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A Neutral DEA Model for Cross-Efficiency Evaluation

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Abstract

The cross-efficiency method in Data Envelopment Analysis (DEA) has widely been used as a suitable utility for ranking decision-making units. In this paper, to overcome the issue of the existence of multiple optimal solutions in cross-efficiency evaluation, we use the neutral strategy to design a new secondary goal. Unlike the aggressive and benevolent formulations in cross-efficiency evaluation, the neutral cross-efficiency evaluation methods have been developed in a way that is only concerned with their own interests and is indifferent to other DMUs. The proposed secondary goal introduces a new cross-efficiency score by maximizing the sum of the output weights. The first model is then extended to a cross-weight evaluation, which seeks a common set of weights for all the DMUs. Finally, we give two numerical examples to illustrate the effectiveness of the proposed neutral models and the potential applications in ranking DMUs by comparing their solutions with those of alternative approaches.

Keywords: Data envelopment analysis, Cross-efficiency evaluation, Ranking.

1 | Introduction

Data Envelopment Analysis (DEA) was introduced to assess the relative efficiency of a homogeneous group of Decision-Making Units (DMUs), such as banks, industries, police stations, hospitals, tax offices, schools, and university departments [1]-[7]. The traditional DEA methods allow each DMU to generate a set of relative weights. These weights maximize the ratio of aggregated weighted outputs to aggregated weighted inputs while ensuring that the same ratio does not exceed one for all DMUs. The maximum ratio is regarded as the efficiency score for the evaluated DMU. The traditional DEA approaches can separate efficient DMUs from inefficient DMUs, but they cannot discriminate the efficient DMUs with the efficiency score one [8]. In the face of this issue, Sexton et al. [9] suggested the cross-evaluation method as a ranking method in DEA that involves self-evaluation efficiency and peer-evaluation efficiency. The standard cross-efficiency method in the self-evaluation section uses traditional DEA models such as the CCR and BCC models that are constructed based on linear programming.

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These models usually have alternate optimal solutions, so each DMU is assigned a self-evaluation efficiency score and several peer-evaluation efficiency scores. So, regarding the ultimate cross-efficiency score that is calculated based on self-evaluation and peer-evaluation scores for each DMU, the ranking of each DMU can be changed [9]. To overcome this issue and considering that the evaluation strategy and attitude among DMUs significantly impact the weight selection and hence the cross-efficiency scores, Doyle and Green [10] proposed the aggressive and benevolent models which these ideas are widely used in cross-efficiency evaluation. Both models attempt that maximize the efficiency of the DMU under evaluation, but simultaneously the benevolent model maximizes the average efficiency of other DMUs and the aggressive model minimizes the average efficiency of other DMUs. Liang et al. [11] proposed benevolent game cross-efficiency. In this model, a unique set of weights is determined based on the Nash equilibrium and the benevolent strategy. Using the symmetric weight assignment technique, Jahanshahloo et al. [12] suggested a new secondary goal for the evaluation cross-efficiency score. Li et al. [8] considered the reciprocal behaviors among DMUs to address the cross-efficiency evaluation and used a novel threshold value to determine positive or negative reciprocal behaviors by comparing the peer-evaluated efficiency with the threshold value-based efficiency. Chen et al. [13] introduced a meta-frontier analysis framework into a cross-efficiency method to develop a new efficiency evaluation method. Chen and Wang [14] innovatively proposed the definition of cross-efficiency and developed two new target-setting approaches for individual DMU and global optimization to improve the cross-efficiency of DMUs in different decision-making situations. Chen et al. [15] introduced prospect theory to describe the subjective preference of decision-makers in the aggregation process when they face gains and losses, then a new method is constructed to aggregate cross-efficiency. Contreras et al. [16] proposed a new cross-efficiency model based on bargaining problems and the Kalai-Smorodinsky solution. Wu et al. [17] proposed an innovative composite method for ranking DMUs by calculating the Shannon entropy of the obtained cross-efficiency scores derived from the perspectives of satisfaction and consensus.

Another strategy for cross-efficiency evaluation is the neutral strategy that was first proposed by Wang and Chin [18]. Unlike aggressive or benevolent models, the neutral cross-efficiency method attempts to specify a set of weights for the inputs and outputs of each DMU from its profit perspective. Wang et al. [19] proposed a neutral method for cross-efficiency evaluation based on the distance of each DMU from the best DMU (IDMU) or the worst DMU (ADMU). Based upon the method of Wang et al. [19], Carrillo and Jorge [20] proposed a neutral model that determines an optimal set of weights that maximize the efficiency score of the ADMU and minimize the efficiency score of the IDMU simultaneously while keeping the efficiency of the evaluated unit unchanged. Shi et al. [21] utilized an ideal and anti-ideal frontier as evaluation criteria and proposed a new method for evaluating cross-efficiency scores. Using IDMU and ADMU, Liu et al. [22] introduced a prospect value based on prospect theory. They proposed a new secondary goal based on a neutral strategy for evaluating cross-efficiency scores.

Kao and Liu [23] studied two basic network systems, series and parallel, and developed a relational model to measure the cross-efficiencies of the systems and divisions. Based on this model, Örkcü et al. [24] proposed a new neutral model for cross-efficiency evaluation of the basic two-stage network systems. This model can be ranked each DMU based on the efficiency score of sub-stages and the overall efficiency score. In this model, the overall efficiency is the product of those of the stages. Liu et al. [25] proposed the neutral cross-efficiency evaluation method for general parallel systems. Shi et al. [26] proposed a neutral cross-efficiency evaluation method based on the prospect theory, which reflects the bounded rationality of DMUs when facing gain and loss as secondary goals. In addition to developing theoretical models, cross-efficiency has been applied in evaluating efficiency in various fields. For example, Wang et al. [27] conducted a cross-efficiency assessment of energy efficiency in the construction industry. Amin and Hajjami [28] generated cross-efficiency matrices by combining multiple optimal solutions to produce stock portfolios with lower risk and higher expected returns.

This paper proposes a new neutral secondary goal for cross-efficiency evaluation. This model can guarantee the maximum self-evaluation efficiency of the DMU being evaluated and the maximum sum of the output weights. The following model is an extension of the first model to cross-weight evaluation. The remainder of this paper is arranged as follows. In Section 2, we address the cross-efficiency evaluation approach. The new models for evaluating the efficiency score are introduced in Section 3. In Section 4, using two data sets, we compare the results of the proposed models with cross-efficiency models and demonstrate the effectiveness of the proposed models. Concluding is discussed in Section 5.

2 | Cross-Efficiency Evaluation

We consider n DMUs that each $DMU_j (j=1,2,...,n)$ produces s different output indexes $Y_j=(y_{1j},y_{2j},...,y_{sj})\in \mathbb{R}_+^s$ from m different input indexes $X_j=(x_{1j},x_{2j},...,x_{mj})\in \mathbb{R}_+^m$, where $\mathbb{R}_+^s$ and $\mathbb{R}_+^m$ are two sets of nonnegative numbers. The efficiency of DMU_j is as follows:

$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}, \tag{1}$$

where $u_r (r=1,2,...,s)$ and $v_i (i=1,2,...,m)$ are the rth output and ith input weights respectively.

The cross-efficiency evaluation process is often two-step, called self-evaluation and peer-evaluation. Each DMU's efficiency score is evaluated against the weights of all DMUs, not just its own. Suppose $DMU_p=\left(X_p,Y_p\right)$ be the DMU under evaluation. In the first phase of the cross-efficiency process, self-evaluation, the relative efficiency of DMU_p to other DMUs can be calculated using the traditional DEA model, such as the CCR model [1] that has the following form:

$$\begin{aligned}
E_{pp}^{*} &= \text{Max} \sum_{r=1}^{s} u_r^p y_{rp}, \\
\text{s.t.} \quad & \sum_{r=1}^{s} u_r^p y_{rj} - \sum_{i=1}^{m} v_i^p x_{ij} \le 0, \quad j=1,2,...,n, \\
& \sum_{i=1}^{m} v_i^p x_{ip} = 1, \\
& u_r^p \ge 0, \quad r=1,2,...,m, \\
& v_i^p \ge 0, \quad i=1,2,...,s,
\end{aligned} \tag{2}$$

where E_{pp}^{*} is referred the relative efficiency of DMU_p. Let $u_r^{p*}(r=1,2,...,s)$ and $v_i^{p*}(i=1,2,...,m)$ be the optimal solution of the *Model (2)* for evaluation DMU_p, then in the second phase of the cross-efficiency method, peer-evaluation, the cross-efficiency score of DMU_j, using $u_r^{p*}(r=1,2,...,s)$ and $v_i^{p*}(i=1,2,...,m)$, calculates as follows:

$$E_{pj} = \frac{\sum_{r=1}^{s} u_r^{p*} y_{rj}}{\sum_{r=1}^{m} v_r^{p*} x_{ij}}, \ \left(j=1,2...,n\right). \tag{3}$$

In this case, $\bar{E}_j = {}^{1}\!/_{n} \sum_{p=1}^{n} E_{pj}$ is the final score of $DMU_j (j=1,2,...,n)$ for ranking.

Model (2) usually generates alternative optimal solutions, so we have different cross-efficiency scores for each DMU. Sexton et al. [9] introduced a secondary goal in cross-efficiency evaluation to overcome this vagueness. In this regard, Doyle and Green [10] presented new secondary goals called benevolent and aggressive models. As mentioned in the last section these models maximize the efficiency of DMU_p while minimize (maximize) the average cross-efficiency of other DMUs. The benevolent and aggressive formulations are as follows:

$$
\begin{aligned}
&\text{Min} \sum_{r=1}^{s} u_r^p \left(\sum_{j=1, j\neq p}^{n} y_{rj} \right), \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_i^p \left(\sum_{j=1, j\neq p}^{n} x_{ij} \right) = 1, \\
&\qquad \sum_{r=1}^{s} u_r^p y_{rp} - E_{pp}^{*} \times \sum_{i=1}^{m} v_i^p x_{ip} = 0, \\
&\qquad \sum_{r=1}^{s} u_r^p y_{rj} - \sum_{i=1}^{m} v_i^p x_{ij} \leq 0, \quad j = 1,2,\ldots,n, \ j \neq p, \\
&\qquad u_r^p \geq 0, \quad r = 1,2,\ldots,m, \\
&\qquad v_i^p \geq 0, \quad i = 1,2,\ldots,s.
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
&\text{Max} \sum_{r=1}^{s} u_r^p \left(\sum_{j=1, j\neq p}^{n} y_{rj} \right), \\
&\text{s.t.} \quad \sum_{i=1}^{m} v_i^p \left(\sum_{j=1, j\neq p}^{n} x_{ij} \right) = 1, \\
&\qquad \sum_{r=1}^{s} u_r^p y_{rp} - E_{pp}^{*} \times \sum_{i=1}^{m} v_i^p x_{ip} = 0, \\
&\qquad \sum_{r=1}^{s} u_r^p y_{rj} - \sum_{i=1}^{m} v_i^p x_{ij} \leq 0, \quad j = 1,2,\ldots,n, \ j \neq p, \\
&\qquad u_r^p \geq 0, \quad r = 1,2,\ldots,m, \\
&\qquad v_i^p \geq 0, \quad i = 1,2,\ldots,s.
\end{aligned} \tag{5}
$$

Models (4) and *(5)* are accounted as the aggressive and benevolent models for cross-efficiency evaluation, respectively. Due to the different nature of the two models, two models provide different weights. As a result, the two methods usually will produce different rankings. Aiming to avoid aggressive and benevolent strategy in evaluating cross-efficiency score performance, Wang and Chin [18] proposed the following neutral model as a secondary goal to evaluate cross-efficiency in DEA:

$$
\begin{aligned}
&\text{Max} \ \delta = \underset{r=1,2,\ldots,s}{\text{Min}} \left\{ \frac{u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} \right\}, \\
&\text{s.t.} \qquad E_{pp}^{*} = \frac{\sum_{r=1}^{s} u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}},
\end{aligned} \tag{6}
$$

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$$\frac{\sum_{r=1}^{s} u_r^p y_{rj}}{\sum_{i=1}^{m} v_i^p x_{ij}} \le 1, \quad j=1,2,\dots,n, \quad j \ne p,$$
$$u_r^p \ge 0, \quad r=1,2,\dots,s,$$
$$v_i^p \ge 0, \quad i=1,2,\dots.,m.$$

For DMU_p, *Model (6)* finds an optimal set of weights to maximize each of its output's efficiency, $\frac{u_r^p y_{rp}}{\sum_{i=1}^{m} v_i^p x_{ip}} (r=1,2,\dots,r)$, as much as possible while its relative efficiency is kept. In this paper, we proposed a new secondary goal based on the neutral cross-efficiency evaluation idea.

3 | Proposed Model for the Cross-Efficiency Evaluation

In this section, we proposed the following neutral DEA model for the cross-efficiency evaluation of $DMU_p (p=1,2,\dots,n)$:

$$\begin{aligned} & Z_{pu}^* = \text{Max} \sum_{r=1}^{s} u_r^p, \\ & \text{s.t.} \ \sum_{r=1}^{s} u_r^p y_{rj} - \sum_{i=1}^{m} v_i^p x_{ij} \le 0, \quad j=1,2,\dots,n, \\ & \quad \sum_{r=1}^{s} u_r^p y_{rp} = E_{pp}^*, \\ & \quad \sum_{i=1}^{m} v_i^p x_{ip} = 1, \\ & \quad u_r^p \ge 0, \quad r=1,2,\dots,m, \\ & \quad v_i^p \ge 0, \quad i=1,2,\dots,s, \end{aligned} \tag{7}$$

where E_{pp}^* is obtained from *Model (2)* for evaluation DMUP. In this model, the sum of output weights is maximized while the efficiency of DMUP is kept. In other words, the value of each of the output weights in the objective function is considered the same, and this causes the production of non-zero output weights to decrease. On the other hand, the efficiency of each of the outputs of DMUP increases regardless of the amount of these outputs, while in the *Model (6)*, the selection of optimal weights is influenced by the lowest output value of DMUP. *Model (7)* has fewer constraints than Model *(6)*, as a result, its computational complexity is less than *Model (6)*, and based on numerical results, it has the same performance as *Model (6)*. The proposed model discusses a new secondary goal in a way that reduces the influences of the existence of multiple optimal solutions. These secondary goals have nothing to do with the cross-efficiency of other DMUs, so they can be categorized as neutral secondary goals rather than aggressive and benevolent.

3.1 | Extension to the Cross-Weight Evaluation

Similar to cross-efficiency models, *Model (7)* must solve n times, and each time the efficiency of a DMU must be maintained unchanged, so we have n sets of input and output weights. If these weights are comparable, we can form a cross weights matrix and obtain a set of input and output weights using the arithmetic mean to calculate the efficiency of DMU. For this purpose, we propose the following model for the cross-efficiency evaluation of $DMU_p(p=1,2,...,n)$ based on which n sets of generated weights are comparable:

$$
\begin{aligned}
& Z^*_{pu} = \text{Max} \sum_{r=1}^{s} u^p_r, \\
& \text{s.t.} \ \sum_{r=1}^{s} u^p_r y_{rj} - E^*_{jj} \sum_{i=1}^{m} v^p_i x_{ij} \le 0, \quad j=1,2,...,n, \\
& \sum_{r=1}^{s} u^p_r y_{rp} - E^*_{pp} \sum_{i=1}^{m} v^p_i x_{ip} = 0, \\
& \sum_{j=1}^{n} \sum_{i=1}^{m} v^p_i x_{ij} = 1, \\
& u^p_r \ge 0, \quad r=1,2,...,m, \\
& v^p_i \ge 0, \quad i=1,2,...,s.
\end{aligned}
\tag{8}
$$

Theorem 1. *Model (8)* has a feasible solution.

Proof: Suppose $v_i^{p*}(i=1,2,...,m)$ and $u_r^{p*}(r=1,2,...,s)$ be the optimal solution of the *Model (7)* for evaluation DMU_p. We define $\left(\hat{v}_i^p(i=1,2,...,m), \hat{u}_r^p(r=1,2,...,s)\right)$ as follows:

$$
\hat{v}^p_i = \frac{v^{p*}_i}{\sum_{i=1}^{m} v^{p*}_i \left(\sum_{j=1}^{n} x_{ij}\right)}, \quad i=1,2,...,m,
$$

$$
\hat{u}^p_r = \frac{u^{p*}_r}{\sum_{i=1}^{m} u^{p*}_r \left(\sum_{j=1}^{n} x_{ij}\right)}, \quad r=1,2,...,s,
$$

Therefore, we have

$$
\sum_{r=1}^{s} u^{p*}_r y_{rj} - \sum_{i=1}^{m} v^{p*}_i x_{ij} \le 0, \rightarrow E_{pj} = \frac{\sum_{r=1}^{s} u^{p*}_r y_{rj}}{\sum_{i=1}^{m} v^{p*}_i x_{ij}} = \frac{\sum_{r=1}^{s} \hat{u}^p_r y_{rj}}{\sum_{i=1}^{m} \hat{v}^p_i x_{ij}} \le E^*_{jj}, \quad j=1,2,...,n,
$$

$$
\sum_{r=1}^{s} u^{p*}_r y_{rp} = E^*_{pp}, \sum_{i=1}^{m} v^{p*}_i x_{ip} = 1 \rightarrow \sum_{r=1}^{s} \hat{u}^p_r y_{rp} - E^*_{pp} \sum_{i=1}^{m} \hat{v}^p_i x_{ip} = 0,
$$

$$
\sum_{j=1}^{n} \sum_{i=1}^{m} \hat{v}^p_i x_{ij} = 1.
$$

The proof is completed.

Let $\left(\bar{u}_r^p(r=1,2,...,s), \bar{v}_i^p(i=1,2,...,m)\right)$ be the optimal solution of *Model (8)* for the cross-efficiency evaluation of $DMU_p(p=1,2,...,n)$, thus the cross-weight matrix is as follows.

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Table 1. Cross-weight evaluation for n DMUs.

Target DMU	Input Weights				Output Weights			
	$\bar{v}_1^p$	$\bar{v}_2^p$	$\cdots$	$\bar{v}_m^p$	$\bar{u}_1^p$	$\bar{u}_2^p$	$\cdots$	$\bar{u}_s^p$
1	$\bar{v}_1^1$	$\bar{v}_2^1$	$\cdots$	$\bar{v}_m^1$	$\bar{u}_1^1$	$\bar{u}_2^1$	$\cdots$	$\bar{u}_s^1$
2	$\bar{v}_1^2$	$\bar{v}_2^2$	$\cdots$	$\bar{v}_m^2$	$\bar{u}_1^2$	$\bar{u}_2^2$	$\cdots$	$\bar{u}_s^2$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$			$\vdots$
n	$\bar{v}_1^n$	$\bar{v}_2^n$	$\cdots$	$\bar{v}_m^n$	$\bar{u}_1^n$	$\bar{u}_2^n$	$\cdots$	$\bar{u}_s^n$

According to information in *Table 1*, the final values of input and output weights are as follows:

$$v_i^* = \frac{1}{n}\sum_{j=1}^{n}\bar{v}_i^j, \quad i = 1,2,\ldots,m,$$
$$u_r^* = \frac{1}{n}\sum_{j=1}^{n}\bar{u}_r^j, \quad r = 1,2,\ldots,s. \tag{9}$$

Then we can calculate the final score of $DMU_j\,(j = 1, 2, \ldots, n)$ using *Eqs. (9)* and *(1)*.

4 | Numerical Example

In this section, to evaluate the performance of the proposed process, we consider two numerical examples that were used in previous studies in the DEA literature. *Models (4)-(6)* and the proposed models are applied to rank all DMUs and compare their performance.

Example 1. Consider the case of 14 airlines as DMUs with three inputs and two outputs which is adapted from [12], Tofallis [29] and Chiang et al. [30]. The airlines data are summarized in *Table 2*.

Table 2. Data for 14 passenger airlines.

Airline (DMU)	Inputs			Outputs	
	x_1	x_2	x_3	y_1	y_2
1	5723	3239	2003	26677	697
2	5895	4225	4557	3081	539
3	24099	9560	6267	124055	1266
4	13565	7499	3213	64734	1563
5	5183	1880	783	23604	513
6	19080	8032	3272	95011	572
7	4603	3457	2360	22112	969
8	12097	6779	6474	52363	2001
9	6587	3341	3581	26504	1297
10	5654	1878	1916	19277	972
11	12559	8098	3310	41925	3398
12	5728	2481	2254	27754	982
13	4715	1792	2485	31332	543
14	22793	9874	4145	122528	1404

The CCR efficiency scores for 14 passenger airlines are revealed in the second column of *Table 3*. According to the results of the CCR model, 7 of 14 airlines are identified as efficient DMUs. In this case, it is not possible to recognize their superiority over each other; so, we use the results of *Models (4)-(6)* and the proposed models to rank them, which are shown in the third through the twelfth columns of *Table 3*, respectively.

Table 3. Results of Models (4)-(6) and proposed models in Example 1.

DMU	CCR Model (2)	Aggressive Model (4)	Rank	Benevolent Model (5)	Rank	Neutral Model (6)	Rank	Model (7)	Rank	Model (8)	Rank
1	0.8684	0.5990	12	0.7543	12	0.7179	11	0.7200	11	0.7543	12
2	0.3379	0.1652	14	0.1894	14	0.1988	14	0.1956	14	0.1894	14
3	0.9475	0.6226	11	0.7678	9	0.7278	10	0.7332	10	0.7678	9
4	0.9581	0.6734	7	0.8222	6	0.7748	8	0.7796	8	0.8222	6
5	1	0.7983	1	0.8912	3	0.8730	4	0.8802	4	0.8912	3
6	0.9766	0.6385	9	0.7554	11	0.7000	13	0.7078	13	0.7554	11
7	1	0.6478	8	0.8214	7	0.7887	7	0.7869	7	0.8214	7
8	0.8588	0.5855	13	0.7242	13	0.7135	12	0.7107	12	0.7242	13
9	0.9477	0.6309	10	0.7590	10	0.7659	9	0.7611	9	0.7590	10
10	1	0.6813	6	0.7803	8	0.8129	5	0.8114	6	0.7803	8
11	1	0.7742	2	0.9193	1	0.9048	2	0.9048	1	0.9193	1
12	1	0.7314	5	0.8850	4	0.8825	3	0.8815	3	0.8850	4
13	1	0.7503	3	0.9190	2	0.9058	1	0.9041	2	0.9190	2
14	1	0.7316	4	0.8659	5	0.8128	6	0.8205	5	0.8659	5
sum		9.0300		10.8544		10.5791		10.5975		10.8544	

As can be seen in *Table 3*, the sum of efficiencies in the *Model (7)* is higher than the *Model (6)*. The sum of efficiencies in the *Models (8)* and *(5)* is the same. Also, the sum of efficiencies in the *Model (6)* and also the *Model (7)* (neutral models) is higher than *Model (4)* (aggressive model) and lower than the *Model (5)* (benevolent model), which all above results are reasonable with respect the structures of models.

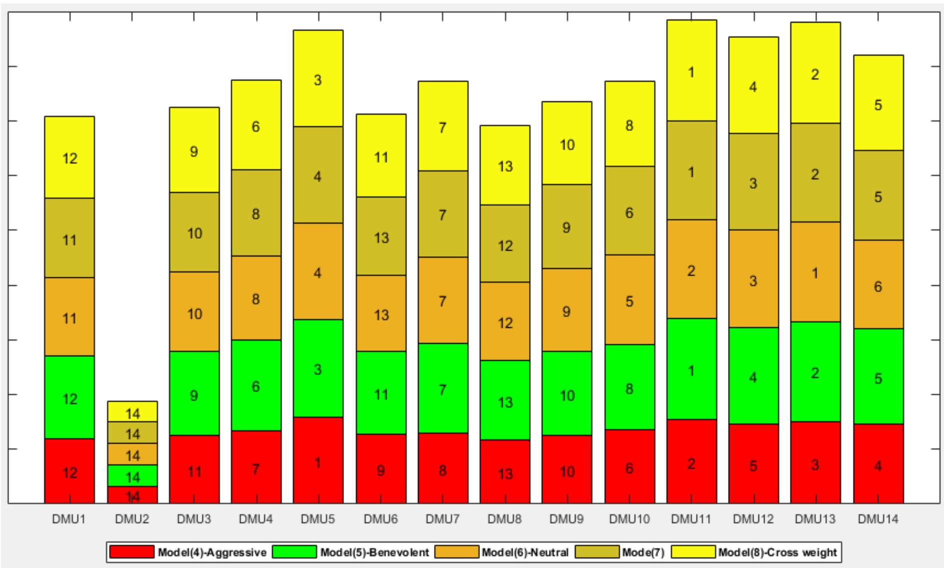

Fig. 1. Illustrative comparison between the ranking results of Models (4)-(6) and proposed models in Example 1.

Fig. 1 provides an illustrative comparison among *Models (4)-(6)* and the proposed models in *Example 1* according to the ranking results shown in *Table 3*. As can be seen, DMU_{11} took first place in *Models (5)*, *(7)* and *(8)*, whereas it gained the second rank in *Models (4)* and *(6)*. Also DMU_{13} and DMU_5 took first place in *Model (6)* and *Model (4)*, respectively. Note that DMU_2 has the worst performance in all methods.

Table 4. Ranking models correlation test in Example 1.

	Spearmen's Rho	Model (4)	Model (5)	Model (6)	Model (7)	Model (8)
Model (4)	Correlation	1.0000	0.9516	0.9033	0.9165	0.9516
	Sig.(bilateral)		0	0	0	0
Model (5)	Correlation	0.9516	1.0000	0.9429	0.9604	1.0000
	Sig.(bilateral)	0		0	0	0
Model (6)	Correlation	0.9033	0.9429	1.0000	0.9912	0.9429
	Sig.(bilateral)	0	0		0	0
Model (7)	Correlation	0.9165	0.9604	0.9912	1.0000	0.9604
	Sig.(bilateral)	0	0	0		0
Model (8)	Correlation	0.9516	1.0000	0.9429	0.9604	1.0000
	Sig.(bilateral)	0	0	0	0	

Table 4 shows the value of Spearman's rank correlation coefficients of the five models in *Table 2* to assess the similarities between the rankings induced from the corresponding values. In all the cases, the values are statistically significant at the 0.0001 level. The test values correlations among *Models (4)-(8)* are all above 0.9. Note that *Model (7)* has the highest correlation with the *Model (6)* (green) and the *Model (8)* has the highest correlation with the *Model (5)* (green). Also, the *Model (7)* has the lowest correlation with the *Model (4)* (red) and the *Model (8)* has the lowest correlation with the *Model (6)* (red).

Example 2. This example is taken from Shi et al. [21] and is about the efficiency evaluation of 20 machinery manufacturing enterprises (DMUs) in 2014 with four inputs and four outputs. The manufacturing data are documented in *Table 5*.

Table 5. Data for 20 machinery manufacturing enterprises.

Enterprises (DMUs)	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4
1	15	1361	222	27	3012	926.51	89	4.2
2	12.2	520	435	205	2144	1146	318	5.6
3	16.54	226.31	226.31	7	2799.76	1118.97	158.63	8.4
4	40	595	74.16	7	198	1554.96	220.35	11.5
5	11.18	0.396	317.3	9	3100	603.6	107.37	3.7
6	12.61	224.6	224.8	119	3436.8	581.6	1177	4.9
7	4.91	349	187	6	3801	1404	193	4.3
8	100	273	285	198	2533	1716	167	8.7
9	12.22	398.89	398.89	8	5192.85	1955.78	125.79	8.8
10	11	532.86	532.86	391	3472.98	3550.89	528.22	9.1
11	11.87	566	566	2	2937	5197	400	0.2
12	14.66	1044.5	961.62	232	11142	4362	827.14	18.8
13	11.48	173.71	173.7	4	1975	1508.96	43.8	12.4
14	24.56	346.04	223.3	11	2605.94	771	173.67	13.5
15	21.18	179.39	179.39	2	1858.3	504	70	5.9
16	10.32	454.71	454.71	31	3300.33	1382.91	532.36	4.9
17	5.33	328	41	4	5549.02	368	987	2.5
18	10.78	547.99	518.68	11	5837	2619.39	285.45	6.4
19	15.48	613.72	573.81	13	4820.71	2621.4	404.94	10.4
20	12.55	615.88	316.79	11	9018.71	384.22	64.99	1.5

The CCR efficiency scores and their rankings for 20 DMUs in the second column in *Table 6* show that 10 of 20 DMUs are efficient, so we cannot find any difference between them for ranking. Thus, we use cross-efficiency for further distinction. The results of *Models (4)-(8)* for evaluations of 20 enterprises are shown in the third through the twelfth columns in *Table 6*.

Table 6. Results of Models (4)-(6) and proposed model in Example 2.

DMU	CCR Model (2)	Aggressive Model (4)	Rank	Benevolent Model (5)	Rank	Neutral Model (6)	Rank	Model (7)	Rank	Model (8)	Rank
1	0.4544	0.1503	20	0.2064	20	0.1778	20	0.1996	20	0.2060	20
2	0.4722	0.2083	18	0.3318	18	0.2670	18	0.2833	18	0.3318	18
3	0.7530	0.4278	10	0.6345	10	0.5365	12	0.5519	12	0.6345	10
4	1	0.3483	15	0.3835	17	0.4367	14	0.4186	14	0.3835	17
5	1	0.5218	5	0.7910	5	0.6067	7	0.6360	8	0.7910	5
6	1	0.4207	12	0.6253	12	0.5417	11	0.5812	10	0.6263	12
7	1	0.5982	4	0.8397	4	0.7031	4	0.7659	4	0.8392	4
8	0.7708	0.1707	19	0.3307	19	0.2182	19	0.2688	19	0.3318	19
9	0.9723	0.5209	6	0.7667	6	0.6174	5	0.6616	5	0.7665	6
10	1	0.4237	11	0.7211	9	0.5531	9	0.5907	9	0.7211	9
11	1	0.6135	3	0.8672	3	0.7458	3	0.8239	3	0.8672	3
12	1	0.4925	8	0.7380	8	0.6139	6	0.6376	7	0.7379	8
13	1	0.7306	2	0.9685	2	0.8862	2	0.8835	2	0.9685	2
14	0.8037	0.3557	13	0.4494	15	0.4409	13	0.4153	15	0.4494	15
15	1	0.3514	14	0.4118	16	0.4172	16	0.3972	16	0.4119	16
16	0.6840	0.3404	16	0.4992	13	0.4350	15	0.4531	13	0.4993	13
17	1	0.7711	1	0.9927	1	0.9915	1	0.9927	1	0.9927	1
18	0.9476	0.5017	7	0.7387	7	0.6018	8	0.6484	6	0.7385	7
19	0.7987	0.4520	9	0.6302	11	0.5468	10	0.5755	11	0.6301	11
20	0.8108	0.2785	17	0.4946	14	0.3124	17	0.3569	17	0.4940	14
sum		8.6780		12.4208		10.6498		11.1415		12.4211	

As can be seen in *Table 6*, the sum of efficiencies in the *Model (7)* is higher than the *Model (6)*. The sum of efficiencies in the *Model (8)* is higher than the *Model (5)*. Also, the sum of efficiencies in the *Model (6)* and also the *Model (7)* (neutral models) is higher than *Model (4)* (aggressive model) and lower than *Model (5)* (benevolent model). Note that in this example, *Model (8)* with the same performance as the *Model (5)* produces the sum of efficiencies than *Model (5)*.

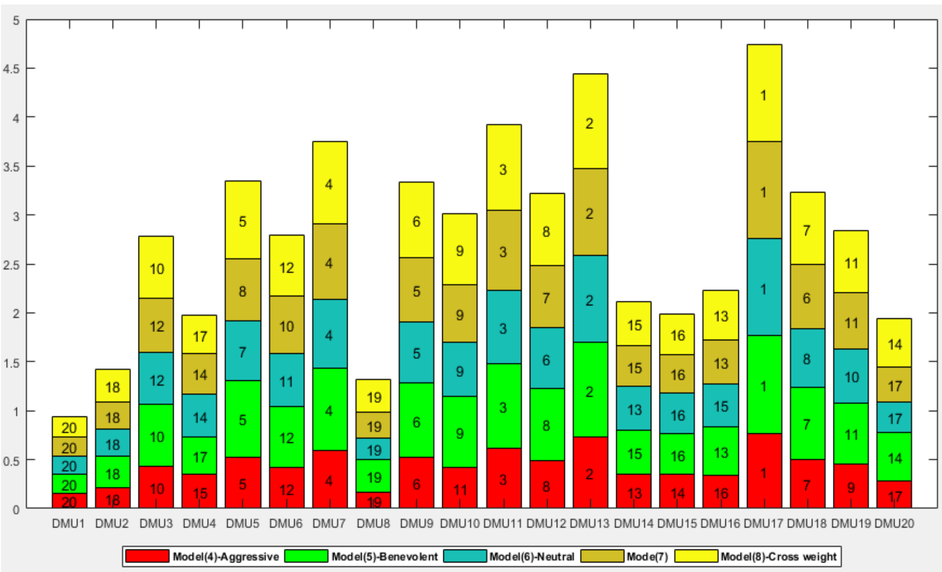

Fig. 2. Illustrative comparison between the ranking results of Models (4)-(6) and proposed models in Example 2.

Fig. 2 provides an illustrative comparison between the results of the *Models (4)-(6)* and the proposed model for ranking of DMUs in *Example 2* according to the rankings that are shown in *Table 6*. As can be seen in *Fig. 2*, in all models, DMU1, DMU8, and DMU2 (inefficient DMUs) are ranked 20th, 19th, and 18th, respectively. Moreover, the DMU17 and DMU13 as efficient DMU have the first and the second rank in the five models, respectively.

Table 7. Ranking models correlation test in Example 2.

	Spearmen's Rho	Model (4)	Model (5)	Model (6)	Model (7)	Model (8)
Model (4)	Correlation	1.0000	0.9714	0.9805	0.9654	0.9714
	Sig.(bilateral)		0	0	0	0
Model (5)	Correlation	0.9714	1.0000	0.9684	0.9714	1.0000
	Sig.(bilateral)	0		0	0	0
Model (6)	Correlation	0.9805	0.9684	1.0000	0.9880	0.9684
	Sig.(bilateral)	0	0		0	0
Model (7)	Correlation	0.9654	0.9714	0.9880	1.0000	0.9714
	Sig.(bilateral)	0	0	0		0
Model (8)	Correlation	0.9714	1.0000	0.9684	0.9714	1.0000
	Sig.(bilateral)	0	0	0	0	

Table 7 shows the value of Spearman's rank correlation coefficients of the five models in *Table 6*. After the Spearman test, the test values of correlations among *Models (4)-(8)* are all above 0.9. In all the cases, the values are statistically significant at the 0.0001 level. Similar to *Example 1*, the *Model (7)* and the *Model (8)* have the highest correlation with the *Model (6)* and *Model (5)*, respectively. Also, the *Model (7)* has the lowest correlation with the *Model (4)* and the *Model (8)* has the lowest correlation with the *Model (6)*.

5 | Conclusion

Cross-efficiency evaluation is a utility to enhance the power of discriminating efficient DMUs in DEA. Although this method is widely used, it also has some drawbacks, such as the existence of multiple optimal weights for DEA models. Secondary goals are proposed based on aggressive, benevolent, and neutral points of view for overcoming this issue. In this paper, we proposed two models. Based on the neutral strategy in DEA, the first model seeks input and output weights that not only undertake the maximum self-assessment efficiency of DMU under evaluation but also maximize the sum of output weights. Therefore, increasing non-zero weights increases the efficiency of each DMU output under evaluation. The second model produces a set of comparable weights. According to these weights and using the matrix of cross weights, input and output weights are generated to calculate the efficiency of the DMUs. We compare the performance of the proposed models by three well-known models with an optimistic, pessimistic and neutral view using two numerical examples. It was found that the first proposed model with the same performance has less computational complexity. Also, the second model (matrix of cross-weights) has the same performance as the optimistic compared model. Existing literature proves that few studies have considered ranking DMUs with network structure using cross-efficiency evaluation. Therefore, the proposed models can be effectively applied to rank DMUs with network structure in DEA.

References

[1] Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European journal of operational research*, 2(6), 429-444.

[2] Ucal Sari, I., & Ak, U. (2022). Machine efficiency measurement in industry 4.0 using fuzzy data envelopment analysis. *Journal of fuzzy extension and applications*, 3(2), 177-191.

[3] Fallah, R., Kouchaki Tajani, M., Maranjory, M., & Alikhani, R. (2021). Comparison of banks and ranking of bank loans types on based of efficiency with dea in Iran. *Big data and computing visions*, 1(1), 36-51. https://doi.org/10.22105/bdcv.2021.142086

[4] Muniz, R. D. F., Andriola, W. B., Muniz, S. M., & Thomaz, A. C. F. (2021). The use of data envelopment analysis (DEA) to estimate the educational efficiency of Brazilian schools. *Journal of applied research on industrial engineering*, 9(4), 374-383. https://doi.org/10.22105/jarie.2021.308815.1388

[5] Ghaziyani, K., Ejlaly, B., & Bagheri, S. F. (2019). Evaluation of the efficiency by DEA a case study of hospital. *International journal of research in industrial engineering*, 8(3), 283-293. http://www.riejournal.com/article_100768_3310e9849f537d1c5ba5dc13afbf6816.pdf

[6] Mirsadeghpour, M., Sanei, M., Tohidi, G., Banihashemi, S., & Modarresi, N. (In Press). Assets performance evaluation with the use of returns distribution characteristics. *Journal of decisions and operations research*. (**In Persian**). https://doi.org/10.22105/dmor.2022.332170.1587

[7] Kayedppour, F., Sayadmanesh, S., Salmani, Y., & Sadeghi, Z. (2021). Measuring the efficiency and productivity of cement companies in Tehran Stock Exchange by data envelopment analysis and Malmquist productivity index in gray environment. *Innovation management and operational strategies*, *1*(4), 363-382. (**In Persian**). https://doi.org/10.22105/imos.2021.276467.1038

[8] Li, F., Wu, H., Zhu, Q., Liang, L., & Kou, G. (2021). Data envelopment analysis cross efficiency evaluation with reciprocal behaviors. *Annals of operations research*, *302*, 173-210. https://doi.org/10.1007/s10479-021-04027-x

[9] Sexton, T. R., Silkman, R. H., & Hogan, A. J. (1986). Data envelopment analysis: critique and extensions. *New directions for program evaluation*, *1986*(32), 73-105.

[10] Doyle, J., & Green, R. (1994). Efficiency and cross-efficiency in DEA: derivations, meanings and uses. *Journal of the operational research society*, *45*, 567-578. https://doi.org/10.1057/jors.1994.84

[11] Liang, L., Wu, J., Cook, W. D., & Zhu, J. (2008). The DEA game cross-efficiency model and its Nash equilibrium. *Operations research*, *56*(5), 1278-1288.

[12] Jahanshahloo, G. R., Hosseinzadeh Lotfi, F., Jafari, Y., & Maddahi, R. (2011). Selecting symmetric weights as a secondary goal in DEA cross-efficiency evaluation. *Applied mathematical modelling*, *35*(1), 544-549. https://doi.org/10.1016/j.apm.2010.07.020

[13] Chen, L., Huang, Y., Li, M. J., & Wang, Y. M. (2020). Meta-frontier analysis using cross-efficiency method for performance evaluation. *European journal of operational research*, *280*(1), 219-229.

[14] Chen, L., & Wang, Y. M. (2020). DEA target setting approach within the cross efficiency framework. *Omega*, *96*, 102072. https://doi.org/10.1016/j.omega.2019.05.008

[15] Chen, L., Wang, Y. M., & Huang, Y. (2020). Cross-efficiency aggregation method based on prospect consensus process. *Annals of operations research*, *288*, 115-135. https://doi.org/10.1007/s10479-019-03491-w

[16] Contreras, I., Lozano, S., & Hinojosa, M. A. (2021). A DEA cross-efficiency approach based on bargaining theory. *Journal of the operational research society*, *72*(5), 1156-1167.

[17] Wu, D., Wang, Y., Liu, Y., & Wu, J. (2021). DEA cross-efficiency ranking method considering satisfaction and consensus degree. *International transactions in operational research*, *28*(6), 3470-3492.

[18] Wang, Y. M., & Chin, K. S. (2010). A neutral DEA model for cross-efficiency evaluation and its extension. *Expert systems with applications*, *37*(5), 3666-3675.

[19] Wang, Y. M., Chin, K. S., & Luo, Y. (2011). Cross-efficiency evaluation based on ideal and anti-ideal decision making units. *Expert systems with applications*, *38*(8), 10312-10319.

[20] Carrillo, M., & Jorge, J. M. (2018). An alternative neutral approach for cross-efficiency evaluation. *Computers & industrial engineering*, *120*, 137-145. https://doi.org/10.1016/j.cie.2018.04.017

[21] Shi, H., Wang, Y., & Chen, L. (2019). Neutral cross-efficiency evaluation regarding an ideal frontier and anti-ideal frontier as evaluation criteria. *Computers & industrial engineering*, *132*, 385-394. https://doi.org/10.1016/j.cie.2019.04.035

[22] Liu, H. H., Song, Y. Y., & Yang, G. L. (2019). Cross-efficiency evaluation in data envelopment analysis based on prospect theory. *European journal of operational research*, *273*(1), 364-375.

[23] Kao, C., & Liu, S. T. (2019). Cross efficiency measurement and decomposition in two basic network systems. *Omega*, *83*, 70-79. https://doi.org/10.1016/j.omega.2018.02.004

[24] Örkcü, H. H., Özsoy, V. S., Örkcü, M., & Bal, H. (2019). A neutral cross efficiency approach for basic two stage production systems. *Expert systems with applications*, *125*, 333-344. https://doi.org/10.1016/j.eswa.2019.01.067

[25] Liu, P., Zhang, Y., & Xu, H. (2022). A neutral cross-efficiency measurement for general parallel production system. *Expert systems with applications*, *205*, 117778. https://doi.org/10.1016/j.eswa.2022.117778

[26] Shi, H. L., Chen, S. Q., Chen, L., & Wang, Y. M. (2021). A neutral cross-efficiency evaluation method based on interval reference points in consideration of bounded rational behavior. *European journal of operational research*, *290*(3), 1098-1110. https://doi.org/10.1016/j.ejor.2020.08.055

[27] Wang, L., Song, X., & Song, X. (2021). Research on the measurement and spatial-temporal difference analysis of energy efficiency in China's construction industry based on a game cross-efficiency model. *Journal of cleaner production*, *278*, 123918. https://doi.org/10.1016/j.jclepro.2020.123918

[28] Amin, G. R., & Hajjami, M. (2021). Improving DEA cross-efficiency optimization in portfolio selection. *Expert systems with applications*, *168*, 114280. https://doi.org/10.1016/j.eswa.2020.114280

[29] Tofallis, C. (1997). Input efficiency profiling: an application to airlines. *Computers & operations research*, *24*(3), 253-258. https://doi.org/10.1016/S0305-0548(96)00067-6

[30] Chiang, C. I., Hwang, M. J., & Liu, Y. H. (2011). Determining a common set of weights in a DEA problem using a separation vector. *Mathematical and computer modelling*, *54*(9-10), 2464-2470.

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