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Designing the Multivariate Nonconforming Proportion Control Charts Considering the Measurement Error

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Abstract

Most qualitative characteristics cannot be easily reported numerically. In such cases, each product is inspected and usually divided into two conforming and nonconforming groups based on their qualitative characteristics. Since nowadays products and processes have generally several interdependent qualitative characteristics, it is necessary to use multivariate quality control methods to make the relationship between variables and their variations. To do this, the sampling of the considered qualitative characteristic is done at the specified time intervals to check the control of the process over time after drawing the considered statistic on the control chart. A common problem while sampling is measurement error. It affects the performance of control charts, impairs their ability to detect changes in the process, and increases the cost and time to search for out-of-control situations. In this paper, the effect of measurement error on the performance of the Multivariate Nonconforming Proportion (MNP) control chart has been evaluated based on the criterion of Average Run Length (ARL) for the first time. The results imply that the measurement error has a considerable impact on the performance of this chart. Also, the results indicate that if the defective items have been wrongly considered as correct items, we would have a higher ARL compared to an ideal and accurate system. On the other hand, if the system considers right items as defective, we will have a lower ARL than the ideal and accurate system. It is proved that if both errors (considering faulty items as correct ones and vice versa) occur simultaneously, the ARL will be reduced like the previous case.

Keywords: Multivariate control chart, Measurement error, Average Run Length (ALR).

1 | Introduction

In today's world, due to the expansion of the competition field and the increase in production costs, and consumers' expectations for the quality of goods, we must always look for a workable solution to improve quality. The control charts of utility tools are in this order. As the products become more complex, it is necessary to consider the quality of the product comprehensively.

Therefore, the quality characteristic will be controlled by more than one characteristic and all the characteristics must be studied simultaneously in order to control the quality of that product properly. To do this, we have to use the information extracted from the samples taken. The measurement system includes human and measuring equipment, so this system is not 100% reliable.

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In most cases, there is a difference between the observed value and the actual value of the measured parameter, known as the measurement error. On the other hand, the design of multivariate graphs in descriptive mode has not been done by considering the measurement error so far and considering the effect of the measurement error on the control chart, it is necessary to consider the measurement error in the design of this graph. Therefore, in this paper, an attempt has been made to investigate the effect of this error on the performance of the control chart of the number of defective items. The paper is organized in such a way that the second part is devoted to a review of the literature. In the third part of the control chart, the number of defective multivariate items is reviewed. In the fourth part, the control chart of the number of defective items in case of measurement error is developed. The fifth part involves solving a numerical example and analyzing its results. Finally, in the sixth section, a summary and conclusion are presented

2 | Literature Review

Today, with modern equipment for collecting information and monitoring processes, the simultaneous control of several quality characteristics is more common than one quality characteristic. For example, consider a box that can be regarded as non-compliant or defective if there is a defect in dimensions or weight. Montgomery [1] first called these processes multivariate processes. Montgomery and Mastrangelo [2] clearly showed that using univariate Shewhart control diagrams in multivariate processes would deviate from simultaneous monitoring of qualitative characteristics. Much research has been done to improve the performance of the Shewhart chart. For example, Champ and Woodall [3] have proposed the use of sensitization rules to increase the power of the graph to detect out-of-control situations. Therefore, it is necessary to use a multivariate control method to consider the internal relationships between qualitative characteristics. In addition, using a multivariate control chart is more economically and operationally cost-effective than using multiple univariate charts. Lowry and Montgomery [4] showed that typically a multivariate control scheme is more sensitive than the univariate mode to control a multivariate process. The use of multivariate control methods to monitor production processes has become increasingly popular. Like many multivariate control diagrams, the Hotelling's T-squared statistics diagram is based on a normal distribution. In the case of multivariate descriptive processes, however, the number of nonconforming units in each qualitative characteristic usually corresponds to a binomial distribution [5].

To use control diagrams, sampling of the process must be done and the common problem in the sampling section is measurement error. Therefore, it can be said that measurement error is one of the common phenomena in the measurement system that is usually not considered and ignoring these errors can have many costs and consequences [6]. Adverse consequences of measurement error include an increase in the number of false alarms such as incorrect detection of signals out of process control and detection of delay in process change [7]. Various researchers have studied the effect of measurement error on the performance of control charts. Bennett used the $Y = X + \epsilon$ model, known as the classical measurement error model, to influence the measurement error on the performance of control charts [8]. Kanazuka [9] used the classical model to investigate the effect of measurement error on $\overline{X} - R$ control diagrams. He showed that the existence of this error reduces the power of these graphs [9]. Linna et al. [10] extended the classical model to the $Y = A + BX + \epsilon$ model. In their study, they investigated the effect of different measurement error parameters on the ARL parameter of $\overline{\mathrm{X}} - S$ and multivariate control charts and reduced the effect of error variance by using several measurements of each characteristic [10]. The effect of measurement error on the performance of weighted moving average control charts and cumulative sum control charts was evaluated by Maravelakis et al. [11]. Xiaohong and Zhaojun [12] investigated the effect of measurement error and autocorrelation on the performance of the CUSUM control chart and obtained the CUSUM control limits using the maximum likelihood method. Abbasi [13] considered the two-component measurement error and presented the model $Y_t = \alpha + \beta X_t e^{\eta_t} + \varepsilon_t$. He examined the performance of the EWMA diagram under the mentioned model [13].

Costa and Castagliola [14] investigated the effect of measurement error and autocorrelation on the performance of the control diagram $\overline{X}$ and showed that the performance of the control diagram is affected by measurement error and self-correlation. They improved the performance of the $\overline{X}$ control chart using multiple measurement and jump strategies. Momeni et al. [15] investigated the effect of measurement error on the control diagram $\overline{X}-R$ in fuzzy mode. Saghaei et al. [16] evaluated the design of the control diagram considering the measurement error. Amiri and Mohebbi [17] reviewed the statistical and economic design of the EWMA multi-objective control chart and in their paper examined the ARL using the Markov chain and then determined the optimal parameters of the chart with a multi-objective genetic algorithm. Noorossana and Zerehsaz [18] investigated the effect of measurement error on the performance of the EWMA control chart for monitoring profiles and showed that measurement error affects whether the process is under control or out of control. Ding and Zeng [19] also investigated the effect of measurement error on multi-stage production processes. They showed that measurement error affects the methods of estimating the regression model coefficients. For the first time, Daryabari et al. [20] investigated the effect of measurement error on the simultaneous control of the mean and variance of the process in a graph by considering the variance of the constant error with the mean time to alert criterion. They showed that the measurement error significantly affects the performance of the graph of the maximum weighted moving average and the mean squared deviations. Sabahno et al. [21] Investigated the effect of measurement error on the performance of the chi-square control chart using the VSI variable sampling distance feature. Sabahno et al. [22] Investigated the effect of measurement error on the performance of the $\overline{X}$ control chart with the VP parameter characteristic.

Table 1. Review of the literature on the subject and comparison of the characteristics of the studies.

Row	Author Names	Year Of Publication	Name Of Journal / Book	Activity Done
1	Costa and Castagliola [14]	2011	Journal of Applied Statistics	The effect of measurement error and autocorrelation on the performance of control chart $\overline{X}$ was investigated and it was shown that the performance of control chart is affected by measurement error and autocorrelation.
2	Saghaei et al. [16]	2014	International Journal of Industrial Engineering	The design of the control diagram was evaluated taking into account the measurement error.
3	Moameni et al. [15]	2012	Engineering , Technology & Applied Science Research	The effect of measurement error on control $\overline{X}-R$ diagram in fuzzy mode was investigated.
4	Amiri and Mohebbi [17]	2014	International Journal of Quality Engineering and Technology	The statistical and economic design of the EWMA multi-objective control chart was examined and the ARL was examined using the Markov chain.
5	Noorossana and Zerehsaz [18]	2015	The International Journal of Advanced Manufacturing Technology	Investigated the effect of measurement error on the performance of the EWMA control chart to monitor the profiles and showed that the measurement error will affect the performance of the chart.
6	Ding and Zeng [19]	2015	Journal of Manufacturing Systems	The effect of measurement error in multi-stage production processes was investigated which showed that measurement error affects the methods of estimating the regression model coefficients.
7	Daryabari et al. [20]	2017	Communications in Statistics-Theory and Methods	Measurement error was shown to significantly affect the performance of the MAX EWMAMS maximum moving average and the mean squared deviation.

Table 1. (Continued).

Row	Author Names	Year Of Publication	Name Of Journal / Book	Activity Done
8	Sabahno et al. [21]	2018	Quality and Reliability Engineering International	The effect of measurement error on the performance of chi-square control chart was investigated using the VSI variable sampling distance feature.
9	Sabahno et al. [22]	2018	Journal of Testing and Evaluation	The effect of measurement error on the performance of $\overline{X}$ control chart was investigated with the parameter characteristic of the VP variable.
10	Current paper			In this study, for the first time, the effect of measurement error on the performance of the multivariate control chart of the number of defective MNP items is investigated by the ARL sequence average length criterion. The results show that measurement error has affected the performance of this diagram.

3 | Multivariate Control Diagram

Most qualitative characteristics cannot be easily reported numerically. In such cases, each inspected product is usually divided into two groups, compliant and non-compliant with the desired quality characteristics. The statistical principles of non-conforming item ratio control charts are based on binomial distribution. Assume that m has a qualitative characteristic and p_i represents the non-conformance ratio of a qualitative characteristic and c_i represents the number of nonconformities in the qualitative characteristic i. The correlation coefficient between qualitative characteristic i and qualitative characteristic j is considered as δ_{ij} [23].

To draw MNP graphs, we need to calculate and draw a statistic called X, which is calculated as the mean and variance as follows [3]:

$$X=\sum_{i=1}^{m}\frac{c_i}{\sqrt{p_i}},\qquad E(X)=n\sum_{j=1}^{m}\sqrt{p_j},$$
$$\mathrm{Var}(x)=n\left\{\sum_{j=1}^{m}\left(1-p_j\right)+2\sum_{i<j}\delta_{ij}\sqrt{(1-p_i)(1-p_j)}\right\}. \tag{1}$$

Using the general principles of control charts, the control limits in MNP charts are obtained according to the following equation [3]:

$$\begin{cases} UCL=n\sum_{j=1}^{3}\sqrt{\bar{p}_j}+3\sqrt{n\left\{\sum_{j=1}^{m}(1-p_j)+2\sum_{i<j}(\delta ij\sqrt{(1-p_i)\,(1-p_j)}\right\}}, \\ CL=n\sum_{j=1}^{m}\sqrt{\bar{p}_j}\ , \\ LCL=n\sum_{j=1}^{3}\sqrt{\bar{p}_j}-3\sqrt{n\left\{\sum_{j=1}^{m}(1-p_j)+2\sum_{i<j}(\delta ij\sqrt{(1-p_i)\,(1-p_j)}\right\}}. \end{cases} \tag{2}$$

4 | MNP Control Chart Considering Measurement Error

4.1 | Parameters and Sets

c_j: Number of non-conformances in sample i.

p_j: Probability of non-conformance due to qualitative characteristic j.

p_i: Probability of non-conformance due to qualitative feature i.

μ_i: Average of sample i.

δ_{ij}: Standard deviation of ith sample.

P_{ej}: Probability of non-compliance in the presence of a measurement characteristic of a qualitative characteristic j.

4.2 | Conditional Bernoulli Error and Distribution

In practice, inspectors may commit two types of inspection errors during the inspection operation. In order to investigate such a situation, the following events are considered.

A_1: The event is that a product is really defective.

A_2: The event is that a product is really intact.

B: The event is that a product is considered defective during an inspection.

$B|A_2$: The event is that a healthy product is considered a defective product.

$B^c|A_1$: The event is that a defective product is considered an intact product.

Assume that P_j and P_{ej} are the proportions of actual and unreal defective items in the accumulations of products, respectively. Then $P_j = p(A_1)$ and $P_{ej} = p(B)$.

Also, suppose $e_1 = (B|A2)$ and $e_2 = (B^c/A1)$ are equal to the probability of classifying an intact product as a defective product and a defective product as an intact product, respectively. As a result, according to the law of total probability, we have:

$$P_{ej} = p\left(A_1\right) p\left(B\middle|A_1\right) + p\left(A_2\right) p\left(B\middle|A_2\right) = P_j(1 - e_2) + (1 - P_j)e_1, \tag{3}$$

$$\begin{cases} UCL = n\sum_{j=1}^{3}\sqrt{P_{ej}} + 3\sqrt{n\left\{\sum_{j=1}^{m}(1-P_{ej}) + 2\sum_{i<j}(\delta_{ij}\sqrt{(1-P_{ej})(1-P_{ej})}\right\}}, \\ CL = n\sum_{j=1}^{m}\sqrt{P_{ej}}, \\ LCL = n\sum_{j=1}^{3}\sqrt{P_{ej}} - 3\sqrt{n\left\{\sum_{j=1}^{m}(1-P_{ej}) + 2\sum_{i<j}(\delta_{ij}\sqrt{(1-P_{ej})(1-P_{ej})}\right\}}. \end{cases}$$

4.3 | MNP Chart Control Interval Considering Measurement Error

Eq. (2) is used to calculate the control limits in MNP diagrams. Due to the inspection error, P_j will no longer be obtained directly, and this value can be calculated from the following probability relation, and instead of P_j, we will use P_{ej} in all relations:

$$P_{ej} = P_j * (1 - e_2) + (1 - P_j) * e_1. \tag{4}$$

4.4 | MNP Control Chart Statistics Considering Measurement Error

The statistics of the multivariate control chart of the number of defective items in case of measurement error is as follows:

$$X = \sum_{j=1}^{m} \frac{c_j}{\sqrt{p_{ej}}}, \quad E(X) = n\sum_{j=1}^{m}\sqrt{P_{ej}},$$

$$Var(x) = n\left\{\sum_{j=1}^{m}(1 - P_{ej}) + 2\sum_{i<j}\delta_{ij}\sqrt{(1-p_{ei})(1-P_{ej})}\right\}. \tag{5}$$

5 | Effect of Measurement Error on the Performance of the Control Chart of the Number of Defective Multivariate MNP Items

Checking the performance of the control chart the samples taken are measured by the ARL to achieve an out-of-control warning. To calculate this important and effective index, MATLAB simulation software has been used, which allows us to simulate our desired conditions by coding. To investigate the effect of the error in the MNP control diagram, because the type of inspections is descriptive and qualitative, we encounter a conditional Bernoulli distribution according to *Eq. (3)*. In order to be able to calculate the ARL index and use it to analyze the results, in the first step we need to calculate some of the parameters used in the formula. As mentioned before, in the case of measurement error in the preparation of MNP diagrams, instead of the p_j parameter, we encounter the p_{ej} parameter, which we will need to determine the effective factors in this regard.

In *Table 2*, different values of p_j, e_1 and e_2 are considered. Once the mentioned values are known, other required parameters can be calculated and by entering the necessary parameters in MATLAB software, we can calculate the ARL value for different values.

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Table 2. Results for ARL (average sequence length) for different values of p_j, e_1 and e_2 .

Assumptions

ARL0 =452.3843, P(P1=.0533, P2=.0933,P3=.1367), e1=good Item is classified bad, P1=P1+Delta, Delta=.01, e2= Bad Item is classified Good

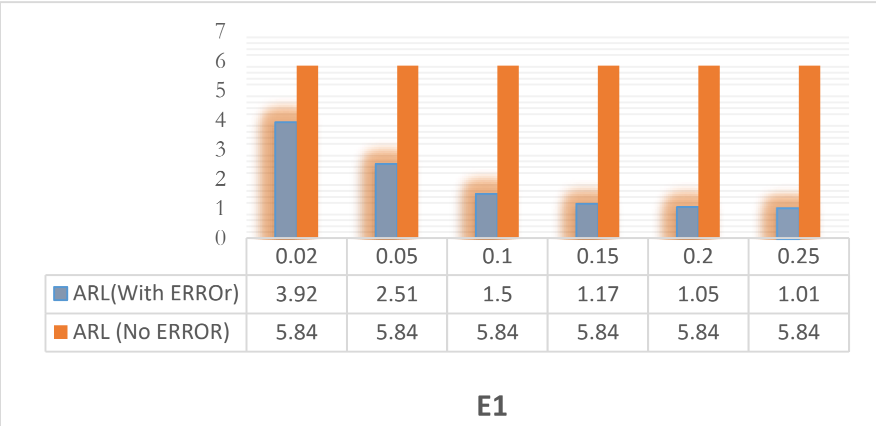
			P=0.0733		P=0.15		P=0.2		p=0.25		p=0.3		0.35		p=0.4		p=0.45		p=0.50	
Error type	e1	e2	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error	ARL Error	ARL No Error
Part One (Only Type I Error)	0.02	0	130.28	295.85	44.83	92.48	25.81	49.84	14.65	31.51	11.65	20.54	8.46	13.87	6.38	10.2	5	7.49	3.92	5.84
	0.05	0	41.51	295.85	17.99	92.48	11.73	49.84	8.31	31.51	5.92	20.54	4.65	13.87	3.68	10.2	3.02	7.49	2.51	5.84
	0.1	0	9.84	295.85	5.54	92.48	4.15	49.84	3.27	31.51	2.63	20.54	2.21	13.87	1.91	10.2	1.67	7.49	1.5	5.84
	0.15	0	3.52	295.85	2.45	92.48	2.06	49.84	1.8	31.51	1.58	20.54	1.43	13.87	1.32	10.2	1.23	7.49	1.17	5.84
	0.2	0	1.81	295.85	1.5	92.48	1.36	49.84	1.27	31.51	1.19	20.54	1.14	13.87	1.1	10.2	1.07	7.49	1.05	5.84
	0.25	0	1.26	295.85	1.17	92.48	1.11	49.84	1.08	31.51	1.05	20.54	1.03	13.87	1.02	10.2	1.01	7.49	1.01	5.84
Part II (only the second type of error)	0	0.02	335.96	295.85	103.33	92.48	56.61	49.84	34.63	31.51	22.24	20.54	15.4	13.87	11.24	10.2	8.4	7.49	6.41	5.84
	0	0.05	373.39	295.85	120.41	92.48	67.5	49.84	41.16	31.51	26.96	20.54	18.3	13.87	13.09	10.2	9.7	7.49	7.57	5.84
	0	0.1	460.95	295.85	159.62	92.48	88.91	49.84	55.04	31.51	35.54	20.54	23.86	13.87	17.77	10.2	12.92	7.49	9.89	5.84
	0	0.15	530.59	295.85	217.06	92.48	118.67	49.84	73.94	31.51	48.52	20.54	33.14	13.87	23.88	10.2	17.52	7.49	13.15	5.84
	0	0.2	536.01	295.85	288.84	92.48	163.95	49.84	103.14	31.51	66.87	20.54	45.03	13.87	32.57	10.2	24.16	7.49	18.27	5.84
	0	0.25	541.6	295.85	395.32	92.48	229.14	49.84	146.5	31.51	96.32	20.54	63.95	13.87	45.6	10.2	33.73	7.49	25.53	5.84
Part III (Existence of the first and second type of error)	0.02	0.02	414.22	295.85	49.05	92.48	28.82	49.84	18.9	31.51	12.99	20.54	9.39	13.87	6.95	10.2	5.37	7.49	4.31	5.84
	0.05	0.05	50.56	295.85	21.96	92.48	13.96	49.84	9.85	31.51	7.28	20.54	5.5	13.87	4.39	10.2	3.61	7.49	2.93	5.84
	0.1	0.1	12.97	295.85	7.6	92.48	5.62	49.84	4.31	31.51	3.55	20.54	2.87	13.87	2.45	10.2	2.15	7.49	1.9	5.84
	0.15	0.15	4.82	295.85	3.31	92.48	2.78	49.84	2.41	31.51	2.07	20.54	1.83	13.87	1.65	10.2	1.53	7.49	1.41	5.84
	0.2	0.2	2.35	295.85	1.98	92.48	1.76	49.84	1.59	31.51	1.48	20.54	1.37	13.87	1.31	10.2	1.24	7.49	1.18	5.84
	0.25	0.25	1.51	295.85	1.35	92.48	1.32	49.84	1.26	31.51	1.19	20.54	1.17	13.87	1.12	10.2	1.1	7.49	1.08	5.84

5.1 | General Results of Reviewing the Results of the Tables

As can be seen in *Table 2*, each section has its own assumptions in different parameters, which consist of three Sections 1, 2 and 3.

In Section 1 of *Table 2*, the error-free mode has a higher ARL than the error mode, which means that the system will need more repetitions to announce the first out-of-control warning in error-free mode. In Section 2, for different p_j values, the error-free state has less ARL than the error state, which means that the system will need fewer repetitions to issue the first out-of-control warning. This is because it treats defective items as healthy items and the system crashes. In Section 3, different states can occur due to the existence of different values for the errors e_1 and e_2, depending on the degree of influence of the relationship of the values e_1 and e_2 , we see different behaviors.

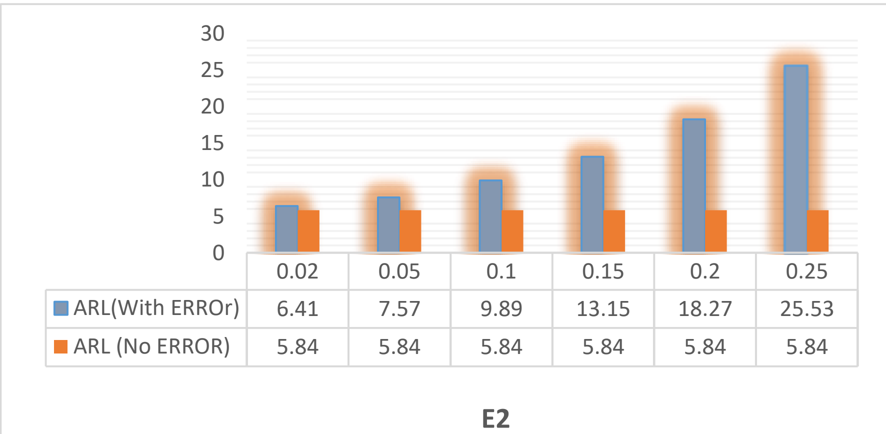
In Section 1, we only have the error e_1 (considering the intact item as a defective item) and by increasing the values of e_1, the ARL values decrease, which means the number of times it takes for the alarm system to go out of control is reduced. This is because the system treats intact items as defective items. *Fig. 1* shows the ARL values for $p = 0.5$ in the error and no error mode.



	0.02	0.05	0.1	0.15	0.2	0.25
ARL(With ERROr)	3.92	2.51	1.5	1.17	1.05	1.01
ARL (No ERROR)	5.84	5.84	5.84	5.84	5.84	5.84

Fig. 1. ARL comparison diagram with error and no error for constant value P = 0.5.

In Section 2, we only have the error e_2 (considering the defective item as a healthy item) and as the values of e_2 increase, the ARL values also increase, which is why This means that the number of times it takes for the alarm system to go out of control increases. *Fig. 2* shows the ARL values for p = 0.0733 in the error and no error mode.



	0.02	0.05	0.1	0.15	0.2	0.25
ARL(With ERROr)	6.41	7.57	9.89	13.15	18.27	25.53
ARL (No ERROR)	5.84	5.84	5.84	5.84	5.84	5.84

Fig. 2. ARL comparison diagram in error and no error mode for constant value P = 0.0733.

In Section 3, we have both the error e_1 and the error e_2, the values of both of which are considered the same because the necessary computational cases are not large and wide, and the value ARL decreases

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with increasing error rates e_1 and e_2. This means that the out-of-control mode is quickly detected due to a measurement error and the system alert is activated. *Fig. 3* shows the ARL values for $E_2 = E_1 = 0.05$ in the error and non-error mode.

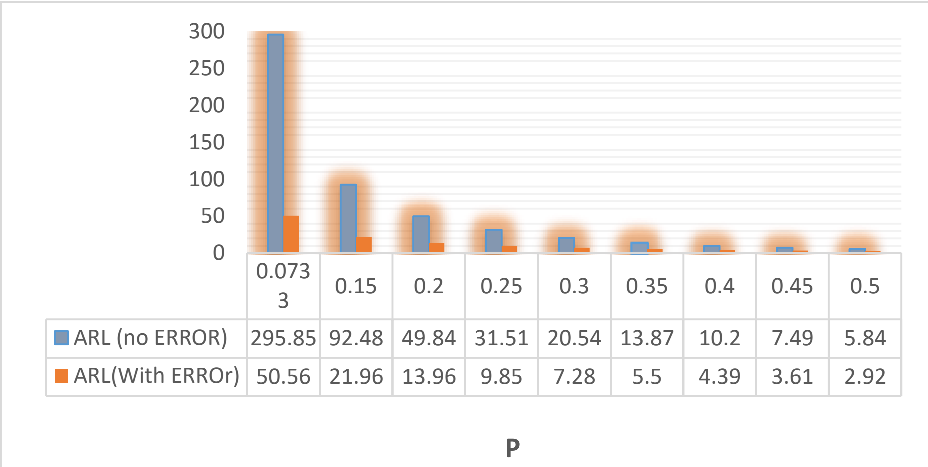


Fig. 3. ARL comparison diagram with error and no error for constant value $E_1 = E_2 = 0.05$.

By changing and increasing the value of p_1 (from $p = 0.0733$ to $p = 0.5$) and increasing the error in different parts, in Sections 1 and 3 of *Table 2*, we see a decrease in ARL and in Section 2 in each section with increasing error we see an increase in ARL. And with increasing the value of p (from p = 0.0733 to p = 0.5) ARL has a decreasing trend.

5.2 | Partial Analysis of Tables

Impact of error e_2, when the ratio of defective process items is almost small, it has a greater effect on the performance of the control graph than large p-cases. For example, consider the case: $p = 0.0733$, $e_2 = 0.02$ and $p = 0.5$, $e_2 = 0.15$ (percentage of error $\frac{e_2}{p}$ is approximately equal with 30%). The difference between ARL in error-free and error-free mode is 40.11 and 7.31, respectively. In other words, the higher the p, the higher the error e_2, even if we increase the error rate by the same ratio $\frac{e_2}{p}$.

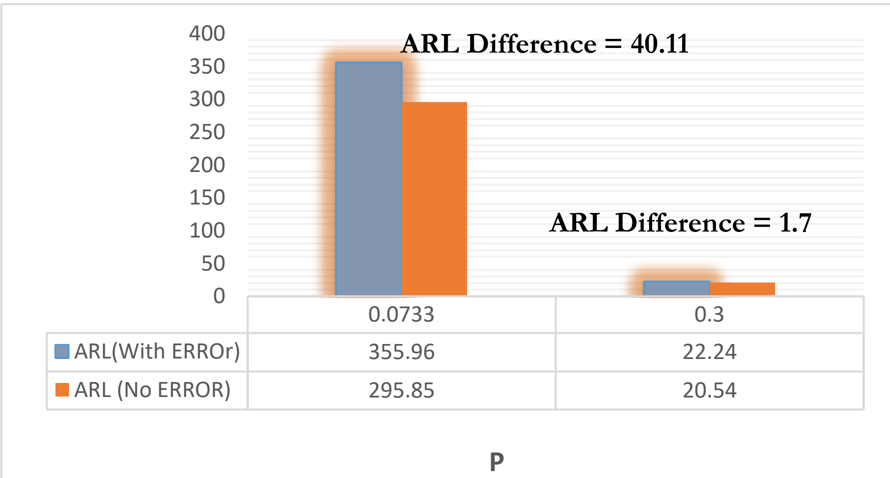


Fig. 4. Comparison diagram of ARL difference with error and no error for constant value $e_2 = 0.02$.

As the error e_2 increases, the ARL increases and the control chart warns later. By increasing p and the error e_2 being the same, the effect of this error can be ignored. For example, as shown in *Fig. 4*, when $e_2 = 0.02, p = 0.0733$ i.e., $\frac{0.02}{0.0733} = \%27$, the rate is ARL = 40.11 and when $e_2 = 0.02, p = 0.3$, i.e., $\frac{0.02}{0.3} = 6\%$, ARL = 1.7 and decreases to 38.41, the effect of this error on the performance of the control chart can be ignored.

As e_1 increases, the proportion of defective items p increases, and the control chart warns earlier than the error-free state. The higher the e_1 value, the lower the ARL and the faster the change can be detected. For example, consider $e_1 = .02, p = 0.0733$ and $e_1 = 0.25, p = .0733$ARL is equal to: 130.28 and 1.26, respectively.

The effect of error e_1 and e_2 for the same values are not equal to each other, and error e_1 has a greater effect on performance than error e_2 The chart will have control. For example, consider $e_1 = 0.02, p = 0.0733$ and $e_2 = 0.02, p = 0.0733$. The state difference (error-free) with ARL is 40.11 and 165.57, respectively, which indicates this.

The effect of error e_1 , when the proportion of defective items in the process is relatively small, it has a greater effect on the performance of the control graph than in the case of large ps, as shown in *Fig. 5*. Compared to $p = 0.5$, it has a larger numerical value.

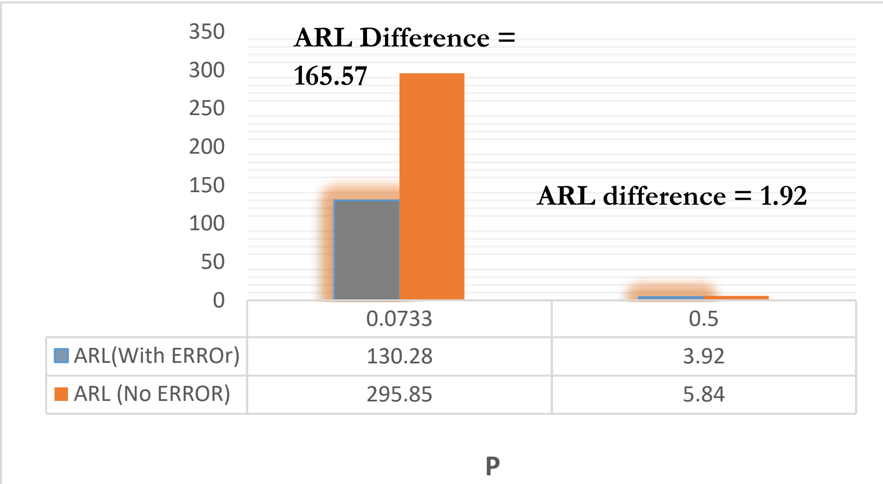
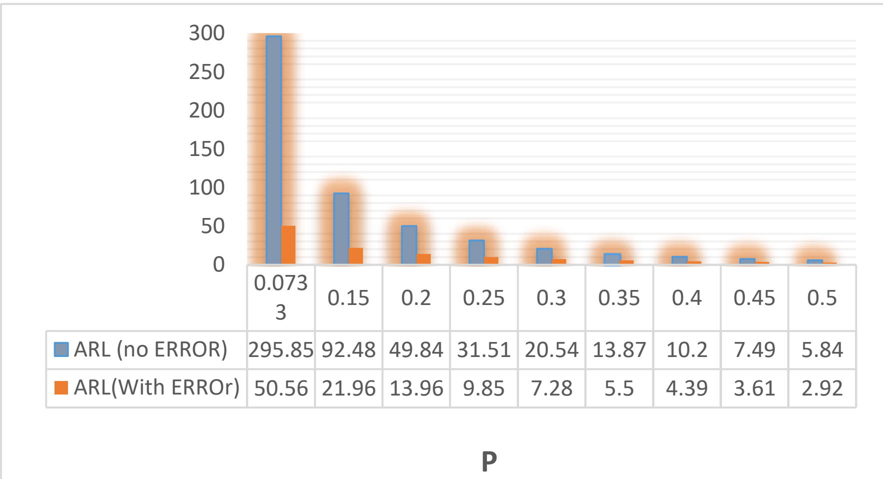
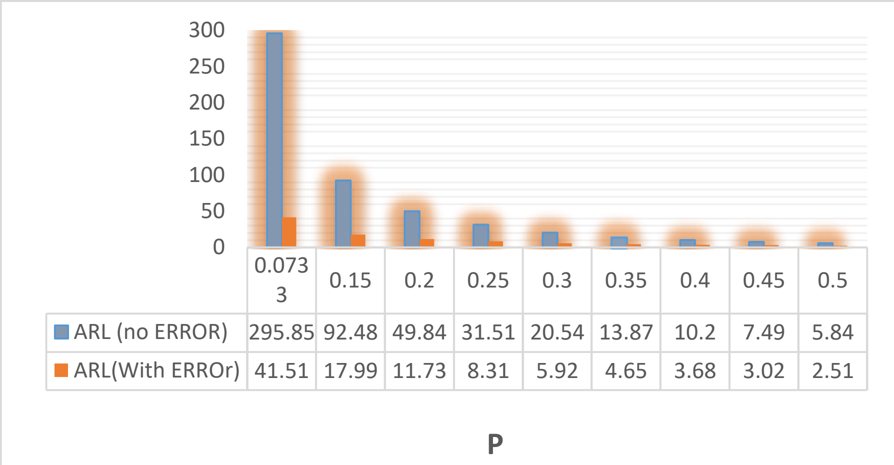


Fig. 5. Comparison diagram of ARL difference with error and no error for constant value $e_1 = 0.02$.

When we have the error e_1 and e_2 at the same time and with the same size, the performance of the graph is almost the same as when we have only the error e_1 . The reason for this is that the error e_2 has little effect on the performance of the control chart, especially at ps greater than 0.0733, and the greatest effect is due to the error e_1 . For example, according to *Fig. 6 (a)* and *(b)*, when p =0.15 and $e_1 = e_2 = 0.05$, the difference is ARL = 70.52 and when we have only the error e_1 and $e_1 = 0.05$, the difference is ARL = 74.49 which are slightly different from each other and almost equal.



(a)



(b)

Fig. 6. Comparison of the state diagram of having both types of errors with the cases having only the error of the first type.

5 | Conclusion

Measurement error is one of the common phenomena in the measurement system that is usually not considered and ignoring this error can have many costs and consequences. Adverse consequences of measurement error include an increase in the number of false alarms, such as incorrect detection of out-of-process signals and delayed detection of changes in the process. This is not consistent with the goal of statistical quality control, which is to identify deviations as soon as possible. Therefore, considering the measurement error in designing multivariate control diagrams is of great importance. In this paper, for the first time, the effect of measurement error on the performance of the MNP control chart was investigated by the ARL criterion. The results show that measurement error affects the performance of this diagram. If the measurement system mistakenly counts only defective items as healthy items, we have an average time to the first warning more than ideally and without system error. On the other hand, if the system mistakenly counts only healthy items as defective items, we will have an average time to the first warning less than ideally and without system error. Also, if both errors occur simultaneously (counting defective items as healthy items and vice versa), the system will behave differently depending on the amount of each error, while the values of both errors are considered the same. It can only be said that as the amount of error increases and the shift increases the probability of the number of defective items decreases, the ARL will decrease. Designing other control charts to monitor processes in the presence of measurement error can be considered as a topic for future studies.

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