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A MOLFP Method for Solving Linear Fractional Programming under Fuzzy Environment

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A B S T R A C T

In this paper, a solution procedure is proposed to solve Fully Fuzzy Linear Fractional Programming (FFLFP) problem where all the variables and parameters are triangular fuzzy numbers. Here, FFLFP problem transformed into an equivalent Multi- Objective Linear Fractional Programming (MOLFP) problem. Then MOLFP converted into an equivalent multi objective linear programming problem by using Mathematical programming approach. The proposed solution illustrated through numerical examples and compared with existing methods.

Keywords: Linear programming problem, triangular fuzzy numbers, fuzzy mathematical programming.

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1. Introduction

Nowadays, the problem of linear fractional programming has significant application in different real life areas such as production planning, financial sector, health care, and all engineering field. In the practical applications, a model involves many parameters whose values are given by experts. However, both experts and decision makers frequently do not know the value of those parameters. To handle this type of situations, one may use the fuzzy numbers in the place of the crisp numbers. Thus, the crisp linear fractional programming problem becomes a fuzzy linear fractional programming (FLFP) problem or fully fuzzy linear fractional programming (FFLFP) problem. Hence, FFLFP is an interesting research area in the recent years. In this paper, we consider the fully fuzzy linear fractional programming (FFLFP) problem.

The concept of fuzzy set and fuzzy number were first introduced by Zadeh [11]. Buckley and Feuring [1] considered the fully fuzzy linear programming (FFLP) problem by establishing all the coefficients and variables of a linear program as being fuzzy quantities. Sapan das [20]

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proposed a method for single stage single constraints linear fractional programming problem by using transforming method. In recently, Pop and Stancu-Minasian [5] proposed a method for solving fully fuzzified linear fractional programming problems where all the parameters and variables are triangular fuzzy numbers. Stnojevic and Stancu-Minasian [9] consider the same problem of [8] for solving fully fuzzy linear fractional programming problem. Recently, Veeramani and Sumathi [16] proposed solution procedure for solving fuzzy linear fractional programming problem by using fuzzy mathematical programming approach. Sapan Das et al. [24-27] have proposed so many methods for solving fuzzy linear fractional programming problem by using various methods. Hatami and Kazemipoor [28] have solved fuzzy linear fractional programming problem by using Big-M method. Saberi Najafi and Edalatpanah [30] have proposed method for solving linear programming problems by using Homotopy perturbation method. Saberi Najafi et al. [31] have proposed method a nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. Hosseinzadeh and Edalatpanah [32] have proposed a method for solving fully fuzzy linear programming by using the lexicography method.

In this paper, we consider the fully fuzzy linear fractional programming problem. First, the FFLFP problem transform into a fuzzy linear fractional programming (FLFP) problem with three crisp objective functions. Then the FLFP problem will be converted into a multi-objective linear fractional programming (MOLFP) problem. We also prove that this solution can be considered as an exact solution of FFLFP problem. Finally, we show the advantages of the proposed method over the existing methods [8-10], numerical example are solved and compared our results with them.

This paper is organized as follows: some basic definitions and notations are present in Section 2. In Section 3, we discuss the LFP problem. In Section 4, we present our proposed method. A real life example is provided to validate the proposed method in Section 5. Finally, the conclusion is given in Section 6.

2. Preliminaries

We have presented some basics concept of fuzzy triangular number, which was very useful in this paper.

Definition 2.1: Let X denotes a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0,1]$; which assigned a real number $\mu_{\tilde{A}}(X)$ in the interval [0, 1], to each element $x \in X$, where the values of $\mu_{\tilde{A}}(X)$ at x shows the grade of membership of x in $\tilde{A}$. A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(X)$ and is often written $\tilde{A} = (x, \mu_{\tilde{A}}(x)) : x \in X$ is called a fuzzy set.

Definition 2.2: A fuzzy number $\tilde{A}=(b,c,a)$ is said to be a triangular fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(X)=\begin{cases}\dfrac{(x-b)}{(c-b)}, & b\le x\le c,\\ \dfrac{(x-a)}{(c-a)}, & c\le x\le a,\\ 0, & else.\end{cases} \tag{1}$$

Definition 2.3: Two triangular fuzzy number $\tilde{A}=(b,c,a)$ and $\tilde{B}=(e,f,d)$ are said to be equal if and only if $b=e, c=f, a=d$.

Definition 2.4: A ranking is a function $R:F(R)\to R$ where *F(R)* is a set of fuzzy number defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A}=(b,c,a)$ is a triangular fuzzy number then $\Re(\tilde{A})=\dfrac{b+2c+a}{4}$.

Definition 2.5: Let $\tilde{A}=(b,c,a)$, $\tilde{B}=(e,f,d)$ be two triangular fuzzy numbers then:

$$\tilde{A}+\tilde{B}=(b,c,a)+(e,f,d)=(b+e,c+f,a+d), \tag{2}$$

$$\tilde{A}-\tilde{B}=(b,c,a)-(e,f,d)=(b-d,c-f,a-e), \tag{3}$$

If $\tilde{A}=(b,c,a)$ be any triangular fuzzy number and $\tilde{B}=(e,f,d)$ be a non-negative triangular fuzzy number then:

$$\tilde{A}\otimes\tilde{B}=\tilde{A}\tilde{B}=\begin{cases}(be,cf,ad) & if\ b\ge 0,\\ (bd,cf,ad) & if\ b<0, a\ge 0,\\ (bd,cf,cd) & if\ c<0,\end{cases} \tag{4}$$

Definition 2.6: Let $\tilde{A}=(b,c,a)$, $\tilde{B}=(e,f,d)$ be two triangular fuzzy numbers. We say that $\tilde{A}$ is relatively less than $\tilde{B}$, if and only if:

i. $b<e$ or
ii. $b=e$ and $(b-c)>(e-f)$ or
iii. $b=e,\ (b-c)=(e-f)$, and $(a+b)=(d+e)$.

Note: It is clear from the definition 2.7 that $b=e,\ (b-c)=(e-f)$ and $(a+b)=(d+e)$ if and only if $\tilde{A}=\tilde{B}$.

3. Linear Fractional Programming

The general form of LFP may be written as:

$$\begin{aligned}
&\text{Max} \frac{c^t z + q}{d^t z + r} = \frac{F(z)}{G(z)} \\
&\text{s.t:} \quad Az \leq = \geq b, \\
&\qquad z \geq 0
\end{aligned} \tag{5}$$

Where,

$$z, c^t, d^t \in R^n \ , \ A \in R^{m \times n}, \alpha, \beta \in R.$$

For some values of z, G (z) may be equal to zero. To avoid such cases, one requires that either:

$\{z \geq 0,\ Az \leq = \geq b, \Rightarrow G(z) > 0\}$, or $\{z \geq 0, Az \leq = \geq b, \Rightarrow G(z) < 0\}$

For satisfaction, we assume that (1) satisfies the condition that:

$$\{z \geq 0,\ Az \leq = \geq b, \Rightarrow G(z) > 0\} \tag{6}$$

Theorem 3.1 [11]: Assume that no point (x, 0) with $x \geq 0$ is feasible for the following linear programming (LP) problem:

$$\begin{aligned}
&\text{Max } c^t x + qt \\
&\text{s.t:} \\
&d^t x + rt = 1, \\
&Ax - bt = 0, \\
&t \geq 0,\ x \geq 0.
\end{aligned} \tag{7}$$

Now assume that the Eq. (6) then the LFP (1) us equivalent into linear programming problem (7).

4. Fully Fuzzy Linear Fractional Programming Problem

Linear fractional programming problem is evidently an uncertain optimization problem due to its variations in the maximum daily requirements. So the amount of each product of ingredient will be uncertain. Hence, we will model the fully fuzzy linear fractional programming problem where all the variables and all the parameters are triangular fuzzy numbers to avoid uncertain. Let us consider a general format of fully fuzzy linear fractional programming problem as follows:

$$\begin{aligned}
&\text{Max } \tilde{Z} = \frac{\tilde{c}^t \tilde{x} + \tilde{q}}{\tilde{d}^t \tilde{x} + \tilde{r}} \\
&\text{s.t:} \\
&\tilde{A}\tilde{x} \leq \tilde{b} \\
&\tilde{x} \geq 0.
\end{aligned} \tag{8}$$

Consider the Eq. (4) and let $\tilde{x}^* = (x^*, y^*, z^*)$ be an optimal solution of this FFLFP. Furthermore, let all the parameters $\tilde{x}$, $\tilde{c}$, $\tilde{q}$, $\tilde{d}$, $\tilde{r}$, $\tilde{b}$ and $\tilde{z}$ are represented by triangular fuzzy numbers (x, y, z), (p, q, r), $(\alpha_1, \alpha_2, \alpha_3), (u, v, w), (\beta_1, \beta_2, \beta_3), (b_1, b_2, b_3)$ and (z_1, z_2, z_3) respectively. Then we can rewrite the mentioned FFLFP as follows:

$$\begin{aligned}
&\text{Max } (z_1, z_2, z_3) = \frac{(p,q,r)^t \otimes (x,y,z) + (\alpha_1, \alpha_2, \alpha_3)}{(u,v,w)^t \otimes (x,y,z) + (\beta_1, \beta_2, \beta_3)} \\
&\text{s.t:} \\
&(b,c,a) \otimes (x,y,z) \le (b_1, b_2, b_3) \\
&(x, y, z) \ge 0.
\end{aligned} \tag{9}$$

The proposed approach for solving FFLFPP can be summarized as follows:

Step 1: The above problem is converted into MOLFP problem as follows:

$$\begin{aligned}
&\text{Max } (z_1, z_2, z_3) = \left\{ \frac{p^t x + \alpha_1}{u^t x + \beta_1}, \frac{q^t y + \alpha_2}{w^t z + \beta_3}, \frac{r^t z + \alpha_3}{v^t y + \beta_2} \right\} \\
&\text{s.t:} \\
&b \otimes x \le b_1, \\
&c \otimes y \le b_2, \\
&a \otimes z \le b_3, \\
&(x, y, z) \ge 0.
\end{aligned} \tag{10}$$

Step 2: With respect to definition 2.2 and 2.3 the problem in Step 1, can be rewritten as:

$$\begin{aligned}
&\text{Max } ((p^t x + \alpha_1), (q^t y + \alpha_2), (r^t z + \alpha_3) \\
&\text{s.t:} \\
&b \otimes x - b_1 \le 0, \\
&c \otimes y - b_2 \le 0, \\
&a \otimes z - b_3 \le 0, \\
&(v^t y + \beta_2) \le 1, \\
&(u^t x + \beta_1) \le 1, \\
&(w^t z + \beta_3) \le 1, \\
&(x, y, z) \ge 0.
\end{aligned} \tag{11}$$

Step 3: Regarding the definition (2.6) the problem in Step 2 is converted to the MOLP problem with three crisp objective functions and the constraints are changed as follows:

$$
\begin{aligned}
&\text{Max } (p^t x+\alpha_1)\\
&\text{Max } (p^t x+\alpha_1)-(q^t y+\alpha_2)\\
&\text{Max } (p^t x+\alpha_1)+(r^t z+\alpha_3)\\
&\text{s.t:}\\
&b\otimes x-b_1\le 0,\\
&b\otimes x-b_1-(c\otimes y-b_2)\le 0,\\
&b\otimes x-b_1+(a\otimes z-b_3)\le 0,\\
&(u^t x+\beta_1 t)\le 1,\\
&(u^t x+\beta_1 t)-(v^t y+\beta_2 t)\le 0,\\
&(u^t x+\beta_1 t)+(w^t y+\beta_3 t)\le 2,\\
&(x,y,z)\ge 0.
\end{aligned} \tag{12}
$$

Step 4: Solving the problem in Step 3 by LINGO software, we get the solution.

5. Numerical Example

In Jamshedpur City, India, A Wooden company is the producer of two kinds of products A and B with profit around (5, 1, 3) and around (4, 1, 6) dollar per unit, respectively. However the cost for each one unit of the above products is around (4, 6, 5) and around (6, 3, 9) dollars respectively. It is assume that a fixed cost of around (1, 2, 6) dollar is added to the cost function due to expected duration through the process of production. Suppose the raw material needed for manufacturing product A and B is about (3, 2, 1) units per pound and about (6, 4, 1) units per dollar respectively, the supply for this raw material is restricted to about (13, 5, 2) dollar. Man-hours per unit for the product A is about (4, 1, 2) hour and product B is about (6, 5, 4) hour per unit for manufacturing but total Man-hour available is about (6, 3, 9) hour daily. Determine how many products A and B should be manufactured in order to maximize the total profit.

This real life problem can be formulated to the following FLFP problem:

$$
\begin{aligned}
&\text{Max } \frac{(5,1,3)(x_1,y_1,z_1)+(4,1,6)(x_2,y_2,z_2)}{(4,6,5)(x_1,y_1,z_1)+(6,3,9)(x_2,y_2,z_2)+(1,2,6)}\\
&\text{s.t:}\\
&(3,2,1)(x_1,y_1,z_1)+(6,4,1)(x_2,y_2,z_2)\le(13,5,2)\\
&(4,1,2)(x_1,y_1,z_1)+(6,5,4)(x_2,y_2,z_2)\le(6,3,9)\\
&(x_1,y_1,z_1),\quad (x_2,y_2,z_2)\ge 0.
\end{aligned} \tag{13}
$$

The problem (13) is converted into the MOLFP problem as follows:

$$
\text{Max } \left\{\frac{5x_1+4x_2}{4x_1+6x_2+1},\frac{y_1+y_2}{6y_1+3y_2+2},\frac{y_1+y_2}{5y_1+9y_2+6}\right\}
$$

s.t:

$$
\begin{aligned}
&3x_1 + 6x_2 \le 13,\\
&2y_1 + 4y_2 \le 5,\\
&z_1 + z_2 \le 2,\\
&4x_1 + 6x_2 \le 6,\\
&y_1 + 5y_2 \le 3,\\
&2z_1 + 4z_2 \le 9,\\
&x_1, x_2, y_1, y_2, z_1, z_2 \ge 0.
\end{aligned} \tag{14}
$$

The problem (14) is transformed into an equivalent multi objective linear programming problem as follows:

$$
\begin{aligned}
&\text{Max } Z_1 = 5y_1 + 4y_2\\
&\text{Max } Z_2 = z_1 + z_2\\
&\text{Max } Z_3 = 3x_1 + 6x_2\\
&\text{s.t:}\\
&4y_1 + 6y_2 + t \le 1,\\
&6z_1 + 3z_2 + 2t \le 1,\\
&5x_1 + 9x_2 + 6t \le 1,\\
&3y_1 + 6y_2 - 13t \le 0,\\
&2z_1 + 4z_2 - 5t \le 0,\\
&x_1 + x_2 - 2t \le 0,\\
&4y_1 + 6y_2 - 6t \le 0,\\
&z_1 + 5z_2 - 3t \le 0,\\
&2x_1 + 4x_2 - 9t \le 0,\\
&x_1, x_2, y_1, y_2, z_1, z_2 \ge 0.
\end{aligned} \tag{15}
$$

The problem (15) can be written as follows:

$$
\begin{aligned}
&\text{Max } Z_1 = 5y_1 + 4y_2\\
&\text{Max } Z_2 = 5y_1 + 4y_2 - z_1 - z_2\\
&\text{Max } Z_3 = 5y_1 + 4y_2 + 3z_1 + 6z_2\\
&\text{s.t:}\\
&4y_1 + 6y_2 + t \le 1,\\
&4y_1 + 6y_2 + t - 6z_1 - 3z_2 - 2t \le 0,\\
&4y_1 + 6y_2 + t + 5x_1 + 9x_2 + 6t \le 2,\\
&3y_1 + 6y_2 - 13t \le 0,\\
&3y_1 + 6y_2 - 13t - 2z_1 - 4z_2 + 5t \le 0,
\end{aligned} \tag{16}
$$

$$3y_1 + 6y_2 - 13t + x_1 + x_2 - 2t \le 0,$$
$$4y_1 + 6y_2 - 6t \le 0,$$
$$4y_1 + 6y_2 - 6t - z_1 - 5z_2 + 3t \le 0,$$
$$4y_1 + 6y_2 - 6t + 2x_1 + 4x_2 - 9t \le 0.$$

Solving the problem (16) we get y_1=0.214, y_2=0, x_1=0, x_2=0.016, z_1=0.067, z_2=0.102, t=0.142

Hence the solution of the problem (13) is Z_1=1.07, Z_2=0.169, Z_3=0.09.

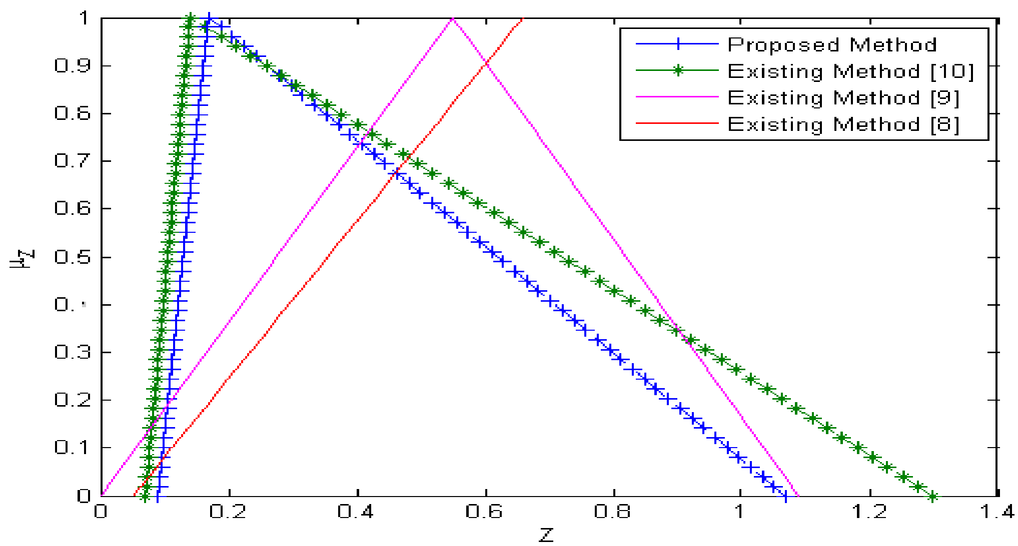

Figure 1. Membership function of the proposed method vs existing methods [8-10].

Example 5.2 [8, 24]. Let us consider the following linear fractional program as follows:

$$\begin{aligned} &\text{Max } z = \frac{x_1 - x_2 + 1}{x_1 + x_2 + 2} \\ &\text{s.t:} \\ &x_1 + x_2 \le 2, \\ &x_1 - x_2 \le 1, \\ &x_1, x_2 \ge 0. \end{aligned} \tag{17}$$

The optimal solution of the problem is x_1=1, x_2=0 and the optimal value of z is 2/3=0.66667.

We attach now to this problem a fully fuzzified problem. Thus, the fully fuzzy linear fractional programming problem which we want to solve is as follows:

$$\begin{aligned} &\text{Max } z = \frac{(0,1,2)\tilde{x}_1 - (-2,-1,0)_2\tilde{x}_2 + (0,1,2)}{(0,1,2)\tilde{x}_1 + (0,1,2)\tilde{x}_2 + (1,2,3)} \\ &\text{s.t:} \\ &(1,2,3)\tilde{x}_1 + (1,2,3)\tilde{x}_2 \le (1,2,3), \end{aligned} \tag{18}$$

$$(0,1,2)\tilde{x}_1 - (-2,-1,0)\tilde{x}_2 \le (0,1,2),$$
$$\tilde{x}_1, \tilde{x}_2 \ge 0.$$

Now the problem is as follows:

$$\begin{aligned}
&\text{Max } \frac{(0,1,2)\otimes((x_1)^p,(x_1)^q,(x_1)^r) - (-2,-1,0)\otimes((x_2)^p,(x_2)^q,(x_2)^r) + (0,1,2)}{(0,1,2)\otimes((x_1)^p,(x_1)^q,(x_1)^r) + (0,1,2)\otimes((x_2)^p,(x_2)^q,(x_2)^r) + (1,2,3)} \\
&\text{s.t:} \\
&(0,1,2)\otimes((x_1)^p,(x_1)^q,(x_1)^r) + (0,1,2)\otimes((x_2)^p,(x_2)^q,(x_2)^r) \le (1,2,3), \\
&(0,1,2)\otimes((x_1)^p,(x_1)^q,(x_1)^r) - (-2,-1,0)\otimes((x_2)^p,(x_2)^q,(x_2)^r) \le (0,1,2), \\
&(x_1)^p,(x_1)^q,(x_1)^r,\ (x_2)^p,(x_2)^q,(x_2)^r \ge 0.
\end{aligned} \tag{19}$$

According to Step 2 and Step 3, we get the following multiple objective linear programming problem:

$$\begin{aligned}
&\text{Max } (-2(y_2)^p,\ (y_1)^q - (y_2)^q + t_2,\ 2(y_1)^r + 2t_3) \\
&\text{s.t:} \\
&-3t_3 \le 0, \\
&(y_1)^q + (y_2)^q - 2t_2 \le 0, \\
&2(y_1)^r + 2(y_2)^r - t_1 \le 0, \\
&-2(y_2)^r - 2t_3 \le 0, \\
&(y_1)^q - (y_2)^q - t_2 \le 0, \\
&2(y_1)^r \le 0, \\
&t_1 \le 1, \\
&(y_1)^q + (y_2)^q + 2t_2 \le 1, \\
&2(y_1)^r + 2(y_2)^r + 3t_3 \le 1, \\
&(y)^q - (y)^p \ge 0, \quad (y)^r - (y)^q \ge 0, \quad (y)^p \ge 0, \quad t_1, t_2, t_3 \ge 0.
\end{aligned} \tag{20}$$

Using steps 3-5 and Step 6, the optimal solution of the problem is:

$$\begin{aligned}
&\tilde{y}^* = \{\tilde{y}_1^* = ((y_1^*)^p,(y_1^*)^q,(y_1^*)^r) = (0,\ 0.2,\ 0.2), \\
&\tilde{y}^* = \{\tilde{y}_2^* = ((y_2^*)^p,(y_2^*)^q,(y_2^*)^r) = (0,\ 0,\ 0), \\
&t^* = \{(t_1,t_2,t_3) = (0.1, 0.2, 0.2)
\end{aligned} \tag{21}$$

Then the solution of the problem is:

$$\begin{aligned}
&\tilde{x}^* = \{\tilde{x}_1^* = ((x_1^*)^p,(x_1^*)^q,(x_1^*)^r) = (0,\ 1,\ 1), \\
&\tilde{x}^* = \{\tilde{x}_2^* = ((x_2^*)^p,(x_2^*)^q,(x_2^*)^r) = (0,\ 0,\ 0),
\end{aligned} \tag{22}$$

The triangular fuzzy number $\tilde{z}^* = (0,\ 0.66667,\ 4)$.

The obtained result is exactly the optimal value of the problem which start with the original problem.

By comparing proposed method results with existing method [8-10, 24], based on Definition 2.7, we conclude that our result is more efficient than other existing method.

$0=(z_1^*)_{\text{proposed method[24]}}=(z_1^*)_{\text{method of [8]}}>(z_1^*)_{\text{method of [9,10]}}=-0.21$

$0.66667=(z_1^*+z_2^*)_{\text{proposed method [24]}}=(z_1^*+z_2^*)_{\text{method of [8]}}=0.66667>(z_1^*+z_2^*)_{\text{method of [9,10]}}=0.55$

$(0, 0.66667, 4)=(z^*)_{\text{proposed method [24]}}>(z^*)_{\text{method of [8]}}=(-0.21, 0.66667, 5.822)>(z_1^*)_{\text{method of [9,10]}}=(0, 0.55, 1.09)$.

6. Conclusions

In the past few years, a growing interest has been shown in Fuzzy linear fractional programming problem and currently there are several methods for solving FFLFP problems with non-negativity restrictions. By using proposed method the FFLFP problem may be transformed into its equivalent three crisp linear fractional programming problems. To show the efficiency of our proposed method a practical problem has been illustrated.

References

[1] Buckley, J. J., & Feuring, T. (2000). Evolutionary algorithm solution to fuzzy problems: fuzzy linear programming. *Fuzzy sets and systems*, *109*(1), 35-53.

[2] Chakraborty, M., & Gupta, S. (2002). Fuzzy mathematical programming for multi objective linear fractional programming problem. *Fuzzy sets and systems*, *125*(3), 335-342.

[3] Charnes, A., & Cooper, W. W. (1962). Programming with linear fractional functionals. *Naval research logistics (NRL)*, *9*(3-4), 181-186.

[4] Dutta, D., Tiwari, R. N., & Rao, J. R. (1992). Multiple objective linear fractional programming—a fuzzy set theoretic approach. *Fuzzy sets and systems*, *52*(1), 39-45.

[5] Pop, B., & Stancu-Minasian, I. M. (2008). A method of solving fully fuzzified linear fractional programming problems. *Journal of applied mathematics and computing*, *27*(1), 227-242.

[6] Ezzati, R., Khorram, E., & Enayati, R. (2015). A new algorithm to solve fully fuzzy linear programming problems using the MOLP problem. *Applied mathematical modelling*, *39*(12), 3183-3193.

[7] Toksarı, M. D. (2008). Taylor series approach to fuzzy multiobjective linear fractional programming. *Information sciences*, *178*(4), 1189-1204.

[8] Stancu-Minasian, I. M., & Pop, B. (2003). On a fuzzy set approach to solving multiple objective linear fractional programming problem. *Fuzzy sets and systems*, *134*(3), 397-405.

[9] Stanojevic, B & Stancu-Minasian, I. M. (2009). On solving fuzzified linear fractional programs. *Advanced modeling and optimization*, *11*, 503-523.

[10] Stanojević, B., & Stancu-Minasian, I. M. (2012). Evaluating fuzzy inequalities and solving fully fuzzified linear fractional programs. *Yugoslav journal of operations research*, *22*(1), 41-50.

[11] Zadeh, L. A. (1996). Fuzzy sets. *Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by lotfi a zadeh* (pp. 394-432).

[12] Ebrahimnejad, A., & Tavana, M. (2014). A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. *Applied mathematical modelling*, *38*(17), 4388-4395.

[13] Kumar, A., Kaur, J., & Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. *Applied mathematical modelling*, *35*(2), 817-823.

[14] Lotfi, F. H., Allahviranloo, T., Jondabeh, M. A., & Alizadeh, L. (2009). Solving a full fuzzy linear programming using lexicography method and fuzzy approximate solution. *Applied mathematical modelling*, *33*(7), 3151-3156.

[15] Maleki, H. R, Tata, M., Mashinchi, M., (1996). Fuzzy number linear programming. *Proceeding of internat confference on intelligent and cognitive systems fss sponsored* (pp. 145-148). Tehran, Iran: IEEE and ISRF.145-148.

[16] Veeramani, C., & Sumathi, M. (2014). Fuzzy mathematical programming approach for solving fuzzy linear fractional programming problem. *RAIRO-Operations research*, *48*(1), 109-122.

[17] Lai, Y. J., & Hwang, C. L. (1992). Fuzzy mathematical programming. *Fuzzy mathematical programming* (pp. 74-186). Springer, Berlin, Heidelberg.

[18] Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions. *Fuzzy sets and systems*, *1*(1), 45-55.

[19] Zimmermann, H. J. (1982). Trends and new approaches in European operational research. *Journal of the operational research society*, 597-603.

[20] Das, S., & Mandal, T. (2015). A single stage single constraints linear fractional programming problem. *International journal of operational research*, *2*, 1-5.

[21] Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management science*, *17*(4), B-141.

[22] Ganesan, K., & Veeramani, P. (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. *Annals of operations research*, *143*(1), 305-315.

[23] Sapan, K. D., Mandal, T., & Edalatpanah, S. A. (2015). A note on "A new method for solving fully fuzzy linear fractional programming with a triangular fuzzy numbers". Retrieved from http://dspace.unimap.edu.my/dspace/handle/123456789/41158.

[24] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A new approach for solving fully fuzzy linear fractional programming problems using the multi-objective linear programming. *RAIRO-Operations research*, *51*(1), 285-297.

[25] Das, S. K., Mandal, T., & Edalatpanah, S. A. (2017). A mathematical model for solving fully fuzzy linear programming problem with trapezoidal fuzzy numbers. *Applied intelligence*, *46*(3), 509-519.

[26] Das, S. K., & Edalatpanah, S. A. (2016c). A general form of fuzzy linear fractional programs with trapezoidal fuzzy numbers. *International journal of data envelopment analysis and *operations research**, *2* (1), 16-19.

[27] Torabi, N., & Najafi, S. E. (2015). New model for ranking based on sum weights disparity index in data envelopment analysis in fuzzy condition. *Journal of applied research on industrial engineering*, *2*(2), 111-119.

[28] Hatami, A., & Kazemipoor, H. (2013). Fuzzy big-M method for solving fuzzy linear programs with trapezoidal fuzzy numbers. *International journal of research in industrial engineering*, *2*(3), 1-9.

[29] Sheikhi, H. (2012). A novel approach for solving fuzzy multi-objective zero-one linear programming problems. *International journal of research in industrial engineering*, *1*(1).

[30] Najafi, H. S., & Edalatpanah, S. A. (2014). Homotopy perturbation method for linear programming problems. *Applied mathematical modelling*, *38*(5), 1607-1611.

[31] Najafi, H. S., Edalatpanah, S. A., & Dutta, H. (2016). A nonlinear model for fully fuzzy linear programming with fully unrestricted variables and parameters. *Alexandria engineering journal*, *55*(3), 2589-2595.

[32] Hosseinzadeh, A., & Edalatpanah, S. A. (2016). A new approach for solving fully fuzzy linear programming by using the lexicography method. *Advances in fuzzy systems*. doi: 10.1155/2016/1538496