Int. J. Res. Ind. Eng. Vol. 5, No. 1-4 (2016) 16-42

International Journal of Research in Industrial Engineering

www.riejournal.com

Solving a Bi-Objective Transportation Vehicle Routing Problem for Waste Collection by a New Hybrid Algorithm

H. Farrokhi-Asl, R. Tavakkoli-Moghaddam*

School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran.

A B S T R A C T

This paper is an extension of the well-known vehicle routing problem (VRP) consisting of two stages. The first and second stages deal with the vehicle routing and transportation problems, respectively. Waste collection is one of the applications of the considered problem in a real world situation. A new mathematical model for this type of the problem is presented that minimizes the waste collection cost and decreases the risk posed to the environment for hazardous wastes transportation simultaneously. According to the NP-hard nature of the problem, a new multi-objective hybrid cultural and genetic algorithm (MOHCG) is proposed to obtain Pareto solutions. A straightforward representation for coding the given model is proposed to help us in reducing the computational time. To validate the proposed algorithm, a number of test problems are conducted and the obtained results are compared with the results of the well-known multi-objective evolutionary algorithm, namely non-dominated sorting genetic algorithm (NSGA-II), with respect to some comparison metrics. Finally, the conclusion is provided.

Keywords: *Waste collection, transportation vehicle routing, multi-objective optimization, cultural algorithm.*

Article history:

Received: 18 December 2015 Revised: 10 August 2016 Accepted: 10 September 2016.

1. Introduction

A wastes collection problem is one of the most important problems in logistics management. Golden et al. [1] introduced this problem as the one of the main applications of a vehicle routing problem (VRP). Sbihi and Eglese [2] introduced the waste collection problem as the one part of green logistics and divided waste collection into two segments, namely hazardous waste transportation and roll-on/roll-off problems. Also, Lin et al. [3] surveyed a waste collection problem and considered it as a subset of the VRP in reverse logistics (VRPRL). Reverse logistics has received close attention in recent years. Dekker et.al [4] defined the reverse logistics as: " The process of planning, implementing and controlling backward flows of raw materials in process inventory, packaging and finished goods, from a manufacturing, or distribution or use point, to a point of recovery of point of proper disposal". Wy et al. [5]

*Corresponding author

E-mail address: *tavakoli@ut.ac.ir*

DOI: 10.22105/riej.2017.49168

considered three major business lines (i.e., residential, commercial and roll-on/roll-off) for a waste collection problem. In the residential waste collection problem, the vehicles pass from streets and collect homemade wastes. In this problem, each street can be seen as one arc that must be passed through it. Bonomo et al. [6] changed a residential waste collection problem into a traveling salesman problem (TSP) and applied it in the real case in Buenos Aires. In a commercial waste collection problem, a goal is to find rational routes in order to collect wastes from location of commercial sites. Sahoo et al. [7] used a heuristic method based on geographical information system (GIS) and clustering methods for the commercial waste collection problem. The roll-on/roll-off waste collection problem considers the collection of wastes from industrial sites and this type of the problem has a specific restriction, in which each tractor can serve only one customer. In this problem, customers demand large container services; however, the tractors can serve only one customer at a time [8]. In this research, we consider the collection of hazardous wastes from commercial location, such as hospitals, shopping centers and industrial companies.

Hazardous waste can be defined as the useless materials that have a bad influence on environment and human's life. Some kinds of these materials cause immediate or long-term risk for humans, plants, animals and in total for our environment. Hence, proper planning for collection of hazardous waste can be advantageous for the environment and can enhance the lives of people. According to Alumur and Kara [9], waste can be characterized as hazardous if it has any of the following attributes: ignitability, corrosiveness, reactivity or toxicity.

Two risk measures are commonly used in the literature of waste collection, societal risk and population exposure [10]. Societal risk is the product of probability of the accident occurrence and the results of the accident. Population exposure is the number of population influenced from hazardous wastes. Giannikos [11] defines risk as the number of products transported per town. Caballero et al. [12] considered three objective functions for transportation risk. The first one takes into account rejection of towns that trucks cross from it in their way. The second one is equity distribution of damage between the towns. The last objective function is about towns that are near to incineration plants, called collective disutility. In this paper, we utilize the risk type 2 meaning that population exposure and dissatisfaction of people in transportation routes are considered as one of criteria to evaluate the solutions.

Martinez-Salazar et al. [13] considered a transportation location-routing problem and presented a new bi-objective mathematical model for this two-stage location-routing problem. They implemented two meta-heuristic algorithms based on scatter and tabu search procedures for non-linear multi-objective optimization (SSPMO) and NSGA-II. Two objectives are considered as decrease of distribution cost and the equitable distribution of workloads for drivers.

The transportation vehicle routing problem belongs to the class of NP-hard problems. The problem size increases heuristic and meta-heuristic methods become the only viable alternative. This paper considers a bi-objective transportation vehicle routing problem, so we use the NSGA-II and proposed a new multi-objective hybrid algorithm based on cultural and genetic algorithms that feature a special solution representation scheme for this problem. Several experiments are conducted for this problem and the results are compared with each other with the respect to some assessment measures.

The remainder of the paper is structured as follows. In Section 2, the problem definition and mathematical model is described. Section 3 explains the details of the proposed algorithm and NSGA-II. Comparisons and discussion on the experimental results are presented in Section 4. The study is finally ended by conclusions and future research in Section 5.

2. Problem description

This research focuses on societal and economic issues of hazardous wastes collection from commercial locations, treatment of these wastes and disposal of wastes residues. There exists specific number of customers and each customer produces some kinds of waste. Customers request for collection, treatment and disposal of these wastes. There exist several treatments facilities and disposal centers with different attributes. Treatment facilities aim to treat hazardous wastes collected from customers' locations and reduce the destructive effects of hazardous wastes. Each treatment facility has its own technology (i.e., each treatment facility can treat only specific kinds of wastes those are compatible with its implemented technology). In addition to these nodes, there exits several depots node and various disposal centers. Trucks start their routes from depots to serve customers and each vehicle must return to the depot, which departs from it. The capacity of each depot is limited and only certain number of trucks can leave the related depot to serve customers. Trucks are considered multi-compartment meaning that each vehicle has specific space for each type of waste; therefore, the capacity of trucks for each type of waste is different. Also the traveling distance must be limited for each truck. Trucks start their routes from depots and move to customers' locations and collect the customers' wastes with regard to capacity and traveling distance limitations. After these actions, each truck must move to treatment facility that has compatible technology with the loads of truck. In other words, trucks must try to find the treatment facilities that can treat all hazardous wastes shipped with them and move to one of these appropriate treatment facilities. Of course, we must regard to capacity limitation of each treatment facility. And the total amount of all types of waste are treated in each treatment, in which facility must not exceed from capacity of treatment facility. After unloading of wastes in location of a treatment facility, trucks must return to its own depot. Thus, the first stage of the problem (i.e., routing phase) is ended at this point.

Treatment facilities eradicate the potential risks of hazardous wastes and each treatment facility according to applied technology can decrease the ratio of wastes' volume. For the

reason of eradication of potential risk and reduction of wastes' volume, residue wastes can be transported from treatment facilities' location to the disposal centers by means of general vehicles. Each disposal center has specific capacity limitations. The second phase is about transportation of waste residues from treatment facilities to disposal centers.
In this research, we consider two objective functions minimizing the economic cost and societal risk. The economic cost includes the routes cost, treatment cost, transportation cost of residue wastes and disposal cost of these residue wastes. The societal risk encompasses the reduction of populations suffering from transportation of hazardous wastes. Trucks pass from towns in their routes to servicing customers and each town has certain population. Hence, we intend to choose the routes that pass from thin towns. Figure 1 depicts an example of a solution to the problem.

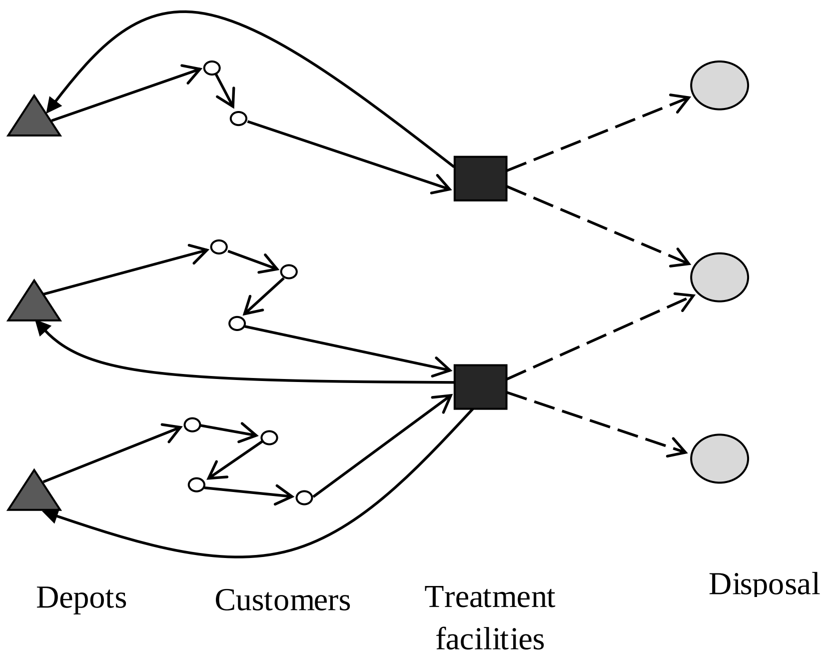

Fig 1. Example of solution to the problem.

The problem can be seen as two distribution stages linked with each other. The first stage also called a vehicle routing problem, which corresponds to the collection of hazardous wastes from customers' locations. The second stage is transportation stage, which corresponds to the transportation of waste residues from treatment facilities to the disposal centers. Figure 2 shows the process of getting rid of hazardous wastes. Each stage of the problem has its own characteristics presented below:

First stage characteristics:

- There exist several depots so the problem can be seen as a multi-depot problem.
- The capacity of depots is limited.
- Several types of wastes are produced in customers' locations.
- Trucks are multi-compartment in this stage.
- The treatment facilities are capacitated.
- Trucks have two types of restrictions: (1) the capacity for each type of waste is limited and (2) the traveling distance must be less than certain amount of length.
- Trucks are homogeneous.
- Several types of technologies are implemented in treatment facilities.
- Each type of waste must be treated at appropriate treatment facilities with compatible technology.

Second stage characteristics:

- There exist different disposal centers.
- Disposal centers are capacitated.
- The percent of mass reduction of each type of waste is deterministic when the waste treated by specific treatment technology.
- The transportation cost of one unit of waste residue between treatment centers and disposal centers is known.

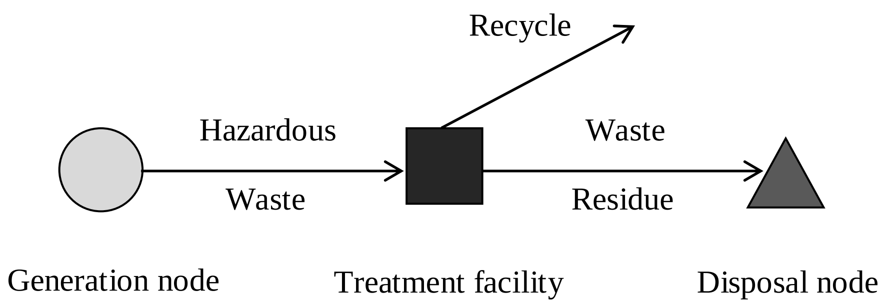

Fig 2. Process of treatment and disposal of wastes.

We propose the following mathematical model for our bi-objective transportation vehicle routing problem.

Sets:

$D=\{1,2,...,d\}$	Set of depots
$T=\{1,2,...,t\}$	Set of treatment facilities
$F=\{1,2,...,f\}$	Set of disposal facilities
$N=\{1,2,...,n\}$	Set of customers
$W=\{1,2,...,w\}$	Set of indexes for types of wastes
$Q=\{1,2,...,q\}$	Set of indexes for treatment technologies

Parameters:

R	Capacity of depots
Tr_t	Capacity of treatment facility *t*
Dis_f	Capacity of disposal facility *f*
Ω_w	Capacity of vehicle for waste type *w*
H	Maximum possible total traveling distance of vehicles in routing stage
d_{iw}	Amount of waste type *w* produced by customer *i*
dis_{ij}	Distance between nodes *i* and *j*
pop_{ij}	Population exposure in arc (*i,j*), $i,j \in D \cup T \cup N$
c_{tf}	Transportation cost of one unit of waste residue from treatment facility *t* to disposal center *f*
α_{wq}	Percent mass reduction of waste type *w* treated with technology *q*
com_{wq}	1 if waste type *w* is compatible with technology *q*
Es_{qt}	1 if treatment facility with technology *q* established in node *t*
V	Cost of traveling one unit of distance by trucks
ct_{wt}	Cost of treatment one unit of waste type *w* at treatment facility *t*
cd_f	Cost of disposing one unit of waste residue in node *f*

Decision variables:

$x_{ij}^{(d)}$	If node *j* is visited just after node *i* in any route of depot *d*, $x_{ij}^{(d)}=1$; otherwise =0
z_{id}	If customer *i* is assigned to depot *d*, $z_{id}=1$; otherwise=0
U_{iw}	Continuous variable that represents the load of waste type *w* after visiting customer *i*
e_i	Continuous variable that represents the distance traveled by truck after visiting node *i*
x_{oi}^{d}	If customer *i* is the first customer in any route of depot *d*, $x_{oi}^{d}=1$; otherwise=0
$x_{i0}^{(d)}$	If treatment facility *i* is the last node in any tour of depot *d*, $x_{i0}^{(d)}$=1; otherwise=0.

y_{wt}	Amount of waste type *w* treated at treatment facility node *t*
o_f	Amount of waste residue disposed at disposal center *f*
ω_{tf}	Amount of waste residue transported from treatment facility *t* to disposal center *f*

Objective functions:

$$\min\ \mathrm{f}_1 = V \sum_{i \in N \cup D \cup T} \sum_{j \in N \cup D \cup T} \sum_{d \in D} dis_{ij} x_{ij}^{(d)} + \sum_{t \in T} \sum_{w \in W} ct_{wt} y_{wt} + \sum_{f \in F} o_f cd_f + \sum_{t \in T} \sum_{f \in F} c_{tf}\, \omega_{tf} \tag{1}$$

$$\min\ \mathrm{f}_2 = \sum_{i \in N \cup T} \sum_{j \in N \cup T} \sum_{d \in D} pop_{ij} x_{ij}^{(d)} \tag{2}$$

s.t.

$$\sum_{i \in N} x_{0i}^{(d)} \le R \qquad \forall i \in N \tag{3}$$

$$\sum_{i \in D \cup N} \sum_{d \in D} x_{ij}^{(d)} = 1 \qquad \forall j \in N \tag{4}$$

$$\sum_{i \in D \cup N \cup T} x_{ij}^{(d)} = \sum_{k \in D \cup N \cup T} x_{jk}^{(d)} \qquad \forall j \in D \cup N \cup T, d \in D \tag{5}$$

$$\sum_{w \in W} y_{wt} \le Tr_t \qquad \forall t \in T \tag{6}$$

$$\sum_{d \in D} z_{id} = 1 \qquad \forall i \in N \tag{7}$$

$$\sum_{i \in D \cup N} x_{il}^{(d)} = z_{ld} \qquad \forall l \in N, d \in D \tag{8}$$

$$u_{iw} - u_{jw} + \Omega_w \sum_{d \in D} x_{ij}^{(d)} \le \Omega_w - d_{jw} \qquad \forall i, j \in N, w \in W \tag{9}$$

$$d_{iw} \le u_{iw} \le \Omega_w \qquad \forall i \in N, w \in W \tag{10}$$

$$\sum_{d \in D} x_{0i}^{(d)} d_{iw} \le u_{iw} \le \Omega_w + \sum_{d \in D} (d_{iw} - \Omega_w) x_{0i}^{(d)} \qquad \forall i \in N, w \in W \tag{11}$$

$$\sum_{i \in N} \sum_{d \in D} x_{ij}^{(d)} u_{iw} = y_{wj} \qquad \forall j \in T \tag{12}$$

$$\sum_{w \in W} y_{wt} \le Tr_t \qquad \forall t \in T \tag{13}$$

$$0 \le x_{ij}^{(d)} \le com_{wq} Es_{qi} (u_{iw} + 1) \qquad \forall i \in N, j \in T, w \in W \tag{14}$$

$$e_i - e_j + (H + dis_{ij}) \sum_{d \in D} x_{ij}^{(d)} + (H - dis_{ji}) \sum_{d \in D} x_{ji}^{(d)} \le H \qquad \forall i, j \in N \cup T \tag{15}$$

$$\sum_{d \in D} dis_{di} x_{0i}^{(d)} \le e_i \le H + \sum_{d \in D} (dis_{di} - H) x_{0i}^{(d)} \qquad \forall i \in N \tag{16}$$

$$e_i \le H - \sum_{d \in D} x_{i0}^{(d)} dis_{id} \qquad \forall i \in T \tag{17}$$

$$\sum_{t \in T} \omega_{tf} = o_f \qquad \forall f \in F \tag{18}$$

$$\sum_{q \in Q} \sum_{w \in W} (1-\alpha_{wq}) Es_{qt} y_{wt} = \sum_{f \in F} \omega_{tf} \qquad \forall t \in T \qquad (19)$$

$$x_{dj}^{(d)} = x_{0j}^{(d)} \qquad \forall d \in D, j \in N \qquad (20)$$

$$x_{id}^{(d)} = x_{i0}^{(d)} \qquad \forall d \in D, i \in T \qquad (21)$$

$$x_{ij}^{(d)}, x_{i0}^{(d)}, x_{0i}^{(d)} \in \{0,1\} \qquad \forall i, j \in D \cup N \cup T, d \in D \qquad (22)$$

$$u_{iw} \geq 0 \qquad \forall i \in N, w \in W \qquad (23)$$

$$e_i \geq 0 \qquad \forall i \in T \cup N \qquad (24)$$

$$y_{wt} \geq 0 \qquad \forall w \in W, t \in T \qquad (25)$$

$$o_f \geq 0 \qquad \forall f \in F \qquad (26)$$

$$\omega_{tf} \geq 0 \qquad \forall t \in T, f \in F \qquad (27)$$

The first objective function considers the economic cost consisting of four terms. The first term calculates the cost of routes and servicing to the customers. The second term relates to the cost of hazardous wastes treatment at treatment facilities. The third terms represents the cost associated to dispose of residue wastes. Finally, the last term considers the cost related to the transportation stage of the problem. The second objective function is about societal rejection of wastes in towns that are crossed by trucks. This objective aims to reduce the people who are suffered from transportation of wastes.

Constraints (3) represent that the number of trucks leaving specific depot must not trespass from the capacity of the depot. Constraints (4) guarantee that each customer must be serviced only by means of one truck. If each truck enters to one node, it must depart from it. This limitation and continuity of routes are considered in constraints (5). Constraints (6) satisfy the capacity restriction for treatment facilities. Constraints (7) ensure that each customer is allocated to only one depot. Constraints (8) guarantee that the routes are established between nodes assigned to same depot. The next three sets of Constraints (9) to (11) are lifted Miller–Tucker–Zemlin (MTZ) sub-tour elimination constraints for the classical VRP, which are first proposed by Desrochers and Laporte [14], and revised by Kara et al. [15]. In our problem, these three constraints are modified to guarantee the sub-tour elimination. Additionally, constraints (10) consider the capacity of vehicles in each route. Constraints (12) calculate the amount of waste treated in each treatment facility. Constraints (13) ensure that the solutions do not exceed the capacity of each facility. Each truck must unload its wastes at treatment facility that are compatible by its load. This restriction is shown in Constraints (14). Constraints (15) to (17) ensure that the length of routes do not trespass from an upper bound of the route length. Constraints (18) specify the waste residues amount that must be disposed in disposal centers. Constraints (19) balance the flow between treatment facilities and disposal centers. Constraints (20) and (21) specify the relation between decision variables. Constraints (23) to (27) define the type of the variable used in this model.

3. Methodology

We design two solution algorithms to find rational Pareto solutions, in which the first one is based on multi-objective hybrid cultural and genetic algorithm (MOHCG) and the other one is based on the non-dominated sorting genetic algorithm (NSGA-II) [16]. Additionally, we present a straight-forward representation to reduce the complexity and decrease of computational time. A new representation, which will be explained in the next section, considers a feasibility of generated solutions and some constraints caused the infeasibility will be managed by introducing penalty function as f_3. It is obvious that we must try to minimize the penalty objective function.

3.1. Order-based representation

This representation is based on order of the problem component. We specify four major components for the problem and the solutions can be extractable by determining the situation of these components. These main components of the problem include customers order, depots order, treatment facilities order, and disposal centers order. Therefore, the representation proposed for this problem consists of:

- Order of customers
- Order of depots
- Order of treatment facilities
- Order of disposal centers

The methodologies, in which we specify the order of the components of the problem, are partly different for the solution approaches investigated in this paper (i.e., MOHCG and NSGA-II) and this difference is related to distinction in nature of these two solution approaches. The nature of NSGA-II is discrete and the nature of MOHCG is continuous. In NSGA-II, we produce a permutation for the integer numbers in the range (1-*n*) and this permutation specifies the priorities of customers. Order of the depots is a permutation for integer numbers in range (1-*d*), order of the treatment facilities is a permutation for integer numbers in range (1-t) and the order of disposal centers is a permutation for integer numbers in range (1-*f*). In MOHCG the manner is partly different with NSGA-II. In MOHCG, we generate random real numbers in range (0-1) for all components of the problem (i.e., customers, depots, treatment facilities and disposal centers). For example in customers ordering, each random real number corresponds to one customer. We can specify the order of customers by sorting of corresponding real numbers. Figure 3 demonstrates a simple example for this type of ordering based on random real numbers between 0 and 1.

Customers	1	2	3	4	5	6
Random real numbers	0.123	0.222	0.809	0.437	0.011	0.943
Numbers rank	5	4	2	3	6	1
Resulted order of customers	6	3	4	2	1	5

Fig 3. Example of customers ordering in MOHCG

After specifying the order of members in each problem component, the construction algorithm can transform this simple representation to meaningful solutions for the problem. At this point, we must produce the routes, so we start with the first rank depot in depots order and assign the first rank customer to the first depot. We keep on with the second customer. If adding this customer to route does not exceed the capacity limitation of truck, we adjoin this customer to the current route and assign this to the first depot. We continue this manner until adding the next customer in customers order to the current route causes trespass from capacity of truck. At this point, we stop assigning customers to current route and current depot, and we continue with the next depot in the depots order. Note that the sequence of customers in each created route is the same as these customers arrangement in customers order. We start with the second depot and perform the previous steps for the second depot. The customers are assigned to the second depot until the adding one customer to the second route exceeds the capacity of truck (the capacity for all types of waste must be investigated). We start from the first depot and repeat these actions till the last depot. If some customers are left unassigned, we return to the first depot and start assignment process from the first depot to the last one. Note that implementation of this policy about assignment of routes to depots prevents any violation about capacity of and we distribute routes between depots in the equitable manner.

Now, the last step of a routing stage (i.e., unloading the waste in compatible treatment facility) must be done to complete this stage. Each truck's load is known at this point and we must decide what treatment facility is appropriate with respect to wastes transported with truck. We start with the first truck (i.e., route) and then search for the first treatment facility in facilities order that has enough capacity and compatible technology for treatment of the loads of this truck. After finding the facility having these two conditions, the truck moves toward this facility. We apply these methods to assign all trucks to treatment facilities. After doing all the previous steps, the first stage is finished and each truck must return to depot that truck starts their routes from it.

The second stage of the problem is the transportation problem, in which the waste residues must transport from treatment facilities to disposal centers. In this stage, we must distribute the produced waste residues in facilities between disposal centers. Order of facilities and disposal centers are known. Hence, we start with the first disposal centers and assign the waste residues associated with the first facility to the first disposal center until fulfilling the capacity of disposal centers. If the remaining capacity of the first unassigned disposal center is less than the waste residues produced in the first unassigned treatment facility, we transport

1. Generate random permutation for customers, depots, treatment facilities and disposal centers
2. Define.
   $\gamma(d,w,r)$ = all demands of waste type *w* assigned to route *r* from depot *d*
   θ_{dr} = set of customers assigned to route *r* from depot *d*
   Φ = set of unassigned customers
3. *d*=1.
4. Start from depot *d*
5. If$_1$ demand(first customer, waste type *w*) + $\gamma(d,w,r) \leq \Omega_w$
   - 5.1. Insert the first customer to θ_{dr} .
   - 5.2. Delete the first customer from customers order and from Φ .
   - 5.3. For all types of waste : $\gamma(d,w,r)$ = $\gamma(d,w,r)$ + demand (first customer, waste type *w*)
   - 5.4. If$_2$ $\Phi = \{\}$
     - 5.4.1. Go to step 8.
   - 5.5. end if$_2$
6. else$_1$
   - 6.1. *d*=*d*+1.
     - 6.1.1. If$_3$ *d*> number of depots
       - 6.1.1.1. *d*=1, *r*=*r*+1.
       - 6.1.1.2. Go to step 4.
     - 6.1.2. end if$_3$
   - 6.2. Go to step 4.
7. end if$_1$
8. r=1, t=1.
9. Start from route r
   - 9.1. If$_4$ treatment facility t was compatible for all wastes of route r && $Tr_t \geq \sum_w \gamma(d,w,r)$
     - 9.1.1. Assign treatment facility t to route r
     - 9.1.2. $Tr_t = Tr_t - \sum_w \gamma(d,w,r)$
     - 9.1.3. t= 1, r=r+1
   - 9.2. else$_4$
     - 9.2.1. t=t+1, and go to step 9.1
   - 9.3. end if$_4$
10. Calculate the volume of waste residue must be transported to disposal center from each treatment facility as Trans(t)
11. for$_1$ every treatment facility t in treatment facility order
    - 11.1. for$_2$ every disposal center f in disposal centers order
      - 11.1.1. if$_5$ Dis(f) >0
        - 11.1.1.1. transportation(t,f)=min {Dis(f), Trans(t)}
        - 11.1.1.2. Dis(f)=Dis(f)-transportation(t,f).
        - 11.1.1.3. Trans(t)=Trans(t)-transportation(t,f)
      - 11.1.2. endif$_5$
      - 11.1.3. if Trans(t)=0
        - 11.1.3.1. Break main loop.
    - 11.2. end for$_2$
12. end for$_1$

Fig 4. Pseudo code of the construction algorithm.

waste residues equal to the remaining capacity of the disposal center and the left waste residues of facility are sent to another disposal centers.

With this representation, every solution generated by a constrction algorithm is treated as a feasible solution. The truck capacity limitation is considered in the generation of solutions; however, as can be seen the length of routes in stage one is not considered in the construction algorithm. To solve this issue, we define a new objective function as penalty (f_3) to be minimized to non-feasible solutions. This representation can be easily implemented in meta-heusristic algorithms and we are not concern about the infeasibility when some operators change the solutions chromosome to generate neighborhood solutions. Because of easiness of this representation to generate solutions this constructive algorithm is used for initial solutions in both algorithms. Figure 4 presents a pseudo code of the construction algorithm.

3.2. Procedure based on MOHCG

The cultural algorithm (CA) was introduced by Reynolds [17], which is an extension of the classical GA. The CA for a single objective problem considers interactions between individuals; however, in the GA, these interactions do not take into consideration. Individuals in the population communicate with each other to generate a belief space (i.e., culture) and this belief space stores knowledge about the problem within itself and use this knowledge to improve produced solutions. Figure 5 shows the cultural algorithm mechanism. Experience of individuals selected from the population space according to some criteria is used to produce problem solving knowledge that impacts on the belief space. The belief space saves and manipulates the knowledge and information to influence on the evolution and improvement of the population space.

Another version of the cultural algorithm is the hybrid version of the CA and GA. In the cultural-based GA, knowledge is used to store and transmit knowledge from one generation to the next. The CGA is composed of two spaces, population space and belief space. The population space is implemented by an extended GA, and in that it uses crossover and mutation operators. Best et al. [18] extended a cultural algorithm framework to handle multi-objective problems. The experimental results showed that MOCA can be used independently or as a supplementary to other MO algorithms.

Belief space includes several main components named as knowledge source. In CA knowledge sources consist of situational knowledge, topographical knowledge, normative knowledge, domain knowledge and historical knowledge. In this research according to the attributes of genetic operator in population space, we consider only two main components of the algorithm, normative knowledge and situational knowledge.

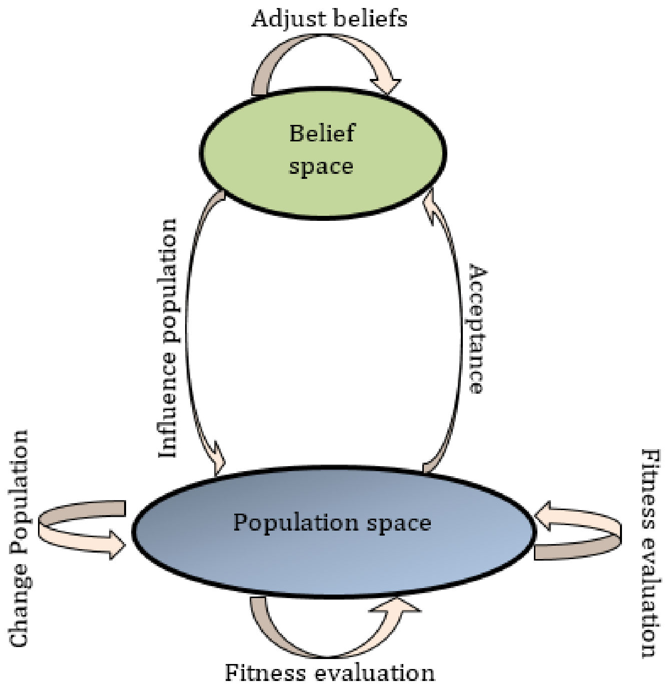

Fig 5. Single objective CA template.

The steps of the proposed MOHCG algorithm are summarized as follows:

Step 1. The initial population of algorithm is generated by means of mentioned constructive algorithm and based on new representation.

Step 2. The solutions are evaluated and a rank is assigned to each solution with regard to its non-domination concept. Level 1 is the best level, 2 is the next best level, and so on. For the solutions of the same rank, a crowding distance is calculated by using a crowded-calculator operator. By this operator, the selection process goes toward a uniformly spread Pareto front at different stages of the algorithm. Among two solutions at the same front, the solution located in a low density region is preferred.

Step 3. Belief space is adjusted by means of specific operators. One operator updates the situational knowledge and another one updates the normative knowledge.

Step 4. Population is influenced from cultural space and knowledge about normative solutions and situational solutions impacts on direction of population component. Population is updated and new population is available to continue algorithm.

Step 5. Genetic operators (i.e., crossover and mutation operators) are applied to create the next generation called the offspring population from the current chromosomes called parents.

Step 6. The new generated population and the old population are merged with each other.

Step 7. The merged populations are sorted according to the rank of solutions and crowding distance criteria.

Step 8. The solutions with the ranked 1 are stored as Pareto solution.

Step 9. The stop criterion in this algorithm is considered as maximum number of iterations. If the criterion is satisfied the algorithm is stopped; otherwise, Steps 3 to 9 are repeated.
The main structure of the proposed MOHCG algorithm is shown in Figure 6.

3.3. Acceptance

In the real world and real society, people who are bold and brilliant can impact on other people or change the conditions or influence on culture and beliefs. Cultural algorithm inspired from this cultural evolution in societies. Population members are eligible to change belief space that have specific characteristics. In single objective CA the sorting of solutions according to their objective functions are simple and phenotype space is sortable regarding to objective values. The solutions which have top rank in order of solutions are accepted. In this proposed algorithm we sort solutions according to their rank and crowding distance. First priority for comparison of solutions is rank criterion and the second priority is crowding distance. Therefore, if two solutions have same rank, we refer to crowding distance to specify the better solution.
After sorting the solutions in population space, the specific number of solutions is accepted to influence on belief space. In this algorithm, number of solutions that are impressing on belief space is dynamic and this number is decreased iteration by iteration. The number of effective members in population impressing on belief space is calculated by:

$$n_B(t) = \left\lceil \frac{n_s \gamma}{t} \right\rceil$$

where $n_B(t)$ is the number of effective members, n_s is the population size and γ is the fixed value between 0 and 1. In this formulation, the number of effective members in population is decreased gradually.

3.4. Belief space adjustment

Situational knowledge stores the records of the population and good solutions in each generation are eligible to change the situational knowledge. This concept is similar to the concept of the leader in the PSO algorithm proposed by Kennedy and Eberhart [19]. In the proposed algorithm in this paper, one solution from Pareto solutions is selected randomly and then this solution is compared with the current situational knowledge solution. If the selected solution dominates the current solution, the situational knowledge will be updated. If the current situational knowledge dominates the selected solution, we do nothing. Finally, if two solutions do not dominate each other, on solution is selected randomly.

$$y(t+1) = \begin{cases} y(t) & \mathrm{y(t)} \prec \mathrm{x} \\ x & \mathrm{x} \prec \mathrm{y(t)} \end{cases}$$

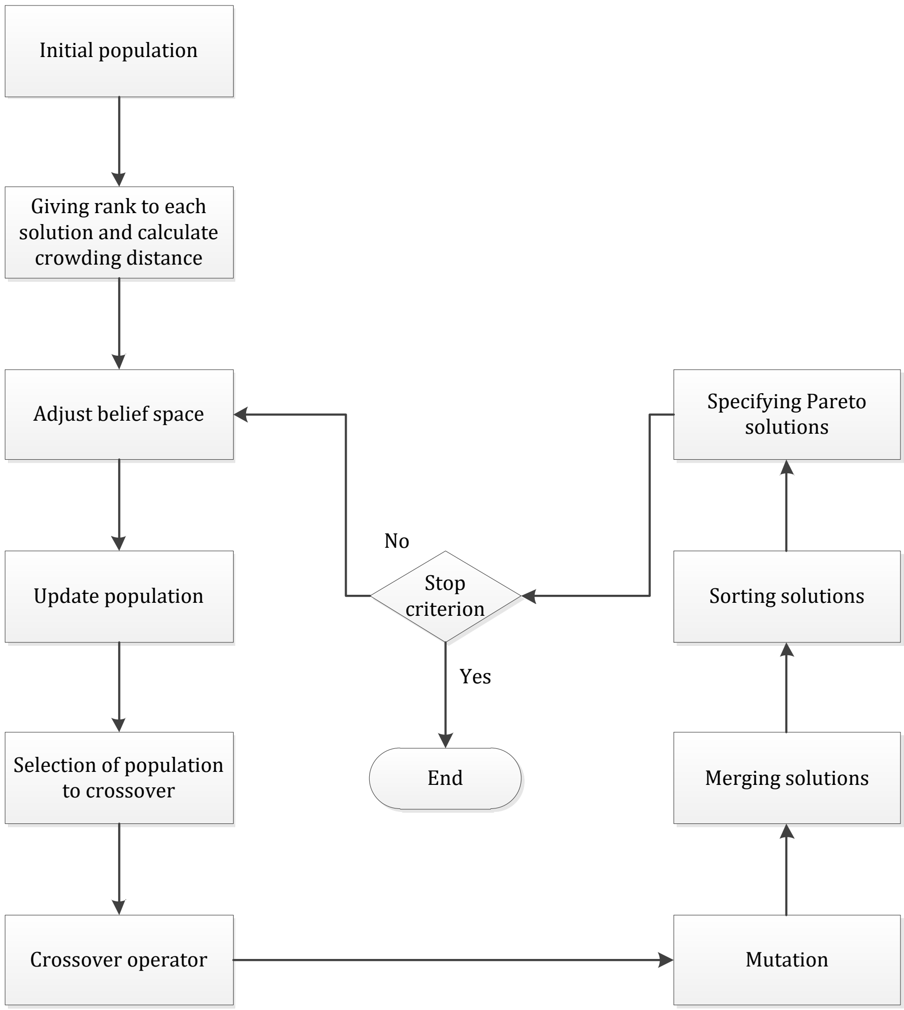

Fig 6. Flowchart of the proposed MOHCG algorithm

Normative knowledge is a set of promising variable ranges that provide norms for the current individuals. Let $x = (x_1, x_2, ..., x_n)$ shows the solution space. In the normative knowledge, we specify a range for each dimension of solution. Each dimension has minimum and maximum value shown $x_i^{\min}$ and $x_i^{\max}$. Solutions, whose minimum or maximum is a part of solutions, have the corresponding objective function values, namely lower and upper bounds. The lower and upper bounds are attributed for minimum and maximum values, respectively. Hence each solution dimension is shown by three factors as follows:

$$
\begin{aligned}
x_i &= (I_i, L_i, U_i) \\
I_i &= (x_i^{\min}, x_i^{\max}) \\
L_i &= (L_i^1, L_i^2, \ldots., L_i^m) \\
U_i &= (U_i^1, U_i^2, \ldots, U_i^m)
\end{aligned}
$$

For the changing normative knowledge in each iteration, we use the following formulation:

$$
x_j^{min} = \begin{cases} x_{lj}(t), & x_{lj}(t) < x_j^{min}(t) \ or \\ & f(x_l) \prec (L_j^1, \ldots, L_j^m) \\ x_j^{min}(t), & Otherwise; \end{cases}
$$

where $x_{lj}(t)$ is the *j*-th component of selected solution from Pareto solutions. If the corresponding $x_j^{\min}$ values is updated, $L_j^i, \forall i = 1, 2, \ldots, m$ will be updated, and new lower bounds are replaced with older ones.

$$
x_j^{max} = \begin{cases} x_{lj}(t), & x_{lj}(t) > x_j^{max}(t) \ or \\ & f(x_l) \prec (L_j^1, \ldots, L_j^m) \\ x_j^{max}(t), & Otherwise; \end{cases}
$$

If the corresponding $x_j^{\max}$ values is updated, $U_j^i, \forall i = 1, 2, \ldots, m$ will be updated, and new upper bounds are replaced with older ones.

3.5. Population change

After updating the belief space, this changes must reflect on the population space and the individuals is population space must conform themselves with the new culture. For the changing the population space, the individuals try to move toward into norms and will be close to situational knowledge. In this proposed algorithm, we use four types of movement and changing individuals. In each iteration, one integer random number is generated between 1 and 4, and one of these methods is selected according to the generated number. These methods are illustrated as follows:

1. In this method, we just use normative knowledge. The following formulations are used to generate new solutions:

$$
\begin{aligned}
x'_{ij}(t) &= x_{ij}(t) + \sigma_j(t) \times N(0,1) \\
\sigma_j(t) &= [x_j^{\max}(t) - x_j^{\min}(t)] \times \alpha \qquad 0 \le \alpha \le 1
\end{aligned}
$$

2. In the second method, we use only situational knowledge for updating the populations. The goal is to close individuals to components of situational knowledge.

$$x'_{ij}(t) = \begin{cases} x_{ij}(t) + \sigma_{ij}|N(0,1)|, & x_{ij}(t) < y_j(t) \\ x_{ij}(t) - \sigma_{ij}|N(0,1)|, & x_{ij}(t) > y_j(t) \\ x_{ij}(t) + \sigma_{ij}N(0,1), & x_{ij}(t) = y_j(t) \end{cases}$$

$$\sigma_{ij}(t) = \alpha \times [x_{ij}(t) - y_j(t)]$$

where $y_j(t)$ is the *j*-th component of situational knowledge in iteration *t*.

3. In the third method, we use situational knowledge for specifying the movement direction and normative knowledge for length of movement. The formulation is the same with the above formula; however for specifying the length of movement, we use following formula:

$$\sigma_{ij}(t) = \alpha \times [x_j^{\max} - x_j^{\min}]$$

4. In the last method, we use only normative knowledge, but in different way with method 1.is one random parameters between 0 and 1. The schematic figure for this β method is shown in Figure 7.

$$x'_{ij}(t) = \begin{cases} x_{ij}(t) + \sigma_j|N(0,1)|, & x_{ij}(t) < x_j^{min}(t) \\ x_{ij}(t) - \sigma_{ij}|N(0,1)|, & x_{ij}(t) > x_j^{max}(t) \\ x_{ij}(t) + \beta\sigma_{ij}N(0,1), & Otherwise; \end{cases}$$

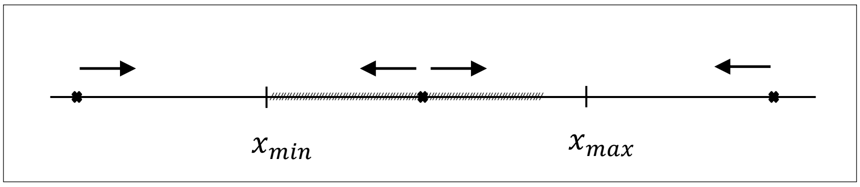

Fig 7. Schematic view of method 4.

3.6. Crossover operator

Before we launch crossover operator, we must choose appropriate parents for applying crossover on these parents. For this reason and for selection appropriate parent for a crossover operator, we use two roulette wheel methods, the first one selects the first parent with respect to the first objective function and the second one selects with respect to the second objective values. As mentioned before, we specify the characteristic of solutions with four components (i.e., depots order, customers order, treatment facilities order and disposal centers order). In this stage, we decide which component must be selected for the crossover operator. For this reason, we generate one integer random number in range (1-15) and specify which policy is chosen to apply in this stage. (15=$\binom{4}{1}+\binom{4}{2}+\binom{4}{3}+\binom{4}{4}$). Because of continuous nature of proposed algorithm, we use a crossover operator that are compatible for continuous problems.

3.7. Mutation operator

In this section, we apply three types of mutation operators to explore better in genotype space. We generate two random numbers, first one is in the range of 1 and 3 and the second one is in range of 1 and 15. If the first number is 1, we use an insertion operation. In this operator, two genes are selected randomly and insert the second genes after the first selected gene. If the first generated random number is 2, we apply the swap operation, which select two genes randomly and swap their location in the chromosome. If the generated random number is 3, we use the reverse operation and reverse genes between randomly selected genes. The second random number is applied for the purpose of specifying which components are selected for mutation.

3.8. Proposed NSGA-II

The genetic algorithm (GA) is a population-based meta-heuristic algorithm, which is widely used to solve optimization problems. It was first introduced by Holland [20] who developed this algorithm on the basis of the natural selection and inheritance. Different extensions of the GA are applying the GA for multi-objective optimization problems that exist in the literature. The GA is one of the most frequent meta-heuristic algorithms for solving vector optimization problems, since it does not need the user to prioritize, scale, or weighing objectives [21]. Fast non-dominated sorting genetic algorithm (NSGA-II) which was presented by Deb et al. [16] is one of the algorithms used in the current study. All the steps for this proposed algorithm is same with the original NSGA-II presented before. The construction method and other steps for the algorithm is illustrated in the previous sections; however, the crossover operator used in this algorithm is different with the MOHCG. For the reason of a discrete nature of the problem, the OX (order crossover) operator is applied [22].

4. Experimental results

The performances of the proposed MOHCG and NSGA-II are compared with each other and the associated results are analyzed. The algorithms are coded in MATLAB R2013a and run on Intel Core i5 2.27 GHz personal computer with 4 GB RAM.

4.1. Test instances

To the best of our knowledge, there are no available benchmark instances in the literature, so the instances solved in this paper are built by a defined instance generator in Matlab software. The generated instances are built from six main components. All components with each other make characteristics of one instance. The generated instances were named ($d\times t\times f\times c\times w\times q$). These characters denote number of depots, number of treatment facilities, number of disposal centers, number of customers, number of waste types and number of available technologies respectively. Instances can be divided into two main categories, small instances, and medium and large scale instances. The instances are generated randomly. This results in a set of 20 instance types for two sizes of problems. Each instance runs 5 times and the results are collected. Instances are available for download from:
https://www.dropbox.com/sh/770opj0ut22jyho/AACsvMzKCHJeCSD2ZaSuYU8ka?dl=0.

4.2. Tuning parameters

The experimental results are implemented in two sections consisting of small, and medium/ large-sized problems. For each kind of problems Taguchi design of experiment are conducted according to the number of Pareto solutions in Minitab software and the tuned values for each parameter of algorithms are summarized as follows :

4.2.1. Small-sized problems

- Number of population size for two algorithms is considered 50.
- Maximum number of iterations is considered 50.
- Mutation and crossover rates in NSGA-II are 0.4 and 0.5, respectively.
- Mutation and crossover rates in MOHCG are 0.4 and 0.6, respectively.
- Alpha and beta parameters in population change methods in MOHCG are considered 0.4 and 0.3, respectively.
- Acceptance fixed coefficient is considered 0.8.

4.2.2. Medium/large-sized problems

- Number of the population size for two algorithms is considered 50.
- Maximum number of iterations is considered 50.
- Mutation and crossover rates in NSGA-II are 0.4 and 0.5, respectively.
- Mutation rate and crossover rate in MOHCG are 0.5 and 0.6, respectively.
- Alpha and beta parameters in population change methods in MOHCG are considered 0.5 and 0.2, respectively.
- Acceptance fixed coefficient is considered 0.7.

4.3. Meta-heuristics comparison

For experimental evaluation when we reference to MOHCG and NSGA-II, we are referring to our implementation of these algorithms. Five run for each instance are performed to gain results. In order to compare the efficiency of the proposed algorithms, we use six performance metrics, named as number of Pareto solutions (NPS), computational time, spacing metric (SM), space covered (SC), Diversification metric (DM) and coverage metric (CM).

The number of Pareto solutions measures the algorithms ability to find an efficient point. Tables 1 and 2 are summarized the results for small and medium/large-sized problems, respectively. In addition, the computational time spent for finding these Pareto solutions is demonstrated in these two Tables. In small-sized problems computational times for both algorithm is in the same range. In large-scale problems, the computational time for NSGA-II is almost better than MOHCG, but this difference is not notable. In most examples, MOHCG is better than NSGA-II in producing Pareto numbers and the average values in small and large-scale problems confirm this assertion.

Table 1. Quantity of Pareto solutions and computational time for small-sized problems

	Quantity of Pareto solution		Computational time(s)	
Problem characters	NSGA-II	MOHCG	NSGA-II	MOHCG
4-4-4-5-2-2	7.2	10.8	24.2	25.8
5-5-4-6-4-3	10.2	14	25	26
5-6-5-20-4-3	9.4	10	24.8	25.4
6-6-6-8-3-4	11.2	8.6	22	24
7-8-8-10-4-3	12.4	12.2	24	24.8
8-7-9-12-2-3	9.8	14.2	24.6	23.4
10-10-10-10-4-4	8	10	25	26.2
10-10-10-5-4-3	8.8	12.4	31	29.8
11-11-11-15-2-2	11.4	11.6	26	27
12-12-10-15-3-4	8.6	10.2	26	27.4
Average	**9.7**	**11.4**	**25.26**	**25.98**

Table 2. Quantity of Pareto solutions and computational time for medium/large-sized problems

	Quantity of Pareto solution		Computational time (s)	
Problem characters	NSGA-II	MOHCG	NSGA-II	MOHCG
15-15-15-25-4-4	7	5	26.2	30
15-15-15-30-5-4	14.8	13.2	27	30.4
20-18-20-30-5-4	11.2	15.4	29.4	32.4
20-20-18-25-5-5	16	15.2	27.4	29.2
20-20-20-40-4-4	7.2	8.6	31	33.4
20-20-20-40-5-5	5.8	9	30.4	33.6
20-25-25-50-3-3	12	14	33.6	35
20-25-25-100-3-3	12.4	11.4	45.8	48.8
30-25-25-100-2-2	14.8	11.8	49	51
30-30-30-150-3-5	13	13.2	58	64.6
Average	**11.42**	**11.68**	**35.78**	**38.84**

Spacing metrics provides us details about the uniformity of the distribution of the solutions obtained by way of each algorithm. This metrics are computed by:

$$SM1 = \sqrt{\frac{1}{N-1} \times \sum_{i=1}^{n} (d_i - \bar{d})^2}$$

where d_i is the Euclidean distance between solution i and the nearest solution is belongs to Pareto sets of solutions.. In some instances, MOHCG d_i is the average value of all $\bar{d}$ operates better and in other instances NSGA-II operates better. This measure does not show a superiority of none of them; however, the average value for NSGA-II is better than MOHCG. The results are shown in Tables 3 and 4.

Table 3. SM measure for small-sized instances

Problem characters	NSGA-II	MOHCG
4-4-4-5-2-2	2.0596e4	1.1687e4
5-5-4-6-4-3	1.5370e4	1.2737e4
5-6-5-20-4-3	2.5077e4	2.3223e4
6-6-6-8-3-4	1.2326e4	1.5078e4
7-8-8-10-4-3	1.8156e5	1.5560e5
8-7-9-12-2-3	0.7814e4	2.3286e4
10-10-10-10-4-4	3.9840e4	5.5144e4
10-10-10-50-4-3	9.0894e4	6.9924e4
11-11-11-15-2-2	1.8823e4	1.4247e4
12-12-10-15-3-4	1.8119e3	2.4534e4
Average	**4.3695e4**	**6.1064e4**

Table 4. SM measure for medium/large-sized instances

Problem characters	NSGA-II	MOHCG
15-15-15-25-4-4	1.445e5	3.1505e4
15-15-15-30-5-4	2.0391e4	3.3161e4
20-18-20-30-5-4	1.8133e4	2.8232e5
20-20-18-25-5-5	1.5975e4	7.1180e4
20-20-20-40-4-4	1.2129e6	2.9243e6
20-20-20-40-5-5	2.1604e5	1.2046e5
20-25-25-50-3-3	3.1683e4	5.135e5
20-25-25-100-3-3	4.4345e4	3.5291e4
30-25-25-100-2-2	5.6636e6	2.4809e7
30-30-30-150-3-5	1.5787e8	1.4095e8
Average	**16.5339e6**	**16.9819e6**

Diversification metric (DM) specifies the spread of solution set and is determined by:

$$DM = \sqrt{\sum_{i=1}^{n} \max(\|x_t^i - y_t^i\|)}$$

where $\max(\|x_t^i - y_t^i\|)$ is the Euclidean distance between the non-dominated solutions x_i^t and y_i^t.

In all small-sized instances, MOHCG operates better than NSGA-II. This is shown in Table 5. In large-scale ones, MOHCG is superior to NSGA-II in seven problems. We can conclude from Tables 5 and 6 that the MOHCG can find the diverse Pareto solutions in the solution space.

Table 5. DM measure for small sized instances

Problem characters	NSGA-II	MOHCG
4-4-4-5-2-2	2.4240e4	2.5923e4
5-5-4-6-4-3	1.0496e3	1.4232e3
5-6-5-20-4-3	1.6538e3	1.8409e3
6-6-6-8-3-4	1.3784e3	1.5226e3
7-8-8-10-4-3	1.7771e4	1.8994e4
8-7-9-12-2-3	1.1243e3	2.3710e3
10-10-10-10-4-4	2.6869e3	3.5351e3
10-10-10-5-4-3	1.4192e3	1.6148e3
11-11-11-15-2-2	1.3227e3	1.5567e3
12-12-10-15-3-4	1.6976e3	1.9243e3
Average	**5.4096e3**	**6.057e3**

Table 6. DM measure for medium/large sized instances

Problem characters	NSGA-II	MOHCG
15-15-15-25-4-4	2.0413e3	1.0493e3
15-15-15-30-5-4	2.04656e3	2.1228e3
20-18-20-30-5-4	1.6284e3	5.9314e4
20-20-18-25-5-5	2.2975e3	2.6382e3
20-20-20-40-4-4	4.7456e4	5.7896e4
20-20-20-40-5-5	2.01888e3	2.2730e3
20-25-25-50-3-3	2.8253e3	2.6624e3
20-25-25-100-3-3	3.7153e3	3.3222e3
30-25-25-100-2-2	6.4085e4	6.5223e4
30-30-30-150-3-5	1.4531e5	1.6572e5
Average	**27.3702e3**	**36.2114e3**

Zitzler and Thiele [23] proposed a comparison metric, named space covered. This metric estimates the size of the space covered by Pareto solutions. For bi-objective problems in phenotype space, each Pareto solution has two values for objective functions (i.e., $f_1(x_i)$, $f_2(x_i)$), so each dominated solution represents a rectangle in the phenotype space.

The results in Tables 7 and 8 show that MOHCG outperforms NSGA-II with respect to space covered measure. In most instances, the proposed MOHCG performs better than NSGA-II.

Table 7. SC measure for small-sized instances

Problem characters	NSGA-II	MOHCG
4-4-4-5-2-2	1.7785e13	1.6343e13
5-5-4-6-4-3	2.4526e9	2.7826e9
5-6-5-20-4-3	6.0411e11	5.9899e11
6-6-6-8-3-4	2.99321e9	3.1132e9
7-8-8-10-4-3	2.3212e10	1.9888e10
8-7-9-12-2-3	1.6863e10	1.8821e10
10-10-10-10-4-4	3.9894e9	2.05050e9
10-10-10-5-4-3	1.012e12	3.9823e12
11-11-11-15-2-2	9.8734e9	1.3223e10
12-12-10-15-3-4	5.8623e9	1.0243e10
Average	**19.4608e11**	**20.9929e11**

Table 8. SC measure for medium/large-sized instances

Problem characters	NSGA-II	MOHCG
15-15-15-25-4-4	1.9523e10	2.3692e10
15-15-15-30-5-4	6.4723e10	1.3034e11
20-18-20-30-5-4	5.3214e10	7.5654e10
20-20-18-25-5-5	3.5634e10	5.6534e10
20-20-20-40-4-4	5.1014e12	5.9432e12
20-20-20-40-5-5	4.5910e10	8.9843e10
20-25-25-50-3-3	1.9994e11	2.1564e11
20-25-25-100-3-3	1.7692e12	3.2224e12
30-25-25-100-2-2	2.4989e15	1.0096e15
30-30-30-150-3-5	5.0567e15	6.3231e15
Average	**756.12e12**	**1634.17e12**

In the coverage metric, two non-dominated sets of solutions obtained by algorithms are compared with each other. Let $x, x' \subseteq x$ is a two sets of Pareto solutions obtained with each algorithm. The function CM maps the pair (x, x') to interval (0, 1) [23]:

$$CM(x, x') = \frac{\left|\{a' \in x' ; a \in x : a \prec a'\}\right|}{|x'|}$$

The results summarizes in Tables 9 and 10 for this measure. For small-sized instances illustrated in Table 9, the MOHCG is able to generate Pareto solutions that dominate 12% of the ones generated by NSGA-II. When MOHCG is compared with NSGA-II, it is able to generate more than 78%, which is better than 12% which is generated by MOHCG. For large scale instances also NSGA-II resulted superior dominating more than 0.89% of the estimated Pareto solutions obtained by MOHCG.

Table 9. Coverage of two sets average value for small-sized problems

x/*x*'	NSGA-II	MOHCG
NSGA-II	0	0.7823
MOHCG	0.121	0

Table 10. Coverage of two sets average value for medium/large sized problems

x/*x*'	NSGA-II	MOHCG
NSGA-II	0	0.8934
MOHCG	0.1667	0

Figures 8 and 9 are the sample Pareto solutions obtained with MOHCG in small and medium/large-sized problems for instance numbers 10 and 17.

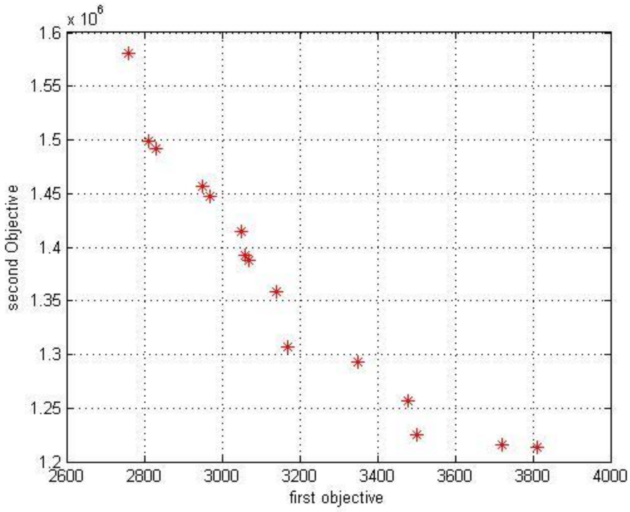

Fig 8. Pareto solutions obtained for problem 10

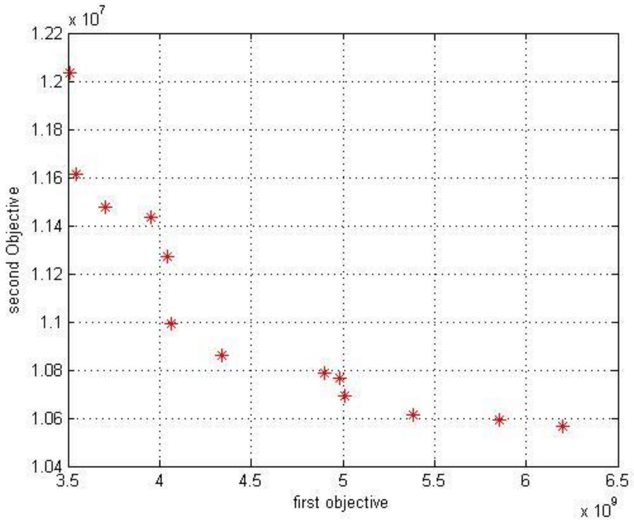

Fig 9. Pareto solutions obtained for problem 17

5. Conclusion

In this work, a bi-objective new mathematical model for a waste collection transportation vehicle routing problem was addressed. An efficient representation for this problem based on order was proposed that decreases computational time and complexity as much as possible. All constraints were addressed in a construction algorithm. For traveling distance constraints that were not addressed in the construction algorithm, a penalty function was defined and one solution penalized if traveling distance limitation was exceed. In this representation, four main components of problem were sorted in chromosomes formation and operators generating new solutions or updating culture were applied on these chromosomes. When all solutions were considered as feasible solutions applying of some operators, such as crossover and mutation, would be easy and executable. This new representation was implemented in two meta-heuristics, namely NSGA-II and MOHCG, which were presented in this paper. In order to show the effectiveness of proposed algorithm several test instances were conducted and the results were compared with each other with respect to six comparison metrics. This experiments showed the effectiveness of the proposed algorithm compared with NSGA-II algorithms which is one of most frequent and effective multi-objective algorithms. For example in diversification metric, all small-sized instances showed that the MOHCG was better than NSGA-II. In other comparison metrics, MOHCG showed the acceptable results compared with NSGA-II. For the future research, we suggest to combine available constraints with other real world constraints. For the reason of importance of time in waste collection problems, time is a critical factor and the time windows constraint can be added to the considered problem. Also, researchers can use other multi-objective evolutionary algorithms and compare their results with each other.

References:

[1] Golden B.L., Assad A.A., Wasil E.A., (2002). Routing vehicles in the real world: applications in solid waste, beverage food, dairy, and newspaper industries. In: Toth, P., Vigo, D.(Eds), The Vehicle Routing Problem. SIAM, Philadelphia, PA, 245-286.

[2] Sbihi A,. Eglese R.W., (2007). Combinatorial optimization and Green Logistics. *4OR*. 5:99-116.

[3] Lin C., Choy K.L., Ho G.T.S., Chung S.H., Lam H.Y., (2014), Survey of Green Vehicle Routing Problem: Past and future trends. *Expert Systems with Applications*. 31: 1118-1138.

[4] Dekker R., Fleischmann M., Inderfurth K., Van Wassenhov L.N., (2004). *Reverse Logistics: quantitative models for closed loop supply chain*. Berlin: Springer.

[5] Wy J., Kim B., Kim S., (2013). The rollon-rolloff waste collection routing problem with time windows, *European Journal of Operational Research*. 224: 466-476.

[6] Bonomo F., Duaran G., Larumbe F., Marenco J., (2012). A method for optimizing waste collection using mathematical programming: A Buenos Aires case study. *Waste Management & Research*. 30(3): 311-324.

[7] Sahoo S., Kim S., Kim B.I., Karas B., Popov Jr., (2005) Routing Optimization for waste management interface. 35(1):24-36.

[8] Kim B.I., Kim S., Sahoo S., (2006). Waste collection vehicle routing problem with time windows. Computers & Operations Research. 33: 3624-3642.

[9] Alumur S., Kara B.Y., (2007), A new model for the hazardous waste location routing problem. *Computers & operations research*; 34:1406-1423.

[10] Erkut E., Verter V., (1995).Hazardous materials logistics. In: Drezner Z, editor. Facility location. New York: Springer

[11] Gianikos I., (1998), A multi-objective programming model for location treatment sites and routing hazardous wastes. *European Journal of Operational Research*. 104: 333-342.

[12] Caballero R., Gonzalez M., Guerrero F.M., Molina J., Paralera C., (2007), Solving a multi-objective location routing problem with a metaheuristic based on tabu search. Application to real case in Andalusia. *European Journal of Operational Research*. 177: 1751-1763.

[13] Martínez-Salazar, I. A., Molina, J., Ángel-Bello, F., Gómez, T., & Caballero, R. (2014). Solving a bi-objective Transportation Location Routing Problem by metaheuristic algorithms. *European Journal of Operational Research*, *234*(1), 25-36.

[14] Desrochers M., Laporte G., (1991). Improvements and extensions to the Miller– Tucker–Zemlin subtour elimination constraints, *Operations Research Letters* 10:27–36.

[15] Kara I., Laporte G., Bektas T., (2004). A note on the lifted Miller–Tucker–Zemlin sub tour elimination constraints for the capacitated vehicle routing problem, *European Journal of Operational Research* 15:8793–795.

[16] Deb K., Agrawal S., Pratab A., Meyunvian T., (2002). A fast elitist non-dominated sorting algorithm for multi-objective optimization NSGA-II. *IEEE Transactions on Evolution Computation*, 2182-2197.

[17] Reynolds, R.G., (1994), An introduction to cultural algorithms. In: A.V. Sebald and L.J. Fogel (Eds.), Proceedings of the Third Annual Conference on Evolutionary Programming, pp. 131–139. World Scientific, River Edge, New Jersey, USA.

[18] Best C., Che X., Reynolds R.G., Liu D., (2010), A multi-objective Cultural Algorithm, Evolutionary Computation (CEC) IEEE Congress on Barcelona, pp 1-9.

[19] Kennedy J, Eberhart R. Particle Swarm Optimization. Proceedings of IEEE International Conference on Neural Networks IV 1995; 1942–1948.

[20] Holland, J. (1975). Adaptation in Natural and Artificial Systems. University of Michigan Press. Second edition, MIT Press, 1992.

[21] Konak, A., Coit, Q. W., and Smith, A. (2006). Multi-objective optimization using genetic algorithms: A tutorial. *Reliability Engineering & System Safety 91(9)*, 992-1007.
[22] Ho W., (2008). A hybrid genetic algorithm for the multi-depot vehicle routing problem, *Engineering Applications of Artificial Intelligence* 21:548–557.
[23] Zitzler E., Thiele L., (1999). Multi-objective evolutionary algorithms: a comparative case study and strength Pareto approaches. *IEEE Transactions on Evolutionary Computation*, 257-271.