1}$$

$$u_r \ge \varepsilon > 0 \quad r = 1,...,s$$
$$v_i \ge \varepsilon > 0 \quad i = 1,...,m.$$

Let E be the set of indexes of efficient DMUs, i.e. $E = \left\{ j \,\middle|\, \theta_j^* = 1.0, j = 1,...,n \right\}$. Indeed, we solve model (2) for efficient alternatives.

$$z^* = \min \quad \sum_{j \in E,\ j \ne o} z_j$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \qquad j \in E,\ \ j \ne o \tag{2}$$
$$\sum_{j=1}^{n} u_j + \sum_{j=1}^{n} v_j = 1$$
$$u_r \ge \varepsilon > 0 \quad r = 1,...,s$$
$$v_i \ge \varepsilon > 0 \quad i = 1,...,m$$

where the constraint $\sum_{r=1}^{s} u_r + \sum_{i=1}^{m} v_i = 1$ is added for normalization purpose. ε is a positive Archimedean infinitesimal constant. The optimal common set of weights $u_r^* (r = 1,...,s), v_i^* (i = 1,...,m)$ to each efficient DMU would be solved. We use definition of the Common weights analysis (CWA) efficiency score of DMU_j that Liu and Peng [16] was defined as following equation:

$$\xi_j^* = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}} \qquad j \in E \tag{3}$$

Therefore, score of relation (3) is 1.0 for efficient DMUs. Based on this, DMU_j is efficient if $z_j^* = 0$ or $\xi_j^* = 1$ otherwise DMU_j is inefficient. By pay attention to optimal function values, a weight vector is obtained.

Capability of these models is illustrated with using numerical examples.

3. Numerical Examples

In this section, we introduce a new application of model (2) in the AHP. Indeed, obtaining the common set of weights is suitable for discriminating between efficient DMUs and ranking them. Consider the following inconsistent comparison matrix, which comes from Saaty [2].

First, we distinguish the efficient units. Then for efficient units, the model (2) is employed. The result is shown in Table 1. According to Table 1, alternatives as DMUs 1, 2 and 5 are efficient i.e. the efficient set is obtained as $E = \{1,2,5\}$, and DMUs 3, 4 and 6 are inefficient. In order to rank the three DMUs by the proposed model, we solve model (2) for each efficient DMU separately. The results of the model and ranking of them can be seen in the two last columns of Table 1.

$$A = \begin{bmatrix} 1 & 4 & 3 & 1 & 3 & 4 \\ \frac{1}{4} & 1 & 7 & 3 & \frac{1}{5} & 1 \\ \frac{1}{3} & \frac{1}{7} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{6} \\ 1 & \frac{1}{3} & 5 & 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 5 & 5 & 1 & 1 & 3 \\ \frac{1}{4} & 1 & 6 & 3 & \frac{1}{3} & 1 \end{bmatrix}$$

TABLE 1. Efficiency scores of alternatives as DMUs and the results of model (3)

Alternatives	*Efficiencies scores*	Λ_j^*	Rank
1	1	2.0315	1
2	1	1.1138	4
3	0.1433	-	6
4	0.6317	-	3
5	1	1.9660	2
6	0.9335	-	5

As it is indicated in Table 1, this procedure produces the best local priorities that are logical and consistent with decision makers (DMs)' individual judgments for inconsistent pairwise comparison matrices. For more illustration of the advantage of the mentioned procedure for determining the priorities of alternatives (i.e., ranking them in the AHP), the following consistent pairwise comparison matrices are considered.

Tables 2 and 3 summarize the result of models (2) and (3) for the data of matrices *B* and *C*. It is clear the model (3) is suitable for consistent pairwise comparison matrices. Indeed, this model is simple in using and it is consistent for any kind of pairwise comparison matrices. It does not limit to DEA. It can be applicable in MCDM problems (e.g., AHP).

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$$B = \begin{bmatrix} 1 & 2 & 5 \\ 0.5 & 1 & 3 \\ 0.2 & 0.33 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 2 & 2 & 2 \\ 0.5 & 1 & 2 & 2 & 2 \\ 0.5 & 0.5 & 1 & 2 & 2 \\ 0.5 & 0.5 & 0.5 & 1 & 2 \\ 0.5 & 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}$$

TABLE 2. Efficiency scores of alternatives and the results of model (2) and (3) for matrix B

Alternatives	*Efficiencies scores*	Λ_j^*	Rank
1	1	1.9799	1
2	0.5625	-	2
3	0.1913	-	3

TABLE 3. Efficiency scores of alternatives and the results of model (2) and (3) for matrix C

Alternatives	*Efficiencies scores*	Λ_j^*	Rank
1	1	1.8364	1
2	0.8333	-	2
3	0.6667	-	3
4	0.5000		4
5	0.3533		5

4. Conclusions and Remarks

Up to this point, we utter a brief review on Jahanshahloo at el.'s approach. In this paper, we show two models which are applicable for determining the priorities the alternatives in the AHP for any kind of pairwise comparison matrices. It contributes to decision maker to employ recent DEA ranking models in his/her decision making. In addition, this approach is feasible in all cases. It is expected the used methodologies can play an important role in the studies and applications in the AHP. Our future research is to see whether this model can be further extended to group decision making in the AHP.

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