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A Mathematical Model of the Location Problem for Central and Secondary Warehouses in the Multi-Level Supply Chain Network of Perishable Products

Seyyed Mostafa Mousavi[1], Majid Motamedi[2,*], Rasoul Karimi[1]

[1] Department of Technical and Engineering, Nowshahr Branch, Islamic Azad University, Nowshahr, Iran; mostafa_mousavi85@yahoo.com; rasoulkarimi2005@gmail.com.

[2] Department of Industrial Management, Nowshahr Branch, Islamic Azad University, Nowshahr, Iran; moatamedi.m@gmail.com.

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Abstract

A supply chain is a network that creates and delivers products and services to customers. In this research, the four-level supply chain in the field of food products, including the levels of supplier, customer, and central and secondary warehouses, has been investigated. A mixed integer mathematical model of the research problem is presented to minimize chain costs, including the setting up and preparation of warehouses, transportation between transfer levels, and holding products. The proposed model is developed based on constraints such as inventory, warehouse capacity, vehicle capacity, and multi-period multi-product. The decision variables are determined after solving the model, which includes the optimal number of central and secondary warehouses, the optimal amount of product transferred between the factory and central warehouse, the central warehouse and secondary warehouse, and the secondary warehouse and customer, the optimal amount of product storage in secondary warehouses, the type of vehicle for transportation between levels, and the capacity level of each product in each central warehouse. To validate the proposed model, experiments were conducted using the Kaleh company's real data in GAMS software. Finally, the sensitivity analysis of the model was carried out on two critical parameters influencing decision-making: demand and the cost of increasing the capacity of the central warehouse. The output results confirmed the validity and efficiency of the proposed model.

Keywords: Mathematical model, Multi-product supply chain, Secondary warehouses, Transportation.

1 | Introduction

Applying the Supply Chain Management (SCM) approach in organizations has led to high profits, remarkable financial savings, and significant progress in the last 40 years. In addition, customers have benefited too. That is why this approach has been especially accepted among different organizations and countries, which itself demands more popularity every day. SCM is the management of supply chain activities to maximize customer value and achieve sustainable competitive advantage. Supply chain activities cover everything from product

Corresponding Author: moatamedi.m@gmail.com

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development, sourcing, manufacturing, and logistics, as well as the information systems needed to coordinate these activities. SCM offers opportunities for the product with a positive approach to integration and management within and between companies. Based on the advantages of commercial processes, SCM provides a new method for managing commercial activities and communicating with the other members of the supply chain. Therefore, the concept of SCM has been redefined from the integration of logistic services to the integration and management of key commercial processes along the supply chain.

One of the most fundamental strategic decisions is the design of the supply chain network. The research aims to design a four-level supply chain, including supplier, customer, and central and secondary warehouses. Transportation is the most expensive distribution factor in many industries. Transportation costs include moving costs from factories to warehouses and from warehouses to customers. Transportation costs comprise an average of 45% of total costs in all industries, and another essential factor in this regard is holding costs, which include approximately 19% of total costs. Therefore, the supply chain design of this research should be performed in a multi-period, multi-product environment to minimize the costs of transportation, preparation or setup of multi-level warehouses, and holding products.

A schematic representation of the research problem is shown in *Fig. 1*. As can be seen, the suppliers transport the products by vehicles to the central warehouses. For the transportation of products, four different types of vehicles with predetermined capacity have been used. The products are then transferred from central warehouses to secondary warehouses and finally to customers. One of the objectives of this research is to determine the type and number of vehicles required to transport products from the factory to the central warehouse, from the central warehouse to the secondary warehouse, and from the secondary warehouse to the customers (movement levels).

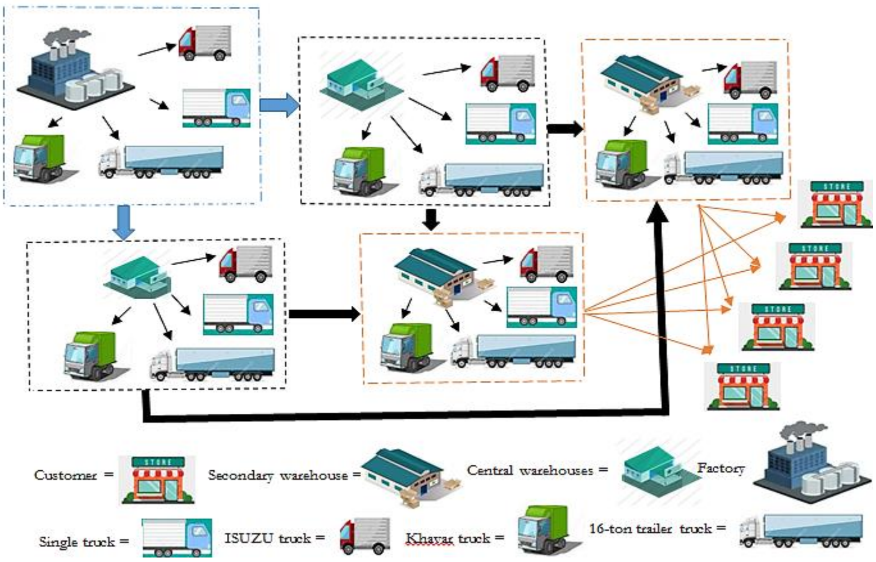

Fig. 1. Research problem schematic.

One of the strategic and vital decisions in supply chain design is determining the optimal number and location of both central and secondary warehouses because it improves processes, reduces costs, and increases customer service. In the research model, it is assumed that there are already warehouses in some places and that it is possible to re-select and change their capacity levels. It should be noted that several capacity levels have been considered for central warehouses, and the optimal capacity level is determined by solving the model. In addition, some new locations have the potential to be warehouses that can be built by investing. Therefore, we have the costs of increasing the capacity level or establishing warehouses. A further goal of this research is to determine the optimal number and location of warehouses (central and secondary) among the previously specified options that minimize the cost of increasing the capacity level, construction, and transportation.

Therefore, the main research question is how to design a multi-period, multi-product supply chain with multi-level warehouses and different vehicles to transfer products from the supplier to the warehouse and finally to the customers to minimize the total chain costs. In this study, we attempted to control the cost components in the supply chain by providing a mathematical model and determining optimal policies to design an efficient and effective supply chain network.

The remainder of the paper is organized as follows. Section 2 provides a review of relevant literature, and at the end, the gap, research model, and assumptions are described. In Section 3, the proposed mathematical model of the research is presented. In Section 4, a case study, implementation, and sensitivity analysis are presented. Finally, Section 5 presents the conclusions and future studies.

2 | Literature Review

The scientific and academic attitude toward the subject of SCM has been very positive, and many papers and books have been written in this field in recent years. The literature review section refers to several instances of papers on multi-level chains with the title of Location Problem (LP) in various fields.

Mayer and Wanger [1] studied an unbounded multiple distribution of LP. They presented a new branch-and-bound method called HubLocator to generate optimal solutions. The lower bound, which was the main component of their method, was determined in two steps. The upper bounds were calculated by considering complementary slackness conditions. Marianov and Serra [2] developed a distribution center location model to minimize total costs, including fixed and transportation costs. Due to the computational complexity of the formulation, the model was solved using a heuristic based on tabu search. Martin and Roman [3] developed a mathematical model that analyzed the competition for hub location when airlines operate in illegal intercontinental markets. The entire process was considered to be dynamically a complete game. Topcuoglu et al. [4] presented a novel and robust solution based on a genetic search framework for the uncapacitated single allocation hub LP.

Yaman et al. [5] presented a 0-1 Mixed Integer Programming (MIP) model for ground-transportation-based parcel-delivery services with stopovers for the latest arrival hub LP. Azadeh et al. [6] presented an integrated hierarchical approach for the location of solar plants using principal component analysis, data envelopment analysis, and numerical taxonomy. The implementation of their proposed approach led to the selection of the best possible location to build a solar power plant at the lowest cost. Contreras et al. [7] presented a hub tree LP that combined several aspects of the investigated problem, including location, network design, and routing. They proposed an MIP formulation to minimize transportation costs through the network. Sender and Clausen [8] presented a new model for the hub LP to design a wagon load traffic network. The strategic objective was to determine the location, size, and function of the formation yards and the connections between them.

Gelareh and Nickel [9] investigated an uncapacitated multiple allocation hub LP for urban transport and liner shipping networks. They described the problem in the form of a mathematical model and proposed an efficient greedy heuristic to solve the NP-hard problem under consideration. Alumur et al. [10] studied hub LPs under uncertainty in setting costs and demands. They presented generic models representing these different sources of uncertainty for the single and multiple allocation versions of the problems. Vera et al. [11] proposed an MIP formulation to determine the location of a fixed number of facilities to maximize their utilization. They applied the heuristic concentration integer procedure to solve larger instances of the problem. Chang et al. [12] investigated the LP of Taiwanese service apartments. The fuzzy Delphi method, analytic network process, and technique for order preference by similarity to the ideal solution were integrated to select the optimal locations effectively.

Darestani and Hemmati [13] designed a supply chain network for perishable products under uncertain conditions. The objective function considered for the proposed model included minimizing total network costs and greenhouse gas emissions. Three multi-criteria decision-making methods were used to solve a two-objective model. Aazami and Saidi-Mehrabad [14] developed a new multi-period production–distribution planning method for fixed-life perishable products in a seller-buyer system. A hierarchical heuristic approach based on the benders' decomposition algorithm and genetic algorithm was proposed to maximize the seller's profit subject to the buyer's optimality in a three-level supply chain, including factories, distribution centers, and retailers. Tavana et al. [15] developed a bi-objective MILP model to solve location-inventory-routing problems in green supply chains with low-carbon emissions under uncertainty. The proposed model was solved using a weighted fuzzy multi-objective solution approach coupled with an intelligent simulation algorithm to ensure the feasibility of the solution space. Shavarani et al. [16] developed a multi-objective mathematical model to calculate the optimal number and location of facilities among a set of candidate locations to simultaneously minimize the total travel distance, costs, and lost demand.

Liu et al. [17] presented a comprehensive study aimed at providing a combined framework for the location of emergency facilities in transportation networks. Ghasemi and Khalili Damghani [18] proposed a robust simulation-optimization approach for multi-period location-allocation-inventory planning before a disaster that determined the location of distribution centers and suppliers and how to allocate them to affected areas. Liu et al. [19] proposed an integrated location-inventory-routing model for perishable products, which considered carbon emission factors and product freshness. A multi-objective planning model was developed to achieve the lowest economic cost and carbon emissions and the highest product freshness. Constraints were established regarding the actual location-inventory-routing situation. Azami et al. [20] developed a bi-objective optimization model for integrated production–distribution planning of perishable products under uncertainty. The objective functions were to maximize profit in a specific supply chain with three levels of plants, distribution centers, and customers and to minimize gas emissions. Robust optimization was used to deal with the operational uncertainty of some cost parameters. Acevedo-Chedid et al. [21] presented a transport model incorporating a cross-dock system to deliver goods from production plants to markets efficiently. The model incorporated a vehicle routing model that considered time windows for pick-ups and deliveries, optimal cross-dock center locations, a heterogeneous vehicle fleet of limited capacity, and scheduling product collections, arrivals, and departures. They proposed a mixed-integer non-linear optimization model to effectively minimize logistics costs and environmental impacts by considering various parameters such as speed, waiting times, loading and unloading times, and costs associated with the entire operation.

The literature review is summarized in *Table 1*. As recently reviewed, the supply chain problem has received more attention because of its academic importance and wide application in the real world. Although there have been studies on the problem of the supply chain with characteristics such as multi-period, multi-product, the number and location of central and secondary warehouses with limited capacity, and holding products individually or a combination of several characteristics mentioned in these years, the advantage of the present research is to consider the mentioned characteristics simultaneously. In addition, several capacity levels are defined for each product in each central warehouse, and the optimal capacity level for each product must be determined. In addition, different types of vehicles are considered between movement levels. To reduce transportation costs, the right vehicle should be chosen according to the amount of product movement. Therefore, the problem is considered more realistically and becomes an integrated SCM problem. The assumptions of the research problem are presented as follows.

The supply chain consists of four levels, including the supplier, central warehouse, secondary warehouse, and customer. In addition, this multi-product and multi-period supply chain is considered. The geographic locations of customer and supplier locations are determined outside the scope of the research model. Still, the locations of the central and secondary warehouses are determined by the model from among the previously specified options. The central warehouse is considered to have multiple capacities, and the model is only allowed to use a maximum of one capacity level. There are several transportation modes for moving products between levels. The capacity of the vehicles is known, but the model determines the number of vehicles needed. In this study, the possibility of a vehicle breakdown is not considered. In addition, there is no shortage of products in this model. It is possible to store products for future periods. The proposed model is single-objective and minimizes costs.

One of the essential issues in the supply chain is how to communicate optimally between the levels of the chain and transfer products between them. To design a supply chain with maximum efficiency, the optimal values of the system parameters at the macro level and the optimal policies at the lower levels of the chain must be achievable. In the following section, the operational research model of the above system is described using mathematical modeling techniques.

Table 1. Summary of the reviewed papers.

First Author and the Year	Problem Statements Period							Objectives		Modeling Approach	Case Study
	Single	Multi	Multi-Product	Holding Products	Warehouse with Multi-Level Capacity	Different Types of Vehicles	Secondary Warehouse	Single	Bi/ Multi		
Mayer and Wagner [1]	×					×		×		Linear programming	
Marianov and Serra [2]	×				×			×		Integer programming optimization	
Topcuoglu et al. [4]	×							×		Mixed integer linear programming	
Yaman et al. [5]	×					×		×		Zero-one MIP	×
Azadeh et al. [6]	×							×		Data envelopment analysis	×
Gelareh et al. [9]	×							×		MIP	×
Alumur et al. [10]	×								×	Robust optimization	×
Aros-Vera et al. [11]	×							×		Mixed linear programming	
Avakh Darestani et al. [13]		×	×	×	×				×	Multi-objective optimization problems	
Tavana et al. [15]		×	×	×					×	Bi-objective mixed integer linear programming	
Aazami et al. [14]		×		×	×			×		Mixed-integer non-linear bi-level programming	×
Shavarani et al. [16]	×								×	Mixed-integer non-linear programming	×
Ghasemi and Khalili-Damghani [18]		×		×	×			×		Multi-period location–allocation-inventory problem	×
Aazami et al. [20]		×	×	×	×				×	Bi-objective optimization model	×
Liu et al. [17]		×	×	×	×	×			×	Multi-objective problem	×
Acevedo-Chedid et al. [21]						×			×	Mixed-integer non-linear optimization	×
This research		×	×	×	×	×	×	×		Mixed integer mathematical model	×

3 | Proposed Mathematical Model

A 0-1 MIP model has been presented to describe the problem in more detail. The purpose of presenting the MIP formulation of the research problem is not to use the formulation in developing a solution procedure but to precisely describe the problem. The indices, input parameters, decision variables, and mathematical model are as follows.

Indices

Indices used to model our problem are listed below:

i	Index for products, $i = 1, 2, 3, \ldots, I$.
d	Index for central warehouses, $d = 1, 2, 3, \ldots, D$.
s	Index for secondary warehouses $s = 1, 2, 3, \ldots, S$.
c	Index for customers, $c = 1, 2, 3, \ldots, C$.
m	Index for the mode of transportation, $m = 1, 2, 3, \ldots, M$.
k	Index for capacity levels, $k = 1, 2, 3, \ldots, K$.
t	Index for periods, $t = 1, 2, 3, \ldots, T$.

Input parameters

$cost_{ikd}^{dist}$	Cost of setting up/increasing capacity for central warehouse d at capacity level k for product i.
$cost_{s}^{strg}$	Cost of setting up a secondary warehouse s.
cap_m	Transport mode capacity m.
cap_{is}^{strg}	Secondary warehouse capacity s for product i.
cap_{ikd}^{max}	Maximum capacity of the central warehouse d for product i at capacity level k.
h_{it}	Cost of holding product i in period t.
dem_{ict}	Demand for product i by customer c in period t.
$trn_{imdt}^{sup-dist}$	Cost of transporting each unit of product i from the factory to the central warehouse d with the mode of transportation m in period t.
$trn_{imdst}^{dist-strg}$	Cost of transporting each unit of product i from the central warehouse d to the secondary warehouse s with transportation mode m in period t.

Decision Variables

X_{ikd}^{dist}	1 if the capacity level k of the central warehouse d is set up for product i and 0 otherwise.
X_{s}^{strg}	1 if secondary warehouse s is set up and 0 otherwise.
$x_{imdt}^{sup-dist}$	Number of product i transferred from factory to central warehouse d by transportation mode m in period t.
$x_{imdst}^{dist-strg}$	Number of product i transferred from central warehouse d to secondary warehouse s by transportation mode m in period t.
$x_{imsct}^{strg-cus}$	Number of product i transferred from secondary warehouse s to customer c by transport mode m in period t.
inv_{ist}	Number of product i stored in secondary warehouse s in period t.
f_m	Number of transport modes m to move the product from the factory to the central warehouse.
z_m	Number of transport modes m to move the product from the central warehouse to the secondary warehouse.
y_m	Number of transport modes m to move the product from the secondary warehouse to the customer.

3.1 | Mathematical Formulation

The problem can now be formulated as

$$\text{Min } Z=\sum_{i,k,d} cost_{ikd}^{dist}\times x_{ikd}^{dist}+\sum_{s} cost_{s}^{strg}\times x_{s}^{strg}+\sum_{i,m,d,t} trn_{imdt}^{sup-dist}\times x_{imdt}^{sup-dist}+\sum_{i,m,d,s,t} trn_{imdst}^{dist-strg}\times x_{imdst}^{dist-strg}$$
$$+\sum_{i,m,s,c,t} trn_{imsct}^{strg-cus}\times x_{imsct}^{strg-cus}+\sum_{i,s,t} inv_{ist}\times h_{it}.$$

$$\sum_{m} x_{imdt}^{sup-dist}+bigm\times(1-x_{ikd}^{dist})\geq cap_{ikd}^{max},\qquad \text{for all } i,k,d,t. \tag{1}$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq cap_{i(k+1)d}^{max}+bigm\times(1-x_{ikd}^{dist}),\qquad \text{for all } i,k,d,t. \tag{2}$$

$$\sum_{k} x_{ikd}^{dist}\leq 1,\qquad \text{for all } i,d. \tag{3}$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq \sum_{k} cap_{ikd}^{max},\qquad \text{for all } i,d,t. \tag{4}$$

$$\sum_{m,s} x_{imdst}^{dist-strg}\leq \sum_{k} cap_{ikd}^{max},\qquad \text{for all } i,d,t. \tag{5}$$

$$\sum_{m,d} x_{imdst}^{dist-strg}\leq cap_{is}^{strg},\qquad \text{for all } i,s,t. \tag{6}$$

$$inv_{ist}=inv_{is(t-1)}+\sum_{m,d} x_{imdst}^{dist-strg}-\sum_{m,c} x_{imsct}^{strg-cus},\ \text{for all } i,s,t>1. \tag{7}$$

$$inv_{is1}=\sum_{m,d} x_{imds1}^{dist-strg}-\sum_{m,c} x_{imsc1}^{strg-cus},\qquad \text{for all } i,s. \tag{8}$$

$$\sum_{m} x_{imdt}^{sup-dist}\geq \sum_{m,s} x_{imdst}^{dist-strg},\qquad \text{for all } i,d,t. \tag{9}$$

$$\sum_{m,s} x_{imsct}^{strg-cus}\geq dem_{ict},\qquad \text{for all } i,c,t. \tag{10}$$

$$\sum_{m} x_{imdt}^{sup-dist}\leq bigm\times\sum_{k} x_{ikd}^{dist},\qquad \text{for all } i,d,t. \tag{11}$$

$$\sum_{m} x_{imdst}^{dist-strg}\leq bigm\times x_{s}^{strg},\qquad \text{for all } i,d,s,t. \tag{12}$$

$$\sum_{i,m,d} x_{imdt}^{sup-dist}\leq \sum_{m} cap_{m}\times f_{m},\qquad \text{for all } t. \tag{13}$$

$$\sum_{i,m,d,s} x_{imdst}^{dist-strg}\leq \sum_{m} cap_{m}\times z_{m},\qquad \text{for all } t. \tag{14}$$

$$\sum_{i,m,s,c} x_{imsct}^{strg-cus}\leq \sum_{m} cap_{m}\times y_{m},\qquad \text{for all } t. \tag{15}$$

The objective function is formulated in the form of cost minimization, which includes the cost of setting up both the central and secondary warehouses, the cost of transferring products between levels, and the cost of holding. *Constraint Sets (1)* and *(2)* determine the level of central warehouse capacity that should be used for each product. *Constraint Set (3)* ensures that approximately one level of central warehouse capacity is used. *Constraint Sets (4)-(6)* ensure that the capacity of the central and secondary warehouses is not exceeded. *Constraint Sets (7)* and *(8)* are related to the inventory balance and the determination of the inventory amount of each product in each period. *Constraint Set (9)* states that the output of each central warehouse should not exceed its input. *Constraint Set (10)* guarantees that customers' demands are satisfied. The condition of product transfer between levels is that the central and secondary warehouses have been set up, which are given in *Constraint Sets (11)* and *(12)*, respectively. Non-exceeding the capacity of the transportation mode (vehicles) and determining the number of each of them between the levels of the supplier to the central warehouse, the central warehouse to the secondary warehouse, and the secondary warehouse to the customer are given in *Constraint Sets (13)-(15)*, respectively.

4 | Numerical Result

For the evaluation of the proposed model, a numerical experiment was performed on the data set of the Kaleh company. The dataset related to Kaleh company is as follows: five categories of frozen products, including dairy, sauces, drinks, ice creams, and meats; four modes of transportation, including 16-ton trailer truck, single truck, Khavar truck, and ISUZU truck; three potential places for central warehouses in the cities of Tehran, Karaj, and Qazvin; four potential places for secondary warehouses in East Tehran, West Tehran, Karaj, and Qazvin; three levels of capacity for the central warehouse; and finally 7 time periods (information about one week). Based on the case study data, the model was coded and solved using GAMS optimization software.

4.1 | Results Based on the Model Output

The output of the model for the objective function is equal to 239092847620 Iranian currency units, and the values of the problem decision variables for this value of the objective function are as follows:

According to the output of the model, the value of decision variables $X_{1.1.1}^{dist}$, $X_{2.2.1}^{dist}$, $X_{3.2.1}^{dist}$, $X_{4.2.1}^{dist}$, and $X_{5.2.1}^{dist}$ is equal to one. These results show that first, the central warehouse located in Tehran (d=1) should be set up. Second, product 1 should use the capacity of level 1, and products 2, 3, 4, and 5 should use the capacity of level 2 of this central warehouse. The value of decision variables X_1^{strg} and X_2^{strg} is equal to one, which means that out of four potential secondary warehouses in East Tehran, West Tehran, Karaj, and Qazvin, two secondary warehouses should be established in East Tehran and West Tehran.

Table 2 shows the amount of each product transferred between the factory and central warehouse levels ($X_{imdt}^{sup-dist}$). From this table, it can be seen that only the central warehouse of Tehran (d=1) should be set up, and only mode 1 of transportation (m=1), i.e., a 16-ton trailer, should be used to transport products from the factory to the central warehouse of Tehran. The number of 16-ton trailers needed to move the products is obtained based on the variable output f_m, which is equal to 29 here ($f_1 = 29$).

Table 3 shows the amount of each product transferred between the levels of the central and secondary warehouses ($X_{imdst}^{sup-dist}$). From this table, it can be seen that two potential secondary warehouses in Tehran East and Tehran West (s=1 and 2) should be set up. Only mode 1 of transportation (m=1), i.e., a 16-ton trailer, should be used to transport products from the central warehouse of Tehran to the secondary warehouses in Tehran East and Tehran West. The number of 16-ton trailers needed to move the products is obtained based on the variable output z_m, which is equal to 27 here ($z_1 = 27$).

Table 4 shows the storage amount of each product transferred in each period in each of the secondary warehouses (inv_{ist}). *Table 5* shows the amount of each product between the secondary warehouse and the customer ($X_{imsct}^{strg-cus}$). From this table, it can be seen that modes 3 and 4 of transportation (m=3 and 4), i.e., Khavar and ISUZU trucks, have been used to transport products from the secondary warehouses in Tehran East and Tehran West to the customer. The number of Khavar trucks (m=3) and ISUZU trucks (m=4) needed to move the products are obtained based on the output of variable y_m, which are equal to 136 and 269, respectively ($y_3 = 136$ and $y_4 = 269$).

Table 2. The amount of product transferred between the factory and the central warehouse.

i	m	d	t						
			1	2	3	4	5	6	7
1	1	1	400000	400000	400000	400000	400000	400000	400000
2	1	1	11000	11000	11000	11000	11000	11000	11000
3	1	1	12200	12200	12200	12200	12200	12200	12200
4	1	1	15100	15100	15100	15100	15100	15100	15100
5	1	1	18200	18200	18200	18200	18200	18200	18200

Table 3. The amount of product transferred between the levels of central and secondary warehouses.

i	m	d	s	t						
				1	2	3	4	5	6	7
1	1	1	1	200000	200000	200000	200000	200000	144881	200000
1	1	1	2	181353	137804	166466	181353	160265	0	0
2	1	1	1	5500	5500	5500	5500	5500	5456	0
2	1	1	2	4225	3227	2566	2051	0	0	0
3	1	1	1	6500	6500	6500	6500	6500	4411	6500
3	1	1	2	4946	4770	4946	4946	4946	0	0
4	1	1	1	7000	7000	7000	7000	7000	7000	65
4	1	1	2	4909	5358	0	0	0	0	0
5	1	1	1	8800	8800	8800	8800	8800	7899	7177
5	1	1	2	6037	5661	6933	4730	8041	0	0

Table 4. The storage amount of products in secondary warehouses in each period.

i	s	t					
		1	2	3	4	5	6
1	1	2226	0	0	27693	0	2368
2	1	0	892	1880	527	0	3688
2	2	0	0	0	835	0	0
3	1	0	2346	2585	1280	0	318
4	1	2650	515	0	2123	1217	0
4	2	0	5320	5044	1085	0	0
5	1	0	966	2568	0	0	0

Table 5. The amount of product transferred between the secondary warehouse and the customer.

i	m	s	c	t						
				1	2	3	4	5	6	7
1	3	1	1	0	0	0	0	0	13820	0
1	3	1	2	10517	0	79065	0	0	0	0
1	3	1	3	0	104083	4281	976	65114	0	95113
1	3	1	4	0	11632	103506	130681	144042	25586	0
1	3	2	1	0	10796	0	0	11856	0	0
1	3	2	2	0	0	0	0	75130	0	0
1	3	2	3	0	0	0	124581	0	0	0
1	3	2	4	0	114584	0	0	0	0	0
1	4	1	1	2904	2300	13148	13059	2527	0	0
1	4	1	2	0	84211	0	27591	16010	79958	0
1	4	1	3	39808	0	0	0	0	23149	0
1	4	1	4	144545	0	0	0	0	0	107255
1	4	2	1	13625	0	0	0	0	0	0
1	4	2	2	79354	0	0	56772	0	0	0
1	4	2	3	88374	12424	110088	0	73279	0	0
1	4	2	4	0	0	6378	0	0	0	0
2	3	1	1	44	0	198	0	0	0	0
2	3	1	2	0	1457	1386	0	178	1033	0
2	3	1	3	0	2455	0	3066	2592	0	1770
2	3	1	4	4109	0	291	0	3043	317	1918
2	3	2	1	207	0	0	0	0	0	0
2	3	2	2	0	0	0	1216	0	0	0
2	3	2	4	0	0	2566	0	0	0	0
2	4	1	1	0	232	0	337	214	137	0
2	4	1	2	232	0	0	259	0	0	0
2	4	1	3	1115	0	2637	0	0	281	0
2	4	1	4	0	464	0	3191	0	0	0
2	4	2	2	1489	0	0	0	835	0	0

Table 5. Continued.

i	m	s	c	t						
				1	**2**	**3**	**4**	**5**	**6**	**7**
2	4	2	3	2529	249	0	0	0	0	0
2	4	2	4	0	2978	0	0	0	0	0
3	3	1	1	0	0	0	293	0	399	0
3	3	1	2	0	0	0	1762	1435	1629	0
3	3	1	3	750	1822	555	883	4972	0	0
3	3	1	4	0	483	0	4867		0	0
3	3	2	1	0	0	115	0	200	0	0
3	3	2	2	1281	0	0	0	0	0	0
3	3	2	3	0	1503	4105	4144	0	0	0
3	3	2	4	0	3267	0	0	4746	0	0
3	4	1	1	303	167	24	0	43	0	0
3	4	1	2	166	1682	1558	0	0	0	0
3	4	1	3	0	0	0	0	0	991	2932
3	4	1	4	5281	0	4124	0	1330	1074	3886
3	4	2	3	3516	0	0	0	0	0	0
3	4	2	4	149	0	726	802	0	0	0
4	3	1	1	554	1085	82	0	373	274	0
4	3	1	2	0	0	39	0	52	0	0
4	3	1	3	0	4182	0	0	4454	3874	31
4	3	1	4	0	0	0	644	3027	4032	34
4	3	2	1	0	0	0	337	0	0	0
4	3	2	2	80	38	0	0	0	0	0
4	3	2	3	3619	0	0	0	0	0	0
4	3	2	4	0	0	0	0	1085	0	0
4	4	1	1	0	0	0	72	0	0	0
4	4	1	2	17	8	0	62	0	37	0
4	4	1	3	771	0	3401	4099	0	0	0
4	4	1	4	3008	3860	3993	0	0	0	0
4	4	2	1	0	0	276	0	0	0	0
4	4	2	4	1210	0	0	3622	0	0	0
5	3	1	1	0	414	0	0	0	0	0
5	3	1	2	0	0	249	0	333	1394	0
5	3	1	3	0	930	0	6265	0	2919	0
5	3	1	4	0	6213	6360	0	7440	3162	0
5	3	2	1	0		0	0	295	0	0
5	3	2	2	0	1298	0	0	1562	0	0
5	3	2	3	0	4363	5367	0	6184	0	0
5	3	2	4	0	0	0	4730	0	0	0
5	4	1	1	583	0	85	493	63	424	0
5	4	1	2	112	277	0	1367	0	0	0
5	4	1	3	1292	0	504	0	964	0	3445
5	4	1	4	6813	0	0	3243	0	0	3732
5	4	2	1	0	0	399	0	0	0	0
5	4	2	2	1525	0	1167	0	0	0	0
5	4	2	3	4512	0	0	0	0	0	0

4.2 | Sensitivity Analysis of the Demand Parameter

Demand is one of the most important parameters whose changes affect supply chain decisions and costs. In this section, a sensitivity analysis is performed on the product demand parameter to examine the validity of the proposed model more precisely. Two categories of scenarios for demand, one based on demand reduction (demands less than the initial value) and another based on demand increase (demands greater than the initial value), are considered for sensitivity analysis.

In the first scenario, it is expected that the value of the objective function will not deteriorate (that is, it will not increase) as demand decreases. The justification for cost reduction is that a decrease in demand decreases the movement of products in the chain. Therefore, the costs of transporting products are reduced accordingly.

For the construction of the first problem of this scenario, the demand for the products is reduced to 95% of the initial value ($0.95 \times dem_{ict}$). The model with new demand information was implemented in the software, and the output of its objective function reached 227189486340. The demand for products is reduced to 90, 85, 80, 75, and 70% of the initial value, respectively, due to the construction of the problems 2, 3, 4, 5, and 6 of the first scenario. The output of the objective function has reached the numbers 215278812940, 203369943320, 191458924220, 179548583840, and 167640015280, respectively.

The change trend in the objective function for the demand reduction scenario is shown in *Fig. 2*. It can be seen that as demand decreases, the value of the objective function decreases, too. Therefore, the results of the model obtained in the sensitivity analysis were consistent with reasonable and logical expectations.

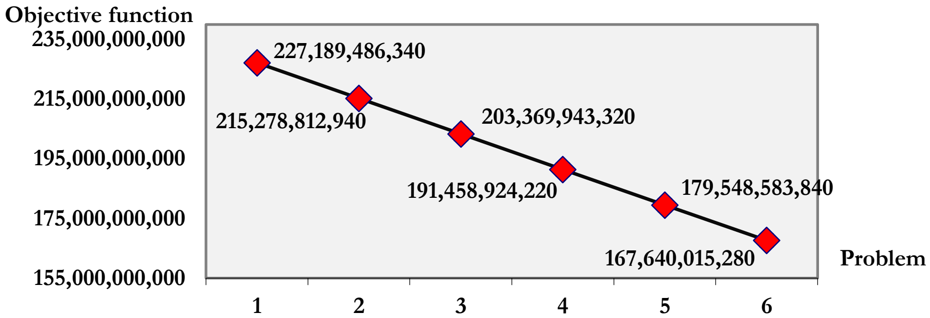

Fig. 2. The value of the objective function in the demand reduction scenario.

In the second scenario, it is expected that the value of the objective function will not improve (that is, it will not decrease) as demand increases. The justification for the rise in cost is that the increase in demand increases the movement of products in the chain. Therefore, the costs of transporting products are increased accordingly.

For the construction of the first problem of this scenario, the demand for the products is increased by 2% of the initial value ($1.02 \times dem_{ict}$). The model with new demand information was implemented in the software, and the output of its objective function reached 243862966880. The demand for products is increased by 4, 6, and 8% of the initial value, respectively, due to the construction of problems 2, 3, and 4 of the second scenario, and the output of the objective function has reached the numbers 248626575100, 253391115640, and 258156712520, respectively. The equivalent model with an increase in demand by 10% of the initial value was recognized as infeasible by the software.

The trend of change in the objective function for the demand increase scenario is shown in *Fig. 3*. It can be seen that as demand increases, the value of the objective function increases, too. Therefore, the results of the model obtained in the sensitivity analysis were consistent with reasonable expectations.

Based on the results of the two categories of proposed scenarios, it was found that by changing the parameters, the model changes logically and correctly. This change shows the effectiveness and guarantees the validity of the proposed model.

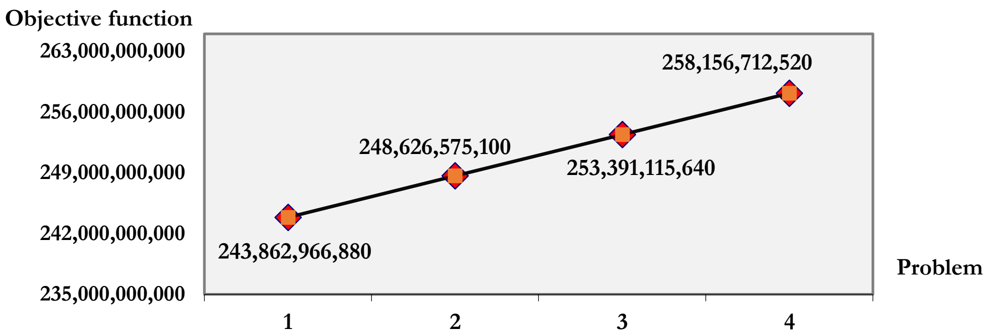

Fig. 3. The value of the objective function in the scenario of increasing demand.

4.3 | Sensitivity Analysis of the Cost Parameter

In this section, sensitivity analysis is performed based on the cost of increasing the capacity of the central warehouse to examine the validity of the proposed model through the cost parameter. To construct problems, the central warehouse capacity costs are increased by 50, 75, 100, 125, 150, 175, and 200% of the initial value to observe how the model behaves with regard to these changes. The model with new cost information was implemented in the software, and the output of the objective function reached 251047490001, 260611203905, 282129560191, 298866259525, 310820701906, and 327557201239, respectively. It can be seen that the increase in the cost of the central warehouse has increased the value of the objective function of the system.

Because the value of the objective function is the sum of several costs, the part related to the cost of increasing the capacity level in the central warehouse has been reduced (due to the higher cost of doing this than before). Instead, the sections related to the costs of setting up the secondary warehouse (due to the construction of a new warehouse), the cost of holding products in the secondary warehouse (due to the completion of the capacity of the central warehouse and the rapid transfer of products from the central warehouse to the secondary warehouse), and the cost of transportation (due to the rapid transfer of products from the central warehouse to the secondary warehouse with vehicles whose capacity level has not been completed) have increased. The changes in costs have been such that the increase in costs was more than the decrease in costs; therefore, the total cost of the whole system has increased.

5 | Conclusions

Supply chain network design is one of the most fundamental strategic decisions. This research aimed to design a four-level supply chain, including supplier, customer, and central and secondary warehouses. The advantage of the present research is that it simultaneously considers characteristics such as multi-period, multi-product, the number and location of central and secondary warehouses with limited capacity, and holding products. In addition, several capacity levels were defined for each product in each central warehouse, and the optimal capacity level for each product in the central warehouse was determined. In addition, different types of vehicles were considered for different movement levels. For reducing transportation costs, the best vehicle should be determined according to the amount of product movement. Therefore, the supply chain design of this research should be performed in a multi-period, multi-product environment to minimize the costs of transportation, preparation, and setting up the multi-level warehouses and holding products. To design a supply chain with maximum efficiency, the optimal values of the system parameters at the macro level and the optimal policies at the lower levels of the chain must be achievable. The set of assumptions, constraints, and objectives of the problem were developed and integrated to achieve the aim of the research. It has been attempted to control the cost components in the supply chain by providing a mathematical model and determining optimal policies so that an efficient and effective supply chain network can be designed. The decision variables were determined after solving the model with the real data of Kaleh company, which include the optimal number of central and secondary warehouses, the optimal amount of product transferred between the factory and central warehouse, the central warehouse and secondary warehouse, and the secondary warehouse and customer, the optimal amount of product storage in secondary warehouses, the type of vehicle for transportation between levels, and the capacity level of each product in each central warehouse. Finally, a sensitivity analysis was performed on the demand and the cost of increasing the capacity of the central warehouse parameters, which is essential in decision-making. In fact, this research has developed a decision support tool for the Kaleh company. Many companies and organizations make critical decisions using the experiences of their managers, some of which may fail and harm the organization. Nowadays, organizations cannot rely only on the genius and experience of managers to make decisions; having support tools can significantly help organizations. In the developed model, it is sufficient to provide updated information about the company as input to the system and to receive the results as output. For future research, it is suggested to consider indicators related to green criteria and sustainable development criteria using multi-objective optimization techniques.

Conflicts of Interest

All co-authors have read and agreed with the contents of the manuscript, and there is no financial interest to report. We certify that the submission is original work and is not under review by any other publication.

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