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Scheduling Project Crashing Time Using Linear Programming Approach: Case Study

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A B S T R A C T

In today's competitive environment completing a project within time and budget, is very challenging task for the project managers. This aim of this study is to develop a model that finds a proper trade-off between time and cost to expedite the execution process. Critical path method (CPM) is used to determine the longest duration and cost required for completing the project and then the time-cost trade–off problem (TCTP) is formulated as a linear programming model. Here, LINDO program is used to determine the solution of the model. To implement the proposed model, necessary data were collected through interviews and direct discussion with the project managers of Chowdhury Construction Company, Dhaka, Bangladesh. The analysis reveals that through proper scheduling of all activities, the project can be completed within 120 days from estimated duration of 140 days. Reduction of project duration by 17% is achieved by increasing cost by 3.73%, which is satisfactory.

Keywords: Linear programming, critical path method, trade-off analysis, crashing.

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1. Introduction

Project management is defined as the process of utilizing resources, techniques etc., to get the job done properly confirming specified time & cost. It also ensures the performance standards to gain attraction to the buyers. In uncertain world to achieve competitive advantages, completing the definite project with limited resource & cost is very challenging issue. Real world is uncertain and there are several factors like labor related delay, political issues, contractor delay and, some unseen delays etc. causing interruptions as well as uncertainty. So, to become leader in the global market, proper planning and scheduling of all activities/jobs is required for the completion of the project on time and within budget. For proper planning and scheduling of large projects, planners use two techniques namely critical path method (CPM) and Project Evaluation and Review Technique (PERT).The objectives of the techniques are to help the project managers in

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monitoring the progress of all stages involved in a project. The expedition of the completing of the project can be done through the reduction of the scheduled execution time by hiring extra labor or using productive equipment. But this activity will incur additional cost. As a result, planners are targeting the trade-off between completion time & project cost for better scheduling and control of a project. The process of trade-off between time & cost is geared through crashing/prolonging the duration of the activities. According to Pour et al. [1], time-cost trade problem (TCTP) is considered as one of the vital decisions in project accomplishment. The main objective of the time-cost trade-off problem (TCTP) is to lessen the original project duration obtained from the critical path analysis with the minimum direct and indirect cost of the project. Direct costs include costs of material, labor, equipment etc. whereas indirect costs are the necessary costs of doing work which can't be related to a particular task.

There are enormous research works in the arena of TCTP. Research on the TCTP was first conducted by Kelly [2] in 1961. Several researchers agreed that from 1961 the research mainly focused on the deterministic cases [3, 4]). A variety of techniques were used to solve the time-cost trade problems which are classified into two areas: mathematical programming method and heuristic methods. Mathematical programming method includes linear programming, integer programming, and dynamic programming whereas heuristic methods use genetic algorithm, fuzzy multi-objective genetic algorithm (FMOGA), cost-loop method etc. In 1991, Shouman et al. [5] constructed a framework using mixed integer linear programming and CPM and utilized in natural gas projects. The value of the study is that, by the use of it, minimum total cost is achieved using crashing concept. Agarwal et al. [6] proposed a Neuro-genetic approach for resource constrained project scheduling problem. Neuro-genetic approach is a hybrid method consisting of genetic algorithm & neural network. The search process relied on genetic iteration for global solution and neural network for local solution. Liu et al. [7] developed a hybrid method using linear and integer programming for time-cost trade-off problem. Several researchers used dynamic programming to adjust between two important aspects of the project [8-10].

Recently, different computational optimization techniques such as genetic algorithms (GA), Evolutionary algorithms (EA), Particle swarm optimization (PSO) etc. have been used for crashing project duration. Nowadays, Genetic algorithm (GA) technique has become very popular for solving optimization problems [11]. Feng et al. [12] adopted genetic algorithms to solve construction time-cost trade-off problems. Li et al. [13] designed machine learning and genetic algorithms based system (MLGAS) in construction project and the system generated better results to nonlinear TCTP. In 2003, Poonambalam et al. [14] applied genetic algorithms for sequencing problems in mixed model assembly lines of industrial arena and found better performance of GA. Genetic algorithm has been also used to optimize multi-objective time-cost-quality trade-off problem [15]. Azaron et al. [16] designed cost trade off problem as a multi-objective optimal problem consisting four objective functions and used genetic algorithm to solve it. Several researchers developed hybrid model based on genetic algorithms and other techniques and applied it to discrete TCTP problems [16-19].

To deal with problems having uncertainties, different researchers have used fuzzy logic. Pathak et al. [20] and Shahsavari Pour et al. [15] applied fuzzy logic theory to consider affecting uncertainty in project quality. Pathak et al. [21] proposed ANN with MOGA (multi-objective genetic algorithm) approaches for solving nonlinear TCTP for better project scheduling. In 2009, Chen et al. [22] applied ant colony optimization algorithm in project scheduling to optimize the discounted cash flows. Zeinalzadeh [23] demonstrated a mathematical model using MILP-Lingo 12 to minimize the total cost of a construction project. Biswas et al. [24] designed a time cost trade off problem and solved it by using Matlab program.

Mokhtari et al. [25] developed a hybrid approach based on cutting plane method and Monte Carlo (MC) simulation for stochastic time–cost trade-off problem (STCTP) in PERT networks of project management. The objective of the study was to improve the project completion probability in a pre-specified deadline from a risky value. Błaszczyk and Nowak [26] developed a project scheduling problem including time-cost trade-off analysis. In this paper, the authors proposed a new technique based on computer simulation and interactive approach. In the first step, simulation experiments were done with a view to evaluating decision alternatives with respect to several criteria. Then, an interactive technique INSDECM was adopted for generating the final solution of the problem.

Nasab et al. [27] developed a Fuzzy Discrete Time-Cost-Quality Trade-off Problem (FDTCQTP) under uncertain condition and solved by using novel genetic algorithm. In this paper, time, cost and quality were considered as fuzzy trapezoidal numbers as well as project network paths were calculated via a new algorithm. Finally, the proposed algorithm was compared with classic GA by ANOVA and the results showed efficiency of the proposed model. Recently, several researchers have worked on time cost trade off problems of project planning such as Su et al. [28] and Zou et al. [29].

The aim of this study is to develop a hybrid a model for identifying the optimum additional cost associated with reduction of project duration. Firstly, critical path method (CPM) is applied to determine the critical path and critical activities. Then cost slope of six jobs are calculated using cost loop method and then TCTP is formulated as a linear programming model. Using LINDO program, objective function of minimizing the additional cost associated with the reduction in project time is obtained.

The rest of this study is arranged as follows: Section 2 frameworks the developed methodology and provides a stepwise depiction of the anticipated steps for project planning and scheduling. Section 3 shows the problem formulation and results & discussions are given with prototype example in Section 4. Finally, in Section 5 conclusion is presented. This section wraps up this study.

2. Methodology

At first, necessary data are taken from Chowdhury Construction Company located in Dhaka, Bangladesh through interviews and direct discussion with the project managers. CPM is applied to find out the critical path and critical activities. The activities are shortened in order to get their lowest cost slopes using the heuristic method. Then project time-cost crash problem is developed as a linear programming (LP) model. Using LINDO software, the model is analyzed in order to minimize the total cost and schedule the project crashing time.

The overall procedure for scheduling project crashing time with the minimum total cost can be summarized as follows:

I. Draw the project network.
II. Perform CPM calculations and identify the critical path, using normal duration and costs for all activities.
III. Compute the cost slope for each activity.
IV. Formulate project time-cost crash problem as a linear programming (LP) model.
V. Analyze the model using LINDO software and optimize the objective function.

3. Problem Formulation

3.1 Problem Description and Notation

The project time-cost crashing problem can be described as follows: Assume that a construction company completes i types of activities/jobs to meet market demand within a time period. The execution of the project can be expedited by crashing duration of all activities. But this operation incurs extra cost. The project time-cost trade-off problem (TCTP) focuses on developing an approach to schedule the activities in such a way that ensures lowest additional cost associated with the reduction of time and expedites the execution of the project. Based on the above characteristics of the considered TCTP problem, the mathematical model herein is developed as follows: Let Z is the total cost of crashing activities. The aim of this study is to optimize the objective function (Z) fulfilling some constraints.

i. Decision variables:

$X_i = Amount\ of\ reduction\ in\ the\ duration\ of\ activity; for\ i = A, B \ldots..F$

$Y_i = Start\ time\ activity\ of\ i; for\ i = C, D \ldots, F$

$Y_{Finish} = Construction\ duration$

ii. Objective function:

Here start time of each activity (including Finish) depends on the start time and duration of each of its immediate predecessors. For example, the immediate predecessor of activity C is activity B then start time of activity C is as follows:

$$Y_c \geq Y_B + 20 - X_B$$

Thus, activity C cannot start until activity B starts and then completes its duration of $20 - X_B$. Including all of these relationships the objective function of the proposed model can be formulated as (Min total cost):

$$Z = 100X_A + 200X_B + 600X_C + 60X_D + 120X_E + 300X_F \tag{1}$$

iii. Constraints (Maximum reduction constraints):

$$X_A \le 20 \tag{2}$$
$$X_B \le 5 \tag{3}$$
$$X_C \le 10 \tag{4}$$
$$X_D \le 10 \tag{5}$$
$$X_E \le 10 \tag{6}$$
$$X_F \le 10 \tag{7}$$

iv. Start time constraints:

$$Y_c - Y_B + X_B \ge 20 \tag{8}$$
$$Y_D - Y_C + X_C \ge 40 \tag{9}$$
$$Y_E - Y_B + X_D \ge 30 \tag{10}$$
$$Y_E - Y_F + X_F \ge 60 \tag{11}$$
$$Y_F - Y_B + X_B \ge 20 \tag{12}$$
$$Y_{Finish} - Y_B + X_A \ge 120 \tag{13}$$
$$Y_{Finish} - Y_E + X_E \ge 50 \tag{14}$$

v. Project duration constraint:

$$Y_{Finish} \le 120 \tag{15}$$

vi. Non-negativity constraints:

$$X_i \ge 0\,; i = A, B \dots \dots, F \tag{16}$$
$$Y_i \ge 0\,; i = C, D \dots \dots, F \tag{17}$$
$$Y_{Finish} \ge 0 \tag{18}$$

4. Results and Discussion

The proposed model has been applied on a construction project to demonstrate the practicality of the proposed methodology. It needs an estimate of how much time each activity takes in the normal way in order to schedule the activities in the network. The project manager wishes to complete the whole operation within 120 days. This means that he needs to crash project by 20 days. The project data of a construction problem provided by the project manager is shown in Table 1. Table 1 presents the details description of all activities required for the completion of the construction project. Here, there are five activities and construction process starts with activity A and ends with activity F.

Table 1. Construction project data.

Activity code	Immediate predecessors	Normal cost ($)	Normal duration (Days)
A	-	12000	120
B	-	1800	20
C	B	16000	40
D	C	1400	30
E	D, F	3600	50
F	B	13500	60

The associated cost in terms of dollar and required number of days to complete individual activity are demonstrated in Table 1. The first column represents the activity identification code, immediate predecessors of each activity are shown in second column, third column shows the estimated normal cost and the fourth column provides the activity duration in days. Daily indirect cost of the project is assumed to be $100. Table 2 depicts a project with hypothetical normal time - cost data and crash time–cost data of necessary activities. Activity C requires highest amount of cost whereas highest normal time duration in days is 120 for activity A.

Table 2. List of normal and crash cost-time data.

Activity code	Immediate predecessors	Normal cost ($)	Normal duration (days)	Crash cost ($)	Crash duration (days)
A	-	12000	120	14000	100
B	-	1800	20	2800	15
C	B	16000	40	22000	30
D	C	1400	30	2000	20
E	D,F	3600	50	4800	40
F	B	13500	60	18000	45

4.1 Determination of Cost-Time Slopes of the Activities

To determine the critical path consisting of activities with zero slack, different variables such as Earliest Start (ES), Earliest Finish (EF), Latest Start (LS), Latest Finish (LF) and Slack are computed. Table 3 shows the computation of determining the critical path of the project. Based on table 3 the total duration for the completion of the project is 140 days and the critical path is B-C-D-E. Project total normal direct cost is equal to sum of normal direct cost of all activities is $48300.

Table 3. Computation of critical path method.

Activity code	Earliest Start (ES)	Earliest Finish (EF)	Latest Start (LS)	Latest Finish (LF)	Slack (LS-ES)	Critical
A	0	120	20	140	20	No
B	0	20	0	20	0	Yes
C	20	60	20	60	0	Yes
D	60	90	60	90	0	Yes
E	90	140	90	140	0	Yes
F	20	80	30	90	10	No

To expedite the project by reducing the expected project duration further down from 140 days, a process of crashing the duration of activities has been anticipated. Reduction of project duration incurs the extra cost. Project duration can be reduced by taking several measures such as overtime, hiring additional workers, using special time-saving materials, and special equipment. Table 4 shows the calculation of the cost-time slopes of the activities by a heuristic method named the cost-lope method.

Table 4. Cost-time slope of the activities.

	Normal		Crash				
Activity Code	Duration (days)	Costs ($)	Duration (days)	Costs ($)	Crash cost-Normal cost (ΔC)	Normal time-Crash time (Δt)	Cost slope (ΔC/Δt)
A	120	12000	100	14000	2000	20	100
B	20	1800	15	2800	1000	5	200
C	40	16000	30	22000	6000	10	600
D	30	1400	20	2000	600	10	60
E	50	3600	40	4800	1200	10	120
F	60	13500	45	18000	4500	15	300

Both the cost slope and the crash ability are shown beneath each activity in the precedence diagram in Figure 1.

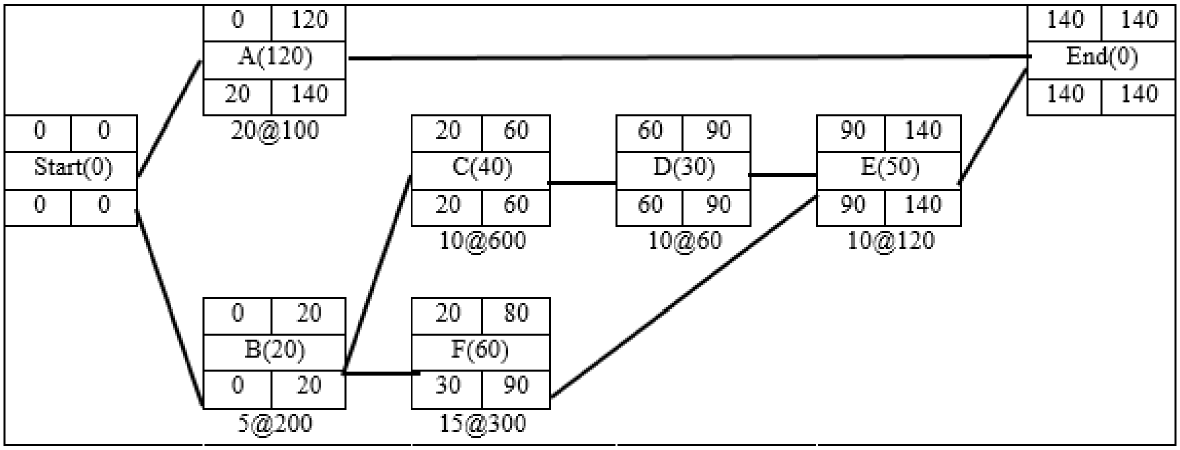

Figure 1. Precedence diagram of all activities.

4.2 Model Implementation

In this study, LINDO software is used to optimize the proposed model and the solution of the model is presented in Table 5. Previously, the total duration and expected total cost for the completion of the project were 140 days and $48300.

Table 5. Solution of the model.

Objective value	Final value	Reduced value
Z	1800	0
X_A	0	100
X_B	0	80
X_C	0	540
X_D	10	0
X_E	10	0
X_F	0	240
Y_A	0	0
Y_B	0	120
Y_C	20	0
Y_D	60	0
Y_E	80	0
Y_F	20	0
Y_{Finish}	120	0

However, Table 5 shows that additional cost for crashing activities and to complete the project by 120 days is $1800. So, through proper scheduling, the activities project completion time is reduced by 20 days which increases the initial expected cost from $48300 to $ 50100.

5. Conclusions

The main goal of this study is to schedule the jobs/activities of a construction project in such a way that expedites the execution of the project. CPM method is used to identify the critical path and estimate the project completion time. Linear programming (LP) approach is suggested to crash the activities of the project. Reduction of 20 days from 140 days estimated by CPM increases the total cost by $1800. The model indicates that about 17% decrease of time can be achieved by increasing cost by 3.73%, which is satisfactory. The contribution of the model is its simplicity of use and project manager can schedule all activities effectively. Different optimization tools like particle swarm optimization (PSO), mixed integer linear programming (MILP) and, fuzzy multi-objective linear programming (FMOLP) etc. can be used to obtain the solution.

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