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A Novel Approach for Solving Fuzzy Multi-Objective Zero-One Linear Programming Problems

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A R T I C L E   I N F O

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A B S T R A C T

Fuzzy multi-objective zero-one linear programming (FMOZOLP) has many applications in various fields such as assembly line balancing, assignment, project portfolio selection and maximal covering location problems. In many of the existing methods for solving FMOZOLP problems, membership degree of different points of a fuzzy number is not considered or by performing α-cut, points with membership function more than or equal to α, are included in calculations. However ,even in this case, membership degree of these points has no effect on optimal solution. In this paper, in addition to modifying defects and failures of Yu and Li method [1] in solving fuzzy zero-one linear programming problems, we develop a novel approach to solve FMOZOLP problems considering membership degree of coefficients. Finally, an illustrative example for the project portfolio selection is included to compare results obtained by the proposed approach with results obtained by the other fuzzy methods.

1. Introduction

Mathematical programming problems with fuzzy data are one of the interested topics in operation research. Fuzzy mathematical programming can be classified into three categories. The first category was initially developed by Bellman and Zadeh[2], Tanaka et al.[3]and Zimmermann[4]. It treats decision making problem under fuzzy goals and constraints. The fuzzy goals and constraints represent the flexibility of the target values of objective functions and the elasticity of constraints. From this point of view, this type of fuzzy mathematical programming is called the *flexible programming* [5]. Numerous papers were devoted to the development of this method. Many of them were overviewed by Zimmermann [6].
The second category in fuzzy mathematical programming treats ambiguous coefficients of objective functions and constraints but does not treat fuzzy goals and constraints. Dubois and Prade [7] treated systems of linear equations with ambiguous coefficients suggesting the possible application to fuzzy mathematical programming for the first time. Some years later, Tanaka et al. [8], Orlovski [9] and Ramik and Rimanek [10] independently proposed

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treatments of linear programming problems with fuzzy coefficients. Since then, many approaches to such kinds of problems have been developed. Since the fuzzy coefficients can be regarded as possibility distributions on coefficient values, this type of fuzzy mathematical programming is usually called the *possibilistic programming* [5].

The last type of fuzzy mathematical programming treats ambiguous coefficients as well as vague decision maker's preference. Negoita et al. [11] were the first who formulated this type of fuzzy linear programming problem. In this model, the vague decision maker’s preference is represented by a fuzzy satisfactory region and a fuzzy function value is required to be included in the given fuzzy satisfactory region. In contrast to the flexible programming, this fuzzy mathematical programming is called the *robust programming* [5].

There are several methods in the literature for solving fuzzy multi-objective linear programming (FMOLP) models [12-16]. In such problems, converted to a crisp model, fuzzy model is optimized by usual techniques [15]. Among them, the fuzzy programming approaches are being increasingly applied. The main advantage of fuzzy approaches is that they are capable of measuring the satisfaction degree of each objective function explicitly. This issue can help the decision maker to make her/his final decision by choosing a preferred efficient solution according to the satisfaction degree and preference of each objective function [15]. Zimmermann developed the first fuzzy approach for solving a multi-objective linear programming (MOLP) called max–min approach [17], but it is well known that the solution yielded by max–min operator might not be unique nor efficient [12,13]. Therefore, after that several methods were proposed to remove this deficiency. Of particular interest, Lai and Hwang [12] developed the augmented max–min approach (LH method), Selim and Ozkarahan [14] presented a modified version of Werner’s approach (MW method), and Li et al. [13] proposed a two-phase fuzzy approach (LZL method)[15].

Fuzzy zero-one linear programming (FZOLP) problems have important role in operation research and management science, especially in assignment [18], assembly line balancing [19], maximal covering location, candidates selection cases [20,21] and etc problems. Many methods have been presented to solve FZOLP problems [13, 15, 22-25]. Most of the proposed methods use α-cut technique to solve FZOLP problems that require iterative processes or utilize arithmetic operations that require tedious computation. Moreover, most of them can solve only problems with fuzzy coefficients in objective function or fuzzy numbers in the right-hand side of constrains. Yu and Li, considering membership degree of coefficients, presented an interesting method to solve FZOLP problems [1]. Existing method except Yu and Li method, do not consider membership function of fuzzy coefficients in their calculations. In other words, fuzzy linear programming problem are solved without considering membership degree of points included in fuzzy numbers that causes concept of membership degree to be ignored in fuzzy numbers. While, one of the main differences between fuzzy and crisp numbers is the value of membership degree of points. Modifying Yu and Li method for solving FZOLP problems, this paper presents a novel approach to solve FMOZOLP problems that can solve a FMOZOLP problem with fuzzy coefficients in the objective function, fuzzy coefficients in the constraint matrix, and fuzzy numbers in the right-hand side of constraints.

The remainder of this paper is organized as follows. Section2 explains Yu and Li method. Defects and failures of Yu and Li method and their correction methods are described in section 3. In section4, we develop a novel approach to solve FMOZOLP problems. Section 5 presents a numerical example in order to compare the proposed method with MW, LH and LZL fuzzy methods. Finally, conclusion and future research directions are drawn in section 6.

2. Description of Yu and Li method

Consider linear programming model (1) that its objective function coefficients, coefficients in the constraint matrix and right-hand side values are triangular fuzzy numbers.

$$\begin{aligned} &\max Z = \sum_{j=1}^{n} \tilde{r}_j x_j \\ &\text{s.t.} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1) \\ &\sum_{j=1}^{n} \tilde{c}_{ij}\, x_j \le \tilde{b}_i \qquad i = 1,2,\dots,m \\ &x_j \in 0\ \ 0r\ \ 1 \qquad\qquad j = 1,2,\dots,n \end{aligned}$$

Yu and Li [1] presented an interesting algorithm to solve this problem (hereafter the YL method). They first presented and proved the following five propositions.

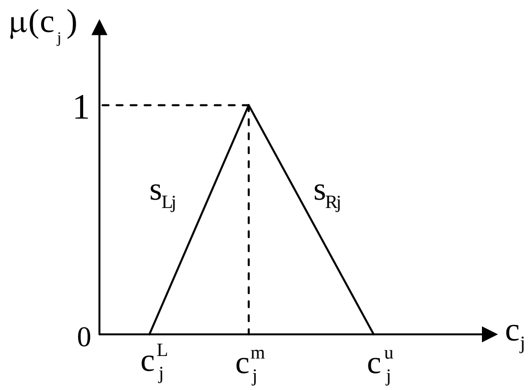

Figure 1. A triangular membership function.

Proposition 1: Let$\mu(c_j)$be a triangular membership function of a fuzzy valuec_j, as depicted in Fig. 1, where c_j^L, c_j^m andc_j^u are, respectively, the possible lowest number, middle number, and highest number,$s_{Lj}$and$s_{Rj}$are the slopes of line segments betweenc_j^m, c_j^L and c_j^u, c_j^m, respectively[1]. So we have:

$$s_{Lj} = \frac{\mu(c_j^m) - \mu(c_j^L)}{c_j^m - c_j^L}$$

$$s_{Rj} = \frac{\mu(c_j^u) - \mu(c_j^m)}{c_j^u - c_j^m} \qquad (2)$$

Therefore $\mu(c_j)$ can be expressed as equation (3).

$$\mu(c_j) = \mu(c_j^L) + s_{Lj}(c_j - c_j^L) + \frac{s_{Rj} - s_{Lj}}{2}(|c_j - c_j^m| + c_j - c_j^m) \qquad (3)$$

Proposition 2: Consider model (4)

$\max z = -(|f(x) - g| + f(x) - g)$
s.t
$x \in F$ (4)

where, F is a feasible set and g is a given non-negative constant. Model (4) can be written as model (5) which is a linear model [1].

$\max z = -2(f(x) - g + d)$
s.t.
$f(x) - g + d \geq 0$
$x \in F$
$d \geq 0$ (5)

Proposition 3: An optimal solution for model (6) is the solution that maximizes of model (7).

$$\max z = (\sum_{j=1}^{n} c_j x_j, \sum_{j=1}^{n} \mu(c_j))$$

s.t.
$c_j \in F$, $c_j \geq 0$ (6)

where, $W_j^+ = \left|1/s_{Lj}\right|$ and $W_j^- = \left|1/s_{Rj}\right|$ are the inverse of slopes as depicted in Figure 1 [1].

$$\max z = \sum_{j=1}^{n} c_j x_j - \sum_{j=1}^{n} (w_j^+ \delta_j^+ + w_j^- \delta_j^-)$$

s.t.
$\mu_j(c_j) - \delta_j^+ + \delta_j^- = 1$
$c_j \in F$, $c_j \geq 0$ (7)

Proposition 4: Consider zero-one linear programming model (8),

$$
\begin{aligned}
& maxz = cx \\
& \text{s.t.} \\
& X \in 0-1 \\
& c \geq 0 \qquad (8)
\end{aligned}
$$

This model is equal to model (9), where, $y = cx$.

$$
\begin{aligned}
& \max z = y \\
& \text{s.t.} \\
& y \leq c + M(1-x) \\
& y \leq Mx \\
& x \in 0-1 \\
& c \geq 0 \qquad (9)
\end{aligned}
$$

where, M is a big value [1].

Proposition 5 Consider zero-one linear programming model (10),

$$
\begin{aligned}
& \min z = cx \\
& \text{s.t.} \\
& x \in 0-1 \\
& c \geq 0 \quad (10)
\end{aligned}
$$

This model is equal to model (11), where, $y = cx$.

$$
\begin{aligned}
& \min z = y \\
& \text{s.t.} \\
& y \geq c + M(x-1) \\
& y \geq 0\,,\ c \geq 0 \quad (11) \\
& x \in 0-1
\end{aligned}
$$

where, M is a big value [1].
Then using these propositions, model (1) first is converted to crisp multi-objective linear programming model (12) and then solved through goal programming model (13) which is weighted by decision maker (DM).

$$\max z_1 = \sum_{j=1}^{n} r_j\, x_j$$

$$\min z_2 = \sum_{j=1}^{n} \mu\,(r_j)$$
$$\max z_3 = \sum_{j=1}^{n}\sum_{i=1}^{m} \mu\left(c_{ij}\right)$$
$$\max z_4 = \sum_{i=1}^{m} \mu\,(b_i)(12)$$
s.t
$$\sum_{j=1}^{n} c_{ij}\, x_j \leq b_i \qquad i = 1,2,\dots,m$$
$$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2,\dots,n$$

Considering $y_j = r_j x_j$ and $z_{ij} = c_{ij} x_j$ for$i = 1,2,\dots,m$ and $j = 1,2,\dots,n$.

$$\max z = \sum_{j=1}^{n} y_j - \sum_{j=1}^{n} (w_{rj}^{+}\,\delta_{rj}^{+} + w_{rj}^{-}\delta_{rj}^{-})$$
$$-\sum_{j=1}^{n}\sum_{i=1}^{m} \beta_{ij}\left(w_{cij}^{+}\delta_{cij}^{+} + w_{cij}^{-}\delta_{cij}^{-}\right) - \sum_{i=1}^{m} \lambda_i\,(w_{bi}^{+}\delta_{bi}^{+} - w_{bi}^{-}\delta_{bi}^{-})$$
s.t.
$$\mu(r_j) - \delta_{rj}^{+} + \delta_{rj}^{-} = 1 \qquad j = 1,2,\dots,n$$
$$\mu(c_{ij}) - \delta_{cij}^{+} + \delta_{cij}^{-} = 1 \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$\mu(b_i) - \delta_{bi}^{+} + \delta_{bi}^{-} = 1 \qquad i = 1,2,\dots,m$$
$$\textstyle\sum_{j=1}^{n} z_{ij} \leq b_i \qquad i = 1,2,\dots,m \quad (13)$$
$$r_j + d_{rj} \geq g_{rj} \qquad j = 1,2,\dots,n$$
$$c_{ij} + d_{cij} \geq g_{cij} \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$b_i + d_{bi} \geq g_{bi} \qquad i = 1,2,\dots,m$$
$$y_j \leq r_j + M\left(1 - x_j\right) \qquad j = 1,2,\dots,n$$
$$y_j \leq M_{xj} \qquad j = 1,2,\dots,n$$
$$z_{ij} \leq c_{ij} + M(1 - x_i) \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$z_{ij} \leq M_{xi} \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2,\dots,n$$
$$y_j, z_{ij}, c_j \geq 0 \qquad j = 1,2,\dots,n$$

where, the weightsw_{rj}^{+} andw_{rj}^{-} are equal to inverse of line slopes of fuzzy numbers of objective function coefficients, the weightsw_{cij}^{+} and w_{cij}^{-} to inverse of line slopes of fuzzy number of constraints coefficients and the weightsw_{bi}^{+} and w_{bi}^{-} to inverse of line slopes of fuzzy number of right-hand side of constraints. λ_j and β_{ij} are trade off weights adjusting among the expected goal, the possible right-hand side values and the possible coefficients of decision variables in the jth constraint [1].

3. Failures of YL method and the correction methods

Failures of YL method include:

1. In this method, both positive and negative deviations of membership degree of fuzzy coefficients are minimized from one, that is:

$$\min z = \sum_{j=1}^{n} (w_{rj}^{+}\,\delta_{rj}^{+} + w_{rj}^{-}\delta_{rj}^{-})$$
$$+ \sum_{j=1}^{n}\sum_{i=1}^{m} \beta_{ij}\left(w_{cij}^{+}\delta_{cij}^{+} + w_{cij}^{-}\delta_{cij}^{-}\right) + \sum_{i=1}^{m} \lambda_i\,(w_{bi}^{+}\delta_{bi}^{+} - w_{bi}^{-}\delta_{bi}^{-})$$

s.t. (14)

$$\mu(r_j) - \delta_{rj}^{+} + \delta_{rj}^{-} = 1 \qquad j = 1,2,\dots,n$$
$$\mu(c_{ij}) - \delta_{cij}^{+} + \delta_{cij}^{-} = 1 \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$\mu(b_i) - \delta_{bi}^{+} + \delta_{bi}^{-} = 1 \qquad i = 1,2,\dots,m$$

But it should be noted that value of membership function of each fuzzy number should be between zero and one $(0 \le \mu(r_j) \le 1, 0 \le \mu(b_i) \le 1, 0 \le \mu(c_{ij}) \le 1)$, and it should never exceed one, so only their negative deviation should be minimized from one. Therefore, in this model, one additional variable is considered for each fuzzy number. So, all δ_j^{+}variables should be deleted from the model. That is, model (14) should be changed into model(15).

$$\min z = \sum_{j=1}^{n} w_{rj}^{-}\,\delta_{rj}^{-} + \sum_{j=1}^{n}\sum_{i=1}^{m} \beta_{ij}\left(w_{cij}^{-}\delta_{cij}^{-}\right) + \sum_{i=1}^{m} \lambda_i\,(w_{bi}^{-}\delta_{bi}^{-})$$

s.t. (15)

$$\mu(r_j) + \delta_{rj}^{-} = 1 \qquad j = 1,2,\dots,n$$
$$\mu(c_{ij}) + \delta_{cij}^{-} = 1 \qquad j = 1,2,\dots,n \qquad i = 1,2,\dots,m$$
$$\mu(b_i) + \delta_{bi}^{-} = 1 \qquad i = 1,2,\dots,m$$

2. By deletingδ_j^{+}variables from model (14), their weighting method (that is:$w_j^{+} = \left|{}^{1}/_{s_{Lj}}\right|andw_j^{-} = \left|{}^{1}/_{s_{Rj}}\right|$) could not be justified and could be corrected as equation (16).

$$w_j = \left|{}^{1}/_{s_{Lj}}\right| + \left|{}^{1}/_{s_{Rj}}\right| \quad (16)$$

That indeed indicates that each fuzzy number which has a bigger spread, would be allocated more weight in objective function in order to decrease uncertainty and increase the possibility of bringing membership degree of that fuzzy number close to one.

3. In order to understand the third failure, it is better to consider example (1) which is presented in their paper as example (3).

Example 1: The board of directors of a large manufacturing firm is considering the investment project illustrated in the following table. The board wishes to maximize the total expected return and investment around the available annual budget. Five projects are being considered for execution over the next three years while the expected return for each project naturally is uncertain. The return, available funds and required yearly investments (in millions dollars) are displayed in Table 1 [1].

Table 1. Available investment information

Project	**Investments for**			**Returns**
	Year 1	**Year 2**	**Year 3**	
1	6	2	(7,8,9)	(18,5,20,23)
2	5	8	(9,10,11)	(38,40,41,5)
3	3	10	(2,5,3,3,5)	(19,20,21,5)
4	7	5	(1,5,2,2,5)	(13,8,15,16,3)
5	9	7	(9,10,11)	(28,2,30,34,5)
Available funds	(22,25,27)	(22,25,27)	(22,25,27)	

The decision problem can be formalized as model (17)

$$\max z = \tilde{r}_1 x_1 + \tilde{r}_2 x_2 + \tilde{r}_3 x_3 + \tilde{r}_4 x_4 + \tilde{r}_5 x_5$$

s.t.

$$6_{x1} + 5_{x2} + 3_{x3} + 7_{x4} + 9_{x5} \leq \tilde{b}_1$$
$$2_{x1} + 8_{x2} + 10_{x3} + 5_{x4} + 7_{x5} \leq \tilde{b}_2 \quad (17)$$
$$\tilde{a}_{13} x_1 + \tilde{a}_{23} x_2 + \tilde{a}_{33} x_3 + \tilde{a}_{43} x_4 + \tilde{a}_{53} x_5 \leq \tilde{b}_3$$
$$x_1, x_2, x_3, x_4, x_5 \in 0 - 1$$

Where the binary variablex_j represent the j[th] project, $j = 1,2, \dots ,5$.

Based on YL method, fuzzy model (17) is finally converted to crisp goal programming model (18).

$$\begin{aligned}\max z = {} & y_1 + y_2 + y_3 + y_4 + y_5 - 1{,}5\delta_1^+ - 3\delta_1^- - 2\delta_2^+ \\ & -1{,}5\delta_2^- - 1\delta_3^+ - 1{,}5\delta_3^- - 1{,}2\delta_4^+ - 1{,}3\delta_4^- - 1{,}8\delta_5^+ \\ & -4{,}5\delta_5^- - 3\lambda_1\delta_6^+ - 2\lambda_1\delta_6^- - 3\lambda_2\delta_7^+ - 2\lambda_2\delta_7^- - \\ & 3\lambda_3\delta_8^+ - 2\lambda_3\delta_8^- - \beta_{13}\delta_{13}^+ - \beta_{13}\delta_{13}^- - \beta_{23}\delta_{23}^+ - \beta_{23}\delta_{23}^- \\ & -0{,}5\beta_{33}\delta_{33}^+ - 0{,}5\beta_{33}\delta_{33}^- - 0{,}5\beta_{43}\delta_{43}^+ - 0{,}5\beta_{43}\delta_{43}^- \\ & -\beta_{53}\delta_{53}^+ - \beta_{53}\delta_{53}^-\end{aligned}$$

s.t.

$$
\begin{aligned}
&6_{x1}+5_{x2}+3_{x3}+7_{x4}+9_{x5}\le b_1\\
&2_{x1}+8_{x2}+10_{x3}+5_{x4}+7_{x5}\le b_2\\
&y_{13}+y_{23}+y_{33}+y_{43}+y_{53}\le b_3\\
&-0{,}3333r_1-d_1+7{,}66605-\delta_1^{+}+\delta_1^{-}=1\\
&-0{,}6667r_2-1{,}1667d_2+27{,}6668-\delta_2^{+}+\delta_2^{-}=1\\
&-0{,}6667r_3-1{,}6667d_3+14{,}334-\delta_3^{+}+\delta_3^{-}=1\\
&-0{,}7692r_4-1{,}6026d_4+12{,}5385-\delta_4^{+}+\delta_4^{-}=1\\
&-0{,}2222r_5-0{,}7778d_5+7{,}6667-\delta_5^{+}+\delta_5^{-}=1\\
&-0{,}49997b_1-0{,}8333d_6+13{,}4992-\delta_6^{+}+\delta_6^{-}=1\\
&-0{,}49997b_2-0{,}8333d_7+13{,}4992-\delta_7^{+}+\delta_7^{-}=1 \quad (18)\\
&-0{,}49997b_3-0{,}8333d_8+13{,}4992-\delta_8^{+}+\delta_8^{-}=1\\
&r_1+d_1\ge 20,\quad r_2+d_2\ge 40,\quad r_3+d_3\ge 20\\
&r_4+d_4\ge 15,\quad r_5+d_5\ge 30,\quad r_6+d_6\ge 25\\
&r_7+d_7\ge 25,\quad r_8+d_8\ge 25,\\
&-a_{13}-2d_9+9-\delta_{13}^{+}+\delta_{13}^{-}=1 \quad , \quad a_{13}+d_9\ge 8\\
&-a_{23}-2d_{10}+11-\delta_{23}^{+}+\delta_{23}^{-}=1 \quad , \quad a_{23}+d_{10}\ge 10\\
&-2a_{33}-4d_{11}+7-\delta_{33}^{+}+\delta_{33}^{-}=1 \quad , \quad a_{33}+d_{11}\ge 3\\
&-2a_{43}-4d_{12}+5-\delta_{43}^{+}+\delta_{43}^{-}=1 \quad , \quad a_{43}+d_{12}\ge 2\\
&-a_{53}-2d_{13}+11-\delta_{53}^{+}+\delta_{53}^{-}=1 \quad , \quad a_{53}+d_{13}\ge 10\\
&y_1\le r_1+M(1-x_1) \quad , \quad y_1\le M(x_1)\\
&y_2\le r_2+M(1-x_2) \quad , \quad y_2\le M(x_2)\\
&y_3\le r_3+M(1-x_3) \quad , \quad y_3\le M(x_3)\\
&y_4\le r_4+M(1-x_4) \quad , \quad y_4\le M(x_4)\\
&y_5\le r_5+M(1-x_5) \quad , \quad y_5\le M(x_5)\\
&y_{13}\le a_{13}+M(1-x_1) \quad , \quad y_{13}\le M(x_1)\\
&y_{23}\le a_{23}+M(1-x_2) \quad , \quad y_{23}\le M(x_2)\\
&y_{33}\le a_{33}+M(1-x_3) \quad , \quad y_{23}\le M(x_3)\\
&y_{43}\le a_{43}+M(1-x_4) \quad , \quad y_{43}\le M(x_4)\text{s}\\
&y_{53}\le a_{53}+M(1-x_5) \quad , \quad y_{53}\le M(x_5)\\
&x_1,x_2,x_3,x_4,x_5\in 0-1
\end{aligned}
$$

Considering allλ_j andβ_{ij}equal to one, Yu and Li solved this problem by LINDO software and presented the following answer in their paper [1].

$$
(x_1,x_2,x_3,x_4,x_5,b_1,b_2,b_3,a_{13},a_{23},a_{33},a_{43},a_{53})\\
=(1,1,1,0,1,25,27,25,8,10,3,2,10)
$$

Substituting obtained answer in the third constraint of model (17), we see that:

$$8(1)+10(1)+3(1)+2(0)+10(1)=31\nleq 25$$

Now this question arises that what was the fault in which obtained answer does not satisfy this constraint?
Answer is the errors in writing constraints (19) as follows:

$$\begin{aligned}
&y_{13} \le a_{13} + M(1-x_1) \quad , \quad y_{13} \le M(x_1)\\
&y_{23} \le a_{23} + M(1-x_2) \quad , \quad y_{23} \le M(x_2)\\
&y_{33} \le a_{33} + M(1-x_3) \quad , \quad y_{33} \le M(x_3) \quad (19)\\
&y_{43} \le a_{43} + M(1-x_4) \quad , \quad y_{43} \le M(x_4)\\
&y_{53} \le a_{53} + M(1-x_5) \quad , \quad y_{53} \le M(x_5)
\end{aligned}$$

In order to obtain correct answer, these constraints could be modified by the following methods.

Method 1: Using constraints (20) instead of constraints (19).

$$\begin{aligned}
&y_{13} \ge a_{13} + M(x_1 - 1) \quad , \quad y_{13} \ge 0\\
&y_{23} \ge a_{23} + M(x_2 - 1) \quad , \quad y_{23} \ge 0\\
&y_{33} \ge a_{33} + M(x_3 - 1) \quad , \quad y_{33} \ge 0 \ (20)\\
&y_{43} \ge a_{43} + M(x_4 - 1) \quad , \quad y_{43} \ge 0\\
&y_{53} \ge a_{53} + M(x_5 - 1) \quad , \quad y_{53} \ge 0
\end{aligned}$$

But combining with poly nominal$-\beta_{13}\delta_{13}^{+} - \beta_{13}\delta_{13}^{-} - \beta_{23}\delta_{23}^{+} - \beta_{23}\delta_{23}^{-} - 0{,}5\beta_{33}\delta_{33}^{+} - 0{,}5\beta_{33}\delta_{33}^{-} - 0{,}5\beta_{43}\delta_{43}^{+} - 0{,}5\beta_{43}\delta_{43}^{-} - \beta_{53}\delta_{53}^{+} - \beta_{53}\delta_{53}^{-}$,in max objective function, these constraints lead to correct answer. So, if in a problem these variables are not considered in objective function, solving that problem will yield wrong answers.

Method 2: In order for our model to obtain correct answer without any doubt and not considering objective function, we should use constraints set (21) instead of constraints set (19) in formulizing the problem.

$$\begin{aligned}
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{23} \ge a_{23} - M(1-x_2) \quad , \quad y \le M(x_2)\\
&y_{33} \ge a_{33} - M(1-x_3) \quad , \quad y \le M(x_3)\\
&y_{43} \ge a_{43} - M(1-x_4) \quad , \quad y \le M(x_4)\\
&y_{53} \ge a_{53} - M(1-x_5) \quad , \quad y \le M(x_5) \quad (21)\\
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{13} \ge a_{13} - M(1-x_1) \quad , \quad y \le M(x_1)\\
&y_{13} \le a_{13} + M(1-x_1)\\
&y_{23} \le a_{23} + M(1-x_2)
\end{aligned}$$

$y_{33} \leq a_{33} + M(1 - x_3)$
$y_{43} \leq a_{43} + M(1 - x_4)$
$y_{53} \leq a_{53} + M(1 - x_5)$

In conclusion, based on the presented suggestions, example (1) is formulated as model (22).

$$\max z = y_1 + y_2 + y_3 + y_4 + y_5 - 4{,}5\delta_1^- - 3{,}5\delta_2^- \\ -2{,}5\delta_3^- - 2{,}5\delta_4^- - 6{,}3\delta_5^- - 5\lambda_1\delta_6^- - 5\lambda_2\delta_7^- - 5\lambda_3\delta_8^- \\ -2\beta_{13}\delta_{13}^- - 2\beta_{23}\delta_{23}^- - \beta_{33}\delta_{33}^- - \beta_{43}\delta_{43}^- - 2\beta_{53}\delta_{53}^- \quad (22)$$

s.t.
$6x_1 + 5x_2 + 3x_3 + 7x_4 + 9x_5 \leq b_1$

Solving this problem by LINDO software obtains the following answer:

$$(x_1, x_2, x_3, x_4, x_5, b_1, b_2, b_3, a_{13}, a_{23}, a_{33}, a_{43}, a_{53}) \\ = (1,1,1,0,1,25,27,25,8,10,3,2,10)$$

As it is seen, this solution in addition to maximizing objective function satisfies all problem constraints.

4. The proposed approach for solving FMOZOLP problems

Consider model (23) which is a bi-objective zero-one linear programming model with fuzzy coefficients in the objective functions, fuzzy coefficients in the constraint matrix, and fuzzy numbers in right-hand side of constraints. We explain the proposed solving approach by model (23) that could be generalized to solve FMOZOLP problems.

$$\max z_1 = \sum_{j=1}^{n} \tilde{r}_j \, x_j$$

$$\min z_2 = \sum_{j=1}^{n} \tilde{d}_j \, x_j$$

s.t. (23)

$$\sum_{j=1}^{n} \tilde{c}_j \, x_j \leq \tilde{b}$$

$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2, \dots, n$

First we turn model (23) into multi-objective linear programming model (24) with certain coefficients, according to YL modified method which was explained in section 3. Considering$y_j = r_j x_j$,$z_j = d_j x_j$ and $p_j = c_j x_j$for$= 1,2, \dots, n$, we obtain model (25).

$$\max z_1 = \sum_{j=1}^{n} r_j\, x_j$$
$$\min z_2 = \sum_{j=1}^{n} d_j\, x_j$$
$$\max z_3 = \sum_{j=1}^{n} \mu(r_j)$$
$$\max z_4 = \sum_{j=1}^{n} \mu(d_j)$$

(24)

$$maxz_5 = \sum_{j=1}^{n} \mu\,(c_j)$$
$$maxz_6 = \mu(b)$$

s.t.

$$\sum_{j=1}^{n} c_j\, x_j \le b$$
$$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2, \dots, n$$

Membership function of a fuzzy number is a scale less number between zero and one. Therefore, objectives which are sum of membership function of fuzzy numbers such asZ_3, Z_4, Z_5 and Z_6 objectives in model (25) are scale less.

$$\max z_1 = \sum_{j=1}^{n} y_j$$
$$\min z_2 = \sum_{j=1}^{n} z_j$$
$$\max z_3 = \sum_{j=1}^{n} \mu\,(r_j)$$
$$\max z_4 = \sum_{j=1}^{n} \mu\,(d_j)$$

(25)

$$\max z_5 = \sum_{j=1}^{n} \mu\,(c_j)$$
$$\max z_6 = \mu(b)$$

s.t.
$$\sum_{j=1}^{n} p_j \le b$$
$$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2,\dots,n$$

But other objectives could have different units (such as $Z_1$and Z_2objectives). So in the first stage we should make them scale less. To this end, we use fuzzy method and obtain membership functions of Z_1andZ_2. Then, we determine the positive ideal solutions (PIS) and negative ideal solutions (NIS) for Z_1 andZ_2 objective functions by solving the corresponding zero-one linear programming model as follow [13, 15].

$$Z_1^{PIS} = max \sum_{j=1}^{n} y_j \; Z_1^{NIS} = min \sum_{j=1}^{n} y_j$$
$$Z_2^{PIS} = min \sum_{j=1}^{n} z_j \; Z_2^{NIS} = max \sum_{j=1}^{n} z_j$$
s.t. (26)
$$\sum_{j=1}^{n} p_j \le b$$
$$x_j \in 0 \;\; or \;\; 1 \qquad j = 1,2,\dots,n$$

$$\mu_1(Z_1) = \begin{cases} 1 & if \quad Z_1 > Z_1^{PIS} \\ \dfrac{Z_1 - Z_1^{NIS}}{Z_1^{PIS} - Z_1^{NIS}} & if \quad Z_1^{NIS} \le Z_1 \le Z_1^{PIS} \\ 0 & if \quad Z_1 < Z_1^{NIS} \end{cases}$$
$$\mu_2(Z_2) = \begin{cases} 1 & if \quad Z_2 < Z_2^{PIS} \\ \dfrac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}} & if \quad Z_2^{PIS} \le Z_2 \le Z_2^{NIS} \\ 0 & if \quad Z_2 > Z_2^{NIS} \end{cases} \qquad (27)$$

Then, linear membership function of each objective function is obtained from equations (27), which are depicted in Figure 2.

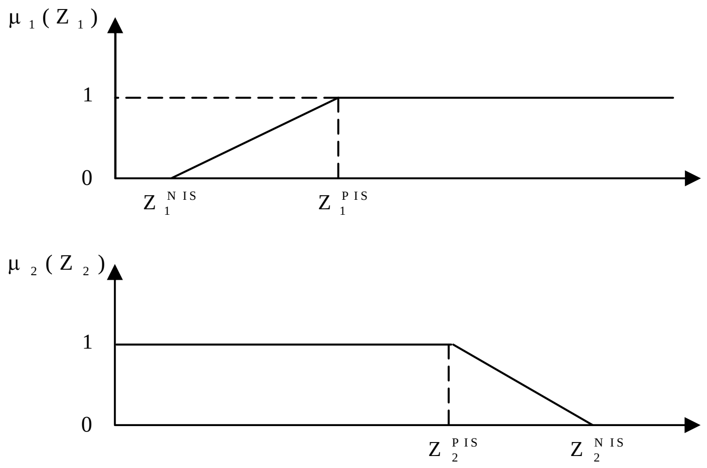

Figure 2. Linear membership functions for Z_1 and Z_2

So we have:

$$\max Z = \theta_1\mu_1(Z_1) + \theta_2\mu_2(Z_2) - \left[\theta_3\sum_{j=1}^{n} w_{rj}\,\delta_{rj}^- + \theta_4\sum_{j=1}^{n} w_{dj}\delta_{dj}^- + \theta_5\sum_{j=1}^{n} w_{cj}\delta_{cj}^- + \theta_6 w_b\delta_b^-\right]$$

s.t.

$$\sum_{j=1}^{n} p_j \le b$$
$$\mu(r_j) + \delta_{rj}^- = 1 \qquad j = 1,2,\dots,n$$
$$\mu(d_j) + \delta_{dj}^- = 1 \qquad j = 1,2,\dots,n$$
$$\mu(c_j) + \delta_{cj}^- = 1 \qquad j = 1,2,\dots,n$$
$$\mu(b) + \delta_b^- = 1 \qquad (28)$$
$$r_j + d_{rj} \ge g_{rj} \qquad j = 1,2,\dots,n$$
$$d_j + d_{dj} \ge g_{dj} \qquad j = 1,2,\dots,n$$
$$c_j + d_{cj} \ge g_{cj} \qquad j = 1,2,\dots,n$$
$$b + d_b \ge g_b$$
$$y_j \le r_j + M(1 - x_j) \qquad j = 1,2,\dots,n$$
$$y_j \le M x_j j = 1,2,\dots,n$$
$$z_j \ge d_j - M(1 - x_j) \qquad j = 1,2,\dots,n$$
$$z_j \le d_j + M(1 - x_j) \qquad j = 1,2,\dots,n$$
$$z_j \le M x_j j = 1,2,\dots,n$$
$$p_j \ge c_j - M(1 - x_j) \qquad j = 1,2,\dots,n$$
$$p_j \le c_j + M(1 - x_j) \qquad j = 1,2,\dots,n$$

$$p_j \leq Mx_j j = 1,2, \dots, n$$

$x_j \in 0 \quad or \quad 1 \qquad j = 1,2, \dots, n$

$y_j, z_j, p_j, r_j, d_j, c_j \geq 0 \qquad j = 1,2, \dots, n$

where, $w_{*j} = \left|{}^{1}/_{s_{Lj}}\right| + \left|{}^{1}/_{s_{Rj}}\right|$for each fuzzy number and$\theta_{is}$are weights which are used by DMs, considering the importance of objectives and its priorities for making balance between objectives and could have a value between 0 and 1, so that $\theta_1 + \theta_2 + \cdots + \theta_6 = 1$.
Optimal solution of model (28) is an efficient solution for model (23).The proposed approach (hereafter the AS method) is actually a hybridization of the modified YL method and LZL method.

5. Numerical example

In this section, we present an example of project portfolio selection problem in pharmaceutical industry (modified from [21]), in order to compare the performance of the AS approach with LZL, LH and MW fuzzy methods. In a pharmaceutical company, 20 R&D projects are candidates. Table (2) shows the uncertain development costs, R&D staffs required, and the fuzzy project values for the company in result of implementation of each R&D projects as triangular fuzzy numbers. The preferred capacity of R&D staff is (in working days) (1916, 2376, 2836). Moreover, 20 R&D projects can be classified into three strategic types: new drug ($s_1 = 13,14,16,17,18,19,20$), derivates of existing drug ($s_2 = 5,6,8,9,10,15$), and incremental improvement to existing drugs ($s_3 = 1,2,3,4,7,11,12$).
Company wants to select at least 3 projects of S_1projects, at least 2 projects of S_2 projects and at least 2 projects of S_3 projects. Company aims to select a proper portfolio of R&D projects which in addition to producing the maximum value for company, has the minimum cost.
General model of this problem is as follows:

$$\max Z_1 = \sum_{j=1}^{20} \tilde{v}_j x_j$$

$$\min Z_2 = \sum_{j=1}^{20} \tilde{c}_j x_j$$

s.t.

$$\sum_{j=1}^{20} \tilde{h}_j x_j \leq \tilde{b} (29)$$

$$x_1 + x_2 + x_3 + x_4 + x_7 + x_{11} + x_{12} \geq 2$$

$$x_5 + x_6 + x_8 + x_9 + x_{10} + x_{15} \geq 2$$

$$x_{13} + x_{14} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \geq 3$$

$x_j \in 0 \quad or \quad 1 \qquad\qquad\qquad j = 1,2,\ldots,20$

LZL, LH and MW fuzzy methods for solving FMOLP problems are briefly presented in appendix A.

Table 2.Fuzzy development costs, required human resource, and project values for 20 candidate projects

Project no.	Fuzzy development cost (in millions)(c_j)	Fuzzy development resource (in working days)(h_j)	Fuzzy project value(v_j)
1	(53,2,62,71,8)	(115,4,128,140,6)	(3,15,27)
2	(83,05,98,112,95)	(126,140,154)	(19,43,67)
3	(157,5,185,212,5)	(169,189,209)	(49,93,137)
4	(204,25,240,275,75)	(164,182,200)	(50,98,146)
5	(259,305,351)	(209,232,255)	(285,407,529)
6	(84,75,100,115,25)	(186,206,226)	(15,18,21)
7	(60,72,84)	(141,157,173)	(16,20,24)
8	(93,75,110,126,25)	(178,197,216)	(6,30,54)
9	(140,5,165,189,5)	(237,264,291)	(27,67,107)
10	(189,223,257)	(257,285,313)	(138,215,292)
11	(58,25,70,81,75)	(147,5,164,180,5)	(11,28,45)
12	(91,107,123)	(144,5,160,175,5)	(8,30,52)
13	(242,75,290,337,25)	(297,330,363)	(143,229,315)
14	(371,435,499)	(337,375,413)	(261,401,541)
15	(166,195,224)	(279,310,341)	(222,317,412)
16	(222,75,260,297,25)	(316,350,384)	(71,136,201)
17	(232,5,277,321,5)	(311,346,381)	(108,181,254)
18	(284,330,376)	(332,368,404)	(237,350,463)
19	(341,405,469)	(365,406,447)	(346,505,664)
20	(452,5,530,607,5)	(399,438,479)	(534,758,982)

Using AS method, model (29) is converted to multi-objective model (30) with crisp coefficients and using LZL, LH and MW fuzzy methods, model (29) is turned into multi-objective model (31) with crisp coefficients.

$$\max Z_1 = \sum_{j=1}^{20} \mu\,(v_j)$$

$$\max Z_2 = \sum_{j=1}^{20} v_j x_j$$

$$\max Z_3 = \sum_{j=1}^{20} \mu\,(c_j)$$

$$\max Z_4 = \sum_{j=1}^{20} \mu\,(h_j)$$

$$\min Z_5 = \sum_{j=1}^{20} c_j x_j \quad (30)$$

$$\text{Max}\, Z_6 = \mu(b)$$

s.t.

$$\sum_{j=1}^{20} h_j x_j \leq b$$

$$x_1 + x_2 + x_3 + x_4 + x_7 + x_{11} + x_{12} \geq 2$$

$$x_5 + x_6 + x_8 + x_9 + x_{10} + x_{15} \geq 2$$

$$x_{13} + x_{14} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \geq 3$$

$$x_j \in 0 \quad or \quad 1 \qquad\qquad j = 1,2,\dots,20$$

This problem is solved by LINGO software using four AS, LZL, LH and MW methods and table (3) presents the obtained solutions. Due to space limitations, the details of the solutions found by the different approaches are not presented here, but can be made available upon request.

$$\min Z_1 = \sum_{j=1}^{20} \left[(v_j^m - v_j^p) x_j \right]$$

$$\max Z_2 = \sum_{j=1}^{20} v_j^m x_j$$

$$\max Z_3 = \sum_{j=1}^{20} \left[(v_j^o - v_j^m) x_j \right]$$

$$\max Z_4 = \sum_{j=1}^{20} \left[\left(c_j^m - c_j^p\right) x_j \right]$$

$$\min Z_5 = \sum_{j=1}^{20} c_j^m x_j \tag{31}$$

$$\min Z_6 = \sum_{j=1}^{20} \left[\left(c_j^o - c_j^m\right) x_j \right]$$

s.t.

$$\sum_{j=1}^{20} h_j^m x_j \leq b^m$$

$$\sum_{j=1}^{20} h_j^o x_j \leq b^o$$

$$x_1 + x_2 + x_3 + x_4 + x_7 + x_{11} + x_{12} \geq 2$$
$$x_5 + x_6 + x_8 + x_9 + x_{10} + x_{15} \geq 2$$
$$x_{13} + x_{14} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} \geq 3$$
$$x_j \in 0 \quad or \quad 1 \qquad\qquad j = 1,2, \dots ,20$$

In order to analyze and compare the performance of these approaches, we have used the well-known distance measure. The distance measure is used for determining the degree of closeness of each solution to the corresponding ideal solution. In this regard, we define the following family of distance functions [12,15]:

$$d_q(Z) = \left[\sum_i \theta_i^q \, (1 - \mu_i(z_i))^q\right]^{\frac{1}{q}} \; ; q \geq 1 \, , q \geq 1 \, , Integer \qquad (32)$$

Since the satisfaction degree of each objective is defined as the relative closeness of the solution to the ideal point or the relative remoteness to the anti-ideal point, they are used explicitly in equation (32). The power q represents a distance parameter and especially q = 1, 2 are operationally important so that q_1 (the Manhattan distance) and q_2 (the Euclidean distance) are the longest and shortest distances in the geometrical sense. Generally speaking, when q increases, the amount of distance d_q and also the credibility of the distance function d_q decreases [12, 15]. It is noted that based on the definition of d_q, the fuzzy approach with minimum d_q (especially for q=1), would be preferred to the other methods. Table 3summarizes the numerical results of the four fuzzy approaches in terms of above-mentioned performance index.

Table 3. Performance comparison of fuzzy approaches for $\boldsymbol{\theta = (0,05, 0,45, 0,05, 0,05, 0,35, 0,05), \delta = 0,01 \; and \; \gamma = 0,4}$

Approaches	**Selected projects** $\boldsymbol{x_j = 1}$	**Number of selected projects**	**Distance measures**	
LZL		8	0,4932	0,2817
LH		8	0,4993	0,2891
MW		7	0,3569	0,2315
AS		7	0,2926	0,2222

As it is seen in table3, among four already-mentioned methods, MW and AS methods lead to a similar project portfolio and also based on distance measure, these methods are superior to LZL and LH methods. On the other hand, considering the differences in solving algorithm and final formulation of model between AS and MW methods, AS method obtains smaller values of distance measure. So, we conclude that AS method is the most appropriate method for solving FMOZOLP problems, compared to above-mentioned fuzzy methods.

6. Conclusion and future research

Many methods have been presented to solve fuzzy programming problems that all of them except YL method ignore membership degree of fuzzy numbers. YL method was presented to solve FZOLP problems, in uni-objective state. There are several faults and failures in their method. In this paper, in addition to address and modify faults and failures of YL method, we developed a novel approach (AS approach) to solve FMOZOLP problems, considering membership function of fuzzy numbers in the calculations. This method can solve FMOZOLP problems with fuzzy coefficients in the objective functions, fuzzy coefficients in constraint matrix and fuzzy numbers in right-hand side of constraints. Advantages of the AS approach compared to other methods are: 1. Membership function of fuzzy coefficients is considered in computations, 2. Minimizing fuzzy number's spread is integrated in solving algorithm as one of the objectives.

The proposed AS method is very promising approach which can produce both unbalanced and balanced efficient solutions based on the decision maker's preferences along with offering appropriate flexibility to provide different solutions to help the decision maker in selecting the final preferred compromise solution. Furthermore, the numerical example indicates that based on distance measure, the AS method is superior to LZL, LH and MW methods.

In future researches, designing computational experiments, we can prove superiority of AS approach to LZL, LH and MW fuzzy methods. The AS proposed approach could also be used to solve other fuzzy multi-objective zero-one linear programming problems. Moreover, we can analyze and compare power and ability of these methods with metaheuristic methods to obtain efficient solutions.

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Appendix A

In this appendix we provide an abstract version of three previously developed approaches (i.e., the LZL, LH and MW methods).

A.1. Li et al. (LZL) two-phase method

$$\max \lambda(z) = \sum_{i=1}^{m} \theta_i \mu_i(z)$$
$$s.t.$$
$$\lambda_i^0 \le \mu_i(z) \quad i = 1,2,\dots,m \tag{1}$$
$$z \in F(z)$$
$$\lambda_i^0, \mu_i(z) \in [0,1]$$

In the above formulation,λ_i^0denotes the minimum satisfaction degree of ith objective function which is found by solving the Zimmermann's max–min approach as follows[13]:

$$\max \lambda$$
$$\text{s.t.}$$
$$\lambda \le \mu_i(z) \qquad i = 1,2,\dots,m \tag{2}$$
$$z \in F(z)$$
$$\lambda \in [0,1]$$

A.2. Lai and Hwang (LH) augmented max–min method

$$max \quad \lambda(z) = \lambda_0 + \delta \sum_{i=1}^{m} \theta_i \mu_i(z)$$
$$\text{s.t.} \tag{3}$$
$$\lambda_0 \le \mu_i(z) \qquad i = 1,2,\dots,m$$
$$z \in F(z)$$
$$\lambda_0 \in [0,1]$$

Here, $\lambda_0$denotes the minimum satisfaction degree of objectives which is determined along with the variables $\mu_i(z)$ viasolving the LH model directly in a single phase. Also,$\delta$is a sufficiently small positive number which is usually set to0.01 [12].

A.3. Selim and Ozkarahan extended Werners (MW) method

$$
\begin{aligned}
&\max \lambda(z) = \gamma\lambda_0 + (1-\gamma)\sum_{i=1}^{m}\theta_i\lambda_i \\
&\text{s.t.} \\
&\lambda_0 + \lambda_i \le \mu_i(z) \qquad i = 1,2,\dots,m \\
&z \in F(z) \\
&\gamma, \lambda_0, \lambda_i \in [0,1]
\end{aligned}
\tag{4}
$$

In this model, $\lambda_0$and$\mu_i(z)$denote the minimum satisfaction degree of objectives and satisfaction degree of objective i, respectively, which simultaneously are determined through solving the MW model [14]. Moreover, γ is the coefficient ofcompensation, and we have set it to 0.4 based on Torabi and Hassini [15] initial tests.