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A New Method for Solving Fuzzy Linear Fractional Programming Problem with New Ranking Function

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A B S T R A C T

Because of uncertainty in the real life applications, reaching to the optimal solution is always time consuming and even sometimes impossible. In order to overcome these limitations the fuzzy set theory is introduced to handle it but not only incomplete information but also indeterminate and inconsistent information which is common in real life conditions. In this paper, we have developed a new ranking function to solve a Fully Fuzzy Linear Fractional Programming (FFLFP). The ranking function is derived by replacing the non-parallel sides of the trapezoidal fuzzy number with non-linear functions. Various numerical examples are included and compared with the pre-existing methods.

Keywords: Trapezoidal Fuzzy Number, Ranking Function, Linear Fractional Programming Problem, Fuzzy Numbers.

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1. Introduction

Due to absence of some certain data, it gives incline to the concept of fuzzy set theory. Let consider a sample, it is unknown to everybody whether tomorrow will be rain; this kind of situation is called uncertainty. If probability is applied to check the possible outcomes, the uncertainty can be known. Assume if it says 75% of sunshine, then there is 25% chance of rain. But there is always some error in calculating the possibility. If the error percentage is 25% then the probability of rain i.e. the possibility of rain tomorrow varies from a certain value to a certain value. Therefore, it is not advisable to consider one single value to predict the outcome. Hence the concept of fuzzy is being introduced.

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LFP problems have various applications like network analysis, operation research, banking and finance etc. It is simple and straight forward when the variables involving the constraints and objective functions are crisp. But in actual case the variables and constraints may not be obtained as crisp. Those are found by conducting some experiment in general. Practically, the values of these parameters in the objective function and in the constraints cannot always be taken as crisp numbers as various factors are associated with them such as non-stochastic uncertainty. So these parameters values are not known specifically to the decision maker and they are taken as fuzzy numerical data which can be expressed as fuzzy numbers. These lead to the development of Fully Fuzzy Linear Programming (FFLFP) techniques. The notion of fuzzy set was pioneered by Zadeh [8] and since then it has found huge applications in various fields.

A large number of researchers have dedicated their endeavor to the area of Fuzzy LFP (FLFP) problems and Fully Fuzzy LFP (FFLFP) problems [9, 13]. Many researchers also keen to interest to using lexicographic technique to solve the problem like FFLP problems [13, 14, 17].

Das et al. [16] proposed a method for solving FFLFP problem in to a Multi-Objective LFP (MOLFP) problem by using lexicographic ordering, which is considered as an exact optimal solution of the FFLFP problem under consideration. By using lexicographic technique the crisp LFP problem is converted into crisp LP problem with the help of Charnes-Cooper method and that can be solved using the standard simplex method. In this paper, we simplify the LFP problems derived from the proposed approach by Das et al. [16]. We consider all the parameters are non-negative triangular fuzzy numbers. Das et al. [15] have proposed another method to solve FFLFP problem based on multi-objective LFP problem. Multi-Objective Fuzzy Linear Fractional Programming (MOFLFP) problem is concerned with simultaneous optimization of multiple objective functions, where every objective function is in the form of a FLFP problem. Das and Mandal proposed a method for solving LFP problem with ranking function [18].

In this paper, we have first discussed about fuzzy number and its arithmetic and l -cut of a fuzzy number. The concept of these has been used for linear programming problems. As such new method is developed here to handle these fuzzy problems in $[0, \infty]$. In special cases the solutions are also compared with the known results that are found in literature.

2. Preliminaries

In this section we include some basic definitions.

Definition 1. (Classical set). A set is a well-defined collection of objects. Classical, or a crisp set, is one which assigns grades of membership of either 0 or 1 to objects.

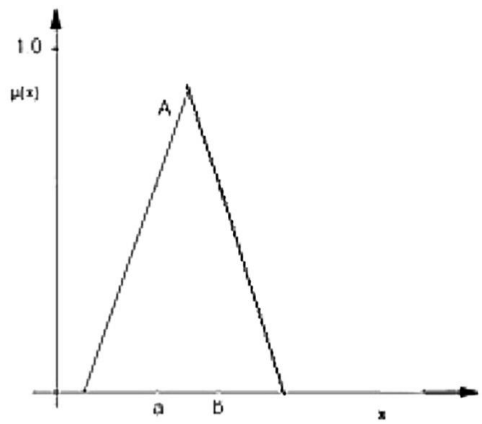

Figure 1. Classical set.

Definition 2. (Fuzzy set). A fuzzy set can be defined as the set of ordered pairs such that A = {(x, m(x))/ x Œ X, m(x) Œ [0, 1]} where m(x) is called the membership function.

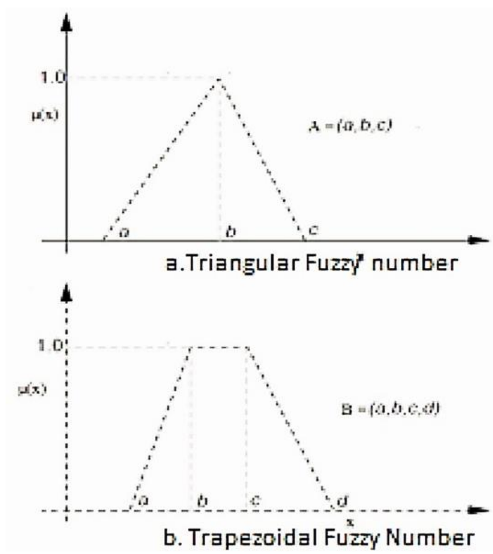

Figure 2. Types of fuzzy numbers.

Definition 3. (l-cut of a fuzzy set). l-cut of a fuzzy set is the crisp set denoted by Al (a crisp interval), for a particular value of membership value l, Al = [a, b] as shown in the ***Figure 1***, and l is in [0,1].

Definition 4. (Fuzzy arithmetic). Generally, a fuzzy number is converted into its λ -cut and the arithmetic operations are carried on as of for an interval.

Definition 5. (Interval arithmetic). For any two intervals [a, b] and [d, e], the arithmetic operations are performed in the following way:

- Addition: [a, b] + [d, e] = [a + d, b + e].
- Subtraction: [a, b] – [d, e] = [a – e, b – d].

- Multiplication: [a, b] · [d, e] = [min (ad, ae, bd, be), max(ad, ae, bd, be)].
- Division: [a, b] / [d, e] = [min (a/d, a/e, b/d, b/e), max (a/d, a/e, b/d, b/e)], provided 0 is not in [d, e].

Definition 6. (L – R flat fuzzy number). A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be an L-R Flat fuzzy number, if,

$$\mu(x) = \begin{cases} L(\frac{m-x}{\alpha}), x \le m, \alpha > 0 \\ R(\frac{x-n}{\beta}), x \ge n, \beta > 0 \\ 1, \quad \text{else.} \end{cases}$$

Definition 7. (Arithmetic operations of L-R flat fuzzy number). Let $\tilde{A}_1 = (m_1, n_1, \alpha_1, \beta_1)$ and $\tilde{A}_2 = (m_2, n_2, \alpha_2, \beta_2)$ be two non-negatives L-R flat fuzzy numbers, then the arithmetic operations are defined as follows:

- Addition/Subtraction:

$$\tilde{A}_1 \pm \tilde{A}_2 = (m_1, n_1, \alpha_1, \beta_1) \pm (m_2, n_2, \alpha_2, \beta_2) = (m_1 \pm m_2, n_1 \pm n_2, |\alpha_1 \pm \alpha_2|, |\beta_1 \pm \beta_2|).$$

- Multiplication: (for $m_1 - \alpha_1 > 0$ and $m_2 - \alpha_2 > 0$):

$$\tilde{A}_1 \times \tilde{A}_2 = (m_1, n_1, \alpha_1, \beta_1) \times (m_2, n_2, \alpha_2, \beta_2) = (m_1 m_2, n_1 n_2, m_1\alpha_2 + m_2\alpha_1, n_1\beta_2 + n_2\beta_1).$$

- Scalar Multiplication:

$$k\tilde{A}_1\ k\tilde{A}_1 = (km_1, kn_1, k\alpha_1, k\beta_1), k \ge 0 = (km_1, kn_1, -k\beta_1, -k\alpha_1), k \le 0.$$

Definition 8. (α - cut of *L-R* flat fuzzy number). Let $\tilde{A} = (m, n, \alpha, \beta)$ be an L-R flat fuzzy number and α be a real number in the interval [0, 1] then the crisp set:

$$A_\lambda = \{x \in X : \mu(x) \ge \lambda\} = [m - \alpha L^{-1}(\lambda), n + \beta R^{-1}(\lambda)].$$

is said to be λ - cut of $\tilde{A}$.

A L-R flat fuzzy number is said to be non-negative if $m - a \ge 0$.

Definition 9. (Ranking function). The concept of ranking function was introduced to compare two fuzzy numbers, which is practically not possible with four coefficients. Hence, the fuzzy number is converted into real number and then the fuzzy numbers are compared.

A ranking function $R_a : F(R) \to R$, maps each fuzzy number into a real line. Now, suppose that $\tilde{A}$ and $\hat{A}$ be two fuzzy numbers. We define Ra as follows:

- $\tilde{A} \ge \hat{A}$ if and only if $Ra(\tilde{A}) \ge Ra(\hat{A})$.

- Ã > Â if and only if Ra (Ã) > Ra (Â).
- Ã = Â if and only if Ra (Ã) = Ra (Â).
- Ã ≤ Â if and only if Â ≥ Ã.

2.1. Existing Ranking Function

2.1.1. Yager's ranking function

Yager proposed for ordering fuzzy sets in which a ranking index R_a. (Ã) is calculated for the fuzzy number $\tilde{A}=(m,n,\alpha,\beta)$ for its λ -cut, $A_\lambda=[m-\alpha L^{-1}(\lambda), n+\beta R^{-1}(\lambda)]$ according to the formula:

$$R_a(\tilde{A})=\frac{1}{2}\int_0^1((m-\alpha L^{-1}(\lambda))+(n+\beta R^{-1}(\lambda)))d\lambda. \quad \textit{(1)}$$

Where $L(x)=\max(0,(1-x^4))$ and $R(x)=\max(0,(1-x^2))$. Then by using ***Eq. (1)*** we have,

$$R_a(\tilde{A})=\frac{1}{2}(m+n+\frac{4}{5}\alpha+\frac{2}{3}\beta).$$

2.1.2. Maleki's ranking function

Here the formula is given as follows:

$$R_a(\tilde{A})=(m+n+\frac{1}{2}(\beta-\alpha)).$$

Where $L(x)=\max(0,(1-x))$ and $R(x)=\max(0,(1-x))$.

3. New Ranking Function

A new ranking function was introduced by Suneela and Chakraverty [26] with L(x) and R(x) as monotonically increasing and decreasing non-linear functions, say parabolas.

For a fuzzy number $\tilde{A}=(m,n,\alpha,\beta)$ the ranking function can be derived using the formula proposed by Yager as in ***Eq. (1)***:

$$R_a(\tilde{A})=\frac{1}{2}\int_0^1((m-\alpha L^{-1}(\lambda))+(n+\beta R^{-1}(\lambda)))d\lambda,$$

where λ in [0,1].

Let us insert parabolas as L(x) and R(x) and consider the fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$, let $m = b, n = c, m - \alpha = a, n + \beta = d.$ Also Let us suppose that L(x) be a parabola with vertex (a, 0) and passing through (b, 1) and R(x) be another parabola with vertex (d, 0) and passing through (c, 1).

Accordingly we get,

$$L(x) \text{ as } y^2 = \frac{x-a}{b-a},$$

$$L(x) \text{ as } y^2 = \frac{x-d}{c-d},$$

$$\text{Let } L(x) = \max(0, \sqrt{\frac{x-a}{b-a}}) \text{ and } R(x) = \max(0, \sqrt{\frac{x-d}{c-d}}).$$

First of all to find the inverse of the above functions for find a new ranking function. We get

$$L^{-1}x = a + x^2 * (b-a). \tag{2}$$

$$R^{-1}x = d + x^2 * (c-d). \tag{3}$$

Now for time to substitute the ***Eqs. (2)*** and ***(3)*** in ***Eq. (1)***, we get

$$R_a\tilde{A} = \frac{1}{2}\int_0^1 ((m - \alpha L^{-1}(\lambda)) + (n + \beta R^{-1}(\lambda)))d = \frac{1}{2}b(b-a) - \frac{(b-a)^2}{3} + c + d(d-c) - \frac{(d-c)^2}{3}.$$

Put $b = m, b - a = \alpha, c = n, d - c = \beta,$ we get

$$R_a\tilde{A} = \frac{1}{2}(m(1-\alpha) + n(1+\beta) + \frac{2}{3}\alpha^2 + \frac{2}{3}\beta^2).$$

This one is proposed new ranking function.

4. Fully Fuzzy Linear Fractional Programming Problem and Algorithm

Consider the following FFLFP problem. We are going to approach m fuzzy equality constraints and n fuzzy variables where all the terms are triangular fuzzy numbers.

$$\begin{aligned} &\text{Maximize } Z = \frac{\tilde{c}^t\tilde{x} + \tilde{\alpha}}{\tilde{d}^t\tilde{x} + \tilde{\beta}}, \\ &\text{Subject to} \quad \tilde{A} \otimes \tilde{x} \le \tilde{b}, \\ &\qquad\qquad\quad \tilde{x} \ge 0. \end{aligned} \tag{4}$$

Where $\tilde{c}^t = [\tilde{c}_j]$ is 1 by n matrix; $\tilde{d}^t = [\tilde{d}_j]$ is 1 by n matrix; $\tilde{x} = [\tilde{x}_j]$ is *n* by 1 matrix; $\tilde{A} = [\tilde{a}_{ij}]$ is m by n matrix; $\tilde{b} = [\tilde{b}_{ij}]$ is a m by 1 matrix; $\tilde{\alpha} = [\tilde{\alpha}_j]$ and $\tilde{\beta} = [\tilde{\beta}_j]$ are the scalars. Here all the parameters $\tilde{c}_j, \tilde{d}_j, \tilde{x}_j, \tilde{a}_{ij}$ are set of fuzzy numbers.

Mention. Let $\tilde{x}$ a fuzzy optimal solution of FFLFP problem. If there exists a fuzzy number $\tilde{y}$ where it satisfies the following conditions:

- $\tilde{y}$ is a non-negative fuzzy number.
- $\tilde{A}\otimes\tilde{y}\leq\tilde{b}$.
- $\Re(\tilde{c}^t\otimes\tilde{x})=\Re(\tilde{c}^t\otimes\tilde{y})$.
- $\Re(\tilde{d}^t\otimes\tilde{x})=\Re(\tilde{d}^t\otimes\tilde{y})$.

Then $\tilde{y}$ is also an exact optimal solution of the problem (4) and is called a substitute optimal solution.

Consider the model (4) and letbe an optimal solution of this FFLFP. $\tilde{x}^*=(x^*,y^*,z^*)$ Furthermore, let all the parameters $\tilde{x}$, $\tilde{c}$, $\tilde{\alpha}$, $\tilde{d}$, $\tilde{\beta}$, $\tilde{b}$ and $\tilde{z}$ are represented by non-parallel trapezoidal fuzzy numbers

(x,y,z,t), (p,q,r,s), $(\alpha_1,\alpha_2,\alpha_3,\alpha_4)$, (u,v,w,q), $(\beta_1,\beta_2,\beta_3,\beta_4)$, (b_1,b_2,b_3,b_4) and (z_1,z_2,z_3,z_4) respectively. Then we can rewrite the mentioned FFLFP as follows:

$$\begin{aligned}
&\text{Max } (z_1,z_2,z_3,z_4)=\frac{(p,q,r,s)^t\otimes(x,y,z,t)+(\alpha_1,\alpha_2,\alpha_3,\alpha_4)}{(u,v,w,q)^t\otimes(x,y,z,t)+(\beta_1,\beta_2,\beta_3,\beta_4)},\\
&\text{Subject to} \qquad (b,c,a,d)\otimes(x,y,z,t)\leq(b_1,b_2,b_3,b_4),\\
&\qquad\qquad (x,y,z,t)\geq 0.
\end{aligned} \tag{5}$$

Step 1. Using the new ranking function, convert model (5) into model (6):

$$\begin{aligned}
&\text{Max } (z_1,z_2,z_3,z_4)=\frac{\Re((p,q,r,s)^t\otimes(x,y,z,t)+(\alpha_1,\alpha_2,\alpha_3,\alpha_4))}{\Re((u,v,w,q)^t\otimes(x,y,z,t)+(\beta_1,\beta_2,\beta_3,\beta_4))},\\
&\text{Subject to} \qquad (b,c,a,d)\otimes(x,y,z,t)\leq(b_1,b_2,b_3,b_4),\\
&\qquad\qquad (x,y,z,t)\geq 0.
\end{aligned} \tag{6}$$

Step 2. Consider that $(b,c,a,d)\otimes(x,y,z,t)=(m,n,o,p)$ the FFLFP problem, obtained in Step 1, we obtain,

$$\begin{aligned}
&\text{Max } (z_1,z_2,z_3,z_4)=\frac{\Re((p,q,r,s)^t\otimes(x,y,z,t)+(\alpha_1,\alpha_2,\alpha_3,\alpha_4))}{\Re((u,v,w,q)^t\otimes(x,y,z,t)+(\beta_1,\beta_2,\beta_3,\beta_4))},\\
&\text{Subject to} \qquad (m,n,o,p)\leq(b_1,b_2,b_3,b_4),\\
&\qquad\qquad (x,y,z,t)\geq 0.
\end{aligned} \tag{7}$$

Step 3. Use arithmetic operation, the FFLFP problem is converted in to the following crisp LFP problem.

$$\begin{aligned}
&\text{Max } (z_1, z_2, z_3, z_4) = \frac{\Re((p,q,r,s)^t \otimes (x,y,z,t) + (\alpha_1, \alpha_2, \alpha_3, \alpha_4))}{\Re((u,v,w,q)^t \otimes (x,y,z,t) + (\beta_1, \beta_2, \beta_3, \beta_4))}, \\
&\text{Subject to} \quad m \le b_1, \\
&\qquad n \le b_2, \\
&\qquad o \le b_3, \\
&\qquad p \le b_4, \\
&\qquad (x,y,z,t) \ge 0.
\end{aligned} \tag{8}$$

Step 4. Now the problem is obtained in Step 3, is Crisp LFP problem. The crisp LFP problem can be solved by any suitable method.

Step 5. After solving the problem, find the optimal solution.

5. Numerical Experiment

Here, we consider two examples to illustrate the effectiveness of our proposed model of FFLFP problems with non-parallel trapezoidal fuzzy number.

Example 1.

$$\begin{aligned}
&\text{Max } Z = \frac{(15,16,6,8)\tilde{x}_1 + (14,17,4,7)\tilde{x}_2}{(8,10,3,4)\tilde{x}_1 + (7,8,1,2)\tilde{x}_2}, \\
&\text{s.t.} \quad (2,3,1,2)\tilde{x}_1 + (3,6,1,2)\tilde{x}_2 \le (5,12,2,3), \\
&\qquad 0\tilde{x}_1 + (1,3,5,7)\tilde{x}_2 \le (9,17,4,5), \\
&\qquad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned}$$

To solve this above problem by using our proposed ranking function with the help of any convenience LFP problem, we get the solution: $\tilde{x}_1 = 0.8, \tilde{x}_2 = 1.4, \tilde{Z} = 38.1$

The above problems are solved in LINGO 18.0 version.

By using Yager ranking function we get $\tilde{x}_1 = 0.6, \tilde{x}_2 = 2.4, \tilde{Z} = 27.4.$

6. Conclusion

In this paper, we have introduced a new ranking function to solve fully fuzzy LFP problem. The concept of new ranking function is giving maximum objective function compare to other ranking function. Finally, a example is also given to illustrate the proposed method and also given our method is more easy and comfortable to apply the FFLFP problem to handle it.

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