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On Solving Two-Person Zero-Sum Fuzzy Matrix Games via Linear Programming Approach

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A B S T R A C T

In this paper, a two-person zero-sum matrix game with $L-R$ fuzzy numbers payoff is introduced. Using the fuzzy number comparison introduced by Rouben's method (1991), the fuzzy payoff is converted into the corresponding deterministic payoff. Then, for each player, a linear programming problem is formulated. Also, a solution procedure for solving each problem is proposed. Finally, a numerical example is given for illustration.

Keywords: Matrix games, $L-R$ fuzzy numbers, Fuzzy payoff, Optimal fuzzy strategy.

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1. Introduction

Game theory is concerned with decision making problem where two or more autonomous decision makers have conflicting interests. They are usually referred to as players who act strategically to find out a compromise solution [16]. Zero-sum games refer to pure conflict. The payoff of one player is the negative of the payoff of the other player. Peski [23] compared the structure information in zero-sum games.

As known, the fuzzy set theory was introduced by Zadeh [33] to deal with fuzziness. Up to now, the fuzzy set theory has been applied to broad fields. The fuzzy set theory was introduced by Zadeh [33], a model has to be set up using data which is approximately known. The fuzzy numerical data can be represented by means of fuzzy subsets of the real line, known as fuzzy numbers). For the fuzzy set theory development, we may referee to the papers of Kaufmann [13]

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and Dubois and Prade [7]; they extended the use of algebraic operations of real numbers to fuzzy numbers by the use a fuzzy faction principle. The fuzzy linear constraints with fuzzy numbers were studied by Dubois and Prade [7]. In real-world problems, uncertainties may be estimated as intervals; Shaocheng [35] studied two kinds of linear programming problems with fuzzy numbers called: Interval numbers and fuzzy number linear programming, respectively. Bellman and Zadeh [2] introduced the concept of a maximizing decision making problem. Zhao et al. [34] introduced the complete solution set for the fuzzy linear programming problems using linear and nonlinear membership functions.

Campos [3] solved the fuzzy matrix game using the fuzzy linear programming. Cevikel and A hlatcioglu [4] introduced new concept of solution for multi-objective two person zero-sum games. Xu [31] discussed two-person zero-sum game with grey number payoff matrix. Dhingra and Roa [6] introduced a new optimization method to herein as cooperative fuzzy games and also solved the multiple objective optimization problems based on a proposed computational technique. An innovative fuzzy logic approach to analyze $n-$person cooperative games is proposed by Espin et al. [10]. Xu and Yao [32] studied rough payoff matrix games. Ein-Dor and Kantar [8] and Takahashi [29] discussed two-person zero-sum games with random payoffs. Applications of game theory may be found in economics, engineering, biology, and in many other fields. Three major classes of games are matrix games, continuous static games, and differential games. In continuous static games, the decision possibilities need not be discrete, and the decisions and costs are related in a continuous rather than a discrete manner. The game is static in the sense that no time history is involved in the relationship between costs and decisions. Elshafei [9] introduced an interactive approach for solving Nash cooperative continuous static games, and also determined the stability set of the first kind corresponding to the obtained compromise solution. Khalifa and ZeinEldein [15] introduced an interactive approach for solving cooperative continuous static games with fuzzy parameters in the objective function coefficients. Navidi et al. [20] presented a new game theoretic-based approach for multi response optimization problem. Osman et al. [21] introduced a new procedure for continuous time open loop stackelberg differential game. Roy et al. [25] solved linear multi-objective programming based on cooperative game approach. Gogodze [12] proposed the game-theoretic approach to solve a multi-objective decision-making problem, where a normalized decision making matrix considered as a payoff matrix for some zero-sum matrix game in which the first and second players choose an alternative and criterion, respectively. Chen and Larboni [5] defined the matrix game with triangular membership function and proved that two person zero-sum game with fuzzy payoff matrices is equivalent to two linear programming problems. Seikh et al. [27] proposed an alternative approach for solving the matrix game. Selvakumari and Lavanya [28] and Thirucheran et al. [30] accelerated the fuzzy game based on the expected value operator and the trust measure of variables under roughness. Nan et al. [19] studied the fuzzy matrix game and a Lexicographic methodology for finding the solution for it. Sahoo [26] proposed a solution methodology for solving the fuzzy matrix game based on signed distance method. Li and Hong

[17] proposed an approach for solving the constrained matrix games with triangular fuzzy numbers payoffs.

In this paper, a two person zero-sum fuzzy matrix game is introduced. The problem is considered by incorporating $L-R$ fuzzy numbers in payoff. Based on the fuzzy number comparison introduced by Rouben's method [24], the fuzzy payoff is converted into the corresponding deterministic payoff. Then, for each player, a linear programming problem is formulated. Also, a solution procedure for solving each problem is proposed.

The remainder of the paper is organized as follows: In Section 2, some preliminaries needs in the paper are presented. In Section 3, a two person zero-sum matrix game with fuzzy payoff is defined. In Section 4, a solution procedure for solving the problems is introduced. A numerical example is given for illustration in Section 5. Finally some concluding remarks are reported in Section 6.

2. Preliminaries

In order to discuss our problem conveniently, we introduce fuzzy numbers and some of the results of applying fuzzy arithmetic on them and also comparison of fuzzy numbers.

Definition 1. [14]. A fuzzy number $\tilde{a}$ is a mapping, defined as $\mu_{\tilde{a}}: R \to [0,1]$, with the following:

- $\mu_{\tilde{a}}(x)$ is an upper semi-continuous membership function.
- $\tilde{a}$ is a convex fuzzy set, i.e., $\mu_{\tilde{a}}(\lambda x+(1-\lambda)y) \geq \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$, for all $x, y \in R, 0 \leq \lambda \leq 1$.
- $\tilde{a}$ is normal, i.e., $\exists x_0 \in R$ for which $\mu_{\tilde{a}}(x_0)=1$.
- $\text{Supp}(\tilde{a})=\{x \in R : \mu_{\tilde{a}}(x)>0\}$ is the support of the $\tilde{a}$, and its closure cl $(\text{supp}(\tilde{a}))$ is compact set.

Definition 2. [7]. The $\alpha-$ level set of the fuzzy number $\tilde{a}$, denoted by $(\tilde{a})_\alpha$ and is defined as the ordinary set:

$$(\tilde{a})_\alpha = \begin{cases} \{x \in R : \mu_{\tilde{a}}(x) \geq \alpha, & 0<\alpha \leq 1 \\ cl\,(\sup p\,(\tilde{a})), & \alpha=0 \end{cases}$$

A function, usually denoted by "L" or "R", is a reference function of a fuzzy number if and only if :

- $L(x)=L(-x)$.
- $L(0)=1$.
- L is nonincreasing on $[0,-\infty[$.

A convenient representation of fuzzy numbers in the $L-R$ flat fuzzy number is defined as:

$$\mu_{\tilde{A}}(x)=\begin{cases} L\left((A^{-}-x)\eta\right), & if\ x\le A^{-},\eta>0,\\ R\left((x-A^{+})\beta\right), & if\ x\ge A^{+},\ \beta>0,\\ 1, & eleswhere\end{cases}$$

Where, $A^{-}<A^{+},\left[A^{-},A^{+}\right]$ is the core of $\tilde{A}$, $\mu_{\tilde{A}}(x)=1;\forall x\in\left[A^{-},A^{+}\right],A^{-},A^{+}$ are the lower and upper modal values of $\tilde{A}$, and $\eta>0,\beta>0$ are the left-hand and right-hand spreads [24].

Remark 1. A flat fuzzy number is denoted by $\tilde{A}=(A^{-},A^{+},\eta,\beta)_{LR}$ Among the various type of $L-R$ fuzzy numbers, trapezoidal fuzzy numbers, denoted by $\tilde{A}=(A^{-},A^{+},\eta,\beta)$, are the greatest importance [24].

Let $\tilde{p}=(p^{-},p^{+},\eta,\beta)$, and $\tilde{q}=(q^{-},q^{+},\gamma,\delta)$, both trapezoidal fuzzy numbers, the formulas for the addition, subtraction, and scalar multiplication are as follows:

- Addition: $\tilde{p}\oplus\tilde{q}=(p^{-}+q^{-},p^{+}+q^{+},\eta+\gamma,\beta+\delta)$.
- Subtraction: $\tilde{p}(-)\tilde{q}=(p^{-}-q^{+},p^{+}-q^{-},\eta+\delta,\beta+\gamma)$.
- Scalar multiplication: $\begin{array}{l} x>0,\ x\in R: x\otimes\tilde{p}=(xp^{-},xp^{+},x\eta,x\beta),\\ x<0,\ x\in R: x\otimes\tilde{p}=(xp^{+},xp^{-},-x\beta,-x\eta).\end{array}$

The main concept of comparison of fuzzy numbers is based on the compensation of areas determined by the membership functions [1 and 18].

Let $\tilde{p},\tilde{q}$ be the fuzzy and numbers, and $S_L(\tilde{p},\tilde{q}),S_R(\tilde{p},\tilde{q})$ be the areas determined by their membership functions according to $S_L(\tilde{p},\tilde{q})=\int_{I(\tilde{p},\tilde{q})}(\inf\tilde{p}_\alpha-\inf\tilde{q}_\alpha)d\alpha$, and $S_R(\tilde{p},\tilde{q})=\int_{S(\tilde{p},\tilde{q})}(\sup\tilde{p}_\alpha-\sup\tilde{q}_\alpha)d\alpha$, where $I(\tilde{p},\tilde{q})=\{\alpha:\inf\tilde{p}_\alpha\ge\inf\tilde{q}_\alpha\}\subset[\theta,1],\theta>0$, and $S(\tilde{p},\tilde{q})=\{\alpha:\sup\tilde{p}_\alpha\ge\sup\tilde{q}_\alpha\}\subset[\theta,1],\theta>0$.

The degree to which $\tilde{p}\ge\tilde{q}$ is defined [24] as $C(\tilde{p},\tilde{q})=S_L(\tilde{p},\tilde{q})-S_L(\tilde{q},\tilde{p})+S_R(\tilde{p},\tilde{q})-S_R(\tilde{q},\tilde{p})$.

Here, let us consider that $\tilde{p}\ge\tilde{q}$ when $C(\tilde{p},\tilde{q})\ge 0$.

Proposition 1. [24]. Let $\tilde{p}$, and $\tilde{q}$ be $L-R$ fuzzy numbers with parameters $(p^{-},p^{+},\eta,\beta),(q^{-},q^{+},\gamma,\delta)$ and reference functions $(L_{\tilde{p}},R_{\tilde{p}}),(L_{\tilde{q}},R_{\tilde{q}})$, where all reference functions are invertible. Then $\tilde{p}\ge\tilde{q}$ if and only if $\sup\tilde{p}_{\alpha_{\tilde{p},R}}+\inf\tilde{p}_{\alpha_{\tilde{p},L}}\ge\sup\tilde{q}_{\alpha_{\tilde{q},R}}+\inf\tilde{q}_{\alpha_{\tilde{q},L}}$.

If $k=\tilde{p}\otimes\tilde{q}$, then $\alpha_{k.R}=R_k\left(\int_0^1 R_k^{-1}(\alpha)d\alpha\right)$, $\quad\alpha_{k.L}=L_k\left(\int_0^1 L_k^{-1}(\alpha)d\alpha\right)$.

Remark 2. $\tilde{p} \geq \tilde{q}$ if and only if

$$p^- + p^+ + \frac{1}{2}(\beta - \eta) \geq q^- + q^+ + \frac{1}{2}(\delta - \gamma). \tag{1}$$

Notation. [11]. The associated real number p corresponding to $\tilde{p} = (p^-, p^+, \eta, \beta)_{LR}$ is $\hat{p} = p^- + p^+ + \frac{1}{2}(\beta - \eta).$

Let $F(R)$ be the set of all trapezoidal fuzzy numbers.

3. Problem Formulation and Solution Concepts

The two-person zero-sum game is the simplest case of game theory in which how much one player receives is equal to how much the other loses. The [22] studied the case when both players gave the pure and mixed strategies. Nevertheless, the noncooperation between players may be vague.

There are three types of two-person zero-sum $L-R$ fuzzy numbers matrix games:

- Two-person zero-sum matrix games with $L-R$ fuzzy numbers goals.
- Two-person zero-sum matrix games with $L-R$ fuzzy numbers payoffs.
- Two-person zero-sum matrix games with $L-R$ fuzzy numbers goals and $L-R$ fuzzy numbers payoffs.

Consider a two player zero-sum game in which the entries in the payoff matrix $\tilde{P}$ are $L-R$ fuzzy numbers. The $L-R$ fuzzy numbers pay-off matrix is:

$$\begin{array}{cc} & \text{Player } II \\ \text{Player } I & \begin{pmatrix} \tilde{p}_{11} & \tilde{p}_{12} \cdots & \tilde{p}_{1j} \ldots & \tilde{p}_{1m} \\ .. & .. & .. & .. \\ \tilde{p}_{i1} & \tilde{p}_{i2} \cdots & \tilde{p}_{ij} \cdots & \tilde{p}_{im} \\ .. & .. & .. & .. \\ \tilde{p}_{n1} & \tilde{p}_{n2} & \tilde{p}_{nj} \cdots & \tilde{p}_{nm} \end{pmatrix} \end{array} \tag{2}$$

Players I, and II have mixed strategies denoted by M_{S_I}, and $M_{S_{ii}}$, respectively, and are defined as:

$$M_{S_I} = \left\{ x \in R^{n:} : x_i \geq 0, \sum_{i=1}^{n} x_i = 1 \right\}, \text{ and} \tag{3}$$

$$M_{S_{ii}} = \left\{ y \in R^{m:} : y_j \geq 0, \sum_{j=1}^{m} y_j = 1 \right\} \tag{4}$$

Definition 3. (Saddle point): If the min-max value equals to the max-min value then the game is called a saddle point (or equilibrium) and the corresponding strategies are said optimum strategies. The amount of payoff at an equilibrium point is the game value.

The fuzzy mathematical expectation for player I is $\tilde{Z} = \sum_{i=1}^{n}\sum_{j=1}^{m}\tilde{p}_{ij}\, x_i y_j$, and for player II is $\tilde{Z} = \sum_{j=1}^{m}\sum_{i=1}^{n}\tilde{p}_{ij}\, x_i y_j$.

Remark 3. It is clear that the two mathematical expectation are the same since the sums are finite.

Because of vagueness of payoffs $L-R$ fuzzy numbers, it is very difficult for the players to choose the optimal strategy. So, we consider how to maximize player's or minimize the opponent's $L-R$ fuzzy numbers payoffs. Upon this idea, let us propose the maximum equilibrium strategy as in the following definition.

Definition 4. In one two-person zero-sum game, player I 's mixed strategy $x^{\bullet}$ player II 's mixed strategy $y^{\bullet}$ are said to be optimal fuzzy strategies if $x^T \tilde{P}\, y^{\bullet} \le x^{\bullet T} \tilde{P}\, y^{\bullet} \le x^{\bullet T} \tilde{P}\, y$ for any mixed strategies x and y.

Remark 4. The optimal fuzzy strategy of player I is the strategy which maximizes Z irrespective of II ' s strategy. Also, the optimal fuzzy strategy of player II is the strategy which minimizes Z irrespective of I ' s strategy.

Let us consider the game with $L-R$ fuzzy numbers payoff matrix (2), and the mixed strategies of players I, and II defined in (3) and (4), respectively. If E is the fuzzy optimum value of the game of a player II , then the linear programming model for player II becomes:

$$\begin{aligned} &\min \mathrm{E} \\ &\text{Subject to} \\ &\sum_{j=1}^{m}\tilde{p}_{ij}\, y_j \le E;\; y_j \ge \tilde{0},\, j = 1,2,\ldots,m. \end{aligned} \tag{5}$$

Putting $y'_j = y_j / \mathrm{E}$, . Then problem (5) becomes:

$$\begin{aligned} &\max_j \left(\sum_{j=1}^{m} y'_j \right) \\ &\text{Subject to} \\ &\sum_{j=1}^{m}\tilde{p}_{ij}\, y'_j \le \tilde{1}; \\ &y'_j \ge \tilde{0}, \forall j \end{aligned} \tag{6}$$

Similarly, the linear programming model for player I is as:

$$\max \Phi$$

Subject to

$$\sum_{i=1}^{n} \tilde{p}_{ij} x_i \geq \Phi; \qquad (7)$$

$$x_i \geq \tilde{0}, i = 1,2,...,n.$$

Putting $x_i^{'} = x_i / \Phi, i = 1,2,...,n$. Then problem (7) becomes:

$$\min_i \left(\sum_{i=1}^{n} x_i^{'} \right)$$

Subject to

$$\sum_{i=1}^{n} \tilde{p}_{ij} x_i^{'} \geq \tilde{1}; \qquad (8)$$

$$x_i^{'} \geq \tilde{0}; \forall i .$$

Where, $\tilde{p}_{ij} = (p_{ij}^{-}, p_{ij}^{+}, \alpha_{ij}, \beta_{ij})_{LR} \in F(R)$.

4. Solution Procedure

In this section, a solution procedure for solving the problem under study is introduced in the following steps:

Step 1. Translate the payoff matrix (2) into the corresponding problems (6) and (8).

Step 2. Based on the operations of $L-R$ fuzzy numbers, problem (6) and problem (8) are converted into the corresponding crisp models as:

Model 1:

$$\max_j \left(\sum_{j=1}^{m} y_j^{'} \right)$$

Subject to

$$\sum_{j=1}^{m} p_{ij} y_j^{'} \leq 1; \qquad (9)$$

$$y_j^{'} \geq 0, \forall j$$

and,

Model 2:

$$\min_i \left(\sum_{i=1}^{n} x_i^{'} \right)$$

Subject to

$$\sum_{i=1}^{n} p_{ij} x_i^{'} \geq 1; \tag{10}$$

$$x_i^{'} \geq 0; \forall i.$$

Step 3. Apply the simplex method or any software (Lingo) to solve the Model 1 and Model 2 to obtain the optimal strategies for players II, and I, respectively.

5. Numerical Example

Consider the following $L-R$ fuzzy numbers payoff matrix game as

$$\tilde{P} = (\tilde{p}_{ij})_{3\times 4} = Player\, I \overset{Player\, II}{\begin{pmatrix} (4,5,3,1) & (2,4,1,1) & (4,5,1,3) & (5,7,1,1) \\ (10,12,5,5) & (9,13,1,5) & (7,10,2,4) & (10,11,3,1) \\ (0,2,1,1) & (2,3,3,1) & (17,21,9,9) & (6,7,1,3) \end{pmatrix}}$$

Referring to the previous notation, the above payoff matrix can be reduced to the corresponding associated ordinary payoff as:

$$\tilde{P} = (\tilde{p}_{ij})_{3\times 4} = Player\, I \overset{Player\, II}{\begin{pmatrix} 8 & 6 & 10) & 12 \\ 11 & 6 & 9 & 10 \\ 1 & 2 & 19 & 7 \end{pmatrix}}$$

For simplicity, let us take 2 as common factor. The optimal strategies will be the same and the game value is just multiplied by 2. The new matrix game is:

$$\tilde{P} = (\tilde{p}_{ij})_{3\times 4} = Player\, I \overset{Player\, II}{\begin{pmatrix} 4 & 3 & 5 & 6 \\ 11 & 6 & 9 & 10 \\ 1 & 2 & 19 & 7 \end{pmatrix}}$$

According to Model 1, we have

$$\max\left(y_1^1 + y_2^{'} + y_3^{'} + y_4^{'}\right)$$

Subject to

$$\begin{aligned} &4y_1^1 + 3y_2^{'} + 5y_3^{'} + 6y_4^{'} \le 1, \\ &11y_1^1 + 6y_2^{'} + 9y_3^{'} + 10y_4^{'} \le 1, \\ &y_1^1 + 2y_2^{'} + 19y_3^{'} + 7y_4^{'} \le 1, \\ &y_1^1, y_2^{'}, y_3^{'}, y_4^{'} \ge 0. \end{aligned}$$

Using the simplex method (Phase 2), we get

Table 1. The optimal strategy of player I.

Variables	Optimal strategy	Game value
$y_1^{'}$	0	
$y_2^{'}$	0	2
$y_3^{'}$	0.231	
$y_4^{'}$	0.769	

Referring to Model 2, we have

$$\min\left(x_1^1 + x_2^{'} + x_3^{'}\right)$$

Subject to

$$\begin{aligned} &4x_1^1 + 11x_2^{'} + x_3^{'} \ge 1, \\ &3x_1^1 + 6x_2^{'} + 2x_3^{'} \ge 1, \\ &5x_1^1 + 9x_2^{'} + 19x_3^{'} \ge 1, \\ &6x_1^1 + 10x_2^{'} + 7x_3^{'} \ge 1; x_1^1, x_2^{'}, x_3^{'} \ge 0. \end{aligned}$$

It is clear from the duality theorem that the optimal strategy of player I is $\left(x_1^1, x_2^{'}, x_3^{'}\right) = (0, 0.923, 0.077)$, which are the coefficients slack variables in the final table of the simplex method.

Thus,

Table 2. The optimal fuzzy strategy.

Player I	Player II	Game value
$x_1^{'} = 0$	$y_1^{'} = 0$	
$x_2^{'} = 0.923$	$y_2^{'} = 0$	$(9.248018, 10.727805, 2.775083, 1.900361)$
$x_3^{'} = 0.077$	$y_3^{'} = 0.231$	
	$y_4^{'} = 0.769$	

6. Concluding Remarks

In this paper, we have considered a two-person zero-sum matrix games with $L-R$ fuzzy numbers payoff. Firstly, we defined the game with $L-R$ fuzzy numbers payoffs and then proposed an equilibrium strategy. Secondly, we proposed a solution procedure for solving each problem with fuzziness in relations. Lastly, a numerical example illustrated our research method. We discussed only one kind of games with uncertain payoffs. But of course, there are games with uncertain payoffs which will be take in our consideration in future.

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