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Ranking Decision Making Units with Negative and Positive Input and Output

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A R T I C L E I N F O

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A B S T R A C T

Data Envelopment Analysis (DEA) is a nonparametric method for measuring the efficiency of Decision Making Units (DMUs) which was first introduced by Charnes, Cooper and Rhodes in1978 as the CCR model .One of the most important topics in management science is determining the efficiency of DMUs. DEA technique is employed for this purpose. In many DEA models, the best performance of a DMU is indicated by an efficiency score of one. There is often more than one DMU with this efficiency score. To rank and compare efficient units, many methods have been introduced .Moreover, the main assumption in all DEA models was that all input and output values are positive, but practically, we encounter many cases that violate this term and we ultimately have negative inputs and outputs.

1. Introduction

Data Envelopment Analysis (DEA) is a nonparametric method for measuring the efficiency of the decision making units (DMU) which was first introduced by Charnes, Cooper and Rhodes in (1978) [1] as the CCR model and then BCC model was introduced by Banker, Charnes and Cooper [2] to the realm of operations research and management science. Theoretically, The main assumption in all DEA models was that all input and output values are positive, but practically, we encounter many cases that violate this term and we ultimately have negative inputs and outputs. Among the proposed methods of dealing with negative data, the following models could be provided. Seiford and Zhu [3], considered a positive and very small value of negative output. Another method was proposed by Halme et al.[4], offering the measurement theory and deference of scale variables and the fraction in order to explain the reason for negative observations and also represented a reliable method for assessing interval scale units .The other method which is pervasive is called RDM introduced by Portela et al.[5]. Modified slack-based measure model, called MSBM was represented by Sharp et al.[6].However, the latest method of behavior with negative data was provided by Emrouznejad et al.[7, 8], which is based on SORM model and considered some variables which are both negative and positive for DMUs. This model by using available variable

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changes was not considered as a reliable method. Consequently, radial methods of DEA were modified for the evaluation of the efficiency of units by negative data. Data Envelopment Analysis (DEA) with integer and negative inputs and outputs has been proposed by Jahanshahloo and piri [9]. Also, Super-efficiency in DEA by effectiveness of each unit in society has been extended by Noura et al. [10]. In this paper, we propose a ranking methodology for DMUs with negative and positive inputs and outputs. This paper is organized as follows. In section 2 we calculate efficiency of decision making units with negative and positive input and output. In section 3, we offer a new method for ranking decision making units with negative and positive input and output. Numerical example is provided in section 4 and the paper concludes in section 5.

2. Efficiency of decision making units with negative and positive input and output

One of the key concerns when we have a variable that takes positive values for some and negative values for other DMUs is that its absolute value should rise or fall for the DMU to improve its performance depending on whether the DMU concerned has a positive or negative value on that variable. For example in the case of an output variable, if the DMU has a positive value the output should rise to improve further but it should fall in absolute value so long as it continues to be negative. To overcome this problem we shall treat each variable that has positive values for some and negative for other DMUs as consisting of the sum of two variables as follows. Let us assume we have n DMUs ($DMU_j \;\; j = 1, \dots, n$) each associated with m inputs; $X_j = (x_{1j}, \dots, x_{mj})$ and s outputs; $Y_j = (y_{1j}, \dots, y_{sj})$. Also, let

$$\begin{aligned}
&I = \{i \in \{1, \dots, m\} : x_{ij} \geq 0 , \; j = 1, \dots, n\} \\
&L = \{l \in \{1, \dots, m\} : \exists\, j \in \{1, \dots, n\} ; \text{for which } x_{lj} < 0 \} \\
&R = \{r \in \{1, \dots, s\} : y_{rj} \geq 0 , \; j = 1, \dots, n\} \\
&K = \{k \in \{1, \dots, s\} : \exists\, j \in \{1, \dots n\} ; \text{for which } y_{kj} < 0 \} \\
&I \cup L = \{1, \dots, m\} , \; R \cup K = \{1, \dots, s\}, I \cap L = \emptyset \; , \; R \cap K = \emptyset
\end{aligned} \tag{1}$$

That is, the set of index of inputs with nonnegative values is indicated by I while L denotes the set of index of inputs which have negative value in at least one DMU. Similarly, R is the set of index of the outputs with nonnegative values and K is the set of index of outputs which have a negative value in at least one observation. Let us take an output variable y_k which is positive for some DMUs and negative for others. Let us define two variables y_k^1 and y_k^2 which for the j-th DMU take values y_{kj}^1 and y_{kj}^2 such that

$$\begin{aligned}
&\forall \; k \in K \qquad y_{kj}^1 = \begin{cases} y_{kj} & \text{if } y_{kj} \geq 0 \\ 0 & \text{if } y_{kj} < 0 \end{cases} \\
&\forall \; k \in K \qquad y_{kj}^2 = \begin{cases} 0 & \text{if } y_{kj} \geq 0 \\ -y_{kj} & \text{if } y_{kj} < 0 \end{cases}
\end{aligned} \tag{2}$$

Note that we have

$y_{kj} = y^1_{kj} - y^2_{kj}$ for each $k \in K$ where $y^2_{kj} \geq 0$, $y^1_{kj} \geq 0$, $(j = 1, \dots, n)$.
Similarly, we define two variables x^1_l and x^2_l which for the jth DMU take values x^1_{lj} and x^2_{lj} such that

$$\forall\ l \in L \quad x^1_{lj} = \begin{cases} x_{lj} & \text{if } x_{lj} \geq 0 \\ 0 & \text{if } x_{lj} < 0 \end{cases}$$

$$\forall\ l \in L \quad x^2_{lj} = \begin{cases} 0 & \text{if } x_{lj} \geq 0 \\ -x_{lj} & \text{if } x_{lj} < 0 \end{cases} \qquad (3)$$

We have $x_{lj} = x^1_{lj} - x^2_{lj}$ for each $l \in L$ where $x^2_{lj} \geq 0$, $x^1_{lj} \geq 0$, $(j = 1, \dots, n)$.
Model (4) represents the general case for an input oriented VRS DEA model which has both inputs and outputs which take positive values for some DMUs and negative for others.

$$\begin{array}{lll} \text{Min} & \theta & \\ \text{s.t} & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{ip} & i \in I \\ & \sum_{j=1}^{n} \lambda_j x^1_{lj} \leq \theta x^1_{lp} & l \in L \\ & \sum_{j=1}^{n} \lambda_j x^2_{lj} \geq (2-\theta) x^2_{lp} & l \in L \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{rp} & r \in R \\ & \sum_{j=1}^{n} \lambda_j y^1_{kj} \geq y^1_{kp} & k \in K \\ & \sum_{j=1}^{n} \lambda_j y^2_{kj} \leq y^2_{kp} & k \in K \\ & \sum_{j=1}^{n} \lambda_j = 1 & \\ & \lambda_j \geq 0 & j = 1, \dots, n \end{array} \qquad (4)$$

Based on this optimal solution, we define a DMU as being SORM-Efficient as follows.
Definition 1: (SORM-Efficient). A DMU (x_p, y_p) is SORM-Efficient, if $\theta^* = 1$.

3. Ranking decision making units with negative and positive input and output

In this section we provide a new method for ranking DMUs with negative and positive input and output. We will deal with n DMUs with the input and output matrices $X = (x_{ij}) \in R^{m\times n}$ and $Y = (y_{rj}) \in R^{s\times n}$,respectively.After specifying SORM-efficient DMUs by using model (4), we'll rank them . At first, we divide the inputs and outputs into two groups, as follows:

$D_i = \{x_{ij} | i \in \{1, \dots, m\} : x_{ij} \geq 0,\ j = 1, \dots, n\}$.

$D_l = \{x_{lj} | l \in \{1, \dots, m\} : \exists\, j \in \{1, \dots, n\}; \text{for which } x_{lj} < 0\}$.

$D_r = \{y_{rj} | r \in \{1, \dots, s\} : y_{rj} \geq 0,\ j = 1, \dots, n\}$.

$D_k = \{y_{kj} | k \in \{1, \dots, s\} : \exists\, j \in \{1, \dots n\}; \text{for which } y_{kj} < 0\}$. (5)

1.We choose upper and lower limits for each input and output among efficient DMUs as follows: $E = \{j | \theta^*_j = 1\}$

$$\{\ x^{*l}_i = \min_{j\in E}\ x_{ij} \quad i \in I \qquad \begin{cases} x^{*u}_l = \max_{j\in E} |x_{lj}| & l \in L \\ x^{*l}_l = \min_{j\in E}\ x_{lj} & l \in L \end{cases}$$

$$\{y_r^{*u} = \max_{j\in E}\ y_{rj} \qquad r \in R \qquad \begin{cases} y_k^{*u} = \max_{j\in E}\ y_{kj} & k \in K \\ y_k^{*l} = \min_{j\in E}\ |y_{kj}| & k \in K \end{cases} \tag{6}$$

2. In this step, the inputs and outputs – regarding to definition of sets D_i , D_l , D_r , D_k -are as follows:

$\overline{x_i} = x_i^{*l} \quad \forall i(i \in D_i)$,

$$\begin{cases} \forall l(l \in D_l) \text{ if input of DMU is positive then } \overline{x_l} = x_l^{*l} \\ \forall l(l \in D_l) \text{ if input of DMU is negetive then } \overline{x_l} = x_l^{*u} \end{cases}$$

$\overline{y_r} = y_r^{*u} \quad \forall r(r \in D_r)$

$$\begin{cases} \forall k(k \in D_k) \text{ if output of DMU is positive then } \overline{y_k} = y_k^{*u} \\ \forall k(k \in D_k) \text{if output of DMU is negetive then } \overline{y_k} = y_k^{*l} \end{cases}$$

3. In this step, we define (d_{ij} , d_{lj} , d_{rj} , d_{kj}) for each (DMU_j S.t. $j\epsilon E$)as follows:

$\forall i \in D_i \quad d_{ij} = \frac{\overline{x_i}}{x_{ij}}$

$$\begin{cases} \forall l(l \in D_l) \text{ if input of DMU is positive then } d_{lj} = \frac{\overline{x_l}}{x_{lj}} \\ \forall l(l \in D_l) \text{ if input of DMU is negetive then } d_{lj} = \frac{|x_{lj}|}{\overline{x_l}} \end{cases}$$

$\forall r \in D_r \quad d_{rj} = \frac{y_{rj}}{\overline{y_r}}$

$$\begin{cases} \forall k(k \in D_k) \text{ if output of DMU is positive then } d_{kj} = \frac{y_{kj}}{\overline{y_k}} \\ \forall k(k \in D_k) \text{ if output of DMU is negetive then } d_{kj} = \frac{\overline{y_k}}{|y_{kj}|} \end{cases}$$

This makes both inputs and outputs dimensionless. Notice, the amount of these fractions are less than or equal one. Now, we have the following formula for ranking of these DMUs.

$$R_j = \sum_{i\in I} d_{ij} + \sum_{l\in L} d_{lj} + \sum_{r\in R} d_{rj} + \sum_{k\in K} d_{kj}$$
$$D_i \cup D_l = \{1, \dots, m\},\ D_r \cup D_k = \{1, \dots, s\} \tag{7}$$

It is possible to rank efficient DMUs with higher R_j.

In the next section, we apply the proposed method to an example to determine rank efficient units.

3. A numerical example

Suppose that there are 10 DMUs with two inputs and two outputs shown in Table (1), second input and second output is a positive value for some of DMUs and a negative value for some. So, we have I={1} , L={1}, R={1} and K={1}.

Table 1. 10 DMUs with two inputs and two outputs

	In I1	In L1	OutR1	OutK1	Efficiency
DMU1	2	-1	2	-3	1
DMU2	3	2	4	-2	0.746
DMU3	4	-2	1	2	0.857
DMU4	3	3	2	-6	0.50
DMU5	5	-3	3	2	1
DMU6	2	4	5	-3	0.810
DMU7	6	2	4	-1	0.478
DMU8	3	1	1	3	1
DMU9	4	-4	3	1	1
DMU10	3	3	4	-2	0.848

Since DMU1, DMU5, DMU8, DMU9 are efficient, in order to select the best alternative among them we rank by the proposed method. Then E ={ DMU1,DMU5 , DMU8 , DMU9}.

$$\{ x_i^{*l} = \min_{j\in E} x_{ij} = 2 \quad i \in I \qquad \begin{cases} x_l^{*u} = \max_{j\in E} |x_{lj}| = 4 & l \in L \\ x_l^{*l} = \min_{j\in E} x_{lj} = 1 & l \in L \end{cases}$$

$$\{ y_r^{*u} = \max_{j\in E} y_{rj} = 3 \quad r \in R \qquad \begin{cases} y_k^{*u} = \max_{j\in E} y_{kj} = 3 & k \in K \\ y_k^{*l} = \min_{j\in E} |y_{kj}| = 3 & k \in K \end{cases}$$

$$D_i = \{x1\}, D_l = \{x2\}, D_r = \{y1\}, D_k = \{y2\}$$

Table 2. The results by new method

DMU	DMU1	DMU5	DMU8	DMU9
$\mathbf{R_j}$	**2.916**	**2.816**	**2.999**	**2.833**
Rank	**2**	**4**	**1**	**3**

Table (2) contains the results by new method, by which the rank of each DMU has been determined. DMU8 has highest rating and DMU4 has the lowest rating.

5. Conclusion

The standard DEA model cannot be used for efficiency assessment of decision making units with negative data. The additive model, undesirable DEA, range directional measures (RDM) and modified slack-based model (MSBM) could be used for this case with some limitations. For example the additive model does not give an efficiency measure. The main drawback of the RDM model is that it cannot guarantee projections on the Pareto efficient frontier, as happens with the classical radial DEA model. The semi-oriented radial measure (SORM) overcomes some of the foregoing difficulties, but not all. The SORM model can be used in cases where some DMUs have positive and others negative values on a variable. Further, it can be used for DMUs with negative input and negative output at the same time. In this paper, we calculate efficiency of decision making units with negative and positive input and output. Then we presented a new methodology for ranking efficient DMUs.

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