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The Methods of Ranking based Super Efficiency

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A B S T R A C T

The science of Data Envelopment Analysis (DEA) evaluates the effectiveness of decision making units. But, one of the problems of Data Envelopment Analysis (DEA) is that, if the number of units with the same efficiency equal to one was more than one, then we couldn't select the best between them. It means that, we can't rank them. Therefore, the need for ranking these units is considered by the managers. Different methods were proposed in this context. Most of these methods are modeled by DEA models. Due to the variety of ranking methods in DEA, this paper will describe ranking methods which are based on super-efficiency. More precisely, we introduced methods that rank using elimination (removing) of decision making units under the evaluation of observations (set). These methods have some advantages and disadvantages such as, model feasibility or infeasibility, stability or instability, being linear or nonlinear, being radial or non-radial, existence or non- existence of bounded optimal solution in objective function, existence or non- existence of multiple optimal solution, non-extreme efficient units ranking, complexity or simplicity of computational processes, that in this paper, Super Efficiency methods are compared with these eight properties.

Keywords: *Data Envelopment Analysis; Ranking Technique; Efficiency; Super Efficiency.*

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1. Introduction

Data Envelopment Analysis was first invented by Charnes, Cooper and Rhodes (1978) examined the effectiveness of an educational unit of United States of America under the title of CCR and later, it was developed by Banker et al (1984) in an article under the title of BCC. Since, more than one unit may be assessed by the efficient DEA models and the

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number of efficient units isn't fewer than the models with returns to variable scale compare to the models with returns to constant scale, the necessity for ranking efficient units is being raised. Rank is better than that DMU which has higher efficiency. In this case, a lot of measures have been proposed to identify this target. In this paper, ranking methods just based on super efficiency is examined. Andersen and Petersen (1993) have introduced super efficiency that is known as the AP model. This model removes DMU under evaluation from observations set. This model has four main problems. Many studies have been done in Iran to eliminate the weakness of AP model. In this regard, Mehrabi et al (1999) presented a model that removes DMU under evaluation from observations set, same as the AP model. This model is known as MAJ model. Sa'ati et al (1999) suggested modified model to eliminate the weakness of MAJ model which is known as MAJ modified model. Jahanshahloo et al (2011) presented JHF model that is a general state of MAJ and MAJ modified model. Jahanshahloo et al (2004a) presented a new model to rank extreme efficient units using norm 1 (L_1). Tavares and Antones (2001) suggested a model to calculate DMUs efficiency using L_∞ (Tchebycheff norm) that Jahanshahloo et al (2004c) presented a more complete state of this model. Ton (2000) presented a model for ranking under the title of SBM which is based on SBM model. Jahanshahloo et al (2004a) used gradient vector to rank efficient units. This model covers the weakness of MAJ and AP models. Amirteymori et al (2005) suggested a model to rank efficient units which was based on using norm 2. Jahanshahloo et al (2006a) presented ranking method using changing the reference set based on Suishi and Hibiki procedure to rank extreme efficient units. Jahanshahloo et al (2006b) suggested a method to rank efficient units based on a full-inefficient frontier. Khodabakhshi et al (2007) introduced super efficiency based on improved outputs that is similar to AP model in the Output-oriented with returns to constant scale. Shanling et al (2007) presented LJK super efficiency model that is always possible and stable. Jahanshahloo et al (2008) presented a ranking model using Context-Dependent DEA. There is also a model of non-DEA that Jahanshahloo et al (2005) presented it to rank non-extreme efficient units that is known as Monte Carlo model (which won't be discussed in this paper).

Since a lot of research works have been done already in the field of ranking decision-making units based on super efficiency, a general classification about previous work about this area is presented in this paper through this, we can use information appropriately and quickly. The features expected these methods had are examined in this paper. Based on these Properties, ranking models are compared with each other. Existence of some Properties causes superiority or inferiority of one method over another one.

Introduction to DEA is expressed in the second part of this paper. In the third part, all of the method based on super efficiency will be expressed. Comparison between the mentioned methods in the second part is done in the fourth part and numerical examples are presented. And in the fifth part, conclusion will be stated.

2. Introduction to DEA

Consider "n" homogenous decision-making units as follow:

$DMU_j; j \in \mathcal{N} = \{1, \dots, n\}$. So that, X_j was an input vector $(X_j \geq 0, X_j \neq 0)$ and Y_j was an output vector for DMU_j. A production possibility set (PPS) is shown by "T" and defined as:

$$T = \{(X, Y) | \text{input vector} X \geq 0 \text{ can produce output vector } Y \geq 0\}$$

Possess the following properties:

Postulate 1 (Nonempty). The observed $(X_j, Y_j) \in T, \forall j \in \mathcal{N} = \{1, \dots, n\}$

Postulate 2 (constant returns to scale). If $(X, Y) \in T$, then $(\lambda X, \lambda Y) \in T$ for all $\lambda \geq 0$.

Postulate 3 (convexity). T is a closed and convex set, i.e. if if $(X'', Y''), (X', Y') \in T$ then for $\lambda \in (0,1)$, $\lambda(X', Y') + (1-\lambda)(X'', Y'') \in T$

Postulate 4 (Plausibility). if $(\hat{X}, \hat{Y}) \in T, X \geq \hat{X} \,\&\, Y \leq \hat{Y}$, then $(X,Y) \in T$.

Postulate 5 (Minimum extrapolation). T is the smallest set that satisfies Postulates1-4.

The above-mentioned postulates define the following unique set:

$$T_c = \{\begin{pmatrix} X \\ Y \end{pmatrix} | X \geq \sum_{j=1}^{n} \lambda_j X_j \,\&\, Y \leq \sum_{j=1}^{n} \lambda_j Y_j \,\&\, \lambda_j \geq 0, \, j = 1, \dots, n\}$$

For more information, see Jahanshahloo et al (1387).

2-1. CCR model:

Suppose that, the objective for performance evaluation of DMU_o , is $o \in \{1, \dots, n\}$. The model is as follows:

$$
\begin{aligned}
& Min\theta \\
& s.t: \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad , \quad i = 1,\dots,m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad , \quad r = 1,\dots,s \qquad (1) \\
& \lambda_j \ge 0 \quad , \quad j = 1,\dots,n
\end{aligned}
$$

Model (1) is envelopment form in input- oriented. Multiply form of the model is as follows:

$$
\begin{aligned}
& Max \sum_{r=1}^{s} u_r y_{ro} \\
& s.t: \sum_{r=1}^{m} u_r y_{rj} - \sum_{i=1}^{n} v_i x_{io} \le 0 \quad , j = 1,\dots,n \qquad (2) \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& u_r \ge 0 \ , v_i \ge 0 \ , r = 1,\dots,s \ , \ i = 1,\dots,m
\end{aligned}
$$

In which, u_r and v_i are corresponding dual variables of “r” output and “i” input in the CRR envelopment form . Problem number 2 is always possible.

2-2. BCC model:

T_V set is produced with eliminating(removing) the Postulate of returns to constant scale from T_C

As follows:

$$T_v = \{\begin{pmatrix} X \\ Y \end{pmatrix} | \sum_{j=1}^{n} \lambda_j X_j \le X_j, \sum_{j=1}^{n} \lambda_j Y_j \ge Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, j = 1,...,n\}$$

BCC model in the input- oriented, evaluates DMU_o efficiency as follows:

$$\begin{aligned} & Min\, \theta \\ & s.t: \sum_{j=1}^{n} \lambda_j X_j \le \theta X_o \\ & \quad \sum_{j=1}^{n} \lambda_j Y_j \ge Y_o \qquad (3) \\ & \quad \sum_{j=1}^{n} \lambda_j = 1 \\ & \quad \lambda_j \ge 0,\ j = 1,...,n \end{aligned}$$

2-3. Additive model (ADD)

Now, a model is introduced that has both input-oriented and output-oriented and it isn't radial. (Jahanshahloo et al 2008)Therefore, additive model is defined as follows:

$$\begin{aligned} & Max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\ & s.t: \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io} \quad i = 1,...,m \qquad (4) \\ & \quad \sum_{j=i}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro} \quad r = 1,...,s \\ & \quad \sum_{j=1}^{n} \lambda_j = 1 \\ & \quad \lambda_j \ge 0 \quad, j = 1,...,n \\ & \quad s_r^+ \ge 0, r = 1,...,s \\ & \quad s_i^- \ge 0, i = 1,...,m \end{aligned}$$

4-2. SBM model

To calculate DMU_O with X_O input and Y_O output, consider following model based on $\lambda_1, \lambda_2, ..., \lambda_n$ and $s_1^-, s_2^-, ..., s_m^-$ and $s_1^+, s_2^+, ..., s_s^+$ variables (Jahanshahloo et al 2008):

$$
\begin{aligned}
& Min\rho_o = \frac{1-\frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}}{1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{y_{ro}}} \\
& s.t: \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io} \quad , i=1,\dots,m \quad (5) \\
& \sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{ro} \quad , r=1,\dots,s \\
& \lambda_j \geq 0 \quad , j=1,\dots,n \\
& s_i^- \geq 0 \quad , i=1,\dots,m \\
& s_r^+ \geq 0 \quad , r=1,\dots,s
\end{aligned}
$$

3. Review of ranking methods based on super efficiency

Anderson and Peterson (1993) introduced super efficiency model that is known as AP model. They removed DMU_o from observation set for ranking, and performed DEA model for DMUs residual. They proposed model for DMU_o ranking is as follows:

$$
\begin{aligned}
& Min\,\theta - \varepsilon(\sum_{i=1}^{m}s_i^- + \sum_{r=1}^{s}s_r^+) \\
& s.t: \sum_{\substack{j=1 \\ j\neq o}}^{n}\lambda_j x_{ij} + s_i^- = \theta x_{io}\, , i=1,\dots,m \\
& \sum_{\substack{j=1 \\ j\neq o}}^{n}\lambda_j y_{rj} - s_r^+ = y_{ro} \, , r=1,\dots,s \quad (6) \\
& s_r^+ \geq 0 , r=1,\dots,s \\
& s_i^- \geq 0 , i=1,\dots,m \\
& \lambda_j \geq 0 , j=1,\dots,n, j\neq 0
\end{aligned}
$$

Due to basic problems of AP model, Mehrabian et al (1999) introduced another model for efficient DMUs ranking. Moving towards frontier was done along with the radial in AP model that might not cross the PPS. In this case, the problem doesn't have feasible solution or might be cross PPS in the far distance. Therefore, the problem is unstable. Moving towards efficiency frontier along with the radial mentioned above doesn't take place in MAJ model; but along parallel inputs (input-oriented) like vertically movement and equal steps. MAJ super efficient model is proposed as follows:

$$\begin{aligned}
&Min\ \ 1+w \\
&s.t: \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \leq x_{io} + w \quad ,i=1,\dots,m \\
&\quad\ \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad ,r=1,\dots,s \qquad (7) \\
&\qquad\quad \lambda_j \geq 0 \qquad ,j=1,\dots,n \ \ ,j\neq o
\end{aligned}$$

MAJ model may be infeasible in some cases. To resolve this problem, Saati et al (1999) proposed a model called Modified MAJ model in which, by use of decreasing the inputs and simultaneously increasing the outputs equally, figured under evaluation DMU on efficiency frontier. Their proposed super efficiency model is as follows:

$$\begin{aligned}
&Min\ \ 1+w_o \\
&s.t: \ \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j X_j \leq X_o + 1w_o \\
&\qquad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j Y_j \geq Y_o - 1w_o \qquad (8) \\
&\quad \lambda_j \geq 0 \quad ,j=1,\dots,n \ \ ,j\neq o
\end{aligned}$$

Jahanshahloo et al (2011) presented a model as JHF model that is much more general than MAJ and Modified MAJ model as follows (Notice $(\alpha > 0, \beta > 0)$) :

$$\begin{aligned}
&Min \sum_{i=1}^{m} \alpha_i w_i + \sum_{r=1}^{s} \beta_r z_r \\
&s.t: \ \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \leq x_{io} + w_i \quad ,i=1,\dots,m \\
&\quad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} - z_r \quad ,r=1,\dots,s \quad (9) \\
&\quad \lambda_j \geq 0 \ ,\ j=1,\dots,n,\ j\neq o \\
&\quad w_i \geq 0 \ ,i=1,\dots,m \\
&\qquad\qquad z_r \geq 0 \qquad\qquad ,r=1,\dots,s
\end{aligned}$$

Jahanshahloo et al (2004a) presented a model for extreme efficient units ranking using norm 1 that likes previous models, doesn't have suggestion for non-extreme efficient units ranking. Suppose that, DMU_o with input and output vectors $(\mathrm{X_o}, \mathrm{Y_o})$ are under evaluation. After removing it from $\mathrm{T_c}$, the new production possibility set is called T_c'. Their proposed model is as follows:

$$
\begin{aligned}
& Min\ \Gamma_c^{\ o}(X,Y) = \sum_{i=1}^{m} |x_i - x_{io}| + \sum_{r=1}^{s} |y_r - y_{ro}| \\
& s.t: \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} \le x_i \quad ,i=1,\dots,m \\
& \qquad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} \ge y_r \quad ,r=1,\dots,s \qquad (10) \\
& \qquad\qquad x_i \ge 0 \quad ,i=1,\dots,m \\
& \qquad\qquad y_r \ge 0 \quad ,r=1,\dots,s \\
& \qquad\qquad \lambda_j \ge 0 \quad ,j=1,\dots,n,\ j\neq o
\end{aligned}
$$

$\Gamma_c^{\ o}(X,Y)$ is the distance between (X_o, Y_o) to (X, Y) through norm one in returns to constant scale model. Now it's defined:

$$
\begin{aligned}
R_i^{\ -} &= \max_j \{x_{ij}\} \quad ,i=1,\dots,m \\
R_r^{\ +} &= \max_j \{y_{rj}\} \quad ,r=1,\dots,s
\end{aligned}
$$

Now by dividing i-th input DMU_j on R_i^- and r-th output DMU_j on R_r^+

(that $\mathrm{j} = 1,2,\dots,\mathrm{n}, \mathrm{j} \neq o$), all data will be normalized. It is obvious that the model (10) is non-linear. To concert the problem into linear model, T_c'' set is defined as follows:

$$T_c'' = T_c' \cap \{(X,Y): X \ge X_o\ ,\ Y \le Y_o\}$$

Therefore, we can rewrite the problem (10) like this:

$$
\begin{aligned}
& Min\Gamma_c^{o}(X,Y) = \sum_{i=1}^{m} x_i - \sum_{r=1}^{s} y_r + \alpha \\
& s.t: \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_r \qquad , r=1,\ldots,s \qquad (11) \\
& \qquad \sum_{\substack{j=1\\ j\neq 0}}^{n} \lambda_j x_{ij} \leq x_i \qquad , i=1,\ldots,m \\
& \qquad x_i \geq x_{io} \qquad , i=1,\ldots, \\
& \qquad 0 \leq y_r \leq y_{ro} \qquad , r=1,\ldots, \\
& \qquad \lambda_j \geq 0 \qquad , j=1,\ldots,n,\ j\neq o
\end{aligned}
$$

Note that this method only ranks DMUs of extreme efficient units.

Tchebycheff norm ranking model is the infinite norm that presented by Jahanshahloo et al (2004c) for extreme efficient DMUs ranking. Tavares and Antones (2001) suggested a model to calculate DMUs efficiency using infinite norm. They used Tchebycheff norm in the objective function. The objective function of model (13) minimizes the distance between DMUo (under evaluating unit) and its projected point on efficiency frontier. The model proposed by Jahanshahloo et al is as follows:

$$
\begin{aligned}
& Min \left\| A - \overline{A} \right\|_{\infty} \\
& s.t: \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j x_{ij} \leq x_{io} \quad , i=1,\ldots,m \qquad (12) \\
& \qquad \sum_{\substack{j=1\\ j\neq o}}^{n} \lambda_j y_{rj} \geq y_{ro} \quad , r=1,\ldots,s \\
& \qquad \lambda_j \geq 0 \qquad , j=1,\ldots,n
\end{aligned}
$$

In which, $A = (X_o, Y_o)$ is the DMU_{o} under evaluation and $\overline{A}$ is a point of T_c'. The problem's objective function is non-linear that will be linear by some changes.

This section is intended to use SBM model for efficient units ranking. Suppose that, T_c' is the set obtained by removing DMU_{o} unit under evaluation from T_C. T''_C subset from T_c' is defined as follows:

$$
T_c'' = T_c' \cap \{(\overline{X}, \overline{Y}); \overline{X} \geq X_o, \overline{Y} \leq Y_o\}
$$

Given that $X > 0, Y > 0$, the above set is non-empty. The distance of (X_o, Y_o) from $(\overline{X}, \overline{Y}) \in T_C''$ is defined as follows:

$$\delta = \frac{(1/m)\sum_{i=1}^{m}(\bar{x}_i / x_{io})}{(1/s)\sum_{r=1}^{s}(\bar{y}_r / y_{ro})}$$

According to T_c'' definition, this distance isn't shorter than 1. Regarding to the mentioned content, super efficiency of DMU_o unit under evaluation is defined as the value of optimal objective function of the following problem:

$$\begin{aligned} & Min\ \delta = \frac{\frac{1}{m}\sum_{i=1}^{m}\frac{\bar{x}_i}{x_{io}}}{\frac{1}{s}\sum_{r=1}^{s}\frac{\bar{y}_r}{y_{ro}}} \\ & s.t:\ \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j X_j \le \bar{X} \qquad (13) \\ & \qquad \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j Y_j \ge \bar{Y} \\ & \qquad \bar{X} \ge X_o \quad , \bar{Y} \le Y_o \quad , \lambda \ge 0 \end{aligned}$$

As it was mentioned before $\delta^* \geq 1$, and moreover $\delta^* = 1$ if and only if, $(X_o, Y_o) \in T_c''$. means that, this model doesn't have any suggestions for non-extreme efficient units, too. For more details, see Ton (2002). Fractional programming problem can be converted to linear programming problem by using Charnes-Cooper conversions. That is used for ranking extreme efficient units.

To resolve difficulties caused by AP and MAJ models infeasibility, Jahanshahloo et al (2004e) used gradient vectors for efficient units ranking. The proposed super efficiency model is as follows:

$$\begin{aligned} & Max\ H_o = -V^t X_o + U^t Y_o \\ & s.t:\ -V^t X_j + U^t Y_j \le 0 \quad , j = 1,\dots,n\ \ j \neq o \\ & \qquad V^t e + U^t e = 1 \qquad (14) \\ & \qquad V \ge \vec{1}\varepsilon \quad , U \ge \vec{1}\varepsilon \end{aligned}$$

Model (14) maximizes $U^tY_o$and minimizes V^tX_osimultaneously.. Suppose that $({U^*}^t, {V^*}^t)$ is the optimal solution. $({U^*}^t, -{V^*}^t)$ shows the gradient of the supporting hyperplane into T_C' (the obtained PPS from removing under evaluation unit from T_c). The intersection of T_c' and this supporting hyperplane is as follows:

$$F = \{(X, Y) : -V^{t*}X + U^{t*}Y = 0\} \cap T_C'$$

That is an efficient surface of T_c'. Model (14) is always feasible. This model doesn't have any suggestions for non-extreme efficient model.

Jahanshahloo et al (2006a) suggested a ranking model based on changing the reference set. Strong efficient(SE) DMU when excluded from the reference set of all the other DMUs allows the efficiency frontier to be closest in relation to the inefficient DMUs should be the strong efficient (SE-DMU). To implement the new method, efficient DMUs (non-strong) are estimated again by following model:

$$\begin{aligned}
&Min\, \partial_{a,b} = \theta - \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) \\
&s.t: -\sum_{j\in J-\{b\}} \lambda_j x_{ij} + \theta x_{ia} - s_i^- = 0 \quad ,i = 1,...,m \quad (15) \\
&\sum_{j\in J-\{b\}} \lambda_j y_{rj} - s_r^+ = y_{ra} \quad ,r = 1,...,s \\
&\lambda_j \geq 0 \quad ,j \in J - \{b\} \\
&s_i^- \geq 0 \quad ,r = 1,...,s \\
&s_r^+ \geq 0 \quad ,i = 1,...,m
\end{aligned}$$

In which, $b \in J_e$, $a \in J_n$, $J = \{1,...,n\}$ that, J_n is the non-strong efficient DMUs set and J_e is the strong efficient DMUs set. After calculating the amount of δ efficiency of for all non-strong efficient DMUs, strong efficient DMUs are obtained as follows:

$$\Omega_b = \frac{\sum_{a\in J_n} \partial_{a,b}}{\tilde{n}}$$

In which, 'b' is evaluated strong efficient DMU, and $\tilde{n}$ is the number of non-strong efficient DMUs.

In this part, another ranking model based on full-inefficient frontier that's suggested by Jahanshahloo et al (2006a) is presented. Based on this, different models can be designed that among them, radial model and a model based on additive variables will be mentioned. In a model considered to evaluate DMU_o , it's assumed that, returns to scale is variable. The resulting content can be easily used in relation to the returns to constant scale is defined as follows.

Definition1.The set

$$F(S)=\{(X,Y)\mid \forall (X',Y')\in R^{m+s}((-X',Y')\underset{\neq}{\leq}(-X,Y)\Rightarrow (X',Y')\notin S)\}$$

is called full-inefficient frontier. Its radial model is as follows:

$$\begin{aligned} & Max\psi \\ & s.t: \sum_{j=1}^{n}\lambda_j X_j = \psi X_o \\ & \quad \sum_{j=1}^{n}\lambda_j Y_j = Y_o \qquad (16) \\ & \quad \sum_{j=1}^{n}\lambda_j = 1 \\ & \quad \lambda_j \geq 0 \quad , j=1,\dots,n \end{aligned}$$

The optimal value of ψ isn't less than unity. Model (16) is similar to BCC(banker et al(1984)) model, except that it compares DMU_o to can inefficient frontier while the BCC model evaluates the distance of DMU_o from an efficient frontier. Therefore model(16) can be considered an optimisitic method in comparison to BCC model. Radial models are used like BCC models for ranking based on full-inefficient frontier. In the followings, non-radial models are used. For this purpose, additive model is used. The objective function of additive model isn't stable against unit changing. Hence, it uses weights to normalize it and presented following model for ranking and using of full-inefficient frontier:

$$
\begin{aligned}
& Max \quad w^{-}S^{-} + w^{+}S^{+} \\
& s.t: \sum_{j=1}^{n} \lambda_j X_j - S^{-} = X_o \\
& \quad \sum_{j=1}^{n} \lambda_j Y_j + S^{+} = Y_o \qquad (17) \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad S^{-} \geq 0 \quad , S^{+} \geq 0 \\
& \quad \lambda_j \geq 0 \quad , j = 1, \dots, n
\end{aligned}
$$

Greater amounts of optimal objective function to evaluate decision making unit means greater its efficiency.

LJK model is invented by Shanling et al (2007). This model is always feasible and stable and its privilege over other similar models is that, it ranks both efficient and inefficient unites with a model solving. LJK model is as follows:

$$
\begin{aligned}
& Min \; z = 1 + \frac{1}{m} \sum_{i=1}^{m} \frac{s^{+}{}_{i2}}{R_i^{-}} \\
& s.t: \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j x_{ij} + s^{-}{}_{i1} - s^{+}{}_{i2} = x_{io} \quad , i = 1, \dots, m \\
& \quad \sum_{\substack{j=1 \\ j \neq o}}^{n} \lambda_j y_{rj} - s^{+}{}_{r} = y_{ro} \quad , r = 1, \dots, s \qquad (18) \\
& \quad s^{-}{}_{i1} \geq 0 \quad , i = 1, \dots, m \\
& \quad s^{+}{}_{i2} \geq 0 \quad , i = 1, \dots, m \\
& \quad s_r^{+} \geq 0 \quad , r = 1, \dots, s \\
& \quad \lambda_j \geq 0 \quad , j = 1, \dots, n, \; j \neq o
\end{aligned}
$$

In which: $R^{-}{}_{i} = Max_{1 \leq k \leq n} \{x_{ik}\}, i = 1, \dots, m$

Khodabakhshi (2007) provided a super efficiency model based on improved outputs which is similar to AP model according to output-oriented with returns to variable scale. The output improvement model is defined blow:

$$
\begin{aligned}
& Max\ \ \varphi_o^{\,s} \\
& s.t: \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j x_{ij} + s^-_{i1} - s^+_{i2} = x_{io} \quad , i=1,\dots,m \\
& \quad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j y_{rj} - \varphi_o^{\,s} y_{ro} - s^+_r = 0 \quad , r=1,\dots,s \qquad (19) \\
& \quad \sum_{\substack{j=1 \\ j\neq o}}^{n} \lambda_j = 1 \\
& \quad s^-_{i1}, s^+_{i2} \geq 0 \qquad , i=1,\dots,m \\
& \quad s^+_r \geq 0 \qquad , r=1,\dots,s \\
& \quad \lambda_j \geq 0 \qquad , j=1,\dots,n,\ j\neq o
\end{aligned}
$$

Model(19) is always feasible.

Jahanshahloo et al(2008), proposed a ranking model based on DEA context-dependent. DEA context-dependent refers to DEA method that evaluates a set of DMUs against Special evaluation context. DEA context-dependent calculates attractiveness and improvement. A set of DMUs index is defined as J^1, that is $J^1 = \{1, \dots, \mathrm{n}\}$ and the set of efficient DMU index in J^1 with CCR model as E^1. Then the sequences of J^l and E^l are defined interactively as $J^{l+1} = J^l - E^l$, the set of E^l can be found as efficient DMUs index in J^l with the following linear programming problem:

$$
\begin{aligned}
& \varphi^l{}_o = \max \varphi \\
& s.t: \sum_{j\in J^l} \lambda_j x_{ij} \leq x_{io} \quad i=1,\dots,m \\
& \quad \sum_{j\in J^l} \lambda_j y_{rj} \geq \varphi y_{ro} \;\; r=1,\dots,s \qquad (20) \\
& \quad \lambda_j \geq o \qquad j \in J^l
\end{aligned}
$$

E^l is called as the lth-evaluation context. Now, based upon these evaluation context, consider $(l = 1, \dots, n)$ E^l , a specific DMU_o from a specific level E^{l_o} $(l_o \in \{1, \dots, l-1\})$. We can obtain the relative relative attractiveness measure of DMU_o whit respect to $(l_o + d)$th-evaluation context, E^{l_o+d} , $(d = 1, \dots, l - l_o)$ by the following context-depended DEA :

$$\begin{aligned} &\Omega_o(d) = \max\ \varphi \\ &\quad s.t: \sum_{j\in E^{l_o+d}} \lambda_j x_{ij} \le x_{io} \qquad i=1,\dots,m \quad d=1,\dots,l-l_o \\ &\qquad \sum_{j\in E^{l_o+d}} \lambda_j y_{rj} \ge \varphi y_{rj} \quad r=1,\dots,s \\ &\qquad \lambda_j \ge o \qquad\qquad j\in E^{l_o+d} \end{aligned} \tag{21}$$

Definition2. d-degree Attractiveness: $A_o(d) = \dfrac{1}{\Omega_o(d)}$ is called d-degree attractiveness of DMU_o from specific level E^{l_o} . Super-efficiency score is obtained from context-dependent DEA, when DMU_o with other DMUs in E^{l_o} were evaluated, as follows:

$$\begin{aligned} &\Omega_o(o) = \max\varphi \\ &s.t: \sum_{j\in E^{l_o-\{o\}}} \lambda_j x_{ij} \le x_{io} \quad ;i=1,\dots,m \\ &\qquad \sum_{j\in E^{l_o-\{o\}}} \lambda_j y_{rj} \ge \varphi y_{ro} \ ;r=1,\dots,s \\ &\qquad \lambda_j \ge o \qquad ; j\in j\in E^{l_o-\{o\}} \end{aligned} \tag{22}$$

Definition3. O-degree Attractiveness: $A_o(o) = \dfrac{1}{\Omega_o(o)}$ is called O-degree attractiveness of DMU_o from specific level E^{l_o} . We have: $\Omega_o(o) < 1$ then $A_o(o) \le 1$.

The progress measure is amount the unattractive DMU_o, when compared to more attractive alternatives DMUs. To obtain progress measure for a specific $DMU_o \in E^{l_o}$ which $l_o \in \{2,\dots,l\}$ DEA context-dependent is used as follows:

$$\begin{aligned} &P_o(g) = \max\varphi \\ &s.t: \sum_{j\in E^{l_o-g}} \lambda_j x_{ij} \le x_{io} \quad ;i=1,\dots,m \ ,g=1,\dots,l_o-1 \\ &\qquad \sum_{j\in E^{l_o-g}} \lambda_j y_{rj} \ge \varphi y_{ro} \ ;r=1,\dots,s \\ &\qquad \lambda_j \ge o \qquad ; j\in E^{l_o-g} \end{aligned} \tag{23}$$

Definition4. g-degree progress: $P_o(g)$ is called g-degree progress of DMU_o from specific level E^{l_o}. Each efficient frontier, E^{l_o-g} , contains a possible target for specific DMU_o in E^{l_o} to improve its performance. The progress here is a level-by-level improvement. For a smaller $P_o(g)$, more progress is expected for DMU_o. Thus, the largest value of $P_o(g)$ is preferred.

Definition5. The concept of attractiveness for $DMU_o \in E^{l_o}$, $l_o \in \{1,...,l_o-1\}$, is defined as follows:

$$\bar{A}_o = \sum_{d=o}^{L-l_o} \frac{A_o(d)}{L-l_o+1}$$

Definition6. The concept of progress for $DMU_o \in E^{l_o}$, $l_o \in \{2,...,L\}$, is defined as follows:

$$\bar{P}_o = \sum_{g=1}^{l_o-1} \frac{P_o(g)}{l_o-1}$$

The more mean attractiveness and less mean progress, the more ranking give. Based upon of factors, we present following measure to rank DMU_o:

$$\gamma_o = \frac{\bar{P}_o}{\bar{P}_o + \bar{A}_o} \qquad (24)$$

The more γ_o, the more ranking is, that is , Measure (24) guarantee the influence mean attractiveness and mean progress to ranking.

4. Ranking method comparison based on super efficiency

In this part, some of the properties expected that ranking models have, are expressed and evaluated. The existence of these properties result in preference or no preference of one method over another one. After that, existence or non-existence of this property in any of the above mentioned methods will be presented in a table and a numerical example, based on the above mentioned methods is given.

One of the basic Properties of ranking model is the models feasibility. If the ranking model isn't feasible, it won't provide any measurements for ranking and therefore, it will be unable to rank. Hence, it's so important that the corresponding model is feasible for all decision-making units. Another property of ranking model is bounded optimal value of the objective function. The feasibility Property of the model can be evaluated in two ways: 1. By bounding of proposed model feasibility area. 2. By dual feasibility of proposed model. Model stability is another property of ranking model. Stability means, little changes in data doesn't cause any dramatic changes. Here, data means input and output values. In fact, instability in super efficiency models occurs by changing the inputs (outputs) of DMUs under evaluation, and for other DMUs, instability wouldn't occur. Other important Properties of ranking is the volume of proposed model's computational process. The volume of proposed model's computational process depends on factors such as: how many models are being resolved in one ranking model to rank DMUs? The number of DMUs, the number of constraints, model linearity or nonlinearity and … . It is obvious that, whatever lower the volume of computational process, higher the consideration of managers will be. But the interesting point is that, in ranking models based on super efficiency, since a variable is removed from the model, the volume of computational process in ranking model based on super efficiency is less than DEA model. Another important property of ranking models is the ranking of efficient units including extreme and non-extreme units. Radial or non-radial is another important property of ranking model. Radial models are models in which, movement of DMU under evaluation toward efficiency frontier are in along one radial and non-radial models are models in which, movement of DMU under evaluation toward efficiency frontier occurs as a form of horizontal-vertical directions and parallel with input axes (in input- oriented). Another property of ranking model is ranking model's linear, in a way that, if model is non-linear, it changes into linear model by using different methods based on the model type. In non-linear models, the volume of computational process is higher than linear models, and also, obtaining dual model is harder than it. Another properties is existence or non-existence of bounded optimal solution, but since all ranking models have unique optimal solution, therefore this

Property isn't included in evaluation and comparison of the methods. Existence or non-existence of these properties in mentioned methods are presented in the following table.

Table1. Express Properties

Bounded optimal solution	Non-extreme efficient unit ranking	Linear	Non-radial	Computational process volume			Stability	Feasibility	← properties
				Number of variables	Number of **constraint**	Number of problem			↓ methods
*		*		n-1	m + s	1			M1
*	*	*		n	m + s+1	1		*	M2
*		*		Depends on the number of non-SE DMU		2	*	*	M3
*	*	*		Depends on the evaluation level		3	*	*	M4
*		*	*	n-1	m+s	1	*		M5
*		*	*	n-1	m+s	1	*	*	M6
*		*	*	n-1	m+s	1	*	*	M7
*		*	*	n	m+s+1	1	*	*	M8
*		*	*	n-1	2m+2s+1	1	*	*	M9
*		*	*	n-1	m+s+1	1	*	*	M10
*		*	*	n-1	m+s	1	*	*	M11
*		*	*	n-1	2m+2s	1	*	*	M12
*		*	*	n-1	m+s	1	*	*	M13
*		*	*	m+s	n	1	*	*	M14

In table 1, M1 is AP method, M2 is ranking based on radial full-inefficient frontier, M3 is ranking method based on changing reference set, M4 is ranking method based on context - dependent DEA, M5 is MAJ method, M6 is modified MAJ method, M7 is JHF method, M8 is ranking method based on non-radial full-inefficient frontier, M9 is SBM method, M10 is ranking method based on improved output, M11 is LJK method, M12 is norm1 method, M13 is Tchebycheff norm method in CCR, and M14 is Gradient vector method. Column number 2, 3, …, and 7 shows seven Property of these methods. Some of the cells from these columns that aren't marked with (*) show that relevant method, doesn't have this Property.

Example 4.1. For better understanding of mentioned implications, consider six decision-making units with three inputs and five outputs:

Table2. Inputs and outputs values

	I1	I2	I3	O1	O2	O3	O4	O5
DMU1	32.73	2725243562	2278270200	1.61587E+11	1.10852E+11	2389618000	241608591	693927977
DMU2	16.19	1776020166	585800359	79617112967	65342101804	1857918400	74029908	181635505
DMU3	16.59	2446352632	1159838458	1.21073E+11	88141979621	1702774112	190551055	578099752

DMU4	13.04	1216942216	6711568830	62973365940	58666708962	553921318	194150645	78280177
DMU5	25.6	1121861587	1975373284	1.6459E+11	99202502284	4827729263	787076695	3672002035
DMU6	22.29	1657621401	7930839753	3.93723E+11	96736740890	493920466	404663066	2207946110

After running CCR model, DMU6, DMU5, DMU2 were efficient. After running mentioned methods on these efficient units in the previous section, the results are presented in table 3:

Table3: Ranking model's objective function values for efficient units

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14
DMU_2	1.1 0	1	0	0.24	1.001	4.1	0.0000 9	0	0.093	21.2	0.00 0898	0.000 898	0.000 86	0.000 86
DMU_5	1.7	1.47	0.011	0.0543 5	1.3431	1.8834	0.5659	1.53E +11	0.375 0	4.62	0.56 569	1.159 56	0.163 6	0.163 6
DMU_6	1.63	1.38	0.007	0.1687 5	1.2229	1.7700	0.4290	3.52E +11	0.187 50	8.35	0.42 903	0.429 03	0.136 7	0.167

According to Table 3, we can calculate achieved ranking for DMUs that is shown in table 4:

Table4. Efficient units ranking

	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M13	M14
DMU_2	3	3	3	1	3	1	3	3	3	1	3	3	3	3
DMU_5	1	1	1	3	1	2	1	2	1	3	1	1	1	2
DMU_6	2	2	2	2	2	3	2	1	2	2	2	2	2	1

5. Conclusions

Seven Properties that expected to include in ranking models were evaluated in part 4. we can divide all ranking methods based on super efficiency into two categories: radial and non-radial. Therefore, in this part, first, the properties of radial methods and then, non-radial methods are enumerated.

Radial methods:

As seen in table 1, the main problem about AP model is model's infeasibility in some Special Conditions. Zero outputs doesn't make AP problem infeasible in T_c, because zero outputs doesn't have any problems. When the output of DMU's under evaluation become zero, therefore possibility area becomes greater and infeasibility doesn't happen. It means that, AP model doesn't have infeasibility problems in output-oriented. The second problem of AP model is model's instability against small changes in data. In AP model, when the data are close to zero, it doesn't obtain correct evaluation; because in this model: $\sum \lambda_j x_j \leq \theta x_o \rightarrow \theta \geq \frac{\sum \lambda_j x_j}{x_o}$ When X_o has a component close to zero, divide by it make the fraction greater. The third problem of AP model is that it calculates the amount of efficiency equal to one for all non-extreme efficient DMUs. So, it can't rank non-extreme efficient DMUs when they're more than one. But, since the possibility of more than one non-extreme efficient DMUs are practically zero, so this weakness isn't really important. AP model is a linear model and it doesn't have unbounded objective function, because it is obtained from envelopment form of CCR model. In this model, movement towards efficiency frontier occurs along with one radial. It can be seen that, calculation's volume of it is less than envelopment form of CCR model, because it has one fewer variable than that one. Since this model is very simple, it resolves a problem for ranking. The number of constraints and variables are equal to (m+s) and (n-1), respectively. Another radial method is ranking based on full-inefficient frontier according to modeled radial models, that isn't super efficient. This model has some advantages such as feasibility, linearity, optimal solution in objective function, and non-extreme efficient units ranking. This model has (m+s+1) constraints, and variable "n", because it isn't supper efficient. Another method is ranking by using reference set. This method is based on eliminating Strong efficient DMUs from observations set in each time. In this way, as explained in part 3, efficiency frontier become closer and a lot of inefficient units become Strong efficient. Since presented model is based on envelopment model of CCR_ε in input-oriented, it has one variable fewer than envelopment CCR_ε model. In this way, feasibility area of CCR_ε multiple form is the subset of multiple forms of presented model. Since CCR_ε multiple form is feasible, then multiple form of presented model is feasible, too. Therefore, its objective function is a bounded optimal solution. At first in this model , non-strong efficient DMUs efficiency is calculated in a separate model. Then, strong efficient DMU efficiency is calculated using non-strong efficient DMUs efficiency. So, two problems are resolved in this model. But the second problem's computational volume depends on the number of non-strong efficient DMUs. This model doesn't rank non-extreme efficient DMUs. This is a linear model and doesn't have instability problem. This model is feasible and has a bounded optimal solution in objective function. context-dependent DEA model is feasible and can rank all the units (Either extreme/non-extreme efficient or inefficient). As it was seen, in this model, ranking is performed relying on the two concepts of Attractiveness and progress. Also in this model, at first, DMUs are arranged using levels, and then, each DMU is compared to all its level DMUs, that for this comparison, its Attractiveness and progress concepts should be calculated using two separated models. So, three problems should be resolved in this method. Since this model is defined in output-oriented of CCR model, it doesn't have instability problem and its objective function has a bounded optimal solution. This is a linear model and its computational process Volume depends on the number of evaluation levels.

Non-radial methods:

In MAJ method, movement toward efficiency frontier is in parallel direction with input axes (in input-oriented). This model doesn't have instability problem, but, it is infeasible. It has bounded optimal solution in objective function. This model solves a problem for ranking and has (m+s) constraints and (n-1) variables. Modified MAJ model brings DMU under evaluation to the efficiency frontier by decreasing input and increasing output to an extent, simultaneously. This model with normalized data is always feasible. The Dual (multiple form) of this model is feasible, too. So, it has bounded optimal solution in objective function. It doesn't have any suggestions for non-extreme efficient units. As mentioned, this model is non-radial, therefore, movement toward efficiency frontier is performed along with the parallel direction of input axes (in input-oriented) and likes movements in horizontal-vertical directions with equal steps, because the amounts of increasing and decreasing are the same. Its computational process is like MAJ model. This model is completely linear and stable. JHF model is given MAJ and modified MAJ model, in a specific condition. This model is non-radial and the only difference with MAJ model is that, the length of steps aren't the same in this method; it means that, the amount of decreasing input and increasing output doesn't occur with the same size. This model is always feasible and stable and has bounded optimal solution in objective function. Also, this model is linear and solves a problem for ranking. This model has (m+1) constraints and (n-1) variables. Another method for ranking is based on full-inefficient frontier using DEA non-radial models. Additive model is used in this model. It's always feasible, linear and stable. It doesn't rank non-extreme efficient units. It has bounded optimal solution in objective function and solves a problem for ranking (This model isn't super efficient but, since it argues about envelopment form, it was included in this paper). Also, it has (m+s+1) constraints and "n" variables. Next model is SBM model. This model mostly used when the number of decision-making units is few (number "n"). In this model, after removing DMUs under evaluation from observations set, its weighted distance to the new efficiency frontier is calculated by SBM objective function. This model is stable. At first, it was a non-liner model, but by Charns-Cooper converting, it turns into linear model. It has bounded optimal solution in objective function and doesn't have any suggestions for non-extreme efficient DMUs ranking. Also, this model solves a problem for ranking. It is feasible and has (2m+2s+1) constraints and (n-1) variables. Improved output model is like AP model in input-oriented with returns to variable scale. So, it is always feasible and has bounded optimal solution in objective function. This model is linear and doesn't have instability problem and it doesn't rank non-extreme efficient. Also, this model solves a problem for ranking. It has (m+s+1) constraints and (n-1) variables. Another non-radial model is LJK that has some advantages such as feasibility, stability, non-radial, linearity, and existence of bounded optimal solution in objective function. This model has (m+1) constraints and (n-1) variables. Also, it solves a problem for ranking units under evaluation. In norm 1 model, the distance between DMU under evaluation from new efficiency frontier is calculated by norm 1, rather than multiplying input vector of DMU under evaluation (in input-oriented) to θ value and bringing it to efficiency frontier. In fact, it looks for a point in new PPS that has the shortest distance from DMU under evaluation of using norm 1. This model doesn't rank non-extreme efficient. But the main problem of this model is about its nonlinearity that by adding the constraints of $X \geq X_O$ and $Y \leq Y_O$ to the basic model's constraints, it turns into linear model. This model is always feasible and stable. Its objective function has bounded optimal solution and it is linear model. It solves a problem for ranking. It has (2m+2s) constraints and (n-1) variables. Tchebycheff norm model is another method for non-radial ranking. At first, this model was linear but by changing variable, it turns to linear form. Linear Tchebycheff norm model is always feasible and doesn't rank non-extreme efficient units. This model doesn't have instability problem and has bounded optimal solution in objective function. If it is consider in BCC, then its computational process volume will increase, because it has one more constraint. In both cases, it solves a problem for ranking. Tchebycheff norm model in T_c has (m+s) constraints and (n-1) variables. Ranking model based on Gradient model is written according to additive model. At first, it was infeasible because it's infinite multiple form of additive model, but by adding normalizing constraint, it turns into a bounded model that is always feasible. So, its objective function has bounded optimal solution. This model doesn't rank non-extreme efficient units. It is stable and has "n" constraints and (m+s) variables. It solves a problem for ranking of under evaluation units.

As seen, infeasibility and instability problems exist in some models in radial models set for ranking. Also, most of these models don't rank non-extreme efficient DMUs. All non-radial ranking models are stable, definitely. It means that, non of non-radial model is instable. Non of this set of models ranks non-extreme efficient. Therefore, the priority of this set of methods is performed according to computational process volume. Methods that were firstly nonlinear, such as SBM and norm 1 methods have high computational volume in this set of methods. Also, methods that solve more than one problem for ranking DMU under evaluation are methods that have higher computational process volume than other methods in this set.

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