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Modified Method for Solving Fully Fuzzy Linear Programming Problem with Triangular Fuzzy Numbers

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A B S T R A C T

The fuzzy linear programming problem has been used as an important planning tool for the different disciplines such as engineering, business, finance, economics, etc. In this paper, we proposed a modified algorithm to find the fuzzy optimal solution of fully fuzzy linear programming problems with equality constraints. Recently, Ezzati et al. (Applied Mathematical Modelling, 39 (2015) 3183-3193) suggested a new algorithm to solve fully fuzzy linear programming problems. In this paper, we modified this algorithm and compare it with other existing methods. Furthermore, for illustration, some numerical examples and one real problem are used to demonstrate the correctness and usefulness of the proposed method.

Keywords: Linear programming problem, fully fuzzy linear programming, multi-objective linear programming, triangular fuzzy numbers.

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1. Introduction

Modeling and optimization under a fuzzy environment is called fuzzy modeling and fuzzy optimization. Fuzzy linear programming is one of the most frequently applied in fuzzy decision making techniques. Although, it has been investigated and expanded for more than decades by many researchers and from the varies point of views, it is still useful to develop new approaches in order to better fit the real world problems within the framework of fuzzy linear programming.

Traditional optimization techniques and methods have been successfully applied for years to solve problems with a well-defined structure. Such optimization problems are usually well formulated by crisply specific objective functions and specific system of constraints, and solved by precise mathematics. Unfortunately, real world situations are often not deterministic. There exist various types of uncertainties in social, industrial and economic systems, such as

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randomness of occurrence of events, imprecision and ambiguity of system data and linguistic vagueness, etc., which come from many ways, including errors of measurement, deficiency in history and statistical data, insufficient theory, incomplete knowledge expression, and the subjectivity and preference of human judgment, etc. As talked about by Zimmermann [1], several types of uncertainties can be categorized as stochastic uncertainty and fuzziness.

Stochastic uncertainty pertains to the uncertainty of events of phenomena or events. Its features lie in the fact that descriptions of information are crisp and well defined, however, they vary in their frequency of occurrence. Therefore, systems with this type of uncertainty are called stochastic systems, which can be solved by stochastic optimization techniques using probability theory.

In some other circumstances, the decision-maker does not think the frequently used probability distribution is always appropriate, particularly when the information is vague, relating to human language and behavior, imprecise/ambiguous system data, or when the information could not be described and defined well due to limited knowledge and deficiency in its understanding. Such types of uncertainty are called fuzziness. It cannot be formulated and solved effectively by traditional mathematics-based optimization techniques nor did probability base stochastic optimization approaches.

The Fuzzy Linear Programming (FLP) problem can be introduced in several ways such as (i) the constraints are inequalities with fuzzy technological coefficients/fuzzy decision variables (ii) the constraints are equalities with fuzzy technological coefficients/fuzzy decision variables equalities (iii) the goals may be fuzzy (iv) all parameters of LP may be in terms of fuzzy numbers.

Many researchers proposed various methods for solving fuzzy linear programming problem [2-11]. Lotfi et al. [7] proposed full fuzzy linear programming problems where all parameters and variables were triangular fuzzy numbers. The proposed method is to find the fuzzy optimal solution with equality constraints. Ebrahimnejad et al. [4] proposed a method for solving fuzzy linear programming problems where all coefficients of objective functions and right hand side are represented by symmetric trapezoidal fuzzy numbers. They convert the fuzzy linear programming problem into an equivalent crisp linear programming and then solved the LP by standard primal simplex algorithm. Kumar et al.[6] proposed a method for solving fully fuzzy linear programming problem with equality constraints. They transformed FLP in to equivalent crisp linear programming problem and used regular simplex method. Iskander [12] introduced a stochastic fuzzy linear multi objective programming problem and transformed it to a stochastic fuzzy linear programming problem using a fuzzy weighted objective function. They used chance-constrained approach as well as the α-level methodology to transform the stochastic fuzzy linear programming problem to its equivalent deterministic-crisp linear programming problem. Veeramani and Duraisamy [10] discussed the fully fuzzy linear programming (FFLP) problems in which all the parameters and variables are triangular fuzzy numbers. They proposed a new approach of solving FFLP problem using the concept of nearest symmetric triangular fuzzy number approximation with preserve expected interval. The dual problem of the LP with trapezoidal fuzzy number and some duality results to solve fuzzy linear programming problem was introduced in [13].

Ezzati et al. [14] introduced a new algorithm to solve fully fuzzy linear programming problem. They have converted the problem to a multi-objective linear programming (MOLP) problem and then it was solved by a new lexicographic ordering method. In this paper, we modified the mentioned algorithm and proposed a new method for finding the fuzzy optimal solution of FFLP problem.

This paper is organized as follows: Some basic definitions and notations are present in Section 2. In Section 3, we present a modified method to solve FFLP problem. Some examples are provided for testing the new proposed method in next section and Kumar et al. method [6] , Ezzati et al. method [14] and the proposed method will be compared each other. Finally, in Section 5, concluding remarks are present.

2. Preliminaries

In this section, we have presented some basics concepts of fuzzy sets and triangular fuzzy number, which was very useful in this paper.

Definition 2.1 [15, 16] Let X denotes a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \rightarrow [0,1]$; which assigned a real number $\mu_{\tilde{A}}(x)$ in the interval [0, 1], to each element $x \in X$, where the values of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x in $\tilde{A}$. A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = (x, \mu_{\tilde{A}}(x)) : x \in X$ is called a fuzzy set.

Definition 2.2 [15, 16]A fuzzy number $\tilde{A} = (b,c,a)$ (where $b \leq c \leq a$) is said to be a triangular fuzzy number if its membership function is given by $\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-b)}{(c-b)}, & b \leq x \leq c, \\ \dfrac{(x-a)}{(c-a)}, & c \leq x \leq a, \end{cases}$

Definition 2.3 [15, 16]A triangular fuzzy number (b,c,a) is said to be non-negative fuzzy number if and only if $b \geq 0$.

Definition 2.4 [15, 16]Two triangular fuzzy number $\tilde{A} = (b,c,a)$ and $\tilde{B} = (e, f, d)$ are said to be equal if and only if $b = e, c = f, a = d$.

Definition 2.5[17] A ranking is a function $R: F(R) \to R$ where *F(R)*is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (b, c, a)$ is a triangular fuzzy number then $\Re(\tilde{A}) = \dfrac{b + 2c + a}{4}$.

Definition 2.6 [15, 16] Let $\tilde{A} = (b, c, a)$, $\tilde{B} = (e, f, d)$ be two triangular fuzzy numbers then:

$$\tilde{A} + \tilde{B} = (b, c, a) + (e, f, d) = (b + e, c + f, a + d),$$
$$\tilde{A} - \tilde{B} = (b, c, a) - (e, f, d) = (b - d, c - f, a - e),$$

Let $\tilde{A} = (b, c, a)$ be any triangular fuzzy number and $\tilde{B} = (e, f, d)$ be a non-negative triangular fuzzy number then,

$$\tilde{A} \otimes \tilde{B} = \tilde{A}\tilde{B} = \begin{cases} (be, cf, ad) & if\ b \geq 0, \\ (bd, cf, ad) & if\ b < 0, a \geq 0, \\ (bd, cf, cd) & if\ c < 0, \end{cases}$$

Definition 2.7 Let $\tilde{A} = (b, c, a)$, $\tilde{B} = (e, f, d)$ be two triangular fuzzy numbers. We say that $\tilde{A}$ is relatively less than $\tilde{B}$, if and only if

$b < e$ or
$b = e$ and $(b - c) > (e - f)$ or
$b = e$, $(b - c) = (e - f)$ and $(a + b) = (d + e)$.

Note: It is clear from the Definition 2.7 that $b = e$, $(b - c) = (e - f)$ and $(a + b) = (d + e)$ if and only if $\tilde{A} = \tilde{B}$.

3. Fully Fuzzy Linear Programming Problem

We consider a standard form of FFLP problem where all parameters and variables are fuzzy triangular numbers as follows:

$$\begin{aligned} &Max \quad \tilde{c}^t x \\ &\text{s.t} \quad \tilde{A}\tilde{x} = \tilde{b}, \\ &\tilde{x} \geq 0. \end{aligned} \tag{1}$$

Where $\tilde{c}^t = [\tilde{c}_j]$ is 1 by *n* matrix; $\tilde{x} = [\tilde{x}_j]$ is *n* by 1 matrix; $\tilde{A} = [\tilde{a}_{ij}]$ is *m* by *n* matrix; $\tilde{b} = [\tilde{b}_{ij}]$ is m by 1 matrix; Here all the parameters $\tilde{c}_j, \tilde{d}_j, \tilde{x}_j, \tilde{a}_{ij}$ are set of fuzzy numbers.

Let $\tilde{x}^* = (x^*, y^*, z^*)$ be an exact optimal solution of the problem (1). Now we are going to introduce a new algorithm to find exact optimal solution. The steps of the proposed algorithm are given as follows:

Step 1. The problem (1) can be written as

$$\begin{aligned}&\text{Max} \quad ((c^t x), (c^t y), (c^t z))\\ &\text{s.t.} \quad ((Ax),(Ay), (Az) = (b_1), (b_2), (b_3)),\\ &x, y, z \geq 0.\end{aligned} \tag{2}$$

Step 2. In problem (2) may be written as

$$\begin{aligned}&\text{Max} \quad ((c^t x), (c^t y), (c^t z))\\ &\text{s.t.} \quad (Ax) = (b_1), ((Ax) - (Ay) = (b_1) - (b_2)), \quad ((Ax) + (Az) = (b_1) + (b_3)),\\ &x - y \geq 0, \; z - y \geq 0, \; x \geq 0.\end{aligned} \tag{3}$$

Step 3. Based on Definition 2.6 the problem (3) can be transformed into MOLP problem with three crisp linear programming problem as follows:

$$\begin{aligned}&\text{Max } c^t x\\ &\text{Max} \quad (c^t x) - (c^t y)\\ &\text{Max} \quad (c^t x) + (c^t z)\\ &\text{s.t.} (Ax) = (b_1),\\ &((Ax) - (Ay) = (b_1) - (b_2)),\\ &((Ax) + (Az) = (b_1) + (b_3)),\\ &x - y \geq 0, \; z - y \geq 0, \; x \geq 0.\end{aligned} \tag{4}$$

Step 4. The lexicographic method will be used to obtain optimal solution of problem (4), we get

$$\begin{aligned}&\text{Max} \quad c^t x\\ &\text{s.t.} (Ax) = (b_1),\\ &((Ax) - (Ay) = (b_1) - (b_2)),\\ &((Ax) + (Az) = (b_1) + (b_3)),\\ &x - y \geq 0, \; z - y \geq 0, \; x \geq 0.\end{aligned} \tag{5}$$

Step 5. Using Steps 3-4 we have,

$$\begin{aligned}&\text{Max} \quad (c^t x) - (c^t y)\\ &\text{s.t.} \quad c^t x = \text{t},\\ &(Ax) = (b_1), ((Ax) - (Ay) = (b_1) - (b_2)),\\ &((Ax) + (Az) = (b_1) + (b_3)),\\ &x - y \geq 0, \; z - y \geq 0, \; x \geq 0.\end{aligned} \tag{6}$$

Where t is the optimal solution of problem (5).

Step 6. Solve this problem used in Step 5 as follows:

$$\begin{aligned}&\text{Max} \quad (c^t x) + (c^t z)\\ &\text{s.t.} (c^t x) - (c^t y) = \text{u},\\ &c^t x = \text{t},\\ &(Ax) = (b_1),\end{aligned}$$

$$((Ax)-(Ay)=(b_1)-(b_2)), \tag{7}$$
$$((Ax)+(Az)=(b_1)+(b_3)),$$
$$x-y\geq 0,\ z-y\geq 0,\ x\geq 0.$$

Where u is the optimal solution of problem (6).
In Step 6, we get an exact optimal solution, which is equivalent to the problem (2).

Theorem 3.1 If $\tilde{x}^* = (x^*),(y^*),(z^*)$ be an optimal solution of problems (5-7), then it is also an exact optimal solution of problem (2).

Proof. By the method of contradiction, let $\tilde{x}^* = (x^*),(y^*),(z^*)$ be an optimal solution of (5)-(7), but it is not the exact optimal solution of problem (2).
Let us we consider $\tilde{x}^0 = (x^0),(y^0),(z^0)$, such that in the case of maximization
$(c^t x^*),(c^t y^*),(c^t z^*) \prec (c^t x^0),(c^t y^0),(c^t z^0)$.

Based on Definition 2.7, we have three conditions as follows:

Case (i) In case of maximization, we consider let $(c^t x^*) \prec (c^t x^0)$. Also, with respect to the assumption we have:

$$(Ax^0)=(\tilde{b}_1),$$
$$(Ax^0)-(Ay^0)=(\tilde{b}_1)-(\tilde{b}_2),$$
$$(Ax^0)+(Az^0)=(\tilde{b}_1)+(\tilde{b}_3),$$
$$x^0-y^0\geq 0,\ z^0-y^0\geq 0,\ x^0\geq 0.$$

Therefore, $(x^0),(y^0),(z^0)$ is a feasible solution of problem (5) in which the objective value in (x^0), (y^0), (z^0) is greater than the objective value in (x^*), (y^*), (z^*). However, it is contradiction.

Case(ii) In case of maximization, let us we consider $(c^t x^*) = (c^t x^0)$, and $(c^t x^*)-(c^t y^*) \succ (c^t x^0)-(c^t y^0)$ Also, with respect to the assumption we have:

$$(Ax^0)=(\tilde{b}_1),$$
$$(Ax^0)-(Ay^0)=(\tilde{b}_1)-(\tilde{b}_2),$$
$$(Ax^0)+(Az^0)=(\tilde{b}_1)+(\tilde{b}_3),$$
$$x^0-y^0\geq 0,\ z^0-y^0\geq 0,\ x^0\geq 0.$$

Therefore, $(x^0),(y^0),(z^0)$ is a feasible solution of problem (6) in which the objective value in (x^0), (y^0), (z^0) is less than the objective value in (x^*), (y^*), (z^*). However, it is contradiction.

Case (iii) In case of maximization, let us we consider $(c^t x^*) = (c^t x^0)$,

$(c^t x^*) - (c^t y^*) = (c^t x^0) - (c^t y^0)$ and $(c^t x^*) + (c^t z^*) \prec (c^t x^0) + (c^t z^0)$. In addition, with respect to the assumption we have:

$$(Ax^0) = (\tilde{b}_1),$$
$$(Ax^0) - (Ay^0) = (\tilde{b}_1) - (\tilde{b}_2),$$
$$(Ax^0) + (Az^0) = (\tilde{b}_1) + (\tilde{b}_3),$$
$$x^0 - y^0 \geq 0,\ z^0 - y^0 \geq 0,\ x^0 \geq 0.$$

Therefore, $(x^0),(y^0),(z^0)$ is a feasible solution of problem (7) in which the objective value in $(x^0),\ (y^0),\ (z^0)$ is greater than the objective value in $(x^*),\ (y^*),\ (z^*)$. However, it is contradiction.

Therefore $\tilde{x}^* = (x^*),(y^*),(z^*)$ is an exact optimal solution of problem (2).

The flow chart describes the procedure of the proposed method as shown in Figure 1.

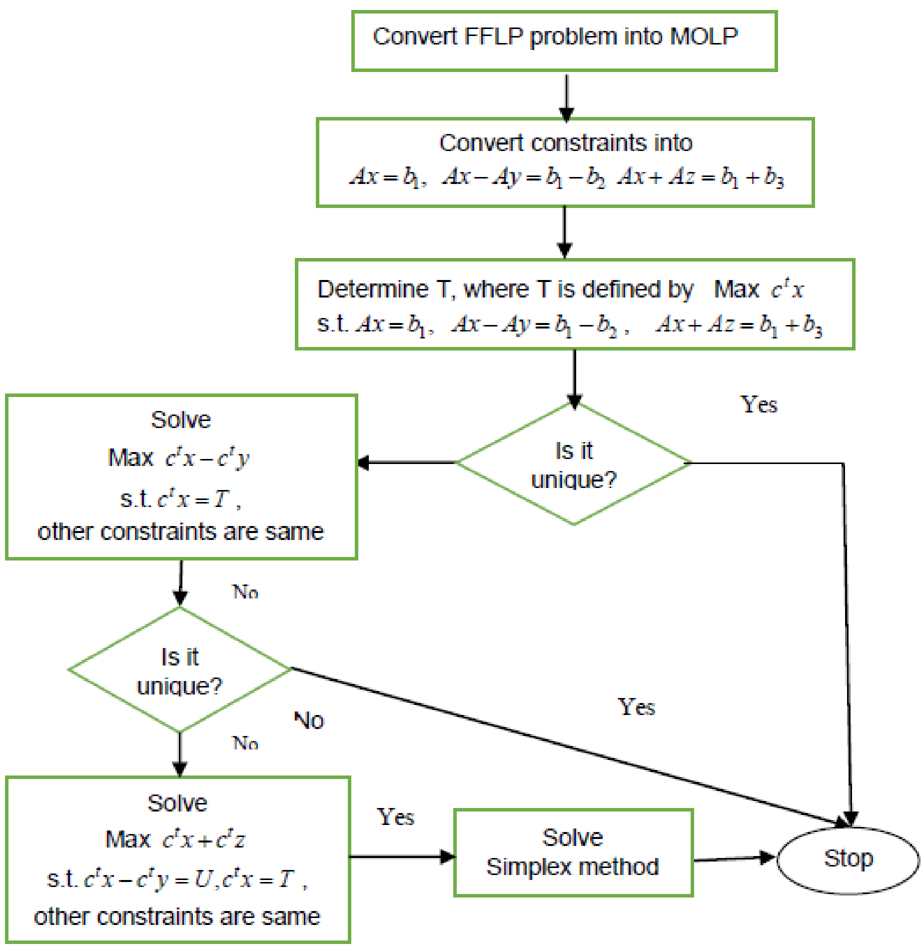

Figure 1. Flowchart for solving FFLP problem.

4. Numerical Example

In this section we consider some examples to illustrate our proposed method and compare it.

Example 4.1. [6]: Let us consider the following FFLP problem and solve it.

$$\begin{aligned}
&\text{Maximize } (1,6,9)\otimes\tilde{x}_1 \oplus (2,3,8)\otimes\tilde{x}_2,\\
&\text{subject to } (2,3,4)\otimes\tilde{x}_1 \oplus (1,2,3)\otimes\tilde{x}_2 = (6,16,30),\\
&(-1,1,2)\otimes\tilde{x}_1 \oplus (1,3,4)\otimes\tilde{x}_2 = (1,17,30),\\
&\tilde{x}_1,\tilde{x}_2 \geq 0.
\end{aligned} \tag{8}$$

Solution: Let us consider $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$ then the given FFLP problem (8) can be written as follows:

$$\begin{aligned}
&\text{Maximize } (1,6,9)\otimes(x_1,y_1,z_1) \oplus (2,3,8)\otimes(x_2,y_2,z_2),\\
&\text{Subject to } (2,3,4)\otimes(x_1,y_1,z_1) \oplus (1,\ 2,3)\otimes(x_2,y_2,z_2) = (6,16,30),\\
&(-1,1,2)\otimes(x_1,y_1,z_1) \oplus (1,3,4)\otimes(x_2,y_2,z_2) = (1,17,30),\\
&(x_1,y_1,z_1),(x_2,y_2,z_2) \geq 0.
\end{aligned} \tag{9}$$

Using Step 2, the problem (9) can be converted in to MOLP problem as follows:

$$\begin{aligned}
&\text{Max } x_1 + 2x_2\\
&\text{Max } 6y_1 + 3y_2\\
&\text{Max } 9z_1 + 8z_2\\
&\text{Subject to } 2x_1 + x_2 = 6,\\
&-x_1 + x_2 = 1,\\
&2x_1 + x_2 - 3y_1 - 2y_2 = -10,\\
&-x_1 + x_2 - y_1 - 3y_2 = -16,\\
&2x_1 + x_2 + 4z_1 + 3z_2 = 36,\\
&-x_1 + x_2 + 2z_1 + 4z_2 = 31,\\
&y_1 - x_1 \geq 0, \quad z_1 - y_1 \geq 0, \quad y_2 - x_2 \geq 0,\\
&z_2 - y_2 \geq 0, \ x_1 \geq 0, \ x_2 \geq 0.
\end{aligned} \tag{10}$$

Using Step 3, the problem (10) can be converted in to MOLP problem as follows:

$$\begin{aligned}
&\text{Max } x_1 + 2x_2,\\
&\text{Max } x_1 + 2x_2 - 6y_1 - 3y_2,\\
&\text{Max } x_1 + 2x_2 + 9z_1 + 8z_2,\\
&\text{Subject to } 2x_1 + x_2 = 6,\\
&-x_1 + x_2 = 1,\\
&2x_1 + x_2 - 3y_1 - 2y_2 = -10,\\
&-x_1 + x_2 - y_1 - 3y_2 = -16,
\end{aligned} \tag{11}$$

$$2x_1 + x_2 + 4z_1 + 3z_2 = 36,$$
$$-x_1 + x_2 + 2z_1 + 4z_2 = 31,$$
$$y_1 - x_1 \ge 0, \quad z_1 - y_1 \ge 0, \quad y_2 - x_2 \ge 0,$$
$$z_2 - y_2 \ge 0, \quad x_1 \ge 0, \; x_2 \ge 0.$$

Using Steps 4- 6, the optimal solution of problem (11) is

$\tilde{x}_1 = (x_1, y_1, z_1) = (1.6, 2, 3),$
$\tilde{x}_2 = (x_2, y_2, z_2) = (2.6, 5, 6),$

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ proposed method= $((c^t x), (c^t y), (c^t z))$= (6.8, 27, 75).

In Ezzati's method [14] the optimal solution of the objective functions are

$\tilde{x}_1 = (x_1, y_1, z_1) = (1.6, 2, 3), \tilde{x}_2 = (x_2, y_2, z_2) = (2.6, 5, 6).$

The objective value is

$(\tilde{c}\tilde{x}^*) = ((c^t x), (c^t y), (c^t z))$= (6.8, 27, 75).

In Kumar's method [6] the optimal solution of the objective functions are

$\tilde{x}_1 = (x_1, y_1, z_1) = (1, 2, 3), \tilde{x}_2 = (x_2, y_2, z_2) = (4, 5, 6).$

The objective value is:

$(\tilde{c}\tilde{x}^*) = ((c^t x), (c^t y), (c^t z))$ = (9, 27, 75).

By comparing the results of proposed method with Ezzati's and Kumar's method, based on Definition 2.7, we can conclude that our result is equal to Ezzati result, because:

$9 = (c^t x)_{\text{Kumar's method}} > (c^t x)_{\text{proposed method}} = (c^t x)_{\text{Ezzati method}} = 6.8,$

$84 = (c^t x)_{\text{Kumar's method}} > (c^t x) + (c^t z)_{\text{proposed method}} = (c^t x) + (c^t z)_{\text{Ezzati method}} = 81.8,$

$(\tilde{c}\tilde{x}^*)_{\text{Ezzati method}} = (\tilde{c}\tilde{x}^*)_{\text{proposed method}} = (6.8, 27, 75) < (c^t x)_{\text{Kumar's method}} = (9, 27, 75).$

Example 4.2 [8]: Consider the following FFLP problem and find the objective value functions.

$$\begin{aligned}
&\text{Min } (1,3,9) \otimes \tilde{x}_1 \oplus (1,2,8) \otimes \tilde{x}_2, \\
&\text{Subject to } \quad (1,3,5) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 = (1,9,22), \\
&\qquad\qquad (1,2,3) \otimes \tilde{x}_1 \oplus (2,3,4) \otimes \tilde{x}_2 = (1,8,18), \\
&\qquad\qquad \tilde{x}_1, \tilde{x}_2 \ge 0.
\end{aligned} \tag{12}$$

Solution: Let us consider $\tilde{x}_1 = (x_1, y_1, z_1)$ and $\tilde{x}_2 = (x_2, y_2, z_2)$ then the given FFLP problem (12) can be written as follows:

Maximize $(1,3,9) \otimes (x_1, y_1, z_1) \oplus (1,2,8) \otimes (x_2, y_2, z_2),$

$$\text{Subject to } (1,3,5)\otimes(x_1,y_1,z_1)\oplus(2,3,4)\otimes(x_2,y_2,z_2)=(1,9,22),$$
$$(1,2,3)\otimes(x_1,y_1,z_1)\oplus(2,3,4)\otimes(x_2,y_2,z_2)=(1,8,18), \tag{13}$$
$$(x_1,y_1,z_1),(x_2,y_2,z_2)\geq 0.$$

Using Step 2, the problem (13) can be converted in to MOLP problem as follows:

$$\begin{aligned}
&\text{Max} && x_1+x_2,\\
&\text{Max} && x_1+x_2-3y_1-2y_2,\\
&\text{Max} && x_1+x_2+9z_1+8z_2,\\
&\text{Subject to} && x_1+2x_2=1,\\
& && x_1+2x_2=1,\\
& && x_1+2x_2-3y_1-3y_2=-8,\\
& && x_1+2x_2-2y_1-3y_2=-7,\\
& && x_1+2x_2+5z_1+4z_2=23,\\
& && x_1+2x_2+3z_1+4z_2=19,\\
& && y_1-x_1\geq 0,\quad z_1-y_1\geq 0,\quad y_2-x_2\geq 0,\\
& && z_2-y_2\geq 0,\ x_1\geq 0,\ x_2\geq 0.
\end{aligned} \tag{14}$$

Using Steps 4-6, the optimal solution of problem (14) is

$\tilde{x}_1=(x_1,y_1,z_1)=(0, 1, 2)$,

$\tilde{x}_2=(x_2,y_2,z_2)=(0.5, 2, 3)$.

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ proposed method= $((c^t x), (c^t y), (c^t z))$ = (1, 7, 42).

In Ezzati's method the optimal solution of the objective functions are:

$\tilde{x}_1=(x_1,y_1,z_1)=(0, 1, 2)$,

$\tilde{x}_2=(x_2,y_2,z_2)=(0.5, 2, 3)$.

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ Ezzati's method= $((c^t x), (c^t y), (c^t z))$ = (1, 7, 42).

In Kumar's method the optimal solution of the objective functions are:

$\tilde{x}_1=(x_1,y_1,z_1)=(0, 1, 2)$,

$\tilde{x}_2=(x_2,y_2,z_2)=(0.5, 2, 3)$.

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ Kumar's method= $((c^t x), (c^t y), (c^t z))$ = (1, 7, 42).

By comparing the results of proposed method with Ezzati's and Kumar's method, based on Definition 2.7, we can conclude that our result is more effective, because:

$(c^t x)$ proposed method$=(c^t x)$ Ezzati method$=(c^t x)$ Kumar's method$=1$

$(c^t x)+(c^t z)$ proposed method$=(c^t x)+(c^t z)$ Ezzati method$=(c^t x)$ Kumar's method$=43$

$(\tilde{c}\tilde{x}^*)$Ezzati method $=(\tilde{c}\tilde{x}^*)$proposed method$=(c^t x)$ Kumar's method$=(1, 7, 42)$.

Example 4.3 [14]: Consider the FFLP problem

$$\begin{aligned} &\text{Max } \tilde{c}^t\tilde{x} \\ &\text{s.t. } \tilde{A}\tilde{x} = \tilde{b}, \\ &\quad\;\; \tilde{x} \geq 0. \end{aligned}$$

where $\tilde{c}$, $\tilde{A}$ and $\tilde{b}$ are given as follows:

$$\tilde{c} = \begin{bmatrix} (10,15,17) \\ (10,16,20) \\ (10,14,17) \\ (10,12,14) \end{bmatrix}, \ \tilde{A} = \begin{bmatrix} (8,10,13) & (10,11,13) & (9,12,13) & (11,15,17) \\ (12,14,16) & (14,18,19) & (14,17,20) & (13,14,18) \end{bmatrix},$$

$$\tilde{b} = \begin{bmatrix} (271.75, 411.75, 573.75) \\ (385.5, 539.5, 759.5) \end{bmatrix},$$

Using Step 1, the problem may be written as follows:

$$\begin{aligned} &\text{Max } 10x_1 + 10x_2 + 10x_3 + 10x_4,\ 15y_1 + 16y_2 + 14y_3 + 12y_4, 17z_1 + 20z_2 + 17z_3 + 14z_4 \\ &\text{s.t. } 8x_1 + 10x_2 + 9x_3 + 11x_4 = 271.75 \\ &\quad 12x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5, \\ &\quad 10y_1 + 11y_2 + 12y_3 + 15y_4 = 411.75, \\ &\quad 14y_1 + 18y_2 + 17y_3 + 14y_4 = 539.5, \\ &\quad 13z_1 + 13z_2 + 13z_3 + 17z_4 = 573.75, \\ &\quad 16z_1 + 19z_2 + 20z_3 + 18z_4 = 759.5, \\ &\quad x_j - y_j \geq 0,\ x_j, y_j, z_j \geq 0,\ \ j = 1,2,3,4. \end{aligned}$$

Using Step 2, problem may be written as

$$\begin{aligned} &\text{Max } 10x_1 + 10x_2 + 10x_3 + 10x_4, \\ &\text{Max } 15y_1 + 16y_2 + 14y_3 + 12y_4, \\ &\text{Max } 17z_1 + 20z_2 + 17z_3 + 14z_4, \\ &\text{s.t.} \quad 8x_1 + 10x_2 + 9x_3 + 11x_4 = 271.75 \\ &\quad 12x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5, \\ &\quad 8x_1 + 10x_2 + 9x_3 + 11x_4 - 10y_1 - 11y_2 - 12y_3 - 15y_4 = -140, \\ &\quad 12x_1 + 14x_2 + 14x_3 + 13x_4 - 14y_1 - 18y_2 - 17y_3 - 14y_4 = -154, \\ &\quad 8x_1 + 10x_2 + 9x_3 + 11x_4 + 13z_1 + 13z_2 + 13z_3 + 17z_4 = 845.5, \\ &\quad 12x_1 + 14x_2 + 14x_3 + 13x_4 + 16_1 + 19z_2 + 20z_3 + 18z_4 = 1145,, \\ &\quad x_j - y_j \geq 0,\ x_j, y_j, z_j \geq 0,\ \ j = 1,2,3,4. \end{aligned}$$

Using Step 3, problem may be written as

$$
\begin{aligned}
&\text{Max } 10x_1 + 10x_2 + 10x_3 + 10x_4,\\
&\text{Max } 10x_1 + 10x_2 + 10x_3 + 10x_4 - 15y_1 - 16y_2 - 14y_3 - 12y_4,\\
&\text{Max } 10x_1 + 10x_2 + 10x_3 + 10x_4 + 17z_1 + 20z_2 + 17z_3 + 14z_4,\\
&\text{s.t.} \quad 8x_1 + 10x_2 + 9x_3 + 11x_4 = 271.75\\
&\qquad 12x_1 + 14x_2 + 14x_3 + 13x_4 = 385.5,\\
&\qquad 8x_1 + 10x_2 + 9x_3 + 11x_4 - 10y_1 - 11y_2 - 12y_3 - 15y_4 = -140,\\
&\qquad 12x_1 + 14x_2 + 14x_3 + 13x_4 - 14y_1 - 18y_2 - 17y_3 - 14y_4 = -154,\\
&\qquad 8x_1 + 10x_2 + 9x_3 + 11x_4 + 13z_1 + 13z_2 + 13z_3 + 17z_4 = 845.5,\\
&\qquad 12x_1 + 14x_2 + 14x_3 + 13x_4 + 16z_1 + 19z_2 + 20z_3 + 18z_4 = 1145,\\
&\qquad x_j - y_j \geq 0,\ x_j, y_j, z_j \geq 0,\quad j = 1,2,3,4.
\end{aligned}
$$

After solving this problem by using Steps 4-6, we have

$$
\tilde{x} = \begin{bmatrix} (17.27, 17.27, 17.27) \\ (2.6, 2.6, 2.6) \\ (4.12, 9.52, 15.85) \\ (6.49, 6.49, 6.49) \end{bmatrix},
$$

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ proposed method= $((c^t x), (c^t y), (c^t z))$= (304.8, 511.81, 705.9).

In Ezzati's method the optimal solution of the objective functions are

$$
\tilde{x} = \begin{bmatrix} (17.27, 17.27, 17.27) \\ (2.16, 2.16, 2.16) \\ (4.64, 9.97, 16.36) \\ (6.36, 6.36, 6.36) \end{bmatrix},
$$

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ Ezzati's method= $((c^t x), (c^t y), (c^t z))$ = (304.58, 509.79, 704.37).

In Kumar's method the optimal solution of the objective functions are

$$
\tilde{x} = \begin{bmatrix} (15.28, 15.28, 15.28) \\ (2.4, 2.4, 9.1) \\ (6, 11.25, 11.25) \\ (6.49, 6.49, 6.49) \end{bmatrix},
$$

$(\tilde{c}\tilde{x}^*)$ Kumar's method= $((c^t x), (c^t y), (c^t z))$ = (301.83, 503.23, 724.15).

By comparing the results of proposed method with Ezzati's and Kumar's method, based on Definition 2.7, we can conclude that our result is effective, because

$304.80 = (c^t x)$ proposed method $> (c^t x)$ Ezzati method $=304.58> (c^t x)$ Kumar's method $=301.83$

$(c^t x) + (c^t z)$ Kumar's method $>1010.7= (c^t x) + (c^t z)$ proposed method $> (c^t x) + (c^t z)$ Ezzati method $=1008.95$,

(301.83, 503.23, 724.15) $=(\tilde{c}\tilde{x}^*)$ Kumar's method >(304.58, 509.79, 704.37)=

$(\tilde{c}\tilde{x}^*)$Ezzati method$>(\tilde{c}\tilde{x}^*)$proposed method=(304.8, 511.81, 705.9).

In Figure 2, we compare the membership function for the proposed method and existing methods. It shows that the proposed method is better than the existing methods.

Example 4.4 [14]. Dali Company is the leading producer of soft drinks and low-temperature foods in Taiwan. Currently, Dali plans to develop the south-East Asian market and broaden the visibility of Dali products in the Chinese market. Notably, following the entry of Taiwan to the world trade Organization, Dali plans to seek strategic alliance with prominent international companies, and introduced international bread to lighten the embedded future impact. In the domestic soft drinks market, Dali produces tea beverages to meet demand from four distribution centers in Taichung, Chiayi, Kaohsiung, and Taipei, with production being based at three plants in Changhua, Touliu, and Hsinchu. According to the preliminary environmental information, Table 1 summarizes the potential supply available from these three plants, the forecast demand from the four distribution centers, and the unit transportation costs for each route used by Dali for the upcoming season. The environmental coefficients and related parameters generally are imprecise numbers with triangular possibility

distributions over the planning horizon due to incomplete or unobtainable information. For example, the available supply of the Changhua plant is (7.2, 8, 8.8) thousand dozen bottles, the forecast demand of the Taichung distribution center is (6.2, 7, 7.8) thousand dozen bottles, and the transportation cost per dozen bottles from Changhua to Taichung is ($8, $10, $10.8). Due to transportation costs being a major expense, the management of Dali is initiating a study to reduce these costs as much as possible.

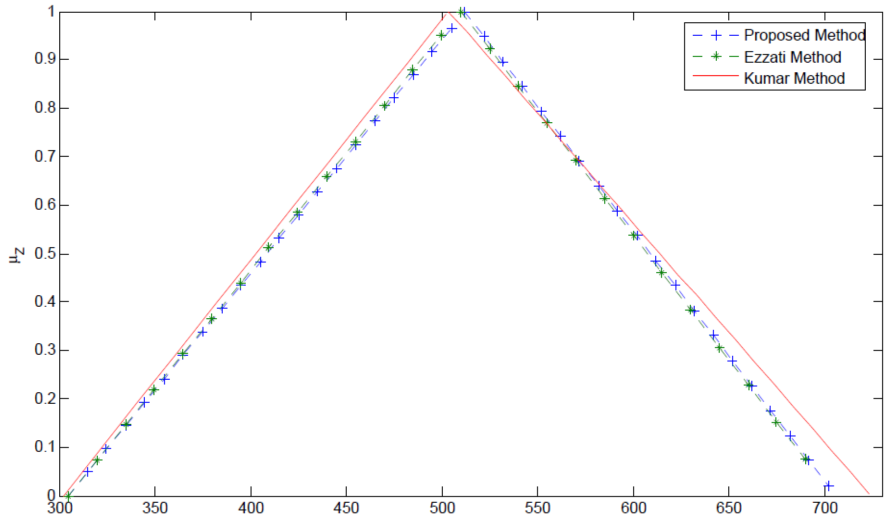

Figure 2. Membership function: Proposed method vs existing methods.

Table 1. Data of Example 4.4 (in U. S. dollar)

Source	Destination				Supply (000dozenbottles)
	Taichung	Chiayi	Kaohsiung	Taipei	
Changhua	(\$8, \$10, \$10.8)	(\$20.4, \$22, \$24)	(\$8, \$10, \$10.6)	(\$18.8, \$20, \$22)	(7.2,8, 8.8)
Touliu	(\$14, \$15, \$16)	(\$18.2, \$20, \$22)	(\$10, \$12, \$13)	(\$6, \$8, \$8.8)	(12, 14, 16)
Hsinchu	(\$18.4,\$20,\$21)	(\$9.6,\$12,\$13)	(\$7.8,\$10,\$10.8)	(\$14,\$15,\$16)	(10.2,12,13.8)
Demand	(6.2,7,7.8)	(8.9,10,11.1)	(6.5,8,9.5)	(7.8,9,10.2)	

This real world problem can be formulated to the following FFLP problem:

$$\text{Min}((8,10,10.8)\tilde{x}_{11}+(20.4,22,24)\tilde{x}_{12}+(8,10,10.6)\tilde{x}_{13}+(18.8,20,22)\tilde{x}_{14}+(14,15,16)\tilde{x}_{21}$$
$$+(18.2,20,22)\tilde{x}_{22}+(10,12,13)\tilde{x}_{23}+(6,8,8.8)\tilde{x}_{24}+(18.4,20,21)\tilde{x}_{31}+(9.6,12,13)\tilde{x}_{32}+(7.8,10,10.8)$$
$$\tilde{x}_{33}+(14,15,16)\tilde{x}_{34})$$

$$\text{s.t. } \tilde{x}_{11}+\tilde{x}_{12}+\tilde{x}_{13}+\tilde{x}_{14}=(7.2,8,8.8),$$
$$\tilde{x}_{21}+\tilde{x}_{22}+\tilde{x}_{23}+\tilde{x}_{24}=(12,14,16),$$
$$\tilde{x}_{31}+\tilde{x}_{32}+\tilde{x}_{33}+\tilde{x}_{34}=(10.2,12,13.8),$$
$$\tilde{x}_{11}+\tilde{x}_{21}+\tilde{x}_{31}=(6.2,7,7.8),$$
$$\tilde{x}_{12}+\tilde{x}_{22}+\tilde{x}_{32}=(8.9,10,11.1),$$
$$\tilde{x}_{13}+\tilde{x}_{23}+\tilde{x}_{33}=(6.5,8,9.5),$$
$$\tilde{x}_{14}+\tilde{x}_{24}+\tilde{x}_{34}=(7.8,9,10.2),$$

Using Step 1, problem may be written as

Min

$$8x_{11}+20.4x_{12}+8x_{13}+18.8x_{14}+14x_{21}+18.2x_{22}+10x_{23}+6x_{24}+18.4x_{31}+9.6x_{32}+7.8x_{33}+14x_{34}$$
$$,10y_{11}+22y_{12}+10y_{13}+20y_{14}+15y_{21}+20y_{22}+12y_{23}+8y_{24}+20y_{31}+12y_{32}+10y_{33}+15y_{34},$$
$$10.8z_{11}+24z_{12}+10.6z_{13}+22z_{14}+16z_{21}+22z_{22}+13z_{23}+8.8z_{2.4}+21z_{31}+13z_{32}+10.8z_{33}+16z_{34}$$

$$\begin{aligned}
\text{s.t. } & x_{11}+x_{12}+x_{13}+x_{14}=7.2,\\
& y_{11}+y_{12}+y_{13}+y_{14}=8,\\
& z_{11}+z_{12}+z_{13}+z_{14}=8.8,\\
& x_{21}+x_{22}+x_{23}+y_{24}=12,\\
& y_{21}+y_{22}+y_{23}+y_{24}=14,\\
& z_{21}+z_{22}+z_{23}+z_{24}=16,\\
& x_{31}+x_{32}+x_{33}+x_{34}=10.2,\\
& y_{31}+y_{32}+y_{33}+y_{34}=12,\\
& z_{31}+z_{32}+z_{33}+z_{34}=13.8,\\
& x_{11}+x_{21}+x_{31}=6.2,\\
& y_{11}+y_{21}+y_{31}=7,\\
& z_{11}+z_{21}+z_{31}=7.8,\\
& x_{12}+x_{22}+x_{32}=8.9,\\
& y_{12}+y_{22}+y_{32}=10,\\
& z_{12}+z_{22}+z_{32}=11.1,\\
& x_{13}+x_{23}+x_{33}=6.5,\\
& y_{13}+y_{23}+y_{33}=8,\\
& z_{13}+z_{23}+z_{33}=9.5,\\
& x_{14}+x_{24}+x_{34}=7.8,\\
& y_{14}+y_{24}+y_{34}=9,\\
& z_{14}+z_{24}+z_{34}=10.2,\\
& y_{ij}-x_{ij}\ge 0,\ \forall i=1,2,3, \quad \forall j=1,2,3,4,\\
& z_{ij}-y_{ij}\ge 0,\ \forall i=1,2,3, \quad \forall j=1,2,3,4,\\
& x_{ij}\ge 0,\ \forall i=1,2,3, \quad \forall j=1,2,3,4.
\end{aligned}$$

After solving this problem by using Steps 2-6, we have:

$$\tilde{x} = \begin{bmatrix} (6.2, 7, 7.8), \\ (0, 0, 0), \\ (1,1,1), \\ (0, 0, 0), \\ (0, 0, 0), \\ (0, 0, 0.3), \\ (4.2, 5,5.5), \\ (7.8, 9, 10.2), \\ (0,0,0), \\ (8.9,10,11), \\ (1.3, 2, 2.8), \\ (0, 0, 0), \end{bmatrix}.$$

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ proposed method= $((c^t x), (c^t y), (c^t z))$ = (241.98, 340, 435.94).

In Ezzati's method the optimal solution of the objective functions are

$$\tilde{x} = \begin{bmatrix} (6.2, 7, 7), \\ (0, 0, 0), \\ (1,1,1), \\ (0, 0, 0.8), \\ (0, 0, 1.1), \\ (4.2, 5, 5.9), \\ (7.8, 9, 9), \\ (0, 0, 0.8), \\ (8.9,10,10), \\ (1.3, 2, 2.6), \\ (0, 0, 0.4). \end{bmatrix},$$

Now the optimal value of the problem (2) may be written as

$(\tilde{c}\tilde{x}^*)$ Ezzati's method= $((c^t x), (c^t y), (c^t z))$ = (241.98, 352, 433.46).

In Kumar's method the optimal solution and optimal valu of the problem is

$$\tilde{x} = \begin{bmatrix} (6.2, 7, 7.8), \\ (0, 0, 0), \\ (1, 1, 1), \\ (0, 0, 0), \\ (0, 0, 0), \\ (0, 0, 0), \\ (4.2, 5, 5.8), \\ (7.8, 9, 10.2), \\ (0, 0, 0), \\ (8.9, 10, 11.1), \\ (1.3, 2, 2.7), \\ (0, 0, 0), \end{bmatrix}$$

$(\tilde{c}\tilde{x}^*)$ Kumar's method= $((c^t x), (c^t y), (c^t z))$= (241.98, 352, 433.46).

By comparing the results of proposed method with Ezzati's and Kumar's method, based on Definition 2.7, we can conclude that our result is effective, because:

$(c^t x)$ Proposed method= $(c^t x)$ Ezzati method= $(c^t x)$ Kumar's method=241.98

677.92= $(c^t x)+(c^t z)$ proposedmethod> $(c^t x)+(c^t z)$ Ezzati method= $(c^t x)+(c^t z)$ Ezzati method=675.44

$\Rightarrow$(241.98, 352, 433.46)=$(\tilde{c}\tilde{x}^*)$ Kumar's method=(241.98, 352, 433.46)=$(\tilde{c}\tilde{x}^*)$Ezzati method

>$(\tilde{c}\tilde{x}^*)$proposed method=(241.98, 340, 435.94).

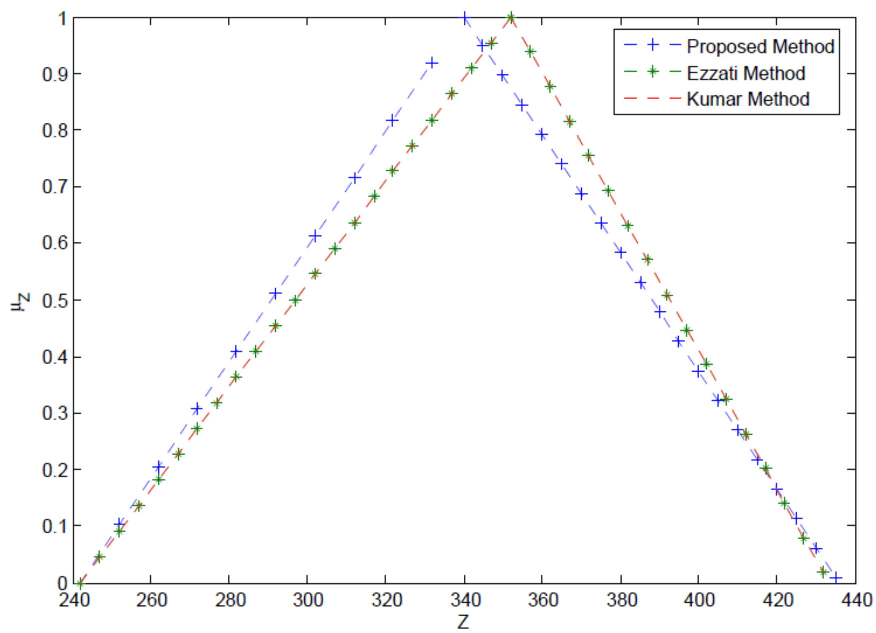

Figure 3. Membership function: Proposed method vs existing methods.

In Figure 3, we compare the membership function for the proposed method and existing methods. It shows that the proposed method is better than the existing methods.

5. Advantages of the Proposed Method

The proposed method has been studied in FFLP problems with fuzzy coefficients. It is pointed out that the proposed method is no restriction of all variables and parameters and the obtained results are satisfied all the constraints. We used in fuzziness of the objective function is neglected by linear ranking function in decision makers. Hence, it is very easy and comfortable for applied in real life application as compared as to existing methods.

6. Conclusions

In this paper, we proposed a modified technique to solve the FFLP problem in order to obtain the fuzzy optimal solution. The main idea behind this paper is LP problem with triangular fuzzy numbers is converted into MOLP problem and solved as a crisp linear programming problem. We compare the proposed technique with existing of two method and also draw graph we shown the easy, applicability of our proposed method which we studied in the above result of two examples and real life problems. The proposed method can be extended to multi-objective linear programming problem, fractional programming problem and multi-objective linear fractional programming problem etc. Moreover, a stochastic approach of the above problem can be studied and the comparison between the approached can be carried out.

References

[1] Zimmermann, H. J. (2011). *Fuzzy set theory—and its applications*. Springer Science & Business Media.

[2] Baykasoğlu, A., & Subulan, K. (2015). An analysis of fully fuzzy linear programming with fuzzy decision variables through logistics network design problem. *Knowledge-Based systems*, *90*, 165-184.

[3] Dehghan, M., Hashemi, B., & Ghatee, M. (2006). Computational methods for solving fully fuzzy linear systems. *Applied mathematics and computation*, *179*(1), 328-343.

[4] Ebrahimnejad, A., & Tavana, M. (2014). A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. *Applied mathematical modelling*, *38*(17), 4388-4395.

[5] Khan, I. U., Ahmad, T., & Maan, N. (2013). A simplified novel technique for solving fully fuzzy linear programming problems. *Journal of optimization theory and applications*, *159*(2), 536-546.

[6] Khan, I. U., Ahmad, T., & Maan, N. (2013). A simplified novel technique for solving fully fuzzy linear programming problems. *Journal of optimization theory and applications*, *159*(2), 536-546.

[7] Lotfi, F. H., Allahviranloo, T., Jondabeh, M. A., & Alizadeh, L. (2009). Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied mathematical modelling*, *33*(7), 3151-3156.

[8] Kumar, A., Kaur, J., & Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. *Applied mathematical modelling*, *35*(2), 817-823.

[9] Shamooshaki, M. M., Hosseinzadeh, A., & Edalatpanah, S. A. (2014). A new method for solving fully fuzzy linear programming with lr-type fuzzy numbers. *International journal of data envelopment analysis and* Operations Research*, *1*(3), 53-55.

[10] Veeramani, C., & Duraisamy, C. (2012). Solving fuzzy linear programming problem using symmetric fuzzy number approximation. *International journal of operational research*, *15*(3), 321-336.

[11] Wu, H. C. (2008). Using the technique of scalarization to solve the multiobjective programming problems with fuzzy coefficients. *Mathematical and computer modelling*, *48*(1), 232-248.

[12] Iskander, M. G. (2003). Using different dominance criteria in stochastic fuzzy linear multiobjective programming: A case of fuzzy weighted objective function. *Mathematical and computer modelling*, *37*(1-2), 167-176.

[13] Mahdavi-Amiri, N., & Nasseri, S. H. (2006). Duality in fuzzy number linear programming by use of a certain linear ranking function. *Applied mathematics and computation*, *180*(1), 206-216.

[14] Ezzati, R., Khorram, E., & Enayati, R. (2015). A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem. *Applied mathematical modelling*, *39*(12), 3183-3193.

[15] Dubois, D. J. (1980). *Fuzzy sets and systems: theory and applications* (Vol. 144). Academic press.

[16] Kauffman, A., & Gupta, M. M. (1991). *Introduction to fuzzy arithmetic*: *Theory and application*. Mathematics of Computation.

[17] Liou, T. S., & Wang, M. J. J. (1992). Ranking fuzzy numbers with integral value. *Fuzzy sets and systems*, *50*(3), 247-255.

[18] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO-operations research*, *51*(1), 285-297.

[19] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied intelligence*, *46*(3), 509-519.

[20] Das, S. K., & Mandal, T. (2015). A single stage single constraints linear fractional programming problem: An approach. *Operations research and applications: An international journal (ORAJ)*, *2*(1), 9-14.