



## **Solving Monetary (MIU) models with Linearized Euler Equations: Method of Undetermined Coefficients**

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### **ABSTRACT**

This paper attempts to solve a benchmark money in utility model by first order Taylor approximation to the policy function. After a brief summary of recent development in first order Taylor approximation in solving dynamic stochastic general equilibrium models, we choose Sidrauski's Money in utility model as a standard model and follow the approach proposed by Uhlig [1] to solve for the recursive law of motion at first order.

**Keywords:** DSGE Model, Calibration, monetary models

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## **1. Introduction**

In this paper I will present another method to solve monetary models or dynamic stochastic general equilibrium models, called the method of undetermined coefficients. The method is proposed by [1]. He has many papers on this method and nice codes to implement the method easily. Please visit his web page for more information on the method. This paper is basically the summary of [1]. The method is similar to the method by [2] in the sense that the method crucially depends on linearizing the equations that characterize the solution. In this sense, both methods are categorized as the linearizing Euler equation method. Moreover, both methods are local method. The (potentially) non-linear equations that characterize the solution of the model are linearized around some state, most likely the steady state of the model. Remember that the approximation is valid only around the steady state. Besides, the methods necessarily imply certainty equivalence. We are going to use the monetary model with [3]. We apply these methods to the Sidrauski model. But rather than assuming that

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utility depends just on consumption and money holdings, we also allow utility to depend on the representative agent's consumption of leisure. This introduces a labor supply decision into the analysis, an important and necessary extension for studying business-cycle fluctuations since employment variation is an important characteristic of cycles. For the production function, we can develop approximations around the steady state that can be solved numerically. We also need to add a source, or sources, of exogenous shocks that disturb the system from its steady-state equilibrium. The two types of shocks we will consider will be productivity shocks, the driving force in real-business-cycle models, and shocks to the growth rate of the nominal stock of money. First, we solve the model using the baby version of the solution method. The model is used to show the basic concept of the solution method. Then we show the general formulation of the method.

## 2. A Money-in-the-Utility (MIU) Model

Let's start by describing the social planner's problem in the economy of [3]. We follow the standard specification in dynamic general equilibrium models by assuming that output is produced using capital and labor according to a Cobb-Douglas, constant returns to scale production function. Consistent with the real business cycle literature, we incorporate a stochastic disturbance to total factor productivity, so that

$$y_t = e^{z_t} k_{t-1}^\alpha n_t^{1-\alpha} \quad 0 \leq \alpha \leq 1 \quad \text{and} \quad z_t = \rho z_{t-1} + e_t$$

is the process followed by the productivity shock.

For the utility function, assume a nested CES specification given by

$$u(C_t, m_t, 1 - n_t) = \frac{[aC_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\phi}{1-b}}}{1-\phi} + \psi \frac{(1-n_t)^{1-\eta}}{1-\eta}$$

Where the maximization is subject to the budget constraint

$$y_t + \tau_t + (1 - \delta)k_{t-1} + \frac{(1 + i_{t-1})b_{t-1} + m_{t-1}}{(1 + \pi_t)} = c_t + k_t + m_t + b_t$$

### 3. The Procedure

You need to take the following steps to solve a model using linearized Euler equations and matrix decomposition. The remaining part of the note explains the procedure to the details step by step [4].

- A. Find the system of (potentially non-linear) equations that characterize the solution of the model.
- B. Find the steady state of the model.
- C. Approximate the non-linear equations in the system around the steady state, using 1st order Taylor approximation.
- D. Fit the system of equations into some matrix representation. Since the representation is closely related to the solution method, the representation differs for each solution method.
- E. Derive the optimal decision rules (linear functions from the state variables to the control variables) and the laws of motion for endogenous state variables (linear function from the state variables to the state variables in the next period). Once the system of equations is fit into the matrix representation associated with the method of undetermined coefficients, the solution is automatically obtained (solution method doesn't depend on the characteristics of the model).

### 4. Characterizing the Solution

The solution of the social planner's problem can include, most importantly, the first order conditions (including the Euler equation), and laws of motion for state variables. Other equations might be included depending on the state and control variables chosen. In general, if we have  $k$  exogenous state variables,  $m$  endogenous state variables, and  $n$  control (jump) variables, we have  $k + n + m$  equations or more [5].

In the current example, we choose  $k = 2 (Z, G)$ ,  $m = 2 (K, M)$ , and  $n = 8 (C, N, Y, \pi, \lambda, X, R, \text{ and } D)$ . Obviously, there is a degree of freedom in how to choose the control variables.

For our current examples, the following system of equations characterize the solution to the social planner's problem:

$$\max_{\{C_t, n_t, m_t, b_t, k_t\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left[ \frac{[aC_t^{1-b} + (1-a)m_t^{1-b}]^{\frac{1-\Phi}{1-b}}}{1-\Phi} + \psi \frac{(1-n_t)^{1-\eta}}{1-\eta} \right] \right]$$

$$\text{s.t.} \quad y_t + \tau_t + (1-\delta)k_{t-1} + \frac{(1+i_{t-1})b_{t-1} + m_{t-1}}{(1+\pi_t)} = c_t + k_t + m_t + b_t$$

$$y_t = e^{z_t} k_{t-1}^\alpha n_t^{1-\alpha}$$

Using the assumed functional forms, and letting  $X_t = aC_t^{1-b} + (1-a)m_t^{1-b}$ , with the maximization problem now an unconstrained one over  $C_t, n_t, m_t, b_t$  and  $k_t$ . The first order necessary conditions for this problem are

$$(X_t)^{\frac{b-\Phi}{1-b}} a(c_t)^{-b} = \lambda_t \quad (1)$$

$$\frac{\psi(1-n_t)^{-\eta}}{\lambda_t} = (1-\alpha) \frac{y_t}{n_t} \quad (2)$$

$$(X_t)^{\frac{b-\Phi}{1-b}} (1-a)(m_t)^{-b} = \lambda_t - \beta E \left( \lambda_{t+1} \frac{1}{(1+\pi_{t+1})} \right) \quad (3)$$

$$\lambda_t = \beta E \left( \lambda_{t+1} \frac{1+i_t}{(1+\pi_{t+1})} \right) \quad (4)$$

$$\lambda_t = \beta E \left( \lambda_{t+1} \left( \alpha \frac{y_{t+1}}{k_t} + (1-\delta) \right) \right) \quad (5)$$

Then

$$\frac{u_m}{u_c} = \left( \frac{1-a}{a} \right) \left( \frac{m_t}{c_t} \right)^{-b} = \frac{i_t}{1+i_t} \quad (6)$$

$$\frac{u_l}{u_c} = \frac{\psi(1-n_t)^{-\eta}}{aX_t^{\frac{b-\Phi}{1-b}} c_t^{-b}} = (1-\alpha) \frac{y_t}{n_t} \quad (7)$$

$$aX_t^{\frac{b-\Phi}{1-b}} c_t^{-b} = \beta E_t \left( aR_t X_{t+1}^{\frac{b-\Phi}{1-b}} c_{t+1}^{-b} \right) \quad (8)$$

$$R_t = \alpha \frac{E_t y_{t+1}}{k_t} + (1-\delta) \quad (9)$$

## 5. Finding Steady State

I will skip the details. For more details, please see the lecture note for [2] method. The key of this step is to assume that  $Z$  stays at its unconditional mean ( $Z = Z' = \bar{Z}$ ). With this assumption, we can solve for the steady state values of all the other variables ( $\bar{K}, \bar{N}, \bar{Y}, \bar{C}, \bar{\pi}, \bar{R}$  and  $\bar{I}$ )

In the steady state for equation (8),  $\bar{R} = \beta^{-1}$ . This condition together with (9) implies that the steady-state output-capital ratio is equal to  $\left( \frac{\bar{y}}{\bar{k}} \right) = \frac{1}{\alpha} \left( \frac{1}{\beta} - 1 + \delta \right)$ . From the production function,

$$\left( \frac{\bar{y}}{\bar{k}} \right) = \left( \frac{\bar{n}}{\bar{k}} \right)^{1-\alpha}, \text{ or}$$

$$\left( \frac{\bar{n}}{\bar{k}} \right) = \left( \frac{\bar{y}}{\bar{k}} \right)^{\frac{1}{1-\alpha}} = \left[ \frac{1}{\alpha} \left( \frac{1}{\beta} - 1 + \delta \right) \right]^{\frac{1}{1-\alpha}}$$

It follows from the aggregate resource constraint that

$$\left(\frac{\bar{c}}{\bar{k}}\right) = \left(\frac{\bar{y}}{\bar{k}}\right) - \delta = \frac{1}{\alpha} \left(\frac{1 - \beta}{\beta}\right) + \left(\frac{1 - \alpha}{\alpha}\right) \delta$$

Since  $(1 + \bar{i}) = \bar{R}(1 + \bar{\pi})$  and  $1 + \bar{\pi} = \Theta$ ,

$$\frac{\bar{i}}{1 + \bar{i}} = \frac{\Theta - \beta}{\Theta}$$

Therefore, using the utility function to evaluate (6) in the steady state yields

$$\left(\frac{\bar{m}}{\bar{c}}\right) = \left(\frac{a}{1 - a}\right)^{-\frac{1}{b}} \left(\frac{\Theta - \beta}{\Theta}\right)^{-\frac{1}{b}}$$

and

$$\left(\frac{\bar{m}}{\bar{k}}\right) = \left(\frac{a}{1 - a}\right)^{-\frac{1}{b}} \left(\frac{\Theta - \beta}{\Theta}\right)^{-\frac{1}{b}} \left(\frac{\bar{c}}{\bar{k}}\right)$$

### 6. Calibration

Thirteen parameters appear in the equations that characterize behavior around the steady state:  $\alpha, \delta, \rho_M, \sigma_M^2, \beta, a, b, \eta, \Phi, \Theta, \rho_Z, \sigma_Z^2$

**Table 1.** Baseline Parameter Values

Parameter:	Definition	Value	Source
$\alpha$	the share of capital income in total income	0.412	[13]
$\delta$	the rate of depreciation of physical capital	0.042	[14]
$\beta$	the subjective rate of time discount in the utility function	0.98	[3]
$\eta$	leisure parameter	2.17	[5]
$a$	weight on consumption in composite good definition	0.95	[3]
$b$	interest elasticity	1.32	[9]
$\Theta$	1 plus quarterly rate of nominal money growth	1.23	[8]
$\Phi$	coefficient of relative risk aversion	1.5	[12]
$\rho_M$	Autocorrelation of money shock	0.562	[8]
$\rho_Z$	the autoregressive coefficient in the productivity process	0.72	[8]
$\sigma_M$	the standard deviation of innovations to the money growth	0.062	[8]
$\sigma_Z$	the standard deviation of productivity innovations	0.045	[8]
$\bar{n}$	Steady state employment	0.33	[11]

**Table 2.** Steady-State Values at Baseline Parameter Values

Value	Definition	Value
$\bar{R}$	Steady state real interest rate	0.072
$\frac{\bar{y}}{\bar{k}}$	Steady state output to capital	0.084
$\frac{\bar{c}}{\bar{k}}$	Steady state consumption to capital	0.065
$\frac{\bar{m}}{\bar{k}}$	Steady state money to capital	0.089
$\frac{\bar{n}}{\bar{k}}$	Steady state employment to capital	0.021

## 7. Log-linearization

Next, we need to linearize the model around the steady state. With the exception of interest rates and inflation, variables will be expressed as percentage deviations around the steady state. Percentage deviations of a variable  $X$  around its steady-state value will be denoted by  $\hat{x}$ ,

$$X_t = X^a (1 + a\hat{x}_t)$$

$$X_t^a = \bar{X}^a (1 + a\hat{x}_t)$$

$$X_t^a Y_t^\beta = \bar{X}^a \bar{Y}^\beta (1 + a\hat{x}_t + \beta\hat{y}_t)$$

$$f(X_t) = f(X) (1 + \eta\hat{x}_t)$$

Where  $\eta = \frac{\partial f(X_t)}{\partial X} \cdot \frac{X}{f(X)}$

If we apply the rules to log-linearize to the system of equations for our current model, we obtain the following system of log-linearized equations:

$$\left(\frac{\bar{y}}{\bar{k}}\right) \hat{y}_t = \left(\frac{\bar{c}}{\bar{k}}\right) \hat{c}_t + \hat{k}_t - (1 - \delta)\hat{k}_{t-1} \quad (10)$$

$$\hat{r}_t = \alpha \left(\frac{\bar{y}}{\bar{k}}\right) (E_t \hat{y}_{t+1} - \hat{k}_t) \quad (11)$$

$$0 = E_t [\Omega_1 (\hat{c}_{t+1} - \hat{c}_t) + \Omega_2 (\hat{m}_{t+1} - \hat{m}_t)] - \hat{r}_t \quad (12)$$

$$\hat{y}_t - \Omega_1 \hat{c}_t + \Omega_2 \hat{m}_t = \left(1 + \eta \frac{\bar{n}}{1 - \bar{n}}\right) \hat{n}_t \quad (13)$$

$$\hat{i}_t = E_t \hat{\pi}_{t+1} + \hat{r}_t \quad (14)$$

$$\hat{m}_t = \hat{M}_t - \hat{p}_t = \hat{c}_t - \left(\frac{1}{b}\right) \hat{i}_t \quad (15)$$

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + u_t \quad (16)$$

$$z_t = \rho_z z_{t-1} + e_t \quad (17)$$

$$u_t \equiv \rho_M u_{t-1} + \varphi_t \quad (18)$$

Where  $\gamma\Phi + (1 - \gamma)b = \Omega_1$  ,  $(b - \Phi)(1 - \gamma) = \Omega_2$

$$\gamma = [1 + a^{-1}(1 - a) \times (\bar{m}/\bar{c})^{1-b}]^{-1}$$

Equations (11)–(18) constitute a linearized version of Sidrauski’s MIU model. These equations represent a linear system of difference equations involving expectational variables. The endogenous state variables are  $k$  and  $m$ ; the endogenous jump variables are  $y$ ;  $c$ ;  $n$ ;  $p$ ;  $i$ ;  $r$ , and  $l$ ; the exogenous variables are  $z$  and  $u$ .

### 8. Solving Linear Rational Expectations Models with Forward-Looking Variables

This sections provides a brief overview of the approach used to solve linear rational expectations models. This discussion follows [1], to which the reader is referred for more details. General discussions can be found in Farmer (1993, chapter 3) or the user’s guide in [6]. See also [7].

Let  $x_t = (k_t, m_t)'$  be the vector of endogenous state variables, and let  $y_t = (y_t, c_t, n_t, i_t, \pi_t, r_t, \lambda_t)'$  be the vector of other endogenous variables. The equilibrium conditions of the MIU model can be written in the form

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t \tag{19}$$

$$0 = E_t[Fx_{t+1} + Gx_t + Hx_{t-1} + Jy_{t+1} + Ky_t + Lz_{t+1} + Mz_t] \tag{20}$$

$$z_{t+1} = Nz_t + \varepsilon_{t+1}; E_t[\varepsilon_{t+1}] = 0 \tag{21}$$

Where  $x$  is a vector of endogenous state variables (size  $m * 1$ ),  $y$  is a vector of control variables (size  $n * 1$ ),  $z$  is a vector of exogenous state variables (size  $k * 1$ ).  $\varepsilon_{t+1}$  is a vector of shocks (size  $k * 1$ ).  $C$  is of size  $n * n$ .  $F$  is of size  $m * n$ , and  $N$  has only stable eigenvalues. It is assumed that  $C$  is of full column rank and that the eigenvalues of  $N$  are all within the unit circle. For this paper

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & -\Omega_2 \\ \kappa & 0 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} \delta - 1 & 0 \\ \alpha & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \delta - 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ \xi & 0 \\ 0 & 0 \end{bmatrix}$$

$$\kappa = \alpha(\alpha - 1) \left(\frac{\bar{y}}{\bar{k}}\right) \eta \left(\frac{\bar{n}}{1 - \bar{n}}\right), \xi = \alpha \left(\frac{\bar{y}}{\bar{k}}\right) \left(1 + \eta \left(\frac{\bar{n}}{1 - \bar{n}}\right)\right) \rho_z, \Omega_2 = (b - \Phi)(1 - \gamma)$$

$C$  is 8\*8 then,

$$\begin{aligned} c_{11} &= -\frac{\bar{y}}{\bar{k}}, c_{12} = \frac{\bar{c}}{\bar{k}}, c_{21} = -1, c_{23} = 1 - \alpha, c_{32} = -1, c_{35} = \frac{1}{b}, c_{47} = 1, c_{51} = 1, c_{53} \\ &= -\left(1 + \eta \left(\frac{\bar{n}}{1 - \bar{n}}\right)\right), c_{54} = 1, c_{62} = \Omega_1, c_{64} = 1, c_{74} = \alpha(1 - \alpha) \left(\frac{\bar{y}}{\bar{k}}\right), c_{76} \\ &= -\left(\delta + \eta \left(\frac{\bar{n}}{1 - \bar{n}}\right) + \alpha(1 - \alpha) \left(\frac{\bar{y}}{\bar{k}}\right)\right), c_{88} = -\delta \end{aligned}$$

$$\Omega_1 = \gamma\Phi + (1 - \gamma)b, \gamma = [1 + a^{-1}(1 - a) \times (\bar{m}/\bar{c})^{1-b}]^{-1}$$

$$F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, H = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, N = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_m \end{bmatrix}$$

Then if an equilibrium solution to this system of equations exists, it takes the form of stable laws of motion:

$$x_t = Px_{t-1} + Qz_t \quad (21)$$

$$y_t = Ry_{t-1} + Sz_t \quad (22)$$

### 9. General Solution Method: Solving the Matrix Equations

As we did for the simple case, let's substitute out  $x_t, x_{t+1}, y_{t+1}, y_t$  and  $z_{t+1}$  using (20), (21), and (22). Then the equations (19) and (20) become the followings:

$$0 = A(Px_{t-1} + Qz_t) + Bx_{t-1} + C(Rx_{t-1} + Sz_t) + Dz_t$$

$$0 = (AP + B + CR)x_{t-1} + (AQ + CS + D)z_t \quad (23)$$

$$0 = E_t[F(Px_t + Qz_{t+1}) + G(Px_{t-1} + Qz_t) + Hx_{t-1} + J(Rx_t + Sz_{t+1}) + K(Rx_{t-1} + Sz_t) + L(Nz_t + \varepsilon_{t+1}) + Mz_t]$$

$$= F(P(Px_{t-1} + Qz_t) + QNz_t) + G(Px_{t-1} + Qz_t) + Hx_{t-1} + J(R(Px_{t-1} + Qz_t) + SNz_t) + K(Rx_{t-1} + Sz_t) + LNz_t + Mz_t$$

$$= [(FP + G + JR)P + H + KR]x_{t-1} + [(FQ + JS + L)N + (FP + G + JR)Q + KS + M]z_t \quad (24)$$

Collecting terms and using  $E_t[\varepsilon_{t+1}] = 0$ . Since the two equations have to be satisfied for any  $x_{t-1}$  and  $z_t$ :

$$0 = AP + B + CR \quad (25)$$

$$0 = AQ + CS + D \quad (26)$$

$$0 = FP^2 + GP + JRP + H + KR \quad (27)$$

$$0 = FPQ + FQN + GQ + JRQ + JSN + KS + LN + M \quad (28)$$

Notice (25) and (26) contain only  $P$  and  $R$ . Solve (25) for  $R$  and substitute into (27) and we obtain:



$$FP^2 + GP + J(-C^{-1}AP - C^{-1}B)P + H + K(-C^{-1}AP - C^{-1}B) = 0$$

Collecting terms:

$$(F - JC^{-1}A)P^2 - (JC^{-1}B - G + KC^{-1}A)P - KC^{-1}B + H = 0 \tag{29}$$

To simplify the notation, let's express (29) as follows:

$$\Psi P^2 - \Gamma P - \Theta = 0 \tag{30}$$

This is a matrix quadratic equation of  $P$ . There are many ways to solve the equation in general, but an often-used method is to use the generalized eigenvalue problem (also called the QZ decomposition). One of the attractive features for the method is that the method does not require invertibility of  $\Psi$  matrix.

In general, suppose we have two matrices of the same size  $X$  and  $Y$ . The generalized eigenvalue problem is to find the generalize eigenvalues  $\lambda_i$  and generalized eigenvectors  $d_i$  satisfying:

$$Xd_i = \lambda_i Yd_i$$

If we use  $Y = I$ , we go back to the standard eigenvalue problem.

How do we apply the generalized eigenvalue problem to our matrix quadratic equation? Let's define the following two matrices:

$$\Xi = \begin{bmatrix} \Gamma & \Theta \\ I_{m,m} & 0_{m,m} \end{bmatrix}, \quad \Delta = \begin{bmatrix} \Psi & 0_{m,m} \\ 0_{m,m} & I_{m,m} \end{bmatrix}$$

Where  $I_{m,m}$  represents the identity matrix of size  $m \times m$  and  $0_{m,m}$  represents the  $m \times m$  matrix with only zero entries. Since both  $\Gamma$  and  $\Psi$  are  $m \times n$  matrices, both  $\Xi$  and  $\Delta$  are  $2m \times 2m$  matrices. Now, apply the generalized eigenvalue problem to the pair of matrices.

$$\begin{bmatrix} \Gamma & \Theta \\ I_{m,m} & 0_{m,m} \end{bmatrix} \begin{bmatrix} d_{i,1} \\ d_{i,2} \end{bmatrix} = \lambda_i \begin{bmatrix} \Psi & 0_{m,m} \\ 0_{m,m} & I_{m,m} \end{bmatrix} \begin{bmatrix} d_{i,1} \\ d_{i,2} \end{bmatrix}$$

If we separate the top and bottom half of the equation, we get:

$$\Gamma d_{i,1} + \Theta d_{i,2} = \lambda_i \Psi d_{i,1}$$

$$d_{i,1} = \lambda_i d_{i,2}$$

The second equation can be used to substitute out  $d_{i,1}$ . Now we have only one equation (To clean up the notation, let's redefine  $d_i = \lambda_i d_{i,2}$ ):

$$\Gamma \lambda_i d_i + \Theta d_i = \lambda_i \Psi \lambda_i d_i \tag{31}$$

Suppose we can find  $m$  eigenvalues corresponding  $m$  linearly independent eigenvectors. Then we have the counterpart of [2] condition.

**Proposition 1**

If all of the  $m$  eigenvalues are inside the unit circle (i.e.,  $\max_i |\lambda_i| < 1$ ), the solution is stable [9].

Suppose the condition is satisfied. We can combine (30) for all  $i$  as follows:

$$\Psi\Omega\Lambda^2 - \Gamma\Omega\Lambda - \Theta = 0 \tag{32}$$

Where  $\Omega$  is  $m*m$  matrix which contains all the eigenvectors in each column, and  $\Lambda$  is  $m*m$  diagonal matrix with  $m$  eigenvalues. Specifically:

$$\Omega = [d_1 \ d_2 \ \dots \ d_m] \tag{33}$$

$$\Lambda = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_m \end{bmatrix} \tag{34}$$

Multiply (32) by  $\Omega^{-1}$  from the right, and we get:

$$\Psi\Omega\Lambda^2\Omega^{-1} - \Gamma\Omega\Lambda\Omega^{-1} - \Theta = 0 \tag{35}$$

Compare (35) with (35). The equation (35) implies that  $P = \Omega\Lambda\Omega^{-1}$ . In sum, once we implement the generalized eigenvalue decomposition with respect to  $\Xi$  and  $\Delta$ , we basically got  $P$ .

$$P = \begin{bmatrix} 0.95578 & 7.57E - 15 \\ 0.56194 & 4.40E - 15 \end{bmatrix}$$

Once  $P$  is obtained, (25) is used to obtain  $R$  as

$$R = -C^{-1}(AP + B) \tag{36}$$

$$R = \begin{bmatrix} 0.38499 & 9.2117E - 016 \\ 0.55298 & 4.607E - 015 \\ -0.045928 & -6.0761E - 016 \\ -0.82958 & -6.88E - 015 \\ -0.011835 & 1.3448E - 015 \\ -0.036684 & -2.9164E - 016 \\ -0.56194 & 1 \\ -0.052865 & 1.8014E - 013 \end{bmatrix}$$

The remaining two equations, (26) and (28), contains  $Q$  and  $S$ . Solve (26) for  $S$  and we get:

$$S = -C^{-1}(AQ + D) \tag{37}$$

Plugging into (28), and we get:

$$(FP + JR + G - KC^{-1}A)Q + (F - JC^{-1}A)QN - JC^{-1}DN + LN - KC^{-1}D + M = 0 \quad (38)$$

The equation contains only  $Q$  as unknown, but it's not trivial as  $Q$  is sandwiched in some terms. In this case, we can use the vectorization. The following is useful:

$$vec(AXB) = (B^T \otimes A)vec(X)$$

Where  $\otimes$  denotes the Kronecker product of the two matrices. Apply vectorization to (38) and we get:

$$(N^T \otimes (F - JC^{-1}A) + I \otimes (FP + JR + G - KC^{-1}A))vec(Q) = vec((JC^{-1}D - L)N + KC^{-1}D - M) \quad (39)$$

Once  $Q$  is obtained, we can use (37) to compute  $S$ .

$$\begin{aligned} (N' \otimes (F - JC^{-1}A) + I_k \otimes (FP + JR + G - KC^{-1}A))vec(Q) \\ = vec((JC^{-1}D - L)N + KC^{-1}D - M) \end{aligned} \quad (40)$$

$$Q = \begin{bmatrix} 0.13941 & -0.000245 \\ 0.16816 & -0.31805 \end{bmatrix}$$

$$S = -C^{-1}(AQ + D)$$

$$s = \begin{bmatrix} 1.0476 & 3.1683E - 005 \\ 0.1760 & 0.0022904 \\ 0.08094 & 5.388E - 005 \\ -0.264 & 0.0004899 \\ 0.010466 & 0.42284 \\ 0.0417722 & 1.055E - 005 \\ -0.16816 & 1.318 \\ 3.3193 & 0.0058558 \end{bmatrix}$$

## 10. Conclusion

In this paper, we have practiced the approach of first order approximation to the policy function in a Money in utility model. We have theoretically solving the approach of approximation to the policy function with the usual approach at the first order.

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