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Optimal Design Of Maintenance Service Contract Considering Preventive Maintenance, Random Repair Cost And Time Value Of Money

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Abstract

Offering Maintenance Service Contracts (MSCs) for production equipment can be a good source of revenue for the providers of these services. To do this, designing an optimal MSCwith a minimum price will be of great interest to service providers. In this paper, for a given equipment item, the effect of conducting periodic preventive maintenance on the failure process of the equipment and its corresponding cost is studied. Assuming minimal repair at failure with a random repair cost, the expected cost of maintenance service is estimated. Due to the time delay between selling the contract and paying the repair bills, the time value of service cost at the time of selling the contract is derived. Then, the cost-plus approach is used to determine the price of the MSC. In the presented model, the service provider determines the number of preventive maintenance and the improvement level to minimize the expected price of the MSC. A numerical example with comprehensive sensitivity analysis is presented to illustrate the model and its parameters' effect. The result shows that the presented model helps the service provider to design a MSC with a minimum price while assuring the profit margin.

Keywords: Maintenance service contract, Preventive maintenance, Pricing, Random repair cost, Time value of money.

1 | Introduction

Maintenance plays a key role in delivering reliable business operations. Maintenance of equipment and facilities can be set up by an in-house maintenance team, Original Equipment Manufacturer (OEM), or third-party maintenance service provider. Since

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outsourcing the maintenance to OEM or a third party is a reliable and cost-effective solution, it makes perfect sense to outsource the maintenance through a Maintenance Service Contract (MSC) [1]. A MSC is a legal agreement between a company and a maintenance service provider. It specifies the terms and conditions of the agreement between the two parties. MSC offers several advantages for both the manufacturer and the service provider. These include potential cost savings, improved quality control, and confidence, increased lifespan and performance of equipment, and more importantly peace of mind [2]. Selling MSC can also be a business opportunity for the service provider. However, designing and pricing a MSC more specifically for a complex and expensive industrial plant is very challenging [3]. Lack of information about the maintenance history and reliability of the equipment, the random nature of failures and corresponding costs, the time delay between selling the contract and paying the repair cost, and the effectiveness of maintenance interventions are among the factors that may affect the maintenance service plan. Besides, the risk attitude [4] and the intention of the manufacturer to outsource the maintenance [5] are also affect the purchasing of the MSC.

In this paper, a periodic preventive maintenance policy, random repair cost, and the time value of money are considered to design the optimal MSC from a service provider's perspective. The presented model can be seen as valuable research because, for the first time in the maintenance literature, the cost of a given repair is assumed to be random and a comprehensive sensitivity analysis is conducted to illustrate the effect of random repair cost parameters on the price of MSC. Besides, considering periodic preventive maintenance during the contract period, the effects of the number and level of preventive maintenance on the failure process of the product and its corresponding cost are studied. Moreover, the time value of costs is addressed to provide a more realistic cost estimation and pricing of MSC.

The organization of the paper is as follows. In the next section, a brief literature review is presented to study the reported research, clarify the research gap, and highlight the contribution of the paper. In Section 3, the general pricing model of the MSC is presented, and for a special case where the time to the first failure of equipment follows from Weibull distribution, the mathematical optimization model is derived. In Section 4, the numerical example is presented to illustrate the model. A comprehensive sensitivity analysis is carried out in Section 5. In the final section, a conclusion is drawn.

2 | Literature review

Literature on the MSC is vast, so we briefly review the reported research from 2010 up to now. For a MSC between a customer and an OEM, Wang [3] optimized the contract parameter of a customer for three contract scenarios under reliability and availability constraints. For a case that a manufacturer outsources preventive maintenance, Wu [6] studied the effect of preventive maintenance parameters and bonus function on the expected lifecycle cost rate of the equipment. As valuable research, Murthy et al. [1] studied and reviewed the maintenance outsourcing issues and challenges. Tantardini et al. [7] studied the effect of maintenance re-scheduling effect on the MSC. To do this, they applied empirical analysis to identify re-scheduling variables, and then they validated the model through real industrial data. For equipment sold with a two-dimensional warranty, Iskandar et al. [8] presented a non-cooperative game model to determine the optimal price and optimal parameters of the maintenance contract for the OEM and the manufacturer respectively. Wang et al. [9] studied the upgrade action and the preventive maintenance effect on the equipment reliability under the usage-based lease contract. In this model, the number and level of PM actions and upgrade degree were determined to minimize the total expected lease servicing cost. For more information about the two-dimensional warranty and two-dimensional MSC, the interested reader can refer to Wang and Xie [10]. Esmaeili et al. [11] proposed a three-level service contract between a manufacturer, an agent, and a customer. They derived non-cooperative and semi-cooperative game models to determine the optimal sale price and maintenance cost of the contract by maximizing the profit of the manufacturer and agent as well as the customer's satisfaction. Husniah et al. [12] developed a non-cooperative game model between a manufacturer and OEM considering the availability target of the equipment. In this research, the optimal service option of the manufacturer and the optimal price structure of the OEM were derived. Hong et al. [13] applied mechanism design theory to design and optimize the MSC between a manufacturer and a service provider. To do this, they derived a menu of service contracts that the manufacturer will accept one of the contract alternatives, and at the same time, the service provider maximizes the expected profit of his own. Santana [14] used a Stackelberg game model to address the interaction between a manufacturer and multiple customers and derive the optimal maintenance service strategies for these agents. For a wind turbine, Zheng et al. [15] studied three maintenance scenarios of wind turbines from the service provider's perspective when a minimum pre-specified availability of the equipment during the service period has to be achieved. In this research, the optimal preventive maintenance programs were derived to minimize the contract service cost.

In the most recent research, Liu et al. [16] proposed a cooperative game model between a lessees and a lessor. In the presented model, the optimal effort level of the lessee's and preventive maintenance degree of the lessor were derived to maximize the revenue of leasing parties. Deprez et al. [17] calibrated predictive models to derive the optimal price of the MSC. In this research, for full-service maintenance contracts, the authors proposed a data-driven tariff plan of the service provider to determine which machine profiles at which price should be attracted. Jackson and Pascual [18] applied the cooperative game model to derive the price of the MSC through bargaining. They also studied the effect of maintenance strategy on the availability-based contract.

In all reviewed research, the number of equipment failures over the MSC period was assumed to be random and then assuming a repair cost as a fixed value (the expected cost of each repair) the cost of maintenance service was determined. However, not only the number of failures over the MSC is random, but also for a given failure, the cost of repair is also random and it depends on the failed part, type of repair, repair time, failure time, labor cost, cost of spare part, etc. Therefore, considering the repair cost as a random variable helps to more accurately estimate the service cost. In addition, the failure time may also affect the repair cost, and since there may be a considerable time delay between the time of MSC sale and repair payment time, incorporating the time value of money helps to more realistic pricing of the MSC. Moreover, conducting preventive maintenance during the contract period may help the service provider to control the degradation process of the equipment and its corresponding cost. So, assuming the periodic preventive maintenance, the optimal number and the level of preventive maintenance are derived to minimize the price of the MSC. As expected, the presented model provides a good insight into the servicing cost of the MSC and its price from the service provider's perspective.

3 | Optimal maintenance service contract model

3.1 | General mathematical optimization model

For a given equipment item with a past age of A the manufacturer decides to outsource the maintenance of the equipment to a maintenance service provider and L is the length of the service period. According to the maintenance agreement, the service provider: 1) conducts periodic adjustable preventive repairs to control the degradation process of the equipment, and 2) minimally repairs all failures during the MSC period. Suppose that failures over the past life were minimally repaired. The preventive and corrective repair times are negligible, and the service provider pays the repair bills over the MSC period.

Let the random variable T be the time to the first failure of the equipment and $f_T(t)$ as well as $h_T(t)$ are its PDF and the hazard rate function respectively. The service provider carries out N periodic preventive maintenance at $t_j = A + j\Delta$, for $j = 1, \dots, N$ where $\Delta = \frac{L}{N+1}$ is the time between two successive PMs. (t_{j-1}, t_j) denotes the j^{th} $(j = 1, \dots, N + 1)$ PM interval where $t_0 = A$ and $t_{N+1} = A + L$. According to Malik's PM model [19], when an imperfect preventive repair is carried out at the time of t, the age of the equipment reduces from t to $\frac{t}{\gamma}$ where $\gamma \geq 1$, and γ is the improvement level. For $\gamma = 1$ the repair is minimal and $\gamma \to \infty$ means that the repair is perfect, and restores the equipment to the as good as new state. The hazard function of the equipment after a preventive repair will be $h_T(\frac{t}{\gamma})$. In this paper, we assume that the number of preventive repairs as well as the improvement level are the service provider's decision variables.

Since failures over the *i*^th^ PM interval, are minimally repaired at a negligible time, therefore failures follow from a Non-Homogenous Poisson Process (NHPP) with the intensity function of $\lambda(t)$ [20] where $\lambda(t) = h_j(t)$ and $h_j(t)$ can be obtained as follows:

$$h_j(t) = h_T\left(\frac{A + \Delta \sum_{s=1}^{j-1} \gamma^{s-1}}{\gamma^{j-1}} + t - A - (j-1)\Delta\right) \quad for\ j = 1, \dots, N+1 \tag{1}$$

It is worth noting that, *Eq. (1)* can be simply obtained by inductive reasoning. The expected number of failures over the j^{th} PM interval is also given by:

$$E\left[N_f\left(t_{j-1}, t_j\right)\right] = \int_{t_{j-1}}^{t_j} h_j(t)dt \quad , \text{for } j = 1, \dots, N+1 \tag{2}$$

The expected number of failures during the MSC period will be obtained by summing over all intervals.

$$\mathrm{E}[\mathrm{N_f}(\mathrm{A}, \mathrm{A} + \mathrm{L})] = \sum_{\mathrm{j}=1}^{\mathrm{N}+1} \int_{\mathrm{t_{j-1}}}^{\mathrm{t_j}} \mathrm{h_j}(\mathrm{t})\mathrm{dt} \tag{3}$$

In the case of equipment failure, the service provider carries out a minimal repair to restore the equipment instantly. For a given failure, the repair cost is uncertain and it depends on the age of the equipment, failed part, the severity of the failure, repair time and labor, the price of spare parts, etc. Therefore, the corrective repair cost can be modeled as a random variable $C = c_l + (c_u - c_l) \times I$, where c_l and c_u are minimum and the maximum possible repair cost respectively. I is a random variable where $0 \leq I \leq 1$. Since Beta is a flexible distribution defined on the interval [0, 1], without loss of generality, let I following from a Beta distribution $I \sim Beta(\alpha, \beta)$ and α and β are shape parameters. Therefore, the PDF of C will be as follows:

$$f_C(c) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)(c_u - c_l)} \left(\frac{c - c_l}{c_u - c_l}\right)^{\alpha - 1} \left(1 - \left(\frac{c - c_l}{c_u - c_l}\right)\right)^{\beta - 1} \tag{4}$$

It is worth noting that, C is a function of random variable I, and consequently C will be a random variable. To compute the PDF of C one can compute the CDF of C. Then, the first order derivative of CDF relative to c will be the PDF of C.

As stated before, in this paper, we model the uncertainty of a repair cost as a random variable. However, one can use fuzzy numbers [21], [22], hesitant fuzzy numbers [23], [24] as well as the interval-valued fuzzy numbers [25] to address the repair cost.

Considering two *Eqs. (3)* and *(4)* expected cost of failure repair over the MSC period can be obtained as follows:

$$E\big[C_f(A, A+L)\big] = \sum_{j=1}^{N+1} \int_{c_l}^{c_u} \int_{t_{j-1}}^{t_j} c h_j(t) f_C(c) dt dc \quad (5)$$

Using *Eq. (5)*, one can estimate the expected cost of failure repairs over the maintenance service period. However, the cost of PM also needs to be estimated. As expected, the cost of PM increases when 1) the equipment deteriorates over time and 2) the service provider puts much effort to improve the health state of the equipment. Therefore, the cost of a preventive repair can be modeled as a function of the equipment's age and improvement level γ as follows:

$$C_p(\gamma, t) = c_0 + c_1 \gamma^u t^v \quad (6)$$

where c_0 is the fixed cost of a PM, c_1, u and v are the parameters of the cost function and they can be estimated using historical data. It is worth to note that the cost of a PM assumed to be deterministic, and the presented cost function is an extension of the Chattopadhyay and Murthy [26] cost model.

Considering all PM, the total cost of PM during the MSC period will be:

$$TC_p = \sum_{k=1}^{N} [c_0 + c_1 \gamma^u (A + k\Delta)^v] \quad (7)$$

As seen, in *Eqs. (5)* and *(7)* the effect of repair cost inflation, as well as the time value of repair cost, are not addressed. To estimate the time value of maintenance service cost at the time of MSC selling, let i be the inflation rate of a repair cost and r be the discount rate. Introducing the time value of cost to *Eqs. (5)* and *(7)*, the expected cost of failure and preventive repairs at the beginning of the service period can be given by *Eqs. (8)* and *(9)* respectively.

$$TVE\big[C_f(A, A+L)\big] = \sum_{j=1}^{N+1} \int_{c_l}^{c_u} \int_{t_{j-1}}^{t_j} \frac{c(1+i)^{t-A}}{(1+r)^{t-A}} h_j(t) f_C(c) dt dc \quad (8)$$

where $TVE\big[C_f(A, A+L)\big]$ denotes the time value of failure repair cost at the time of MSC selling. The time value of the total PM cost will also be:

$$TVTC_p = \sum_{k=1}^{N} \frac{(1+i)^{k\Delta}}{(1+r)^{k\Delta}} [c_0 + c_1 \gamma^u (A + k\Delta)^v] \quad (9)$$

where $TVTC_p$ denotes the time value of the total preventive repairs cost.

Suppose that the service provider decides to offer MSC with m marginal profit ($0 \leq m < 1$). In such a situation, the expected price of MSC will be as follows:

$$P_{MSC}(\gamma, N) = (1+m)\, \big[TVE\big[C_f(A, A+L)\big] + TVTC_p\big] \quad (10)$$

where P_{MSC} is the expected price of the MSC from the service provider's perspective. In the presented model, the service provider chooses the number of PM during the MSC period (N) and the improvement level (γ) to minimize the expected price of MSC. Therefore, the final mathematical optimization model from the service provider's perspective will be as follows:

$$\begin{aligned} &\min P_{MSC}(\gamma, N) \\ s.t \quad &\gamma \geq 1 \\ &N \in \{0,1,2,\dots\} \end{aligned} \tag{11}$$

In the presented model, along with N and γ, the random nature of failure repair cost, cost inflation, time value of money, and the profit margin of the service provider are considered to derive an optimal design of MSC.

In the following subsection, for a special case where the time to the first failure of the equipment follows from Weibull distribution, the mathematical model will be derived.

3.2 | Mathematical optimization model for a special case

In this subsection for a special case where the time to the first failure of the equipment follows from Weibull distribution, we extract the mathematical optimization model. To do this, let $T{\sim}Weibull(a, b)$ and a and b denote the scale and shape parameters respectively. The hazard function of T is given by:

$$h_T(t) = \frac{b}{a^b} t^{b-1} \tag{12}$$

We introduce *Eq. (12)* into *Eqs (8)* and *(9)*, then into *Eq. (10)*, to derive the closed form of the mathematical model as follows:

$$\begin{aligned} \min \quad &(1+m) \sum_{j=1}^{N+1} \int_{c_l}^{c_u} \int_{t_{j-1}}^{t_j} \frac{c(1+i)^{t-A}}{(1+r)^{t-A}} \times \frac{b}{a^b} \left(\frac{A + \Delta \sum_{s=1}^{j-1} \gamma^{s-1}}{\gamma^{j-1}} + t - A - (j-1)\Delta \right)^{b-1} \\ &\times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)(c_u - c_l)} \left(\frac{c - c_l}{c_u - c_l} \right)^{\alpha-1} \left(1 - \left(\frac{c - c_l}{c_u - c_l} \right) \right)^{\beta-1} dtdc \\ &+ (1+m) \sum_{k=1}^{N} \frac{(1+i)^{k\Delta}}{(1+r)^{k\Delta}} [c_0 + c_1 \gamma^u (A + k\Delta)^v] \\ s.t \quad &\gamma \geq 1 \\ &N \in \{0,1,2,\dots\} \end{aligned} \tag{13}$$

As seen, the presented model is a mixed-integer nonlinear programming problem and can be solved using conventional methods like the Interior-Point method. For more information about the solution, please see [27].

4 | A numerical Example and Sensitivity Analysis

Suppose that for a given equipment item, a manufacturer outsources the maintenance to a maintenance service provider. According to the service agreement, the service provider is responsible for 1) conducting periodic preventive maintenance, and 2) minimally repairing the equipment failures over the service period. Let the time to the first failure of equipment following from Weibull distribution and the parameters of the model are as shown in *Table* (1).

Table 1: Model parameters and their values

The parameter of the Model	*The value of the parameter*
Equipment age at the time of selling MSC	$A = 5$ (years)
Length of MSC	$L = 2$ (year)
Time to the first failure of equipment	$T \sim Weibull(a = 1.2\,, b = 1.5)$
Parameters of the failure repair cost	$\alpha = 5, \quad \beta = 4,$ $c_l = 200$ (Dollar), $c_u = 1000$ (Dollar)
Parameters of the preventive maintenance cost	$c_0 = 50$ (Dollar), $c_1 = 20\,, u = 1.2\,, v = 1.1$
Parameters of the time value of cost	$i = 0.15\,, r = 0.20$
Service provider's profit margin	$m = 0.15$

Considering the above parameters, the optimal design of the MSC contracts is determined by solving the *Eq. (13)*. The results show that if the service provider plans to carry out PM every eight months i.e. ($N^* = 2$, $\Delta^* = 0.667\ years$, $t_1^* = 5.667$, $t_2^* = 6.333$) with the optimal improvement level $\gamma^* = 2.007$, the price of MSC can be minimized to 4125.6 (Dollars). For different numbers of PM (N) during the MSC period, and for different values of the improvement factor (γ), the price of MSC is derived and presented in *Fig 1*.

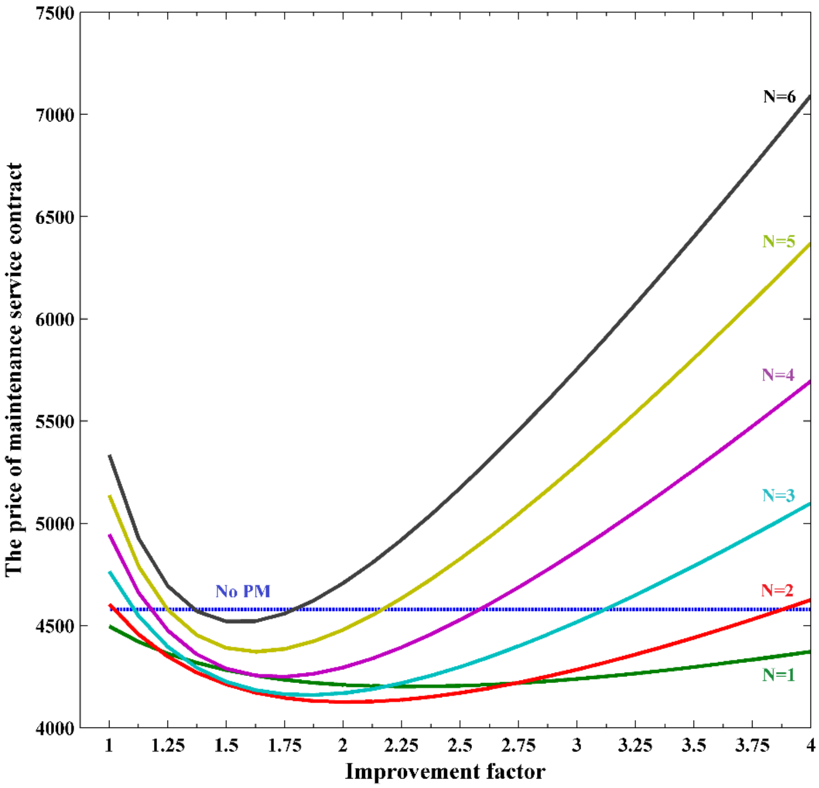

Fig. 1. The price of MSC for a different number of PMs and different values of improvement factor

As seen in Fig. 1, if the service provider decides to offer MSC with no PM, the price will be 4579.3 (Dollars). However, designing optimal MSC with two PMs and the optimal improvement level $\gamma^* = 2.007$ can 10 percent reduce the price. As seen, improper PM policy will also increase the service cost of MSC and its price.

In the next section, a comprehensive sensitivity analysis is presented.

5 | Comprehensive Sensitivity Analysis

In order to study the effect of parameters on the optimal price of the MSC, a comprehensive sensitivity analysis is carried out. In the first step, the effect of the parameters of $T \sim Weibull(a, b)$ i.e the shape parameter (a) and the scale parameter (b) on the price of MSC are conducted. The result is shown in *Table* 2.

Table 2: The effect of time to first failure (Weibull distribution) parameters on the price of MSC

Values of scale parameter (a)	Values of shape parameter(b)										
	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
1	1421.4	1924.0	2582.9	3443.1	4329.3	5086.2	5749.6	6340.8	6898.0	7465.2	8099.6
1.1	1291.5	1731.9	2303.5	3041.8	3851.3	4575.3	5179.4	5725.4	6233.2	6722.3	7238.2
1.2	1184.5	1574.4	2074.6	2716.3	3453.6	4125.8	4704.4	5214.1	5682.6	6123.5	6556.8
1.3	1093.0	1441.0	1884.9	2448.4	3127.0	3752.3	4292.4	4767.9	5201.8	5613.6	6011.4
1.4	1014.9	1328.3	1725.0	2223.0	2849.7	3427.0	3944.3	4391.9	4794.4	5166.3	5530.5
1.5	947.3	1231.7	1587.0	2032.1	2587.5	3130.8	3622.3	4048.3	4429.9	4788.3	5121.0
1.6	888.0	1147.0	1469.7	1868.8	2363.3	2878.6	3347.9	3757.8	4121.4	4443.3	4753.8
1.7	835.8	1073.0	1366.2	1727.3	2171.2	2659.8	3110.7	3493.4	3829.6	4142.0	4440.0
1.8	789.4	1007.6	1275.4	1603.1	2004.5	2470.2	2881.2	3248.0	3577.7	3880.5	4143.8
1.9	747.8	949.4	1194.9	1495.0	1858.4	2297.7	2676.4	3035.1	3356.5	3628.4	3889.9

As seen in *Table* 2, the scale and shape parameters have inverse effects on the price of MSC. In other words, increasing the scale parameter (a) the MSC price decreases, while increasing the shape parameter (b) leads to an increase in the price. This behavior relies on the fact that, with increasing the scale parameter of Weibull distribution, one can expect that the time to first failure increases, and in turn, the equipment experiences less failure.

In the next step, the effect of repair cost parameters on the MSC price is studied. To do this for a different combination of shape parameters α and β, the price of the MSC are determined (see *Table* 3).

Table 3: The effect of random repair cost (Beta distribution) parameters on the price of MSC

Values of scale parameter α	Values of shape parameter β									
	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5
3	4,306.2	4,084.4	3,898.6	3,740.6	3,603.2	3,477.4	3,360.3	3,255.6	3,163.0	3,081.3
3.5	4,451.8	4,238.2	4,056.1	3,898.6	3,761.3	3,640.0	3,532.6	3,425.8	3,329.2	3,241.8
4	4,571.0	4,368.4	4,190.2	4,034.6	3,898.6	3,778.0	3,668.6	3,571.8	3,477.4	3,387.9
4.5	4,663.4	4,478.4	4,306.2	4,153.9	4,018.4	3,898.6	3,790.1	3,692.6	3,603.2	3,520.1
5	4,743.2	4,571.0	4,406.9	4,259.2	4,125.8	4,006.3	3,898.6	3,800.8	3,710.7	3,628.6
5.5	4,812.5	4,646.3	4,494.8	4,350.5	4,220.8	4,103.3	3,995.7	3,898.6	3,809.6	3,727.0
6	4,872.8	4,712.9	4,571.0	4,432.6	4,306.2	4,190.2	4,084.4	3,987.3	3,898.6	3,817.3
6.5	4,926.2	4,772.5	4,635.0	4,507.0	4,382.7	4,270.1	4,164.9	4,068.4	3,979.8	3,898.6
7	4,973.2	4,825.5	4,692.1	4,571.0	4,451.8	4,341.4	4,238.2	4,143.3	4,056.1	3,973.9

The presented result in *Table* 3 shows that similar to Weibull parameters, the parameter β inversely affects the price of MSC while increasing the shape parameter α causes an increase in MSC price. It is worth noting that for $\alpha = 1$ and $\beta = 1$ the Beta distribution will be equivalent to the Uniform distribution $U(0,1)$, and represents the situation that the cost of a given repair can uniformly take any value between the c_l and c_u. As α increases the bulk of the PDF will shift to the right, and equivalently the cost of a given repair increases. An increase in β shifts the distribution to the left and it means that the cost of a given repair decreases. For the case that both α and β increase, the distribution will narrow, and the variance of repair cost reduces. For more information about the properties of Beta distribution, the interested reader can refer to [28].

As one can expect, the equipment age at the beginning of the service period (A) and the length of the contract period (L) may also affect the price of MSC. To provide a good insight into the effects of A and L, for different values of the equipment age and contract length, the price of MSC was obtained and presented in *Table 4*.

Table 4: The effects of the equipment's past age and the length of the contract on the price of MSC

Equipment age at the time of MSC purchase (A years)	The length of the MSC period (L years) 0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0	342.7	816.0	1,340.8	1,898.3	2,482.2	3,089.5	3,712.8	4,363.8	5,039.2	5,735.4
1	552.8	1,186.2	1,809.8	2,448.7	3,104.0	3,772.6	4,450.6	5,138.0	5,842.8	6,563.6
2	693.5	1,470.1	2,204.3	2,925.8	3,646.2	4,373.9	5,105.6	5,842.9	6,578.4	7,328.6
3	809.9	1,718.3	2,554.9	3,358.0	4,149.8	4,931.3	5,711.5	6,492.2	7,277.3	8,052.8
4	911.6	1,940.1	2,880.9	3,760.4	4,610.5	5,450.5	6,284.8	7,109.5	7,929.2	8,754.3
5	1,003.0	2,110.3	3,159.3	4,125.8	5,048.1	5,950.0	6,825.9	7,695.8	8,562.1	9,409.4
6	1,086.9	2,266.7	3,422.3	4,475.1	5,469.8	6,418.5	7,345.3	8,264.6	9,160.9	10,052.3
7	1,165.0	2,412.7	3,671.0	4,811.1	5,862.6	6,863.9	7,849.0	8,801.2	9,741.3	10,670.9
8	1,237.4	2,550.7	3,910.1	5,115.0	6,230.7	7,300.7	8,331.6	9,324.7	10,312.3	11,265.6
9	1,306.4	2,681.8	4,119.3	5,396.1	6,587.6	7,724.2	8,789.3	9,837.3	10,849.9	11,847.2
10	1,372.0	2,807.2	4,298.7	5,669.0	6,939.0	8,123.8	9,235.8	10,336.5	11,379.6	12,422.3
11	1,434.1	2,926.8	4,471.2	5,931.6	7,277.7	8,500.0	9,679.5	10,808.2	11,900.8	12,966.0
12	1,493.9	3,041.8	4,636.8	6,187.7	7,591.5	8,869.8	10,108.5	11,267.8	12,410.3	13,501.6

According to *Table 4*, as the age of the equipment increases, the equipment is more worn out. The number of failures and corresponding repair costs rise, and consequently the price of maintenance service increases. Similarly, with increasing the length of the MSC period, the number of covered failures and the price of the MSC increases sharply.

As seen in *Eqs. (8)* and *(9)*, the parameters of the time value of money also affect the expected cost of maintenance service and in turn the price of the MSC. Therefore, to address the effect of changes in the inflation rate (i) and discount rate (r), we conduct a sensitivity analysis. The results are shown in *Table 5*.

Table 5: The effects of inflation and discount rates on the price of MSC

Inflation rate (%)	Discount rate (%) 0	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
0	4,292.6	4,194.6	4,101.5	4,013.8	3,930.4	3,850.6	3,775.5	3,703.7	3,634.9	3,569.0	3,506.7
2.5	4,393.6	4,292.6	4,196.7	4,106.5	4,020.5	3,938.5	3,860.0	3,786.1	3,715.4	3,647.5	3,583.4
5	4,495.4	4,390.2	4,292.6	4,198.6	4,110.3	4,025.9	3,946.4	3,869.1	3,796.3	3,727.7	3,660.7
7.5	4,596.4	4,490.1	4,388.1	4,292.6	4,200.4	4,114.8	4,031.9	3,952.6	3,877.8	3,806.2	3,738.5
10	4,699.4	4,589.7	4,486.0	4,386.2	4,292.6	4,203.4	4,118.3	4,036.8	3,959.9	3,886.3	3,816.8
12.5	4,803.3	4,689.7	4,582.7	4,479.8	4,384.2	4,292.6	4,205.1	4,121.6	4,042.7	3,967.0	3,894.4
15	4,905.6	4,789.7	4,679.1	4,575.0	4,476.1	4,382.4	4,292.6	4,206.8	4,125.8	4,048.2	3,973.8
17.5	5,009.7	4,891.3	4,777.4	4,670.0	4,569.0	4,471.1	4,379.6	4,292.6	4,208.4	4,128.7	4,052.5
20	5,114.4	4,992.9	4,876.4	4,765.7	4,662.3	4,562.5	4,468.8	4,377.9	4,292.6	4,209.8	4,131.6
22.5	5,221.5	5,094.7	4,975.2	4,862.0	4,755.4	4,653.7	4,557.7	4,465.3	4,376.2	4,292.6	4,212.5
25	5,325.3	5,196.6	5,075.3	4,958.8	4,849.1	4,744.3	4,646.5	4,551.0	4,460.5	4,374.7	4,292.6

As seen in *Table 5*, the inflation of the service cost causes an increase in the price of the MSC. In contrast, increasing the discount rate will decrease the present value of service cost and in turn, decreases the price of the MSC. It is worth noting that the inflation rate of repair costs can be interpreted as a rate of increase in general repair bills. The discount rate is the rate the service provider expects to return spending on paying repair bills. As expected, the discount rate is a function of the inflation rate, risk of return, external rate of return, etc. Therefore one can expect that $r > i$.

In the final sensitivity analysis, we study the effect of changes in the parameters of the preventive maintenance cost function on the price of the MSC. To do this, for a different values of c_0 and c_1, we derive the minimum price of MSC. The results are presented in *Table 6*.

Table 6: The effects of preventive maintenance cost function parameters on the price of MSC

		c_1 ($)								
		0	**5**	**10**	**15**	**20**	**25**	**30**	**35**	40
c_0 ($)	0	1,672.4	2,659.8	3,284.3	3,707.9	3,994.4	4,189.1	4,332.9	4,411.6	4,483.5
	12.5	1,768.9	2,756.3	3,365.1	3,763.0	4,035.7	4,216.7	4,346.7	4,425.4	4,497.3
	25	1,865.4	2,852.7	3,434.0	3,813.6	4,070.7	4,244.2	4,360.5	4,439.2	4,511.0
	37.5	1,961.8	2,949.2	3,501.9	3,854.9	4,098.2	4,271.8	4,374.2	4,452.9	4,524.8
	50	2,058.3	3,034.7	3,557.0	3,896.3	4,125.8	4,299.3	4,388.0	4,466.7	4,538.6
	62.5	2,154.7	3,115.6	3,612.1	3,937.6	4,153.4	4,314.7	4,401.8	4,480.5	4,552.4
	75	2,251.2	3,184.4	3,667.3	3,978.9	4,180.9	4,328.5	4,415.6	4,494.3	4,566.1
	87.5	2,347.6	3,253.3	3,709.3	4,010.3	4,208.5	4,342.3	4,429.3	4,508.1	4,579.3
	100	2,444.1	3,310.8	3,750.7	4,037.9	4,236.0	4,356.1	4,443.1	4,521.8	4,579.3
	112.5	2,540.5	3,366.0	3,792.0	4,065.4	4,263.6	4,369.8	4,456.9	4,535.6	4,579.3
	125	2,635.8	3,421.1	3,833.4	4,093.0	4,284.7	4,383.6	4,470.7	4,549.4	4,579.3

As seen in *Table 6*, with increasing both cost parameters i.e. c_0 and c_1, the price of MSC increases. However, changes in the value of the parameter c_1, sharply increase the price, while increasing the fixed cost of preventive repair c_0 has a mild effect. This relies on the fact that, in the preventive maintenance cost function, the value of $c_1$is multiplied by the equipment age and improvement level, and its effect is intensified.

In the next section, the conclusion will be presented.

6 | Conclusion

Due to the complex and stochastic nature of maintenance issues, estimating and pricing MSC can be challenging. As expected, offering MSC at an affordable price will increase the demand and help the service provider to remain in a competitive market while ensuring a profit. In this paper, a mathematical optimization model was derived to determine the minimum price of the MSC from the service provider's perspective. In the presented model, the number and improvement level of preventive maintenance was the service provider's decision variables. The major contributions of the paper were; 1) modeling preventive maintenance during the MSC, 2) considering a given failure repair cost as a random variable, and 3) incorporating the time value of money.

Comparing findings with previously published research reveals that considering the time value of money will help the service provider to address the effect of service cost inflation on the price of the MSC. This could be a major concern, especially in economies with high inflation rates. The research also showed that modeling the repair cost as a random variable, and incorporating its characteristics into the process of service cost estimation, helps the service provider to understand the possible behaviors of service cost as well as the risk of violating service cost from its expected value. As one can expect, higher dispersion of the repair cost can considerably affect the profitability of offering MSC. Results also demonstrated that the price of the MSC heavily depends on the length of the contract and the age of the equipment at the time of MSC purchase. Moreover, a preventive maintenance needs to be carefully planned and improper PM policy not only does not help to reduce the failure repair cost but also it can impose an additional cost.

Although the model provides good insight into the pricing MSC, it can be enriched from different aspects. Considering incomplete repair, upgrade, inspection as well as taking into account the risk attitude of the manufacturer can make the model a more realistic, and they can be seen as future research directions. Moreover, Addressing different maintenance modalities in the MSC model based on the equipment service life can be another interesting future research direction.

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