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Development of a Multi-Period Multi-Attribute Group Decision-Making Method Using Type- 2 Fuzzy Set of Linguistic Variables

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Abstract

Recently, fuzzy linguistic variables have gained a great deal of attention from researchers in many decision-making problems. In this problems, type-2 fuzzy sets have been used to better cover linguistic data uncertainty. However, many of research works in this regard have been performed in type-2 fuzzy domain and in static mode. Since the decision-making problems in the real world usually fluctuate over time, so it needs to use decision making models in multi period of time. In the present research, a Multi-Period (Dynamic) Multi-Attribute Group Decision-Making (MPMADM) method is presented based on type-2 fuzzy sets where decision-making attributes are first expressed in linguistic terms and then incorporated, as interval type-2 fuzzy numbers, into problem solving where a new integrating operator called Multi-Period Trapezoidal Interval Type-2 Fuzzy Number Weighted Arithmetic averaging (MPTIT2FNWA) is defined on type-2 interval fuzzy numbers to integrate decision-making information in multiple periods of time. Once finished with explaining the proposed method, a numerical example is given to evaluate the proposed method in terms of effectiveness and applicability, with the results compared to those of other methods.

Keywords: Interval type-2 fuzzy sets, Linguistic variables, Multi-period trapezoidal interval type-2 fuzzy number weighted arithmetic operator, Multi-period multi-attribute group decision-making.

1 | Introduction

Today, Multi-Attribute Decision-Making (MADM) methods have found numerous applications in solving different management problems. However, in many real-world problems, decision-making process has its basis on decision information of several time periods, and one should include the trend of change in the values induced over the course of time in the decision-making process because information is subject to change with time and these changes can seriously affect the trend of decision-making process and prioritization of alternatives; in this case, Multi-Period Multi-Attribute Decision-Making (MPMADM) comes into play.

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Even though MPMADM problems are more complicated than MADM, their application can be associated with better results. Numerous research works have been done in this respect. In a study, Xu [1] presented a MPMADM method where Simple Additive Weighting (SAW) was used to undertake MPMADM. He used distance data to develop a model where a Multi-Period Weighted Averaging (DWA) operator was used to integrate values of the attributes in different periods of time, and further evaluated an investment method selection problem. Lin et al. [2] proposed a MPMADM method wherein TOPSIS method was used to rank alternatives in different periods of time. In this study, values of attributes were considered as Triangular Fuzzy Numbers (TriFN) and Minkowski Distance was used to compare these numbers. They finally implemented the proposed method for contractor selection. In a MPMADM problem, Xu and Yager [3] first introduced Multi-Period Intuitionist Fuzzy Weighted Averaging (DIFWA) and Uncertain Multi-Period Intuitionist Fuzzy Weighted Averaging (UDIFWA) operators. In this study, first, these operators were employed to integrate decision matrixes for all periods of time. Then TOPSIS technique was used to have available alternatives ranked. Yong et al. [4] proposed TOPSIS technique to solve MPMADM on gray numbers. The began with obtaining weighted Minkowski distance function of gray number by using the Euclidian distance between two gray numbers and the concept of weighted Minkowski distance function. They then implemented different steps of TOPSIS technique. Chen and Li [5] proposed a MPMADM method based on Triangular Intuitionist Fuzzy Numbers (TriIFN) where fuzzy entropy method and averaging operator were used to determine weights of attributes. They obtained a general ranking of alternatives by integrating, using TOPSIS, the results obtained from different periods of time, and finally analyzed an investment problem. Zhu and Hipel [6] proposed a MPMADM method based on linguistic variables and then evaluated performance of vendors of an electronic navigation system in terms of economic and production development of aviation industry in China, in this study, decision information was used as 2-tuple linguistic terms and solved MPMADM based them. Hu and Yang [7] proposed a MPMADM method on the basis of probability theory where attributes had their values expressed in terms of discrete random variables. They used this method for risk evaluation. Sadeghian and Forootan [8] proposed a MPMADM method using a regression model. In their study, they presented a regression model for each element in the decision-making matrix and then used TOPSIS technique to rank alternatives. They used this method to rank investments in textile industry. Park et al. [9] used VIKOR method to develop a MPMADM approach. In this study, Interval Triangular Fuzzy Numbers (ITFN) to express decision information and used the proposed method to evaluate university professors for promotions. Liu et al. [10] used extendible interval numbers in MPMADM method. In this study, they introduced a distance function for extendible interval numbers, and following the determination of comparative value of each alternative in each time period, TOPSIS technique was used to rank the alternatives to select one city among three candid cities for investment on public transportation development. Li et al. [11] presented a MPMADM method where used data was expressed in terms of TriFNs. In this research, the authors used mathematical programming to determine weights of different attributes in different periods of time, followed by ranking the alternatives utilizing TOPSIS technique. Bai et al. [12] presented a MPMADM method proposed a MPMADM based on TOPSIS technique and representation of available data in terms of Trapezoidal Fuzzy Numbers (TraFNs); the proposed method was then implemented it to rank a set of suppliers. Bera et al. [13] presented a two phase MPMADM Approach for supplier evaluation and order allocation considering multi-objective, multi-product and multi-period. In this research, in first phase, the ranking of supplier was performed by using fuzzy MULTIMOORA method with regard to the important criteria. In the second phase, multi-objective linear programming (MOLP) method in fuzzy environment was proposed to allocate orders to the preferred suppliers. Li et al. [14] presented a MPMADM method for supplier selection problem. In this research, data of decision problem was expressed in generalized fuzzy numbers and the weights of different periods are determined by a mathematical programming method. Fei and Feng [15] proposed a novel framework for dynamic MADM in Pythagorean fuzzy environments based on Dempster–Shafer Theory (DST). In this research, the period weight from the dual dimensions of information and consistency were determined and the period weight from the dual dimensions of information and consistency were determined and the dynamic MADM was completed by combining the decision information of all attributes in each period, fusing the decision information of different periods, and calculating the ranking index of each alternative.

Decision-making information, which are expressed by group of experts, are usually inaccurate and ambiguous because of, for example, lack of accurate data, shortage of time, or failure to pay adequate attention or inadequate knowledge of the members of expert group. In many problems, it is difficult to access accurate values of data for decision-making [16]. In such situations, opinions of the expert group (originally expressed in words) serve as criteria for valuation. However, words are always associated with ambiguities, so that researchers have tended to use fuzzy sets theory to address this ambiguity, where values of variables are specified with a membership degree. However, complexities in some of decision-making problems have made it difficult to determine exact value of membership degree [17]. The fuzzy linguistic approach is an approximate technique appropriate to deal with the qualitative aspects of decision-making problems [6]. Zadeh [18] introduced Type-2 Fuzzy Sets (T2FS) as an extension to Type 1 Fuzzy Sets (T1FS). T2FSs tend to exhibit better performance in reducing the effect of uncertainty in fuzzy rules. Due to fuzzy nature of membership functions in T2FSs, the possibility to model linguistic uncertainties is effectively improved. Turksen [19] introduced the application of T2FSs to support word calculation reasoning. Based on the reasoning that words are associated with more complex uncertainties than that of T1FSs, Mandel [20] recognized the use of T1FSs for word modelling as being inappropriate and believed that T2FSs can better model word uncertainties. Since computation in type-2 fuzzy domain has numerous complexities, the use of Interval Type-2 Fuzzy Sets (IT2FS), for which numerous functions and operators have been proposed, has recently gain a large deal of attention, so that it is now developed as an efficient theory in domains of high uncertainty [21] and [22].

In MADM problems, there are typically approaches to dealing with linguistic models as, T1FS [11], [12], [23]-[25], hesitant fuzzy [26]-[28], intuitionistic fuzzy [3], [5], [9], type-2 fuzzy [17], [29]-[36], [41]-[43], fuzzy 2- tuple [6], Neutrosophic Sets (NS) [37] and etc.

Even though many research works have been performed on MPMADM, this method is yet to be addressed in type-2 fuzzy environment and its application to solve of different problems in domain of management can have interesting results. As such, the present research aims to present MPMADM in type-2 fuzzy domain. In this method, a new operator Multi-Period Trapezoidal Interval Type-2 Fuzzy Number Weighted Arithmetic Averaging (MPTIT2FNWA) based on Basic Unit-interval Monotonic (BUM) probability distribution function is defined to integrate decision information in multiple time periods. This operator was found to provide required flexibly to select any trend of time series for the specified weights of time periods depending on the problem characteristics. To determine the efficiency of the proposed method, we will solve a numerical example in [6] and compare the results with their method. Accordingly, in Section 2, T2FSs are defined together with respective functions and operators. Then in Section 3 we proceed to introduce MPTIT2FNWA operator. Section 4 explains the framework of MPMADM in type-2 fuzzy domain, and Section 5 gives a numerical example to better understand the proposed method. Finally, conclusions are drawn in Section 6.

2 | Type-2 Fuzzy Sets

First introduced by Zadeh in 1965, fuzzy sets theory serves as a modeling tool for complicated systems [38] and [39]. Original concepts within the scope of fuzzy sets theory were formulated under the name of Type 1 Fuzzy Sets (T1FSs). These then found numerous applications, particularly in MADM problems. In T1FSs, each set is determined by its elements and their membership function which gives a real number between 0 and 1 for each member in the set. Zadeh [18] introduced T2FS as an extension to T1FSs. In T2FSs, membership function of the elements in the set is itself a fuzzy set. Mandel and Wu [21] presented a new concept of T2FSs with a simple calculation process where superior and inferior limits are considered for the membership functions, with each of these membership functions resembling a membership function in T1FSs. Later on, Mandel et al. [22] further proposed a new concept called IT2FS, where membership function of each element was a fuzzy set in the interval of [0, 1]. In the following sections, definitions of some of concept and operators related to IT2FS are given.

2.1 | Type-2 Fuzzy Sets

Explained in this section are definitions of some of the concept and operators related to IT2FS, followed by a presentation of the IT2FS method along with the required operators.

Definition 1. [30] and [31]. If $\tilde{A}$ is a T2FS on the universe of discourse X, it can be defined as follows:

$$\tilde{A}=\left\{\left((x,u),\mu_{\tilde{A}}(x,u)\right);\forall x\in X,\forall u\in J_x\subseteq[0,1]\right\}. \tag{1}$$

where $0\geq\mu_{\tilde{A}}(x,u)\geq 1$, and we have:

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/(x,u)=\int_{x\in X}\left(\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/u\right)/x. \tag{2}$$

Where $\int$ represents the sum of all combinations of (x, u), x is the primary variable with its membership function being $J_x\subseteq[0,1]$, and u is the secondary variable with the membership function $\int_{u\in J_x}\mu_{\tilde{A}}(x,u)/u$ on X.

Definition 2. [30], [31]. Let $\tilde{A}$ is a T2FS where in all $\mu_{\tilde{A}}(x,u)$ are equal to 1, then $\tilde{A}$ is referred to as an IT2FS, in which case we have:

$$\tilde{A}=\int_{x\in X}\int_{u\in J_x}1/(x,u)=\int_{x\in X}\left(\int_{u\in J_x}1/u\right)/x. \tag{3}$$

where x is the primary variable with its membership function being $J_x\subseteq[0,1]$, and u is the secondary variable with the membership function $\int_{u\in J_x}1/u$ and foot print of uncertainty in the set $\tilde{A}$ is defined as follows:

$$FOU(\tilde{A})=\cup_{x\in X}J_x. \tag{4}$$

which includes the sum of primary membership function over the reference set X.

Definition 3. [21]. In general case, a Trapezoidal Interval Type-2 Fuzzy Number (TIT2FN) is defined as $\tilde{A}=\left(\tilde{A}^U,\tilde{A}^L\right)=\left(\left(a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(a_1^L,a_2^L,a_3^L,a_4^L;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right)$ where $\tilde{A}^U$ and $\tilde{A}^L$ are type-1 fuzzy numbers (T1FN) and, $a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right),a_1^L,a_2^L,a_3^L,a_4^L;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)$ are real numbers and is establishing the inequality $a_1^U\leq a_2^U\leq a_3^U\leq a_4^U, a_1^L\leq a_2^L\leq a_3^L\leq a_4^L$, as can be seen on *Fig. 1*, $H_i\left(\tilde{A}^U\right)$ is the membership value of the element a_{i+1}^U in the upper trapezoidal membership function (UMF) and $H_i\left(\tilde{A}^L\right)$ is membership value of the element a_{i+1}^L in the lower trapezoidal membership function (LMF) where, $0\leq H_i\left(\tilde{A}^U\right)\leq 1$, $0\leq H_i\left(\tilde{A}^L\right)\leq 1$ and $1\leq i\leq 2$.

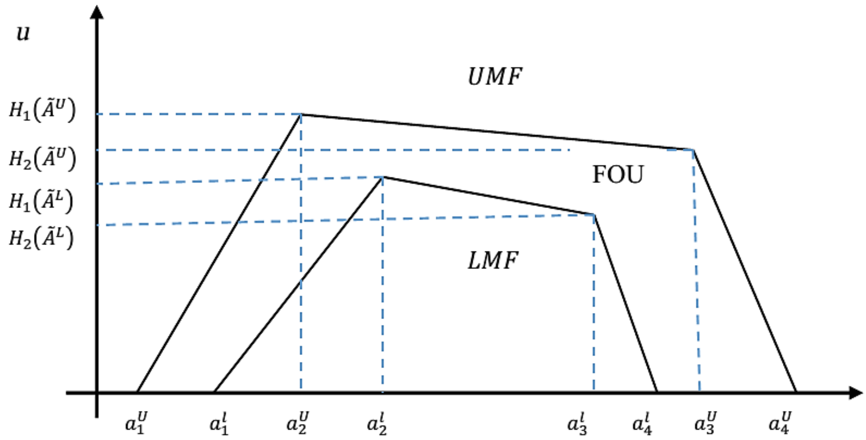

Fig. 1. A trapezoidal interval type-2 fuzzy number.

Definition 4. [30] and [31]. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two TIT2FNs defined as follows:

$$\tilde{A}_1=\left(\tilde{A}_1^U,\tilde{A}_1^L\right)=$$

$$\left(\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_1^U\right)\right),\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_1^L\right)\right)\right),$$

$$\tilde{A}_2=\left(\tilde{A}_2^U,\tilde{A}_2^L\right)=$$

$$\left(\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1\left(\tilde{A}_2^U\right),H_2\left(\tilde{A}_2^U\right)\right),\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1\left(\tilde{A}_2^L\right),H_2\left(\tilde{A}_2^L\right)\right)\right).$$

Then, summation operator on these numbers can be defined as follows:

$$\tilde{A}_1\oplus\tilde{A}_2=\left(\tilde{A}_1^U,\tilde{A}_1^L\right)\oplus\left(\tilde{A}_2^U,\tilde{A}_2^L\right)=$$

$$\left(\left(a_{11}^U,a_{12}^U,a_{13}^U,a_{14}^U;H_1\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_1^U\right)\right),\left(a_{11}^L,a_{12}^L,a_{13}^L,a_{14}^L;H_1\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_1^L\right)\right)\right)\oplus$$

$$\left(\left(a_{21}^U,a_{22}^U,a_{23}^U,a_{24}^U;H_1\left(\tilde{A}_2^U\right),H_2\left(\tilde{A}_2^U\right)\right),\left(a_{21}^L,a_{22}^L,a_{23}^L,a_{24}^L;H_1\left(\tilde{A}_2^L\right),H_2\left(\tilde{A}_2^L\right)\right)\right)= \tag{5}$$

$$\left(\left(a_{11}^U+a_{21}^U,a_{12}^U+a_{22}^U,a_{13}^U+a_{23}^U,a_{14}^U+a_{24}^U;\min\left(H_1\left(\tilde{A}_1^U\right),H_1\left(\tilde{A}_2^U\right)\right),\right)\right)$$

$$\min\left(H_2\left(\tilde{A}_1^U\right),H_2\left(\tilde{A}_2^U\right)\right)a_{11}^L+a_{21}^L,a_{12}^L+a_{22}^L,a_{13}^L+a_{23}^L,a_{14}^L+a_{24}^L;$$

$$\min\left(H_1\left(\tilde{A}_1^L\right),H_1\left(\tilde{A}_2^L\right)\right),\min\left(H_2\left(\tilde{A}_1^L\right),H_2\left(\tilde{A}_2^L\right)\right).$$

Definition 5. [30] and [31]. Let $\tilde{A}$ be a TIT2FN and

$$\tilde{A}=\left(\tilde{A}^U,\tilde{A}^L\right)=\left(\left(a_1^U,a_2^U,a_3^U,a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(a_1^l,a_2^l,a_3^l,a_4^l;H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right).$$

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Then we will have:

$$\lambda\times\tilde{A}=\left(\lambda\times\tilde{A}^{U},\lambda\times\tilde{A}^{L}\right)$$

$$\left(\left(\lambda\times a_1^U,\lambda\times a_2^U,\lambda\times a_3^U,\lambda\times a_4^U;H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(\lambda\times a_1^l,\lambda\times a_2^l,\lambda\times a_3^l,\lambda\times a_4^l;H_1\left(\tilde{A}^L\right),\right)\right) \quad (6)$$

$$H_2\left(\tilde{A}^L\right)).$$

$$\frac{\tilde{A}}{\lambda}=\left(\frac{\tilde{A}^U}{\lambda},\frac{\tilde{A}^L}{\lambda}\right)=$$

$$\left(\left(\frac{a_1^U}{\lambda},\frac{a_2^U}{\lambda},\frac{a_3^U}{\lambda},\frac{a_4^U}{\lambda};H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),\left(\frac{a_1^l}{\lambda},\frac{a_2^l}{\lambda},\frac{a_3^l}{\lambda},\frac{a_4^l}{\lambda};H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right)\right). \quad (7)$$

and

$$(\tilde{A})^{\lambda}=\left((a_1^U)^{\lambda},(a_2^U)^{\lambda},(a_3^U)^{\lambda},(a_4^U)^{\lambda};H_1\left(\tilde{A}^U\right),H_2\left(\tilde{A}^U\right)\right),$$

$$\left((a_1^l)^{\lambda},(a_2^l)^{\lambda},(a_3^l)^{\lambda},(a_4^l)^{\lambda};H_1\left(\tilde{A}^L\right),H_2\left(\tilde{A}^L\right)\right). \quad (8)$$

Definition 6. [30]. If $\tilde{A}$ is a TIT2FN, magnitude of the rank of $\tilde{A}$, $Rank(\tilde{A})$ is defined as follows:

$$\mathrm{Rank}(\tilde{A})=M_1\left(\tilde{A}^U\right)+M_1\left(\tilde{A}^L\right)+M_2\left(\tilde{A}^U\right)+M_2\left(\tilde{A}^L\right)+M_3\left(\tilde{A}^U\right)+M_3\left(\tilde{A}^L\right)-1/4(S_1\left(\tilde{A}^U\right)$$

$$+S_1\left(\tilde{A}^L\right)+S_2\left(\tilde{A}^U\right)+S_2\left(\tilde{A}^L\right)+S_3\left(\tilde{A}^U\right)+S_3\left(\tilde{A}^L\right)+S_4\left(\tilde{A}^U\right)+S_4\left(\tilde{A}^L\right))+H_1\left(\tilde{A}^U\right) \quad (9)$$

$$+H_1\left(\tilde{A}^L\right)+H_2\left(\tilde{A}^U\right)+H_2\left(\tilde{A}^L\right).$$

where $M_p(\tilde{A}^q)$ is the average between the elements a_p^q and a_{p+1}^q, that is $M_p(\tilde{A}^q)=\frac{\left(a_p^q+a_{p+1}^q\right)}{2}$, $S_p(\tilde{A}^q)$ represents the standard deviation of the elements a_p^q and a_{p+1}^q, that is $S_p(\tilde{A}^q)=\sqrt{\frac{1}{2}\sum_{k=p}^{p+1}\left(a_k^q-\frac{1}{2}\sum_{k=p}^{p+1}a_k^q\right)^2}$, and $S_4(\tilde{A}^q)$ represents standard deviation of the elements a_1^q, a_2^q, a_3^q, and a_4^q, that is $S_4(\tilde{A}^q)=\sqrt{\frac{1}{4}\sum_{k=1}^{4}\left(a_k^q-\frac{1}{4}\sum_{k=1}^{4}a_k^q\right)^2}$. Moreover, $H_p(\tilde{A}^q)$ denotes membership degree of the element a_{p+1}^q where $1\le p\le 3$ and $q\in\{U,L\}$.

For example, let $\tilde{A}=\{(0.34, 0.4, 0.42, 0.48; 1, 1),(0.36, 0.38, 0.4, 0.44;0.95, 0.95)\}$, then we have:

$$\mathrm{Rank}(\tilde{A})=M_1\left(\tilde{A}^U\right)+M_1\left(\tilde{A}^L\right)+M_2\left(\tilde{A}^U\right)+M_2\left(\tilde{A}^L\right)+M_3\left(\tilde{A}^U\right)+M_3\left(\tilde{A}^L\right)-1/4$$

$$\left[S_1\left(\tilde{A}^U\right)+S_1\left(\tilde{A}^L\right)+S_2\left(\tilde{A}^U\right)+S_2\left(\tilde{A}^L\right)+S_3\left(\tilde{A}^U\right)+S_3\left(\tilde{A}^L\right)+S_4\left(\tilde{A}^U\right)+S_4\left(\tilde{A}^L\right)\right]$$

$$H_1\left(\tilde{A}^U\right)+H_1\left(\tilde{A}^L\right)+H_2\left(\tilde{A}^U\right)+H_2\left(\tilde{A}^L\right)=0.37+0.37+0.41+0.39+0.45+0.42-1/4[0.03$$

$$+0.01+0.01+0.012247+0.03+0.02+0.05+0.02958]+1+0.95+1+0.95=6.2620$$

In decision making problems where data is expressed in terms of linguistic variables by qualitative terms such as very poor (VP), poor (P), poor to medium (PM), fair (F), medium to good (MG), good, (G), and very good (VG), according to *Table 1*, one can implement corresponding TIT2FNs in the problem [17].

Table 1. Linguistic variables and corresponding TIT2FNs [17].

Linguistic variables	TIT2FN
Very poor (VP)	((0, 0, 0, 0.1; 1, 1), (0, 0, 0, 0.05; 0.95, 0.95))
Poor (P)	((0, 0.01, 0.15, 0.3; 1, 1), (0.05, 0.1, 0.1, 0.2; 0.95, 0.95))
Poor to medium (PM)	((0.15, 0.3, 0.35, 0.5; 1, 1), (0.2, 0.25, 0.3, 0.4; 0.95, 0.95))
Fair (F)	((0.3, 0. 5, 0.55, 0.7; 1, 1), (0.4, 0.45, 0.5, 0.6; 0.95, 0.95))
Medium to good (MG)	((0.5, 0.7, 0.75, 0.9; 1, 1), (0.6, 0.65, 0.7, 0.8; 0.95, 0.95))
Good (G)	((0.7, 0.9, 0.95, 1; 1, 1), (0.8, 0.85, 0.9, 0.95; 0.95, 0.95))
Very good (VG)	((0.9, 1, 1, 1; 1, 1), (0.95,1, 1, 1;0.95,0.95))

3 | MPTIT2FNWA Operator

Integration of the information collected from different sources is a necessary and important process in multi-period problems, representing a highlighted research topic [9]. Previous studies have proposed operators for integrating TriT1FNs [9], [11]-[12], intuitionistic fuzzy data [3] and [5], and interval data [10]. Therefore, this section defines MPTIT2FNWA operator for integrating MPMAGDM information.

Definition 7. Let $\tilde{A}_t=\left(\tilde{A}_t^U,\tilde{A}_t^L\right)$ be a set of variables at p periods ($t=t_1,t_2,\ldots,t_p$), with $\eta_t=\left(\eta_{t_1},\eta_{t_2},\ldots,\eta_{t_p}\right)^T$ being the function for evaluating its weights. Then, MPTIT2FNWA operator is defined as follows:

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\tilde{A}_{t_1},\tilde{A}_{t_2},\tilde{A}_{t_3},\ldots,\tilde{A}_{t_p}\right)=\frac{1}{\sum_{k=1}^{p}\eta_{t_k}}\left(\eta_{t_1}\left(\tilde{A}_{t_1}\right)\oplus\eta_{t_2}\left(\tilde{A}_{t_2}\right)\oplus\ldots\oplus\eta_{t_p}\left(\tilde{A}_{t_p}\right)\right). \tag{10}$$

Where,

$$\sum_{k=1}^{p}\eta_{t_k}=1\ ,\ \eta_{t_k}\geq 0 \text{ for } k=1,2,\ldots,p. \tag{11}$$

Therefore, using *Eqs. (6), (9), (10)* and *(11)* one can write:

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\tilde{A}_{t_1},\tilde{A}_{t_2},\tilde{A}_{t_3},\ldots,\tilde{A}_{t_p}\right)=\left(\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{A}_{t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{A}_{t_k}^L\right)\right)=$$

$$\left(\begin{array}{c}\left(\begin{array}{c}\sum_{k=1}^{p}\eta_{t_k}\left(a_{1t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{2t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{3t_k}^U\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{4t_k}^U\right);\\ \min_k\left(H_1\left(\tilde{A}_{t_k}^U\right)\right),\min_k\left(H_2\left(\tilde{A}_{t_k}^U\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\sum_{k=1}^{p}\eta_{t_k}\left(a_{1t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{2t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{3t_k}^L\right),\sum_{k=1}^{p}\eta_{t_k}\left(a_{4t_k}^L\right);\\ \min_k\left(H_1\left(\tilde{A}_{t_k}^L\right)\right),\min_k\left(H_2\left(\tilde{A}_{t_k}^L\right)\right)\end{array}\right)\end{array}\right) \tag{12}$$

According to the *Definition 7*, the following properties can be extracted:

Properties. Let $\widetilde{A}_{t_p},\ldots,\widetilde{A}_{t_3},\widetilde{A}_{t_2},\widetilde{A}_{t_1}$ be a set of TIT2FNs at p periods ($t=t_1,t_2,\ldots,t_p$), with $\eta_t=\left(\eta_{t_1},\eta_{t_2},\ldots,\eta_{t_p}\right)^T$ such that $\sum_{k=1}^{p}\eta_{t_k}=1$, and we have $\eta_{t_k}\geq 0$ ($k=1,2,\ldots,p$) weight vectors, then:

- *Idempotency: If all $\widetilde{A}_{t_k}$s are equal for $k=1,2,\ldots,p$, so that $\widetilde{A}_{t_k}=\widetilde{A}_t$, then we have:*

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)=\widetilde{A}_t.$$

- *Boundedness: we have, $\widetilde{A}_t^-\leq MPTIT2FNWA_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)\leq\widetilde{A}_t^+$, where $\widetilde{A}_t^-=\min_k\left(\widetilde{A}_{t_k}\right)$ and $\widetilde{A}_t^+=\max_k\left(\widetilde{A}_{t_k}\right)$.*
- *Monotonicity: Let $\widetilde{A}_{t_k}^*$ be a set of TIT2FNs at p periods ($t=t_1,t_2,\ldots,t_p$), and $\widetilde{A}_{t_k}^*\leq\widetilde{A}_{t_k}$ for all k values, then we have:*

$$\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1},\widetilde{A}_{t_2},\widetilde{A}_{t_3},\ldots,\widetilde{A}_{t_p}\right)\leq\mathrm{MPTIT2FNWA}_{\eta_t}\left(\widetilde{A}_{t_1}^*,\widetilde{A}_{t_2}^*,\widetilde{A}_{t_3}^*,\ldots,\widetilde{A}_{t_p}^*\right).$$

In order to apply MPTIT2FNWA operator, determination of the vector of weights (η_t) is an important step. In general, this vector (η_t) can be determined by various methods such as decision-maker's opinion [7], or on the basis of different types of probability distribution functions such as BUM probability distribution function [3] and [11], Gamma distribution function [33], normal distribution function [1], [3], etc. Relying on the applied example provided in [40], application of BUM probability distribution function for determining the vector of weights of time periods (η_t) is explained in the following [1] and [3]:

BUM function: the function $Q:[0,1]\longrightarrow[0,1]$ of the following properties:

- $Q(0)=0.$
- $Q(1)=1.$
- $Q(x)\leq Q(y)$ *if* $x>y.$

The function Q is referred to as general monotonic unit distance function. Using this function, one can calculate vector of weights (η_t) as follows:

$$\eta_{t_k}=Q\left(\frac{k}{p}\right)-Q\left(\frac{k-1}{p}\right);\ k=1,2,\ldots,p. \tag{13}$$

Now, letting $Q(x)=(x)^r$ and $r>0$ gives:

$$\eta_{t_k}=Q\left(\frac{k}{p}\right)-Q\left(\frac{k-1}{p}\right)=\left(\frac{k}{p}\right)^r-\left(\frac{k-1}{p}\right)^r=\left(\frac{k}{p}\right)^r-\left(\frac{k}{p}-\frac{1}{p}\right)^r;k=1,2,\ldots,p. \tag{14}$$

Then

$$\frac{\partial\left(\eta_{t_k}\right)}{\partial\left(\frac{k}{p}\right)}=r\left(\frac{k}{p}\right)^{r-1}-r\left(\frac{k}{p}-\frac{1}{p}\right)^{r-1}=r\left(\left(\frac{k}{p}\right)^{r-1}-\left(\frac{k}{p}-\frac{1}{p}\right)^{r-1}\right). \tag{15}$$

Therefore:

- *If $r>1$, then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}>0$*, so* η_{t_k} *is a monotonic increasing function.*
- *If $r=1$, then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}=0$*, so* η_{t_k} *is a constant function.*
- *If $r<1$, then* $\frac{\partial(\eta_{t_k})}{\partial(\frac{k}{p})}<0$*, so* η_{t_k} *is a monotonic decreasing function.*

4 | MPTIT2FNWA based MPMAGDM Method

Assumed in this section is a MPMAGDM problem wherein values of attributes are expressed in MPTIT2FNWA in multiple periods of time. Consider a decision-making problem with n attributes $C_j;j=1, 2, 3, \dots, n$, m alternatives $A_i;i=1, 2, 3, \dots, m$, q decision-makers $DM_l;l=1, 2, 3, \dots, q$, in p periods $t_k;k=1, 2, 3, \dots, p$. Suppose that $x_{ijl}^{(t_k)}$ is the corresponding TIT2FN to a linguistic term expressing the value of jth attribute of the ith alternative determined by lth decision-maker in kth period of time, such that:

$$\tilde{x}_{ijq}^{(t_k)}=\left(\tilde{x}_{ijq}^{(t_k)U},\tilde{x}_{ijq}^{(t_k)L}\right)=\begin{pmatrix}\left(x_{ijl1}^{(t_k)U},x_{ijl2}^{(t_k)U},x_{jil3}^{(t_k)U},x_{jil4}^{(t_k)U};H_1\left(\tilde{x}_{ijl}^{(t_k)U}\right),H_2\left(\tilde{x}_{ijl}^{(t_k)U}\right)\right),\\ \left(x_{ijl1}^{(t_k)L},x_{ijl2}^{(t_k)L},x_{jil3}^{(t_k)L},x_{jil4}^{(t_k)L};H_1\left(\tilde{x}_{ijl}^{(t_k)L}\right),H_2\left(\tilde{x}_{ijl}^{(t_k)L}\right)\right)\end{pmatrix}.$$

Now, with this information, we can implement the proposed method by following steps.

Step 1. Calculation of average decision matrix in each time period.

If $\varphi(t_k)=\left(\varphi_1(t_k),\varphi_2(t_k),\dots,\varphi_q(t_k)\right)^T$ such that $\sum_{l=1}^{q}\varphi_l(t_k)=1$, and $\varphi_l(t_k)\geq 0$ (for $l=1, 2,\dots,q$) represents vector of the weights of decision-makers in k th period, then, decision matrix for each time period, t_k, is formed as follows:

$$\tilde{D}^{(t_k)}=\begin{array}{c} \\ A_1\\ A_2\\ \vdots\\ A_i\\ \vdots\\ A_m\end{array}\begin{array}{c}\begin{array}{cccc}\tilde{c}_1 & \tilde{c}_2 & \cdots & \tilde{c}_n\end{array}\\ \begin{bmatrix}\tilde{x}_{11}^{(t_k)} & \tilde{x}_{12}^{(t_k)} & \cdots & \tilde{x}_{1n}^{(t_k)}\\ \tilde{x}_{21}^{(t_k)} & \tilde{x}_{22}^{(t_k)} & \cdots & \tilde{x}_{2n}^{(t_k)}\\ \vdots & \vdots & \cdots & \vdots\\ \tilde{x}_{i1}^{(t_k)} & \tilde{x}_{i2}^{(t_k)} & \cdots & \tilde{x}_{in}^{(t_k)}\\ \vdots & \vdots & \cdots & \vdots\\ \tilde{x}_{m1}^{(t_k)} & \tilde{x}_{m2}^{(t_k)} & & \tilde{x}_{mn}^{(t_k)}\end{bmatrix}\end{array};i=1,2,\dots,m\quad;j=1,2,\dots,n\quad;k=1,2,\dots,p.$$

Where,

$$\tilde{x}_{ij}(t_k)=\oplus_{l=1}^{q}(\varphi_l(t_k)\times\tilde{x}_{ijl}(t_k)). \tag{16}$$

Where $\tilde{x}_{ij}^{(t_k)}$ is weighted average of the opinions of the q decision-makers on the value of the j th attribute of the i th alternative in the k th period of time. Now, letting $W(t_k)=(w_1(t_k),w_2(t_k),\dots,w_n(t_k))^T$ be the vector of weights of attributes in k th period of time and $\eta_t=\left(\eta_{t_1},\eta_{t_2},\dots,\eta_{t_p}\right)^T$ be the vector of weights of time periods, the proposed method is presented in terms of the following steps.

Step 2. Calculation of weighted decision matrix in each time period.

Decision matrix $\tilde{R}(t_k)=[\tilde{r}_{ij}(t_k)]_{m\times n}$ is the weighted decision matrix in k the period of time, such that:

$$\tilde{r}_{ij}(t_k)=w_j(t_k)\times\tilde{x}_{ij}(t_k);\ i=1,2,\ldots,m\ ;j=1,2,\ldots,n;k=1,2,\ldots,p.$$

Step 3. Integration of decision matrices using MPTIT2FNWA operator and formation of general decision matrix.

In this step, the values of the weighted attributes in *p* periods of time are integrated using the MPTIT2FNWA operator as follows:

$$\tilde{r}_{ij}=\mathrm{MPTIT2FNWA}_{\eta_t}\left(\tilde{r}_{i1}(t_k),\tilde{r}_{i2}(t_k),\tilde{r}_{i3}(t_k),\ldots,\tilde{r}_{in}(t_k)\right)=\left(\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{r}_{t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(\tilde{r}_{t_k}^{L}\right)\right)=$$

$$\left(\begin{array}{l}\left(\sum_{k=1}^{p}\eta_{t_k}\left(r_{1t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{2t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{3t_k}^{U}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{4t_k}^{U}\right);\min_k\left(H_1\left(\tilde{r}_{t_k}^{U}\right)\right),\min_k\left(H_2\left(\tilde{r}_{t_k}^{U}\right)\right)\right),\\ \left(\sum_{k=1}^{p}\eta_{t_k}\left(r_{1t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{2t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{3t_k}^{L}\right),\sum_{k=1}^{p}\eta_{t_k}\left(r_{4t_k}^{L}\right);\min_k\left(H_1\left(\tilde{r}_{t_k}^{L}\right)\right),\min_k\left(H_2\left(\tilde{r}_{t_k}^{L}\right)\right)\right)\end{array}\right)$$

$$;\ i=1,2,\ldots,m\ ;\ j=1,2,\ldots,n.$$

In this case, general decision matrix is obtained as $\tilde{R}=[\tilde{r}_{ij}]_{m\times n}$.

Step 4. Determination of Positive Ideal Solution (PIS), and Negative Ideal Solution (NIS).

Letting $\tilde{r}^{+}=(\tilde{r}_1^{+},\tilde{r}_2^{+},\ldots,\tilde{r}_n^{+})$ and $\tilde{r}^{-}=(\tilde{r}_1^{-},\tilde{r}_2^{-},\ldots,\tilde{r}_n^{-})$ denote PIS and NIS, respectively, we will have:

$$\tilde{r}^{+}=\begin{Bmatrix}\max_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_b\\ \min_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_c\end{Bmatrix}, \tag{17}$$

$$\tilde{r}^{-}=\begin{Bmatrix}\max_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } j=1,2,\ldots,n;\ j\in X_c\\ \min_i\{\mathrm{Rank}(\tilde{r}_{ij})\}\ ,\ \text{for } i=1,2,\ldots,n;\ j\in X_b\end{Bmatrix}. \tag{18}$$

where X_b is the set of all positive attributes (e.g. profit) and X_c is the set of all negative attributes (e.g. cost).

Step 5. Calculation of distances of alternatives from PIS and NIS.

In order to calculate the distance from the *i*th alternative to PIS and NIS, one could act as follows:

$$d^{+}(A_i)=\sqrt{\sum_j\left(\mathrm{Rank}(\tilde{r}_{ij})-\tilde{r}^{+}\right)^2};\ i=1,\ 2,\ \ldots,\ m, \tag{19}$$

$$d^{-}(A_i)=\sqrt{\sum_j\left(\mathrm{Rank}(\tilde{r}_{ij})-\tilde{r}^{-}\right)^2};\ i=1,2,\ldots,m. \tag{20}$$

where $d^{+}(A_i)$ denotes the distance from *i*th alternative to PIS and $d^{-}(A_i)$ is the distance from the *i* th alternative to NIS.

Step 6. Ranking the alternatives using closeness coefficient of the alternatives.

$$CC(A_i)=\frac{d^-(A_i)}{\left(d^-(A_i)+d^+(A_i)\right)};\ i=1,2,\ldots,m. \tag{21}$$

where $CC(A_i)$ is the closeness coefficient of the ith alternative. In order to rank the alternatives, values of $CC(A_i)$ $(i=1,2,\ldots,m)$ should be sorted in decreasing order. It is obvious that the best alternative will be that with highest closeness coefficient. Accordingly, A^* will be chosen as the best alternative if and only if $CC\left(A^*_i\right)=max\{CC(A_i)\}$.

5 | Numerical Example

In this example, the numerical example previously presented by Zhu and Hipel [6] is used, with all data and coefficients being used in the same sense as they were used in the reference. In 2008, Chinese government decided to undertake a research and development plan in commercial aviation industry to enhance economic and production capacities of the industry. One of the most important parts of this plan was vendor evaluation which is based on criteria whose performances were considered over multiple periods of time. On this basis, five vendors (A_1, A_2, A_3, A_4, A_5) of an electronic navigation system are studied in this research. Schedule (C_1), quality (C_2), technology (C_3), and level of service (LOS) (C_4) were considered as the problem attributes; these were evaluated along a three years period by three experts (*Table 2*). In this research, the attributes had their weights expressed in terms of the vector $W(t_k)=(0.45,0.40,0.1,0.05)^T$ such that $W(t_1)=W(t_2)=W(t_3)$. Weights of the time periods were defined as the vector $\eta_t=(1/3,1/3,1/3)^T$ and weights of the decision-makers were considered as $\varphi(t_k)=(0.2,0.40,0.40)^T$.Accordingly, the proposed method was undertaken by taking the following steps.

Step 1. Calculation of average decision matrix in each time period.

By using *Eq. (18)*, weighted average of the decision-makers' opinions are considered as the value of each attribute, as can be seen in *Table 3*.

Table 2. Linguistic values of the evaluated attributes by three decision-makers in three periods of time.

DM₁																	
t_1						t_2						t_3					
C_1	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_2	F	P	VP	P	MP	C_1	MG	F	P	MG	MG	C_1	VG	MG	P	G	G
C_3	P	F	P	MP	VP	C_2	F	G	MP	P	MP	C_2	P	G	P	VG	MG
C_4	VP	P	F	VP	MP	C_3	P	P	MG	MG	MP	C_3	VP	G	F	P	G
C_5	VP	VP	MP	F	P	C_4	P	MP	P	G	G	C_4	MG	MP	G	G	P
DM₂																	
t_1						t_2						t_3					
	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_1	P	P	P	P	MP	C_1	MP	F	F	MP	P	C_1	MG	F	MP	MG	F
C_2	P	MP	P	MP	P	C_2	P	MG	P	F	MG	C_2	VP	VG	VP	G	MG
C_3	VP	P	F	P	MP	C_3	MP	MP	G	F	MP	C_3	MP	P	MG	P	MP
C_4	MP	VP	MP	F	P	C_4	F	MP	MP	MG	P	C_4	P	MP	G	MG	G
DM₃																	
t_1						t_2						t_3					
	A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5		A_1	A_2	A_3	A_4	A_5
C_1	P	P	P	MP	P	C_1	P	F	P	MP	MP	C_1	VP	G	MP	F	MG
C_2	P	MP	P	F	VP	C_2	VP	MG	MP	G	P	C_2	VP	VG	P	MG	MP
C_3	VP	P	MP	P	VP	C_3	MP	MP	MG	MP	P	C_3	G	P	F	MP	P
C_4	P	VP	MP	F	P	C_4	F	MP	MP	MG	VP	C_4	P	MP	MG	G	G

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Table 3. Weighted average of the attributes evaluated by decision-makers in three periods of time.

	C_1	C_2	C_3	C_4
t_1				
A_1	((0.06,0.108,0.23,0.38;1,1),(0.12,0.17,0.18,0.28;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0,0,0,0.1;1,1),(0,0,0,0.05;0.95,0.95))	((0.03,0.066,0.16,0.3;1,1),(0.07,0.11,0.12,0.21;0.95,0.95))
A_2	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0.18,0.34,0.39,0.54;1,1),(0.24,0.29,0.34,0.44;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0,0,0,0.1;1,1),(0,0,0,0.05;0.95,0.95))
A_3	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))	((0.21,0.38,0.43,0.58;1,1),(0.28,0.33,0.38,0.48;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_4	((0.09,0.184,0.27,0.42;1,1),(0.14,0.19,0.22,0.32;0.95,0.95))	((0.24,0.42,0.47,0.62;1,1),(0.32,0.37,0.42,0.52;0.95,0.95))	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0.3,0.5,0.55,0.7;1,1),(0.4,0.45,0.5,0.6;0.95,0.95))
A_5	((0.06,0.126,0.23,0.38;1,1),(0.11,0.16,0.18,0.28;0.95,0.95))	((0,0.002,0.03,0.14;1,1),(0.01,0.02,0.02,0.08;0.95,0.95))	((0.06,0.12,0.14,0.26;1,1),(0.08,0.1,0.12,0.19;0.95,0.95))	((0,0.01,0.15,0.3;1,1),(0.05,0.1,0.1,0.2;0.95,0.95))
t_2				
	C_1	C_2	C_3	C_4
A_1	((0.13,0.206,0.31,0.46;1,1),(0.19,0.24,0.26,0.36;0.95,0.95))	((0.06,0.102,0.14,0.26;1,1),(0.09,0.11,0.12,0.19;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.24,0.402,0.47,0.62;1,1),(0.33,0.38,0.42,0.52;0.95,0.95))
A_2	((0.3,0.5,0.55,0.7;1,1),(0.4,0.45,0.5,0.6;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_3	((0.06,0.108,0.23,0.38;1,1),(0.12,0.17,0.18,0.28;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))
A_4	((0.22,0.38,0.43,0.58;1,1),(0.28,0.33,0.38,0.48;0.95,0.95))	((0.48,0.642,0.71,0.8;1,1),(0.66,0.71,0.72,0.76;0.95,0.95))	((0.25,0.42,0.47,0.62;1,1),(0.32,0.37,0.42,0.52;0.95,0.95))	((0.54,0.74,0.79,0.92;1,1),(0.67,0.72,0.76,0.84;0.95,0.95))
A_5	((0.19,0.322,0.39,0.54;1,1),(0.25,0.3,0.34,0.44;0.95,0.95))	((0.13,0.206,0.31,0.46;1,1),(0.19,0.24,0.26,0.36;0.95,0.95))	((0.06,0.126,0.23,0.38;1,1),(0.11,0.16,0.18,0.28;0.95,0.95))	((0.14,0.182,0.22,0.32;1,1),(0.2,0.22,0.22,0.27;0.95,0.95))
t_3				
	C_1	C_2	C_3	C_4
A_1	((0.28,0.34,0.35,0.44;1,1),(0.31,0.33,0.34,0.39;0.95,0.95))	((0,0.002,0.03,0.14;1,1),(0.01,0.02,0.02,0.08;0.95,0.95))	((0.45,0.6,0.64,0.72;1,1),(0.61,0.65,0.66,0.69;0.95,0.95))	((0.1,0.148,0.27,0.42;1,1),(0.16,0.21,0.22,0.32;0.95,0.95))
A_2	((0.58,0.78,0.83,0.92;1,1),(0.77,0.82,0.84,0.88;0.95,0.95))	((0.86,0.98,0.99,1;1,1),(0.95,1,1,1;0.95,0.95))	((0.14,0.188,0.31,0.44;1,1),(0.23,0.28,0.28,0.36;0.95,0.95))	((0.15,0.3,0.35,0.5;1,1),(0.2,0.25,0.3,0.4;0.95,0.95))
A_3	((0.12,0.242,0.31,0.46;1,1),(0.17,0.22,0.26,0.36;0.95,0.95))	((0,0.008,0.12,0.26;1,1),(0.04,0.08,0.08,0.17;0.95,0.95))	((0.34,0.54,0.59,0.74;1,1),(0.44,0.49,0.54,0.64;0.95,0.95))	((0.58,0.78,0.83,0.94;1,1),(0.74,0.79,0.82,0.88;0.95,0.95))
A_4	((0.42,0.62,0.67,0.8;1,1),(0.55,0.6,0.64,0.72;0.95,0.95))	((0.62,0.8,0.84,0.94;1,1),(0.74,0.79,0.82,0.88;0.95,0.95))	((0.09,0.184,0.27,0.42;1,1),(0.14,0.19,0.22,0.32;0.95,0.95))	((0.66,0.86,0.91,0.98;1,1),(0.88,0.93,0.94,0.96;0.95,0.95))
A_5	((0.5,0.7,0.75,0.88;1,1),(0.63,0.68,0.72,0.8;0.95,0.95))	((0.29,0.46,0.51,0.66;1,1),(0.36,0.41,0.46,0.56;0.95,0.95))	((0.17,0.246,0.35,0.48;1,1),(0.26,0.31,0.32,0.4;0.95,0.95))	((0.56,0.722,0.79,0.86;1,1),(0.77,0.82,0.82,0.84;0.95,0.95))

Table 4. Weighted decision matrix in three periods of time.

t_1				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.027,0.049,0.104,0.171;1,1),(0.054,0.077,0.081,0.126;0.95,0.95))	((0,0.004,0.06,0.12;1,1),(0.02,0.04,0.04,0.08;0.95,0.95))	((0,0,0,0.01;1,1),(0,0,0,0.005;0.95,0.95))	((0.002,0.003,0.008,0.015;1,1),(0.004,0.006,0.006,0.011;0.95,0.95))
A_2	((0,0.005,0.068,0.135;1,1),(0.023,0.045,0.045,0.09;0.95,0.95))	((0.072,0.136,0.156,0.216;1,1),(0.096,0.116,0.136,0.176;0.95,0.95))	((0,0.001,0.015,0.03;1,1),(0.005,0.01,0.01,0.02;0.95,0.95))	((0,0,0,0.005;1,1),(0,0,0,0.003;0.95,0.95))
A_3	((0,0.004,0.054,0.117;1,1),(0.018,0.036,0.036,0.077;0.95,0.95))	((0,0.004,0.06,0.12;1,1),(0.02,0.04,0.04,0.08;0.95,0.95))	((0.021,0.038,0.043,0.058;1,1),(0.028,0.033,0.038,0.048;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_4	((0.041,0.083,0.122,0.189;1,1),(0.063,0.086,0.099,0.144;0.95,0.95))	((0.096,0.168,0.188,0.248;1,1),(0.128,0.148,0.168,0.208;0.95,0.95))	((0,0.001,0.012,0.026;1,1),(0.004,0.008,0.008,0.017;0.95,0.95))	((0.015,0.025,0.028,0.035;1,1),(0.02,0.023,0.025,0.03;0.95,0.95))
A_5	((0.027,0.057,0.104,0.171;1,1),(0.05,0.072,0.081,0.126;0.95,0.95))	((0,0.001,0.012,0.056;1,1),(0.004,0.008,0.008,0.032;0.95,0.95))	((0.006,0.012,0.014,0.026;1,1),(0.008,0.01,0.012,0.019;0.95,0.95))	((0,0.001,0.008,0.015;1,1),(0.003,0.005,0.005,0.01;0.95,0.95))
t_2				
	C_1	**C_2**	**C_3**	**C_4**
A_1	((0.059,0.093,0.14,0.207;1,1),(0.086,0.108,0.117,0.162;0.95,0.95))	((0.024,0.041,0.056,0.104;1,1),(0.036,0.044,0.048,0.076;0.95,0.95))	((0.012,0.024,0.031,0.046;1,1),(0.017,0.022,0.026,0.036;0.95,0.95))	((0.012,0.02,0.024,0.031;1,1),(0.017,0.019,0.021,0.026;0.95,0.95))
A_2	((0.135,0.225,0.248,0.315;1,1),(0.18,0.203,0.225,0.27;0.95,0.95))	((0.216,0.296,0.316,0.368;1,1),(0.268,0.288,0.304,0.336;0.95,0.95))	((0.012,0.024,0.031,0.046;1,1),(0.017,0.022,0.026,0.036;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_3	((0.027,0.049,0.104,0.171;1,1),(0.054,0.077,0.081,0.126;0.95,0.95))	((0.048,0.097,0.124,0.184;1,1),(0.068,0.088,0.104,0.144;0.95,0.95))	((0.054,0.074,0.079,0.092;1,1),(0.067,0.072,0.076,0.084;0.95,0.95))	((0.006,0.012,0.016,0.023;1,1),(0.009,0.011,0.013,0.018;0.95,0.95))
A_4	((0.099,0.171,0.194,0.261;1,1),(0.126,0.149,0.171,0.216;0.95,0.95))	((0.192,0.257,0.284,0.32;1,1),(0.264,0.284,0.288,0.304;0.95,0.95))	((0.025,0.042,0.047,0.062;1,1),(0.032,0.037,0.042,0.052;0.95,0.95))	((0.027,0.037,0.04,0.046;1,1),(0.034,0.036,0.038,0.042;0.95,0.95))
A_5	((0.086,0.145,0.176,0.243;1,1),(0.113,0.135,0.153,0.198;0.95,0.95))	((0.052,0.082,0.124,0.184;1,1),(0.076,0.096,0.104,0.144;0.95,0.95))	((0.006,0.013,0.023,0.038;1,1),(0.011,0.016,0.018,0.028;0.95,0.95))	((0.007,0.009,0.011,0.016;1,1),(0.01,0.011,0.011,0.014;0.95,0.95))
t_3				
	C_1	C_2	C_3	C_4
A_1	((0.126,0.153,0.158,0.198;1,1),(0.14,0.149,0.153,0.176;0.95,0.95))	((0,0.001,0.012,0.056;1,1),(0.004,0.008,0.008,0.032;0.95,0.95))	((0.045,0.06,0.064,0.072;1,1),(0.061,0.065,0.066,0.069;0.95,0.95))	((0.005,0.007,0.014,0.021;1,1),(0.008,0.011,0.011,0.016;0.95,0.95))
A_2	((0.261,0.351,0.374,0.414;1,1),(0.347,0.369,0.378,0.396;0.95,0.95))	((0.344,0.392,0.396,0.4;1,1),(0.38,0.4,0.4,0.4;0.95,0.95))	((0.014,0.019,0.031,0.044;1,1),(0.023,0.028,0.028,0.036;0.95,0.95))	((0.008,0.015,0.018,0.025;1,1),(0.01,0.013,0.015,0.02;0.95,0.95))
A_3	((0.054,0.109,0.14,0.207;1,1),(0.077,0.099,0.117,0.162;0.95,0.95))	((0,0.003,0.048,0.104;1,1),(0.016,0.032,0.032,0.068;0.95,0.95))	((0.034,0.054,0.059,0.074;1,1),(0.044,0.049,0.054,0.064;0.95,0.95))	((0.029,0.039,0.042,0.047;1,1),(0.037,0.04,0.041,0.044;0.95,0.95))
A_4	((0.189,0.279,0.302,0.36;1,1),(0.248,0.27,0.288,0.324;0.95,0.95))	((0.248,0.32,0.336,0.376;1,1),(0.296,0.316,0.328,0.352;0.95,0.95))	((0.009,0.018,0.027,0.042;1,1),(0.014,0.019,0.022,0.032;0.95,0.95))	((0.066,0.043,0.046,0.049;1,1),(0.044,0.047,0.047,0.048;0.95,0.95))
A_5	((0.225,0.315,0.338,0.396;1,1),(0.284,0.306,0.324,0.36;0.95,0.95))	((0.116,0.184,0.204,0.264;1,1),(0.144,0.164,0.184,0.224;0.95,0.95))	((0.017,0.025,0.035,0.048;1,1),(0.026,0.031,0.032,0.04;0.95,0.95))	((0.028,0.036,0.04,0.043;1,1),(0.039,0.041,0.041,0.042;0.95,0.95))

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Table 5. Integrated decision matrix.

	C_1	Rank	C_2	Rank	C_3	Rank	C_4	Rank
A_1	((0.071,0.098,0.134,0.192;1,1),(0.093,0.111,0.117,0.155;0.95,0.95))	4.575	((0.008,0.015,0.043,0.093;1,1),(0.02,0.031,0.032,0.063;0.95,0.95))	4.084	((0.019,0.028,0.032,0.043;1,1),(0.026,0.029,0.031,0.037;0.95,0.95))	4.074	((0.006,0.01,0.015,0.022;1,1),(0.009,0.012,0.013,0.018;0.95,0.95))	3.972
A_2	((0.132,0.194,0.23,0.288;1,1),(0.183,0.206,0.216,0.252;0.95,0.95))	5.123	((0.211,0.275,0.289,0.328;1,1),(0.248,0.268,0.28,0.304;0.95,0.95))	5.52	((0.009,0.015,0.026,0.04;1,1),(0.015,0.02,0.021,0.031;0.95,0.95))	4.019	((0.005,0.01,0.012,0.018;1,1),(0.007,0.008,0.01,0.014;0.95,0.95))	3.958
A_3	((0.027,0.054,0.099,0.165;1,1),(0.05,0.071,0.078,0.122;0.95,0.95))	4.337	((0.016,0.035,0.077,0.136;1,1),(0.035,0.053,0.059,0.097;0.95,0.95))	4.226	((0.036,0.055,0.06,0.075;1,1),(0.046,0.051,0.056,0.065;0.95,0.95))	4.222	((0.014,0.022,0.025,0.032;1,1),(0.019,0.021,0.023,0.027;0.95,0.95))	4.031
A_4	((0.11,0.178,0.206,0.27;1,1),(0.146,0.168,0.186,0.228;0.95,0.95))	4.961	((0.179,0.248,0.269,0.315;1,1),(0.229,0.249,0.261,0.288;0.95,0.95))	5.391	((0.011,0.02,0.029,0.043;1,1),(0.017,0.021,0.024,0.034;0.95,0.95))	4.036	((0.036,0.035,0.038,0.043;1,1),(0.033,0.035,0.037,0.04;0.95,0.95))	4.116
A_5	((0.113,0.172,0.206,0.27;1,1),(0.149,0.171,0.186,0.228;0.95,0.95))	4.963	((0.056,0.089,0.113,0.168;1,1),(0.075,0.089,0.099,0.133;0.95,0.95))	4.469	((0.01,0.016,0.024,0.037;1,1),(0.015,0.019,0.021,0.029;0.95,0.95))	4.016	((0.012,0.015,0.019,0.025;1,1),(0.017,0.019,0.019,0.022;0.95,0.95))	4.006

Step 2. Calculation of weighted decision matrix in each period.

In this step, by using *Eq. (6)*, weighted decision matrix in each period was calculated, which results came in *Table 4*.

Step 3. Integration of decision matrices using MPTIT2FNWA operator and formation of general decision matrix.

In this step, using MPTIT2FNWA operator, the values of the weighted attributes in three periods of time are integrated, then, their rank were calculated, which came in *Table 5*.

Step 4. Determination of PIS and NIS.

Using *Eqs. (19)* and *(20)*, PIS and NIS vectors were calculated as follows:

$$\tilde{r}^{+}=(5.1230,\ 5.5200,\ 4.2220,\ 4.1160),$$
$$\tilde{r}^{-}=(4.3370,\ 4.0840,\ 4.0160,\ 3.9580). \qquad (21)$$

Step 5. Calculation of distances of alternatives from PIS ($d^{+}(A_i)$) and NIS ($d^{-}(A_i)$).

Eqs. (21) and *(22)* were used to calculate distances of alternatives from PIS and NIS, with the results given in *Table 6*.

Table 6. Distances of alternatives from PIS and NIS.

Distance of the alternative from PIS		Distance of the alternative from NIS	
$d^+(A_1)$	1.550819	$d^-(A_1)$	0.245365
$d^+(A_2)$	0.257241	$d^-(A_2)$	1.63704
$d+(A_3)$	1.516396	$d^-(A_3)$	0.260632
$d^+(A_4)$	0.278354	$d^-(A_4)$	1.457048
$d^+(A_5)$	1.088456	$d^-(A_5)$	0.736482

Step 6. Calculation of relative closeness coefficient and ranking.

Eq. (23) was used to calculate relative closeness of different alternatives, based on which the alternatives were ranked, with the results reported in *Table 7*.

Table 7. Relative closeness coefficient and ranking of different alternatives.

Relative closeness coefficient		Rank
$CC(A_1)$	0.136603	5
$CC(A_2)$	0.864201	1
$CC(A_3)$	0.146667	4
$CC(A_4)$	0.839603	2
$CC(A_5)$	0.596435	3

Considering the results of implementing the proposed method, it is clear that the alternative A_2, i.e. the fourth vendor, should be selected as the best vendor, with the alternatives ranked as follows: $A_2 \prec A_4 \prec A_5 \prec A_3 \prec A_1$. *Table 8* presents a comparison between the results of the proposed method and those of the study by Xu and Hipel [6].

Table 8. Comparison with other methods.

Research work	Ranking of the alternatives
Method of Xu and Hipel	A2≻ A4≻ A5≻ A3≻ A1
Proposed method	A2≻ A4≻ A5≻ A3≻ A1

6 | Conclusion

In this paper, A MPMADM method was presented where in initial data was expressed in linguistic terms. In order to cover with the ambiguity and uncertainty of this information, TIT2FNs were used. In order to complete the proposed process, MPTIT2FNWA operator was defined on TIT2FNs. We utilized the BUM function in this operator and its properties were investigated. This operator was used to integrate information from different periods of time and form a general decision matrix. This operator was found to provide required flexibly to select any trend of time series for the specified weights of time periods depending on the problem characteristics. On this basis, the proposed method was presented in several steps. In order to demonstrate efficiency and applicability of the proposed method, the numerical example presented by Zhu and Hipel [6] was used and the problem of vendors of an electronic navigation system for developing commercial aviation industry was evaluated. The results showed that the ranking is the same in both methods, which confirms the efficiency of the proposed method.

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