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A New Model to Measuring Efficiency and Returns to Scale on Data Envelopment Analysis

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Abstract

We extend the concept of returns to scale in Data Envelopment Analysis (DEA) to the weight restriction environments. By adding weight restrictions, the status of returns to scale, i.e. increasing, constant, and decreasing, may need a change. We first define "returns to scale" underweight restrictions and propose a method for identifying the status of returns to scale. Then, we demonstrated that this addition would usually narrow the region of the Most Productive Scale Size (MPSS). Finally, for an inefficient Decision-Making Unit (DMU), we will present a simple rule for determining the status of returns to the scale of its projected DMU. Here, we carry out an empirical study to compare the proposed method's results with the BCC model. In addition, we demonstrate the change in the MPSS for both models. We have presented different models of DEA to determine returns to scale. Here, we suggested a model that determines the whole status to scale in decision-making units.

Keywords: Data envelopment analysis, Decision-making units, Most productive scale size, Returns to scale.

1 | Introduction

One of the concepts that have gained significant attention from economic and management-science research is the concept of Returns To Scale (RTS). There are several types of research on the right and left RTS behavior, as two specific directions of RTS, based on Data Envelopment Analysis (DEA) models. However, the main weakness of the majority of these methods is that researchers have based them on the defined parameters. Then we demonstrate that leads to the high sensitivity of the models to variations in the magnitudes of the parameters. Thus, unreliable results. Mirbolouki & Allahyar [11], In a paper, proposed a simple procedure for detecting the right and left RTS classification with an important feature that is independent of any predetermined parameters. Besides the RTS type, they suggested a method to determine the right and left RTS value corresponding to each of the efficient DMUs [11].

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The analyses introduced by DEA has redefined the concept of returns to scale. It is now possible to evaluate the returns-to-scale characteristics of each observation in the data set, in contrast to the Cobb-Douglas type production function approaches that usually estimate the characteristics as a whole. Furthermore, Banker and Thrall [3] introduced the concept of Most Productive Scale Size (MPSS) which scholars widely used for analyzing the optimal level of the scale size associated with the set of observations concerned. Meanwhile, weight restrictions have been recognized as an essential factor when applying DEA to actual situations, and scholars have developed several models for this purpose. These include the Assurance Region (AR) model and the Conratio Approach [3] and [4]. Since researchers regard weights for inputs/outputs as associated with costs of inputs and prices of outputs, constraints on weights should preferably reflect the actual costs/prices information. However, exact knowledge of prices and costs is often unavailable or unstable, so we use some substitutes in these models. For example, in the AR model, upper and lower bounds are imposed on the ratio of weights for specific pairs of inputs or outputs. These weight restrictions contribute to avoiding the occurrence of frequently observed zero optimal weights to some inputs/outputs caused by the optimization mechanism of DEA. Hence the results of analysis using weight restrictions are more persuasive than those without restrictions. This paper aims to develop the returns-to-scale concept when we impose weight restrictions and to demonstrate that the status of returns to scale may suffer a change when we add these restrictions. Therefore, the research subject must investigate returns to scale under weight restrictions theoretically and empirically [1].

The knowledge of DEA evaluates the effectiveness of decision-making units. One of the problems of DEA is that if the number of units with the same efficiency equal to one was more than one, they could not select the best between them. It means that we cannot rank them. Therefore, we consider the need for ranking these units by the managers. Different methods are scale proposed in this context. DEA models imitate most of these methods [14].

There are many methods to determine returns to scale. Here, we have not discussed the details of these methods. However, we will discuss the suggested model more thoroughly. This model is a linear programming question. Its solution gives us the returns to scale of the evaluated unit [3].

Despite the massive use of DEA models for efficiency estimations in scientific applications, no paper cared about identifying the DEA model, providing the most accurate efficiency estimates so far. Develop an established method based on a Monte Carlo data generation process to create artificial data. As a user, the trans-log production function, instead of the commonly utilized Cobb Douglas production function, can construct meaningful scenarios for constant returns to scale. Then, we use the decision-making units resulting from the generated data to calculate DEA estimators using different DEA models. Today, the use of data envelopment analysis techniques is expanding rapidly. The researchers use it to evaluate various organizations and industries such as banks, hospitals, training centers, Etc. In real-world problems, the values observed from input and output data are often ambiguous and random. researchers have proposed data envelopment analysis in a stochastic fuzzy environment to solve this problem [12] and [13].

With this procedure, can postulate general statements on parameters that influence the quality of DEA studies in a positive/negative way and determine which DEA model operates in the most accurate way for a range of scenarios. Here, we show that the Assurance Region and Slacks-Based-Measurement models outperform the CCR (Charnes–Cooper–Rhodes) model in constant returns to scale scenarios [15].

The concepts of RTS and Scale Economies (SE) have a crucial position in economics and production theory. Scholars use these concepts to provide valuable information on the optimal size of the firms [22]. In DEA, DMUs are classified into three categories based on their type of RTS: Constant RTS (CRS), Decreasing RTS (DRS), and Increasing RTS (IRS). RTS is applied to recognize whether an efficient production activity can enhance its productivity by changing the scale of its operations [6] and [11].

Eslami and Khoveyni [8] studied determining the type and measuring value right and left returns to scales in data envelopment analysis. Taeb et al. [16] studied to determine the efficiency of time depended on units using data envelopment analysis. This study identifies types and values of right and left RTSs of efficient decision-making units (DMUs) in DEA [7]. Alireza et al.[2] studied Objective identification of technological returns to scale for data envelopment analysis models. This paper considered one of the most critical problems for setting up a data envelopment analysis model: identifying suitable RTS for the data. Referred to it as the Technological Returns To Scale (TRTS) to completely separate the technology's RTS from the DMU's RTS [2].Abri [1] considered investigating the sensitivity and stability radius of returns to scale and efficiency in data envelopment analysis. This paper will study the sensitivity of the RTS classifications in data envelopment analysis using linear programming problems. It is surprising since RTS classifications provide essential information for improving an individual DMU's performance when scale inefficiencies are detected [1].

We should note that the subject of RTS mainly has a clear interpretation only if the DMU under evaluation is efficient. RTS is a characteristic of the frontier at a specific point, so RTS is discussed only for efficient DMUs in this study.

Reedy [17] assessed Thirupati Reddy Comparison and Correlation Coefficient between CRS and VRS models of OC Mines [17]. Hatami-Marbini et al. [8] considered the measurement of returns-to-scale where they used interval data envelopment analysis, models. In this paper, researchers have studied the economic concept of RTS intensively in the context of DEA. The conventional DEA models that researchers use for RTS classification require well-defined and accurate data, whereas in reality, observations gathered from production systems may be characterized by intervals. For instance, the heat losses of the Combined Production of Heat and Power (CHP) systems may be within a specific range, hinging on a wide variety of factors such as external temperature and real-time energy demand [8].

Khodadadi and Haghighi [10] studied Two Methods for Measuring the Environmental Returns to Scale Using Data Envelopment Analysis Approach. Sueyoshi and Wang [19] studied measuring scale efficiency on sizeable commercial rooftop photovoltaic systems in California. They examined managerial sources of operational efficiency or inefficiency on 855 large commercial rooftop PV power systems in California by examining both scale efficiency and RTS. For the research purpose, this study utilizes DEA as a methodology to assess the scale measures. Furthermore, by Paying attention to the effects of those uncontrollable factors, this study discusses how to measure scale efficiency and RTS within the framework of DEA.

Toloo and Allahyar [18], investigated a generalized simplification returns to scale approach for selecting performance measures in data envelopment analysis. Toloo and Tichý [20], to hold the rule of thumb in data envelopment analysis, developed a pair of models that optimally chooses some inputs and outputs among selective measures under variable returns to scale assumption. Their approach involves a lower bound for the input and output weights in the multiplier model and a penalty term in the objective function of the envelopment model [20].

Ghasemi et al. [9], a case study with DEA for Estimate Efficiency and Ranking operating rooms. Czyżewski et al. [5] Assessed the impact of environmental policy on eco-efficiency in country districts in Poland: How does the decreasing return to scale change perspectives? in this study, authors show how changing CRS assumption affects environmental policy effectiveness based on Polish example. The problem revealed in the conducted analysis is in many countries, where the local perspective may efface global threats. The empirical objective of this paper is to assess the cost-effectiveness of environmental policies at the county level under various RTS scenarios [5].

Bernstein [6] investigated an updated assessment of technical efficiency and returned to scale for U.S. electric power plants. This paper utilizes cutting-edge panel stochastic frontier electricity production models to measure the impact of state and federal regulations on United States (U.S.) natural gas-fired power plants from 1994 to 2016. Deploying a trans-log functional form, extract firm-specific information on RTS [6].

We have organized the rest of this manuscript as follows. We will also study the BCC model to determine returns to scale in section 2 and present our suggested model in section 3.

2 | BCC Model in Determining Returns to Scale

Farrel [21] defines the measurement of efficiency and its function by Charnes et al. [4]. It led to the establishment of tools of important methods DEA for evaluating the efficiency. The standard method DEA, which measures technical efficiency with return hypothesis to variable scale in input essence, is done by Banker, Charnes, and Cooper (BCC) [3]:

$$
\begin{aligned}
&\text{Miximize} \quad \theta = \sum_{r=1}^{s} u_r y_{ro} + u_0 \\
&\text{Subject} \quad \text{to.} \\
&\qquad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \qquad j = 1,\dots,n \\
&\qquad u_r \ge \varepsilon \qquad\qquad r = 1,\dots,s \\
&\qquad v_i \ge \varepsilon \qquad\qquad i = 1,\dots,m \\
&\qquad u_0 \qquad\qquad\quad \text{free}
\end{aligned} \tag{1}
$$

In this problem, θ_o is the efficiency level of the evaluated decision-making unit. y_{rj}, x_{ij} are, respectively introduced as ith levels of input of output DMU_j. v_i, u_r are weights related to inputs and outputs, comparable with model variables. It can be interpreted as shadow price; therefore, the price of input and output DMU which will be shown, is the best possible price. ε is a small non archimedes value. It guarantees that all inputs and outputs will be used in calculations for efficient evaluation.

Definition: DMU_o is completely efficient, if and only if the condition $\theta_o = 1$ available in its evaluation by *Model 1*. Banker and Thrall [3] proved this to identify returns to scale with u_0.

3 | Model Generating Returns to Scale GRS

While $L \le 1$ and $U \ge 1$ the resulting model have a Generating Returns to Scale (GRS), this model allows the expansion and limitation of size over a while. *Fig. 1* shows the difference between the GRS boundary and the BCC and CCR boundaries.

Theorem 1. These conditions identify this situation for returns to efficient unit scale in the BCC model.

1. Returns to scale DMU_o is increasing if and only if $u_o \succ 0$ for all optimal solutions.

2. Returns to scale DMU_o is decreasing if and only if $u_0 \prec 0$ for all optimal solutions.

3. Returns to scale DMU_o is constant (MPSS), if and only if $u_o = 0$ for some optimal solutions.

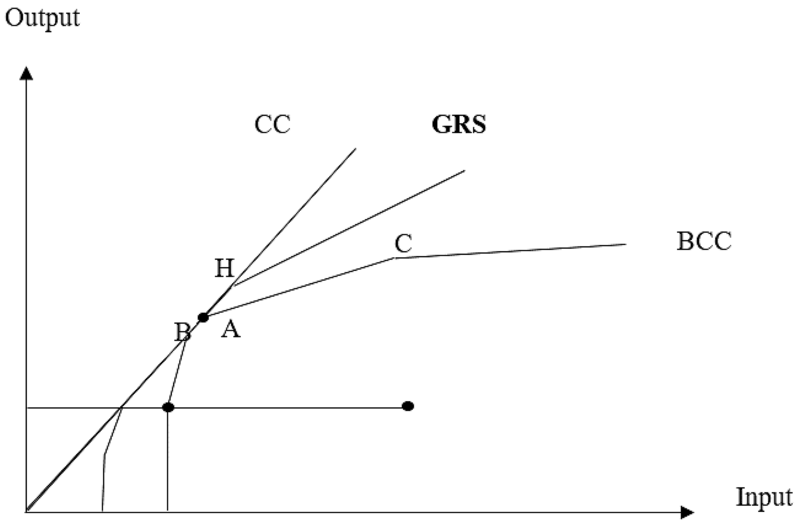

Fig 1. The GRC.

However, Banker and Thrall [3] presented a way to abandon the need to find all the optimal solutions. We are not going to discuss it here.

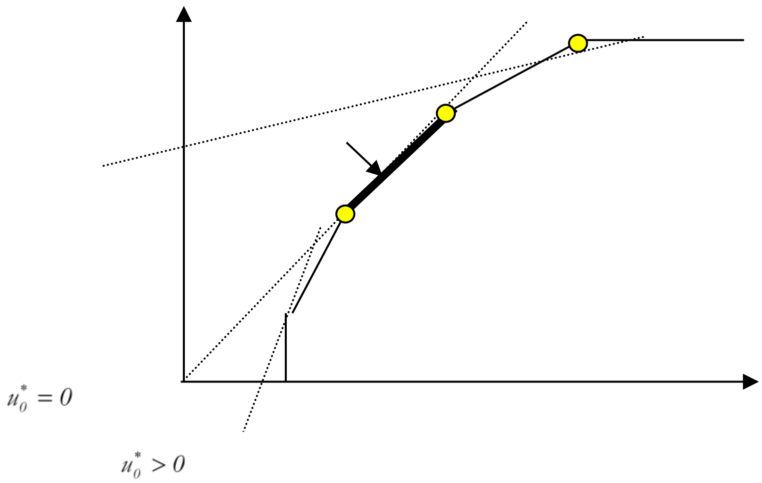

Fig. 2. The identification of returns to scale with u_o

4 | Proposed Model

The additive model which has been provided by Charnes et al. [4] to evaluate decision-making units is defined as follows [2].

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$$\text{Maximize} \quad \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+$$

Subject to.

$$\begin{aligned} &\sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = x_{io} && \forall i, \\ &\sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+ = y_{ro} && \forall r, \\ &\sum_{j=1}^{n} \lambda_j = 1 \\ &S_i^-, S_r^+ \geq 0 && \forall i, r, \\ &\lambda_j \geq 0 && \forall j, \end{aligned} \tag{2}$$

The objective function of the above model is linear which indicates the L_1 norm.

In this model, the objective function calculates the farthest distance of the evaluated unit on dominant efficient units.

Theorem 2. In the integrative model, the evaluated decision-making unit is efficient if and only if the objective function is zero in optimality.

Theorem 3. Suppose that $DMU_o = (x_o, y_o)$ is efficient. then:

1) $0 \prec \xi \prec 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series, if and only if (xo, yo) has decreasing returns to scale.

2) $\xi > 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series, if and only if (xo, yo) has increasing returns to scale.

3) For every $\xi > 0$ so that $(\xi x_0, \xi y_0)$ is the possible efficient production, if and only if (xo, yo) has constant returns to scale (MPSS).

Proof 1. If $0 \prec \xi \prec 1$ so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series. Since (xo, yo) is an efficient unit, there is a supporting hyperplane $U^* y - V^* x + u_0^* = 0$ in possible production series so that it is active on (xo, yo) (because it is supposed that (U^*, V^*, u_0^*) is the optimal solution for the BCC model in (xo, yo) evaluation. Since $(\xi x_0, \xi y_0)$ is inefficient in possible production series, we have

$$U^* \xi y_o - V^* \xi x_o + u_0^* < 0 \Rightarrow \xi (U^* y_o - V^* x_o + u_0) - \xi u_0^* + u_0^* < 0 \Rightarrow u_0^* (1 - \xi) < 0 \Rightarrow u_0^* < 0.$$

Hence, *(xo, yo)* has decreasing returns to scale according to *Theorem 1*.

On the contrary, suppose that (xo, yo) has decreasing returns to scale. Therefore, we should show that $\xi \varepsilon$ (0,1) so that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series.

Since (xo, yo) has decreasing returns to scale, for each optimal solution for the BCC model in its evaluation $u_0 \prec 0$ according to therem1: Supposition breach for every $\xi \varepsilon$(0,1) so that it $(\xi x_0, \xi y_0)$ is the possible production, it is efficient. Therefore, every convex combination of $(\xi x_0, \xi y_0)$ and (xo, yo) is in the possible production of series and is available on the efficient symbol. Therefore, the supporting hyperplane $\overline{U}y - \overline{V}x + \overline{U}o = o$ can be taken on possible production series so that it passes the connecting line $(\xi x_0, \xi y_0)$ and (xo, yo). If $\overline{V}x = a$, $(U^*, V^*, u_0^*) = (a^{-1}\overline{U}, a^{-1}\overline{V}, a^{-1}u_0)$ is an optimal solution in the evaluation of (xo, yo) so that it is active on (xo, yo) and $(\xi x_0, \xi y_0)$ then

$$U^* y_o - V^* x_o + u_0^* = 0.$$

$$U^* \xi y_o - V^* \xi x_o + u_0^* = 0.$$

In the two above equations, if we subtract ε the first equality from the second equality, there is:

$$u_0^*(1 - \xi) = 0 \Rightarrow u_0^* = 0.$$

It means that (x_o, y_o) has constant returns to scale. This is in contrast with our hypothesis. Therefore, the supposition breach is invalid and the axiom is true.

The proof of the conditions 2) and 3) can be done similarly. We tried to make $(\xi x_0, \xi y_0)$ inefficient in our model to determine returns to scale so that the value of ε can be determined in this way. Therefore, if the inefficiency $(\xi x_0, \xi y_0)$ leads to an increase of ε from level, a return to scale is increasing. If the inefficiency of $(\xi x_0, \xi y_0)$ leads to a decrease of ε from leve1, returns to scale are decreasing. If $(\xi x_0, \xi y_0)$ cannot be the possible inefficient production, there will be constant returns to scale. The suggested model is:

$$\begin{aligned}
&\text{Maximize} \quad \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+ \\
&\text{Subject to.} \\
&\qquad \sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = \xi x_{io} \qquad \forall i, \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} - S_r^+ = \xi y_{ro} \qquad \forall r, \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1, \\
&\qquad S_i^-, S_r^+ \geq 0 \qquad \forall i, r, \\
&\qquad \lambda_j \geq 0 \qquad \forall j, \\
&\qquad \xi \geq 0.
\end{aligned} \tag{3}$$

We present the following theorem to identify returns to scale with *Model 3*.

Theorem 4. The following conditions determine returns to scale for DMUo in *Model 3*.

1) The optimal value of the objective function in optimality is non – zero and, $\varepsilon^* \succ 1$ if and only if returns to scale DMUo is increasing.

2) The optimal value of the objective function in optimality is non zero an and, $\varepsilon^* \prec 1$ if and only if returns to scale DMUo decreases.

3) The optimal value of the objective function in optimality is zero, if and only if returns to scale DMUo are constant (MPSS).

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The proof of condition 1. We suppose that the optimal value of the objective function in optimality is nonzero and $\varepsilon^* \succ 1$, therefore $(\xi x_0, \xi y_0)$ is the possible production in series, and it is inefficient. Returns to scale DMUo are increasing according to *Theorem 3*.

On the contrary, if returns to scale of DMUo are increasing $\varepsilon^* \succ 1$ according to *Theorem 3* so that $(\xi x_0, \xi y_0)$ is the possible inefficient production. The inefficiency of $(\xi x_0, \xi y_0)$ in *Model 3* requires the level of the objective function to be above zero. Because this function is a kind of maximizing, function in optimality should be non-zero.

It means that $(\xi x_0, \xi y_0)$ is the possible inefficient production in series.

Returns to scale are increasing, therefore, $\varepsilon^* \succ 1$.

we can prove conditions 2 and 3 similarly.

It means that is the possible inefficient production in series.

Returns to scale are increasing. Therefore,

Example 1. *Table 1* presents a model of the characteristics of three decision-making units to determine returns to scale.

Table 1. RTS data.

RTS	ξ^*	S_1^{+*}	S_1^{-*}	y_1	x_1	DMU#
Increasing	2	0	1	1	1	1
Constant	1	0	0	2	2	2
Decreasing	0/65	0	1	5	4	3

Model 3 to determine returns to scale in decision-making unit in *Example 1* is:

$$\begin{aligned}
&\text{Maximize} \quad S_1^- + S_1^+ \\
&\text{Subject to.} \\
&\qquad 1\lambda_1 + 2\lambda_2 + 4\lambda_3 + S_1^- - 1\xi = 0, \\
&\qquad 1\lambda_1 + 2\lambda_2 + 5\lambda_3 - S_r^+ - 1\xi = 0, \\
&\qquad \lambda_1 + \lambda_2 + \lambda_3 = 1, \\
&\qquad \lambda_1, \lambda_2, \lambda_3 \geq 0, \\
&\qquad \xi \geq 0.
\end{aligned}$$

The optimal response to this question is:

$$(\lambda_1^*, \lambda_2^*, \lambda_3^*, S_1^{-*}, S_2^{+*}, \xi^*) = (0,1,0,1,0,2).$$

$$\xi^* = 2 > 1.$$

The value of the objective function in optimality is 1, which indicates that returns to scale of this decision-making unit cannot be constant. On the other hand, $\varepsilon^* = 2 \succ 1$ therefore, returns to scale are increasing.

Another characteristic of the suggested model is that we can identify both the returns to scale of the evaluated decision-making unit and MPSS as a management objective with its solution. The following theorem indicates this subject.

5 | Conclusions

In this manuscript, we used the DEA method to find the weight restriction environments. Our empirical study compares the results obtained through the proposed method with those of the BCC model. It further demonstrates the change in the MPSS for both models. We have presented different data analysis models to determine returns to scale and have suggested a model here. The suggested model determines the constant returns to scale, increasing returns to scale, and decreasing returns to scale in decision-making units.

This manuscript studies the new method detects both the type and the value of returns to scale.

The advantage of the new method is that it uses no parameter in model formulation.

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