Carbon Based Closed-Loop Supply Chain Design under Uncertainty using an Interval-Valued Fuzzy Stochastic Programming Approach

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ABSTRACT

Recovery of used products is receiving much attention recently due to growing environmental concern. In this paper, we address the carbon footprint based problem arising in closed-loop supply chain where returned products are collected from customers. These returned products can either be disposed or be remanufactured to be sold as new ones again. Given this environment, an optimization model for a closed-loop supply chain in which the carbon emission is expressed in terms of environmental constraints, namely carbon emission constraints, is developed. These constraints aim at limiting the carbon emission per unit of product supplied with different transportation mode. Here, we design a closed-loop network where capacity limits, single-item management and uncertainty on product demands and returns are considered. First, the fuzzy mathematical programming is introduced for uncertain modeling. Therefore, the statistical approach towards possibility to synthesize fuzzy information is utilized. So, using defined possibilistic mean and variance, we transform the proposed fuzzy mathematical model into a crisp form to facilitate efficient computation and analysis. The model is applied to an illustrative example of an uncertain green supply chain (GSC).

1. Introduction

The global economic growth from the 20th to the 21st century has led to rise in consumption of goods. Consequently, large streams of goods all over the world have been founded. In this way, the production and all aspects of logistics such as transportation, warehousing and inventories have created large environmental problems such as global warming and climate changes [1]. Department of the Environment, Transport and the Regions (DETR) estimated that among the greenhouse gases, CO2 is present in the atmosphere in significant quantities and accounts for two thirds of global warming [2]. Integration of SCM concept with the issue of environment protection confirms sharp decline in pollution problem. Research on this
approach has received considerable attention recently and led to create new research agenda; Green supply chain management (GSCM). So, GSCM is a new paradigm where the supply chain will have a direct relation to the environment. Nowadays, most research on GSCM has had a tendency to the reverse logistics and closed-loop supply chains. In the reverse logistics/closed-loop supply chain systems, a product returns to the manufacturer after use and can be repaired or remanufactured to be delivered again to the end consumers. A top environmental issue for an enterprise is how to reduce the utilization of the materials by reusing and remanufacturing the used products. This brings about the GSCM concept and has led to a problem of the closed-loop supply chain management. With well-managed reverse logistics, the environment protection can be achieved with minimizing of total costs in the whole closed-loop supply chain. Some researchers presented the closed-loop models, but they did not consider the relation between forward and reverse flows in their proposed models. These models often assumed the unlimited capacities for the reverse logistics which is not valid assumption for representing the real situations. In real life situations, the DC also plays such role as a collector in a recovery system. So, the capacity of DC is restricted to both distribution and collection. Now, there is an interaction between amounts of the distribution and the collection so that when the amounts of the collection are larger, then the amounts of distribution must decrease under the same capacity. The closed-loop supply chain is characterized with these interactions. With the lack of such kind of relations, the model can be separated into two parts independently and become a supply chain including forward and reverse chains but not a loop.

Reviewing the literature reported in Table 1, on supply chain management, it is concluded that a few studies consider the relations between forward and reverse logistics [12, 14, 15, 18, 19, and 20]. In this study, we extend the Mohajeri and Fallah's model [20] doing more to approach to the realistic decisions. First of all, basis on the best mechanism to reduce carbon emissions come from freight transport concluded in [20] (Emission-constraint problem), we optimize the proposed closed-loop model in uncertain situation. Due to the capability of fuzzy presentation to engage uncertain factors, fuzzy numbers are used to describe these uncertain factors. Thus, the proposed fuzzy closed-loop model is configured. Second, applying the statistical approach towards possibility, we find a way to synthesize fuzzy information. In the framework of fuzzy programming, possibilistic mean and variance are formulated to transform the proposed fuzzy closed-loop model into a crisp form to facilitate efficient computation and analysis. Thus, the proposed interval closed-loop model is configured in this phase.

Third, given this uncertain environment, we find that the development of decision support procedure is an essential for actual management practice. Therefore, the DM's preference is taken into account in the proposed interval model formulation. Thus, the proposed preference closed-loop model is configured.

To our knowledge, this study is the first paper which formulates the comprehensive closed-loop supply chain model introduced by Mohajeri and Fallah [20] under uncertainty to support more realistic decisions of logistics and facility locations. The remainder of our works is organized as follows. The methodology, including procedure to transform the fuzzy numbers
into crisp numbers is fully explained and justified in section 2. Next, a mathematical programming model of the green supply chain logistics in uncertain situation is developed. In section 4, a numerical example of an uncertain GSC to illustrate the effectiveness of the proposed model is given. Finally, conclusions are drawn.

2. Methodology

Here, a procedure to transform the fuzzy numbers into crisp numbers without losing any information is developed. In the following, the details of transforming are described. Figure 1 shows the proposed resolution procedure.

![Figure 1. The resolution procedure](image)

With respect to the Figure 1, a solution procedure to cope with the uncertain environment is discussed below. First, a general fuzzy linear model is introduced to determine the $n$ decision variables, $x$, subjective to $M$ constraints with $m$ " $\geq$ " constraints, and $M - m$ " $\leq$ " constraints.
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2.1. General Fuzzy Programming Model (GFPM)

\[
\begin{align*}
\text{min } & \mathbf{c}^T \mathbf{x} \\
\text{s.t. } & D \mathbf{x} \geq \mathbf{G} \\
& E \mathbf{x} \geq \mathbf{G} \\
& x \geq 0
\end{align*}
\]

(1)

By defining a fuzzy set \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | \mu_{\tilde{A}}(x) \in [0,1], \forall x \in \mathbb{R} \} \) where \( \mu_{\tilde{A}}(x) \) is the membership function. A crisp set of elements which belong to a fuzzy set \( \tilde{A} \) at least to a degree of \( \gamma \) is called a \( \gamma \)-level set of \( \tilde{A} \) defined by \( A_\gamma = \{x \in \mathbb{R} | \mu_{\tilde{A}}(x) \geq \gamma, 0 \leq \gamma \leq 1 \} \). If \( \tilde{A} \) is a fuzzy number, then for each \( \gamma \)-level, \( A_\gamma \) is a closed interval that can be defined by its lower and upper bounds as \([a_L(\gamma), a_U(\gamma)]\).[21] Since the mean of occurrence is always an ideal index for the DMs to make a decision in the uncertain environment, we consider this concept for defuzzification process. When a fuzzy number is defuzzified by its level set, the information is disaggregated into an interval set. Integrating all levels is a way to collect the whole information [22]. With respect to this reference, the possibilistic mean of a fuzzy number is shown below.

\[
\bar{M}(\tilde{A}) = \int_0^1 \gamma [a_L(\gamma) + a_U(\gamma)] 2d\gamma
\]

(2)

\[
\frac{1}{2} \left( \frac{1}{\gamma} \int_0^{a_L(\gamma)} d\gamma + \frac{1}{\gamma} \int_0^{a_U(\gamma)} d\gamma \right) - \frac{1}{2} \left( M_s(\tilde{A}) + M^*(\tilde{A}) \right)
\]

These results in a crisp interval is denoted by \( M(\tilde{A}) = [M_s(\tilde{A}), M^*(\tilde{A})] \) and is closed by the lower and upper posibilistic mean values of \( \tilde{A} \).

Then, the variance of the fuzzy number is given as below [22]:

\[
\text{Var}(\tilde{A}) = \int_0^1 \gamma \left[ \frac{a_L(\gamma) + a_U(\gamma)}{2} - a_L(\gamma) \right]^2 + \left[ \frac{a_L(\gamma) + a_U(\gamma)}{2} - a_U(\gamma) \right]^2 d\gamma
\]

(3)

\[
\frac{1}{2} \int_0^1 \gamma \left[ a_U(\gamma) - a_L(\gamma) \right]^2 d\gamma
\]

If \( \tilde{A} = (a, S_L, S_U) \) is a triangular fuzzy number with center \( a \); left spread from \( a \) is \( S_L \), and right spread from \( a \) is \( S_U \), then,

\[
\bar{M}(\tilde{A}) = \int_0^1 \gamma [a - (1 - \gamma)S_L + a + (1 - \gamma)S_U] d\gamma = a + \frac{S_U - S_L}{6}
\]

(4)
The confidence interval for an asymmetric triangular distribution is 
\[ [a-(1-w)S_L, a+(1-w)S_U] \]
with a \( 1\)-\( w \) confidence level, where, \( w \) is the level of significance.

Thus, the expected crisp interval can be derived by:

\[
M(\tilde{\lambda}) = \left[ \int_0^1 y[a-(1-\gamma)S_L] \, dy \int_0^1 y[a+(1-\gamma)S_U] \, dy \right] = \left[ a - \frac{S_L}{3}, a + \frac{S_U}{3} \right]
\]

If it is a symmetric triangular fuzzy number, \( S_L = S_U \), then, \( \overline{M}(\tilde{\lambda}) = a \), \( \text{Var}(\tilde{\lambda}) = \frac{S^2}{6} \) and

\[
M(\tilde{\lambda}) = \left[ a - \frac{S}{3}, a + \frac{S}{3} \right].
\]

Now, in the following, the general fuzzy programming model (GFPM) is transformed into an interval program to represent the uncertainty by the expected crisp intervals.

### 2.2. General Interval Programming Model (GIPM)

\[
\begin{align*}
\text{Min} & \quad [c_L^T, c_U^T]^T x \\
\text{s.t.} & \quad [D_L, D_U] x \geq [F_L, F_U] \\
& \quad [E_L, E_U] x \leq [G_L, G_U] \\
& \quad x \geq 0
\end{align*}
\]

To transform the interval programming model into a linear crisp form to facilitate efficient computation and analysis, a decision maker's preference is integrated into the model. The parameter \( \theta \) is defined to reflect the DM's preference[23].

So, the interval programming model (GIPM) is transformed into a linear form with the two preference parameters, \( \theta_1 \) and \( \theta_2 \), as shown below.

### 3. The proposed closed-loop modeling for uncertain logistic

This section focuses on the uncertain issues of green supply chain (GSC) management. Demand, landfilling, and recovery rates are the basic factors that contribute to the uncertainty in management [24] and [25]. Basis on the best closed-loop model concluded in [20] (Constraint-based model), fuzzy programming is adopted in the following to cope with the three major uncertain factors.

#### 3.1. The proposed fuzzy programming model

Here, all of the uncertain parameters are first described as fuzzy numbers. Then, an uncertain mathematical programming model (fuzzy programming model) is proposed. In the proposed model, some of the objectives and constraints of the constraint-based closed-loop model [20] are replaced with the following equations to consider the fuzziness issue. In order to formulate this fuzzy model mathematically, the following changes are necessary:

Parameters:

\( \tilde{pr}_k \) Uncertain recovery percentage of customer \( k \)
\( \tilde{P}_m \) Uncertain landfiling rate of dismantler \( m \)
\( \tilde{d}_{ck} \) Uncertain demand of customer \( k \)

Using these definitions, the fuzzy objective of green closed-loop logistics model is defined as follows:

**Objective function:**

\[
\min \tilde{f} = \sum_{i \in I} \alpha_i \cdot FM_i + \sum_{j \in J} \beta_j \cdot FDC_j + \sum_{m \in M} \gamma_m \cdot FD_m + \sum_{i \in I} \sum_{j \in J \cap V_i} \sum_{v \in V_j} y_{MD_{ijv}} \cdot \text{dis}_MD_{ijv} \cdot CMD_{v_i} \\
+ \sum_{j \in J} \sum_{k \in K_j \cap V_j} \sum_{v \in V_j} y_{DC_{jkv}} \cdot \text{dis}_DC_{jk} \cdot CDC_{v_j} + \sum_{k \in K} \sum_{j \in J \cap V_j} \sum_{v \in V_j} y_{CC_{kjk}} \cdot \text{dis}_CC_{kj} \cdot CDC_{v_j} \\
+ \sum_{m \in M} \sum_{i \in I \cap V_m} \sum_{v \in V_{ij}} y_{CD_{ijv}} \cdot \text{dis}_CD_{ij} \cdot CDM_{v_m} \cdot CDM_{v_i} + \sum_{i \in I} PM_i \cdot P_{\text{cost}i} + \\
\sum_{k \in K_j \cap V_j} \sum_{v \in V_j} y_{CD_{ijv}} \cdot R_{kj} + CL \cdot \sum_{m \in M} \left[ \tilde{P}_m \cdot \sum_{j \in J \cap V_j} \sum_{v \in V_j} y_{DD_{ijv}} \right]
\]

where,

- \( P_{\text{cost}i} \) Unit cost of production in manufactory \( i \)
- \( CMD_{v_i} \) Unit cost of transportation from manufactory to DC by vehicle \( v_i \) per km
- \( CDC_{v_j} \) Unit cost of transportation from DC to customer by vehicle \( v_j \) per km
- \( CDM_{v_m} \) Unit cost of transportation from dismantler to manufactory by vehicle \( v_m \) per km
- \( FM_i \) Fixed cost for operating manufactory \( i \)
- \( FDC_j \) Fixed cost for operating DC\( j \)
- \( FD_m \) Fixed cost for operating dismantler \( m \)
- \( CL \) Fixed cost for landfilling per unit
- \( \text{dis}_MD_{ij} \) Distance between manufactory \( i \) and DC \( j \)
- \( \text{dis}_DC_{jk} \) Distance between DC \( j \) and customer \( k \)
- \( \text{dis}_CC_{kl} \) Distance between customer \( k \) and customer \( l \)
- \( \text{dis}_DD_{jm} \) Distance between DC \( j \) and dismantler \( m \)
- \( \text{dis}_DM_{mi} \) Distance between dismantler \( m \) and manufactory \( i \)
- \( R_{kj} \) The recovery cost in DC \( j \) from customer \( k \)
- \( \alpha_i \) \( \begin{cases} 1, & \text{if production takes place on manufactory } i \\ 0, & \text{o.w.} \end{cases} \)
- \( \beta_j \) \( \begin{cases} 1, & \text{if DC } j \text{ is opened} \\ 0, & \text{o.w.} \end{cases} \)
- \( \gamma_m \) \( \begin{cases} 1, & \text{if dismantler } m \text{ is opened} \\ 0, & \text{o.w.} \end{cases} \)
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\[ y_{MD_{ijv}} \] amount shipped by vehicle \( v_j \) from manufactory \( i \) to DC \( j \)

\[ y_{DC_{j lv}} \] amount shipped by vehicle \( v_j \) from DC \( j \) to customer \( k \)

\[ y_{DD_{jmv}} \] amount recovered product shipped by vehicle \( v_j \) from DC \( j \) to dismantler \( m \)

\[ y_{CD_{k lv}} \] recovered amount shipped by vehicle \( v_j \) from customer \( k \) to DC \( j \)

\[ y_{DM_{miv}} \] reused amount shipped by vehicle \( v_m \) from dismantler \( m \) to manufactory \( i \)

\[ y_{CC_{k lv}} \] recovered amount shipped by vehicle \( v_j \) from customer \( k \) to customer \( l \)

\[ PM_j \] quantity produced at manufactory \( i \)

And the fuzzy constraints of green closed-loop logistics model are defined as follows:

\[
cong_{R_k} \geq \left( \sum_{v_j \in V_j} \sum_{l \in k} z_{CC_{k lv}} \cdot cong_{R_l} \right) + \bar{d}_{c_k}, \quad \forall k \in K, \quad (10)
\]

\[
cong_{F_k} \geq \left( \sum_{v_j \in V_j} \sum_{l \in k} z_{CC_{k lv}} \cdot cong_{F_l} \right) + \left( \bar{\tilde{p}}_{r_k} \bar{d}_{c_k} \right), \quad \forall k \in K, \quad (11)
\]

\[
y_{CC_{k lv}} \geq \left( \sum_{j \in J} y_{DC_{j lv}} + \sum_{l \in K} y_{CC_{k lv}} \right) - \left( \left( 1 - \bar{\tilde{p}}_{r_k} \right) \cdot \bar{d}_{c_k} \right) - 1 \cdot M \left( 1 - z_{CC_{k lv}} \right), \quad (12)
\]

\[
\sum_{v_j \in V_j} \sum_{j \in J} y_{DD_{jmv}} \leq \left( \bar{p}_{l_m} \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{DD_{jmv}} \right) + \sum_{v_m \in V_m} \sum_{i \in I} y_{DM_{miv}}, \quad \forall m \in M, \quad (13)
\]

\[
\sum_{v_m \in V_m} \sum_{i \in I} y_{DM_{miv}} + \left( \bar{p}_{l_m} \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{DD_{jmv}} \right) \leq \gamma \cdot Cd_{m}, \quad \forall m \in M, \quad (14)
\]

where,

\[ Cd_m \] Capacity of dismantler \( m \)

\[ z_{CC_{k lv}} \] \[1, \] if a product can be shipped by vehicle \( v_j \) from customer \( k \) to customer \( l \)

\[ 0, \] o.w.

\[ cong_{R_k} \] amount of congested product at customer \( k \)

\[ cong_{F_k} \] amount of congested recovered product at customer \( k \)

The Constraint (10) indicates the possible amount of congested product for supplying other customers by each customer from an uncertain customer demand. The amount of congested product that is uncertain under uncertain recovery rate and customer demand for recovering from other customers by each customer is indicated by Constraint (11). Customer demand is uncertain, and the recovery amount depends on the uncertain recovery percentage of an uncertain demand. Thus, the derived degree of uncertainty in the reverse chain is more than that in the forward chain. The possible amount of flow among customers from both an
uncertain recovery rate and an uncertain customer demand is represented by Constraint (12). The Constraints (13 and 14) pertain to the uncertain landfilling rate that reverts the reuse amount back to the forward chain. The three uncertain factors have a very close relation, which causes the whole closed-loop chain to become a highly uncertain environment.

### 3.2. The proposed interval programming model

In order to represent the uncertainty by the expected crisp intervals of the corresponding fuzzy numbers, the proposed fuzzy programming model is transformed into an interval program as shown below.

**Notations:**
- $I$: set of candidate manufactories
- $J$: set of candidate DCs
- $K$: set of customers
- $M$: set of candidate dismantlers
- $V$: set of transport mode types
- $V_I$: set of transport mode types at manufactary; $V_I \subset V$
- $V_J$: set of transport mode types at DC; $V_J \subset V$
- $V_M$: set of transport mode types at dismantler; $V_M \subset V$

**Parameters:**
- $Cm_i$: Capacity of manufactary $i$
- $Tc_j$: Total capacity of DC $j$ (forward & reverse)
- $Pc_j$: The percentage of total capacity for reverse logistics in DC $j$
- $p^L_k$: The lower bound of possibilistic mean value of fuzzy percentage of recovery of customer $k$
- $p^U_k$: The upper bound of possibilistic mean value of fuzzy percentage of recovery of customer $k$
- $p^L_m$: The lower bound of possibilistic mean value of fuzzy percentage of landfilling of dismantler $m$
- $p^U_m$: The upper bound of possibilistic mean value of fuzzy percentage of landfilling of dismantler $m$
- $d^L_c$: Lower bound of possibilistic mean value of fuzzy demand of customer $k$
- $d^U_c$: Upper bound of possibilistic mean value of fuzzy demand of customer $k$
- $t_{DC,jv_k}$: The time of transportation from DC $j$ to customer $k$ using vehicle $v_j$
- $t_{CC,kv_l}$: The time of transportation from customer $k$ to customer $l$ using vehicle $v_j$
- $a_{-c_k}$: The lower bound of expected time for delivering product at customer $k$
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\[ b_{-c_k} \] The upper bound of expected time for delivering product at customer \( k \)

\[ NVM_{iv_i} \] Number of vehicle \( v_i \) at manufacture \( i \)

\[ NVD_{ij} \] Number of vehicle \( v_j \) at DC \( j \)

\[ NVD_{im} \] Number of vehicle \( v_m \) at dismantler \( m \)

\[ CVM_{vi} \] Capacity of vehicle \( v_i \)

\[ CVD_{vj} \] Capacity of vehicle \( v_j \)

\[ CVD_{vm} \] Capacity of vehicle \( v_m \)

\[ LO_{i}^{max} \] Maximum load for vehicle \( v_i \)

\[ LO_{j}^{max} \] Maximum load for vehicle \( v_j \)

\[ LO_{m}^{max} \] Maximum load for vehicle \( v_m \)

\[ LF_{i} \] Average load factor for vehicle \( v_i \)

\[ LF_{j} \] Average load factor for vehicle \( v_j \)

\[ LF_{m} \] Average load factor for vehicle \( v_m \)

\[ vol \] Volume of product

\[ \rho_{vi} \] Density of product for vehicle \( v_i \)

\[ wp \] Weight of product

\[ capw \] Total capacity of cargo vessel

\[ Q \] The maximum number of nodes a salesman may visit

\[ L \] The minimum number of nodes a salesman must visit

\[ M \] A large number

\[ CEF \] Constant emission factor

\[ VEF \] Variable emission factor

\[ FC_{v_j} \] The fuel consumption for vehicle \( v_j \)

\[ FE_{v_j} \] The fuel emissions for diesel fuel for vehicle \( v_j \)

\[ FC_{M} \] The fuel consumption for semi-trailer stated in manufactory

\[ FE_{M} \] The fuel emissions for diesel fuel for semi-trailer stated in manufactory

\[ FC_{Di} \] The fuel consumption for semi-trailer stated in dismantler

\[ FE_{Di} \] The fuel emissions for diesel fuel for semi-trailer stated in dismantler

\[ T \] The fuel consumption factor for diesel train

\[ FER \] The fuel emissions for diesel train

\[ W_{gr} \] The gross weight of the train

\[ FEW \] The fuel emissions for cargo vessel

\[ FCW \] The fuel consumption for cargo vessel

\[ EM_{Average} \] The average carbon emissions of the entire system

**Decision variables:**
Using these definitions, the interval programming model for the constraint-based closed-loop chain can be described as follows:
Objective function:

\[ f = \sum_{i \in I} \alpha_i \cdot F_{m} + \sum_{j \in J} \beta_j \cdot F_{D} + \sum_{m \in M} y_{m} \cdot F_{D} + \sum_{i \in I} \sum_{j \in J} \sum_{v \in V} y_{MD_{ijv}} \cdot dis_{MD_{ij}} \cdot CMD_{v} + \sum_{j \in J} \sum_{k \in K} \sum_{v \in V} y_{DC_{kfv}} \cdot dis_{DC_{jk}} \cdot CDC_{v} + \sum_{k \in K} \sum_{j \in J} \sum_{v \in V} y_{CC_{kfv}} \cdot dis_{CC_{kl}} \cdot CDC_{v} + \sum_{j \in J} \sum_{m \in M} \sum_{v \in V} y_{DD_{mvj}} \cdot dis_{DD_{jm}} \cdot CDC_{v} + \sum_{m \in M} \sum_{i \in I} \sum_{v \in V} y_{DM_{miv}} \cdot dis_{DM_{mi}} \cdot CDM_{v} + \sum_{i \in I} \sum_{j \in J} \sum_{v \in V} y_{CD_{kfv}} \cdot R_{ck} + CL \cdot \sum_{m \in M} \left[ P_{Lm}^{L} \cdot P_{Lm}^{U} \sum_{j \in J} \sum_{v \in V} y_{DD_{mvj}} \right] \]  

(15)

Constraints:

\[ \sum_{i \in I} \alpha_i \geq 1, \]  

(16)

\[ \sum_{j \in J} \beta_j \geq 1, \]  

(17)

\[ PM_{i} \geq 1 - M(1 - \alpha_{i}), \quad \forall i \in I, \]  

(18)

\[ \sum_{v \in V} \sum_{j \in J} x_{MD_{ijv}} \geq 1 - M(1 - \alpha_{i}), \quad \forall i \in I, \]  

(19)

\[ \sum_{v \in V} \sum_{i \in I} x_{MD_{ijv}} \geq 1 - M(1 - \beta_{j}), \quad \forall j \in J, \]  

(20)

\[ y_{MD_{ijv}} \geq 1 - M\left(1 - x_{MD_{ijv}} \right), \quad \forall i \in I, \forall j \in J, \forall v \in V, \]  

(21)

\[ \sum_{v \in V} \sum_{j \in J} y_{MD_{ijv}} \leq C_{m{i}}, \quad \forall i \in I, \]  

(22)

\[ \sum_{j \in J} x_{MD_{ijv}} \leq NVM_{iv}, \quad \forall i \in I, \forall v \in V, \]  

(23)

\[ \sum_{v \in V} x_{MD_{ijv}} \leq 1, \quad \forall i \in I, \forall j \in J, \]  

(24)

\[ wp \cdot y_{MD_{ijv}} \leq CVM_{vi} \cdot LF_{M_{iv}}, \quad \forall i \in I, \forall j \in J, \forall v \in V, \]  

(25)

\[ \sum_{v \in V} \sum_{j \in J} x_{DC_{kfv}} \geq 1 - M(1 - \beta_{j}), \quad \forall j \in J, \]  

(26)

\[ \sum_{v \in V} \sum_{k \in K} x_{CD_{kfv}} \geq 1 - M(1 - \beta_{j}), \quad \forall j \in J, \]  

(27)

\[ \sum_{v \in V} \sum_{j \in J} x_{DC_{kfv}} + \sum_{v \in V} \sum_{j \in J} x_{CD_{kfv}} \leq 1, \quad \forall k \in K, \]  

(28)
\[
\sum_{v_j \in V} \sum_{e \in E} x_{-DC_{jkv}} + \sum_{v_j \in V} \sum_{e \in E} z_{-CC_{ekv}} = 1, \quad \forall k \in K, \quad (29)
\]

\[
\sum_{v_j \in V} \sum_{e \in E} x_{-CD_{jkv}} + \sum_{v_j \in V} \sum_{e \in E} z_{-CC_{ekv}} = 1, \quad \forall k \in K, \quad (30)
\]

\[
\sum_{l \in K} z_{-CC_{ekv}} + \sum_{j \in J} x_{-CD_{jkv}} = \sum_{l \in K} z_{-CC_{ekv}} + \sum_{j \in J} x_{-DC_{jkv}}, \quad \forall k \in K, \forall v_j \in V, \quad (31)
\]

\[
u(k) = u(l) + (Q \cdot z_{-CC_{ekv}}) + (Q - 1) \cdot z_{-CC_{ekv}} \leq Q - 1, \quad \forall k, l \in K, \forall v_j \in V, \quad (32)
\]

\[
u(k) = \left[ (Q - 2) \sum_{v_j \in V} \sum_{e \in E} x_{-DC_{jkv}} \right] \cdot \sum_{v_j \in V} \sum_{e \in E} x_{-CD_{jkv}} \leq Q - 1, \quad \forall k \in K, \quad (33)
\]

\[
\sum_{v_j \in V} \sum_{e \in E} x_{-DC_{jkv}} + \left[ (2 - L) \sum_{v_j \in V} \sum_{e \in E} x_{-CD_{jkv}} \right] \geq 2, \quad \forall k \in K, \quad (34)
\]

\[
cong_{Rk} - \left( \sum_{v_j \in V} \sum_{e \in E} z_{-CC_{ekv}} \cdot cong_{Rk} \right) \geq \left[ dc_k^L, dc_k^U \right], \quad \forall k \in K, \quad (35)
\]

\[
y_{-DC_{jkv}} \geq 1 - M \left( 1 - x_{-DC_{jkv}} \right), \quad \forall k \in K, \forall v_j \in V, \forall j \in J, \quad (36)
\]

\[
y_{-DC_{jkv}} \geq cong_{Rk}, \quad \forall k \in K, \forall v_j \in V, \forall j \in J, \quad (37)
\]

\[
wp \cdot y_{-DC_{jkv}} \leq CVD_{v_j} \cdot LF_{-D_{v_j}}, \quad \forall k \in K, \forall v_j \in V, \forall j \in J, \quad (38)
\]

\[
\sum_{k \in K} x_{-DC_{jkv}} \leq NVD_{v_j}, \quad \forall v_j \in V, \forall j \in J, \quad (39)
\]

\[
\sum_{v_j \in V} \sum_{e \in E} y_{-MD_{ijv}} = \sum_{v_j \in V} \sum_{e \in E} y_{-DC_{jkv}}, \quad \forall j \in J, \quad (40)
\]

\[
cong_{Fk} - \left( \sum_{v_j \in V} \sum_{e \in E} z_{-CC_{ekv}} \cdot cong_{Fk} \right) \geq \left[ dc_k^L, dc_k^U \right], \quad \forall k \in K, \quad (41)
\]

\[
y_{-CD_{jkv}} \geq 1 - M \left( 1 - x_{-CD_{jkv}} \right), \quad \forall k \in K, \forall v_j \in V, \forall j \in J, \quad (42)
\]

\[
y_{-CD_{jkv}} \geq cong_{Fk}, \quad \forall k \in K, \forall v_j \in V, \forall j \in J, \quad (43)
\]

\[
y_{-CC_{ekv}} = \left( \sum_{j \in J} x_{-DC_{jkv}} + \sum_{h \in K} y_{-CC_{hkv}} \right) + M \left( 1 - y_{-CC_{ekv}} \right) \geq - \left( \left[ pr_k^L, pr_k^U \right] \cdot \left[ dc_k^L, dc_k^U \right] \right), \quad \forall k, l \in K, \forall v_j \in V, \forall j \in J \quad (44)
\]

\[
\sum_{v_j \in V} \sum_{e \in E} x_{-DD_{jmv}} \geq 1 - M \left( 1 - y_{m} \right), \quad \forall m \in M, \quad (45)
\]
\[ \sum_{v_j \in V_j} x_{-DD_{jmv}} \leq 1, \quad \forall j \in J, \forall m \in M, \quad (46) \]

\[ \sum_{v_j \in V_j} \sum_{m \in M} x_{-DD_{jmv}} \geq 1 - M(1 - \beta_j), \quad \forall j \in J, \quad (47) \]

\[ y_{-DD_{jmv}} \geq 1 - M\left[1 - x_{-DD_{jmv}}\right], \quad \forall j \in J, \forall m \in M, \forall v_j \in V_j, \quad (48) \]

\[ \sum_{v_j \in V_j} y_{-CD_{jmv}} = \sum_{v_j \in V_j} \sum_{m \in M} y_{-DD_{jmv}}, \quad \forall j \in J, \quad (49) \]

\[ \sum_{v_j \in V_j} \sum_{k \in K} y_{-DC_{jkv}} = \sum_{v_j \in V_j} \sum_{m \in M} y_{-DD_{jmv}}, \quad \forall j \in J, \quad (50) \]

\[ \sum_{m \in M} x_{-DD_{jmv}} \leq \text{NVD}_{jv}, \quad \forall j \in J, \forall v_j \in V_j, \quad (51) \]

\[ wp \cdot y_{-DD_{jmv}} \leq \text{CVD}_{jv} \cdot \text{LF}_{-D_{jv}}, \quad \forall j \in J, \forall m \in M, \forall v_j \in V_j, \quad (52) \]

\[ \sum_{v_j \in V_j} \sum_{m \in M} x_{-DM_{miv}} \leq 1 - M(1 - \gamma_m), \quad \forall m \in M, \quad (54) \]

\[ x_{-DM_{miv}} \leq 1 \quad \forall m \in M, \forall i \in I, \quad (55) \]

\[ \sum_{v_m \in V_M} \sum_{m \in M} x_{-DM_{miv}} \geq 1 - M(1 - \alpha_i), \quad \forall i \in I, \quad (56) \]

\[ y_{-DM_{miv}} \geq 1 - M\left[1 - x_{-DM_{miv}}\right], \quad \forall m \in M, \forall i \in I, \forall v_m \in V_M, \quad (57) \]

\[ \sum_{v_m \in V_M} \sum_{m \in M} y_{-DM_{miv}} + PM_i = \sum_{v_j \in V_j} \sum_{i \in I} y_{-MD_{ijv}}, \quad \forall i \in I, \quad (58) \]

\[ \left(1 - \left[P^L_{m}, P^U_{m}\right]\right) \sum_{v_j \in V_j} \sum_{i \in I} y_{-DD_{jmv}} - \sum_{v_m \in V_M} \sum_{i \in I} y_{-DM_{miv}} \leq [-5, 5], \quad \forall m \in M, \quad (59) \]

\[ \sum_{v_m \in V_M} \sum_{i \in I} y_{-DM_{miv}} + \left[P^L_{m}, P^U_{m}\right] \sum_{v_j \in V_j} \sum_{i \in I} y_{-DD_{jmv}} - y_{-DM_{miv}} \cdot \text{Cd}_{m} \leq [-5, 5], \quad \forall m \in M, \quad (60) \]

\[ \sum_{i \in I} x_{-DM_{miv}} \leq \text{NVD}_{miv}, \quad \forall m \in M, \forall v_m \in V_M, \quad (61) \]

\[ wp \cdot y_{-DM_{miv}} \leq \text{CVD}_{m} \cdot \text{LF}_{-D_{miv}}, \quad \forall m \in M, \forall i \in I, \forall v_m \in V_M, \quad (62) \]

\[ S_k \geq a_{-c_k}, \quad \forall k \in K, \quad (63) \]
\[ S_k \leq b_k \cdot c_k, \quad \forall k \in K, \] (64)
\[ S_k + t \cdot CC_{klv} - M[1 - z \cdot CC_{klv}] \leq S_J, \quad \forall k, l \in K, \forall v_j \in V_J, \] (65)
\[ S_k + t \cdot CC_{klv} + M[1 - z \cdot CC_{klv}] \geq S_l, \quad \forall k, l \in K, \forall v_j \in V_J, \] (66)
\[ t \cdot DC_{jkv} - M[1 - x \cdot DC_{jkv}] \leq S_k, \quad \forall k \in K, \forall v_j \in V_J, \forall v_j \in J, \] (67)
\[ t \cdot DC_{jkv} + M[1 - x \cdot DC_{jkv}] \geq S_k, \quad \forall k \in K, \forall v_j \in V_J, \forall v_j \in J, \] (68)
\[ EM_{total \cdot MD_{ij}} \geq \{CEF + \langle VEF, 0.801 \cdot dis_{\cdot MD_{ij}} \rangle \} - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (69)
\[ e_{u \cdot MD_{ij}} \geq \left( v \cdot \rho_d \cdot EM_{total \cdot MD_{ij}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{M_{ij}}} \right) - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (70)
\[ EM_{total \cdot MD_{ij}} \geq EM_{total \cdot MD_{ij}} \left( \left[ FE_{\cdot M \cdot FC_{\cdot M \cdot \{ dis_{\cdot MD_{ij}} \} \cdot M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}]} \right) \right), \quad \forall i \in I, \forall j \in J, \] (71)
\[ e_{u \cdot MD_{ij}} \geq \left( v \cdot \rho_r \cdot EM_{total \cdot MD_{ij}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{M_{ij}}} \right) - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (72)
\[ EM_{total \cdot MD_{ij}} \geq EM_{total \cdot MD_{ij}} \cdot \left( \left[ MC_{\cdot FC_{\cdot M \cdot \{ dis_{\cdot MD_{ij}} \} \cdot M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}]} \right) \right), \quad \forall i \in I, \forall j \in J, \] (73)
\[ e_{u \cdot MD_{ij}} \geq \left( v \cdot \rho_d \cdot EM_{total \cdot MD_{ij}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{M_{ij}}} \right) - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (74)
\[ EM_{total \cdot MD_{ij}} \geq \left( v \cdot \rho_r \cdot EM_{total \cdot MD_{ij}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{M_{ij}}} \right) - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (75)
\[ e_{u \cdot MD_{ij}} \geq \left( v \cdot \rho_d \cdot EM_{total \cdot MD_{ij}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{M_{ij}}} \right) - M[1 - x \cdot MD_{ija} - x \cdot MD_{ijr} + x \cdot MD_{ijw}], \quad \forall i \in I, \forall j \in J, \] (76)
\[ EM_{total \cdot DC_{jk}} \geq EM_{total \cdot DC_{jk}} \cdot \left( \left[ FE_{\cdot D_{vk}} \cdot FC_{\cdot D_{vk}} \cdot \{ dis_{\cdot DC_{jk}} \} \right) \right) - M[1 - x \cdot DC_{jkv}], \quad \forall k \in K, \forall v_j \in V_J, \] (77)
\[ e_{u \cdot DC_{jk}} \geq \left( v \cdot \rho_d \cdot EM_{total \cdot DC_{jk}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{D_{vk}}} \right) - M[1 - x \cdot DC_{jkv}], \quad \forall k \in K, \forall v_j \in V_J, \] (78)
\[ EM_{total \cdot CC_{klv}} \geq EM_{total \cdot CC_{klv}} \cdot \left( \left[ FE_{\cdot D_{vk}} \cdot FC_{\cdot D_{vk}} \cdot \{ dis_{\cdot CC_{klv}} \} \right) \right) - M[1 - x \cdot CC_{klv}], \quad \forall k, l \in K, \forall v_j \in V_J, \] (79)
\[ e_{u \cdot CC_{klv}} \geq \left( v \cdot \rho_d \cdot EM_{total \cdot CC_{klv}} \right) \left( L_{\max} \cdot _{a \cdot LF \cdot _{D_{vk}}} \right) - M[1 - x \cdot CC_{klv}], \quad \forall k, l \in K, \forall v_j \in V_J, \] (80)
\[ EM_{total \cdot CD_{kJ}} \geq EM_{total \cdot CD_{kJ}} \cdot \left( \left[ FE_{\cdot D_{vk}} \cdot FC_{\cdot D_{vk}} \cdot \{ dis_{\cdot CD_{kJ}} \} \right) \right) - M[1 - x \cdot CD_{kJ}], \quad \forall k \in K, \forall v_j \in V_J, \] (81)
Equation (15) is the interval-valued objective function which minimizes cost of opening manufactory, distribution center and dismantler, minimizes the total cost of both forward and backward distance and minimizes the total cost of operations. The Constraints (16) and (17) show that there exists at least one activated manufactory and DC in the chain, respectively. The Constraint (18) ensures that each manufactory can produce an amount of product just after it is selected. Each activated manufactory covers at least one DC, and the Constraints (19) represent this goal. On the contrary, each DC receives at least one link from manufactories just after it is selected (Constraints (20)). The Constraint (21) represents the amount of flow between manufactory and DC. The Constraint (22) represents the limit of the capacity for manufactories in forward logistics. The Constraint (23) imposes that the number of traveled
vehicles from manufactory would not exceed the existing vehicles. The Constraint (24) prevents the route between manufactory and DC from accepting its vehicle more than once. The capacity constraint of each vehicle traveled from manufactory to DC is shown by Constraint (25). The Constraint (26) guarantees that each activated DC covers at least one customer. Each activated DC receives at least one link from customers, and the Constraint (27) represents this goal. The Constraint (28) represent a salesman from DC must visit at least two customers. The Constraint (29) requires that any customer be supplied by either DC or other customer. As well as, it either comeback to DC or supply other customer. This concept is represented by constraint (30). Each customer is supplied and supplies by the same vehicle. This is represented by Constraint (31). The Constraints (32), (33) and (34) prevent any sub tour in network. The Constraint (35) is the interval-valued constraint which indicates the amount of congested product for supplying other customers by each customer. The Constraint (36) represents the amount of flow between DC and customer. The Constraint (37) is to satisfy the customer demand. The capacity constraint of each vehicle traveled from DC to customer is shown by Constraint (38). The Constraint (39) imposes that the number of traveled vehicles from DC would not exceed the existing vehicles. The Constraint (40) satisfies the law of the flow conservation by in-flow equal to out-flow. The amount of congested product for recovering from other customers by each customer is indicated by the interval-valued Constraint (41). The Constraints (42-43) represent the amount of flow between customer and DC. The amount of flow among customers is represented by the interval-valued Constraint (44). The Constraint (45) guarantees that each activated dismantler receives at least one link from DCs. The Constraint (46) prevents the route between DC and dismantler from accepting its vehicle more than once. The Constraint (47) guarantees that each activated DC covers at least one dismantler. The amount of flow between DC and dismantler is shown by Constraint (48). The Constraint (49) satisfies the law of the flow conservation by in-flow equal to out-flow. The Constraint (50) indicates that the total flows of forward and backward cannot exceed the total capacity of DC. The Constraint (51) imposes that the number of traveled vehicles from DC to dismantler would not exceed the existing vehicles. The capacity constraint of each vehicle traveled from DC to dismantler is shown by Constraint (52). The Constraint (53) means the reverse limit of the capacity for DCs. The Constraint (54) ensures that each activated dismantler covers at least one manufacturer. The Constraint (55) prevents the route between dismantler and manufacturer from accepting its vehicle more than once. The Constraint (56) guarantees that each activated manufactory receives at least one link from dismantlers. The amount of flow between dismantler and manufacturer is shown by Constraint (57). The Constraint (58) and the interval-valued Constraint (59) satisfy the law of the flow conservation by in-flow equal to out-flow. The interval-valued Constraint (60) means the reverse limit of the capacity for dismantlers. The Constraint (61) imposes that the number of traveled vehicles from dismantler to manufactory would not exceed the existing vehicles. The capacity constraint of each vehicle traveled from dismantler to manufactory is shown by Constraint (62). The Constraints (63-68) satisfy time windows. The Constraints (69-76) show the emissions allocated to one unit of the product for transportation from the i-th manufactory to the j-th DC. where, $x_{MD_{ij}}^{MD_{ijw}}$ are the binary variables to link carbon emissions
constraints to the related types of transport. The Constraints (69-70), (71-72), (73-74), and (75-76) measure carbon emissions of the aircraft, vehicle, diesel train, and vessel based on NTM method for air transport, road transport, rail transport, and water transport.

The Constraints (77) and (78) show the emissions allocated to one unit of the product for transportation from the j-th DC to the k-th customer. The Constraints (79) and (80) show the emissions allocated to one unit of the product for transportation from the k-th customer to the l-th customer. The Constraints (81) and (82) show the emissions allocated to one unit of the product for transportation from the k-th customer to the j-th DC. The Constraints (83) and (84) show the emissions allocated to one unit of the product for transportation from the j-th DC to the m-th dismantler. The Constraints (77-84) measure carbon emissions of the vehicle based on NTM method for road transport. The Constraints (85-88) show the emissions allocated to one unit of the product for transportation from the m-th dismantler to the i-th manufacturer.

where, \( x_{DM_{mi}} \) and \( x_{DM_{mi}} \) are the binary variables to link carbon emissions constraints to the related types of transport. The Constraints (85-86) and (87-88) measure carbon emissions of the vehicle and diesel train based on NTM method for road transport and rail transport. The carbon emissions constraint is shown by constraint (89). The Constraint (90) denotes the binary variables, and the Constraint (91) restricts all other variables from taking non-negative values.

To transform the interval programming model into a crisp form with the two preference parameters, \( \theta_1 \) and \( \theta_2 \), we act out the following transformation for the interval equations. As equation (15) is an interval equation, we turn it into the following equation,

Equation (15) \( \rightarrow \)

\[
\begin{align*}
f &= \sum_{i \in I} \alpha_i \cdot FM_i + \sum_{j \in J} \beta_j \cdot FDC_j + \sum_{m \in M} \gamma_m \cdot FD_m + \sum_{i \in I} \sum_{j \in J} \sum_{v \in V_j} y_{MD_{ijv}} \cdot dis_{MD_{ij}} \\
&+ \sum_{j \in J} \sum_{k \in K^e} \sum_{v \in V_j} y_{DC_{jkv}} \cdot dis_{DC_{jk}} \cdot CDC_{v_j} + \sum_{k \in K^e} \sum_{v \in V_j} y_{CC_{klv}} \cdot dis_{CC_{kl}} \cdot C \\
&+ \sum_{k \in K^e} \sum_{v \in V_j} \sum_{j \in J} y_{DD_{jmv}} \cdot dis_{DD_{jm}} \\
&+ \sum_{m \in M} \sum_{i \in I} \sum_{v \in V_m} y_{DM_{miv}} \cdot dis_{DM_{mi}} \cdot CD_{m_i} + \sum_{i \in I} \sum_{j \in M} \sum_{v \in V_j} y_{DD_{jmv}} \\
&+ \sum_{k \in K} \sum_{j \in J} \sum_{v \in V_j} y_{CD_{kjv}} \cdot RC_{kj} + CL \cdot PM_i \cdot P_{cost_i} + \\
&\left( \sum_{m \in M} \sum_{i \in I} \sum_{v \in V_m} y_{CD_{kjv}} \cdot RC_{kj} + CL \cdot PM_i \cdot P_{cost_i} \right) / 2 \cdot \sum_{j \in J} \sum_{v \in V_j} y_{DD_{jmv}}
\end{align*}
\]

As Constraint (35) is an interval equation, we turn it into the following equations,

Equation (35) \( \rightarrow \)

\[
\begin{align*}
congR_k &= \left( \sum_{v \in V_j} \sum_{l \in k} z_{CC_{klv}} \cdot congR_l \right) \geq dc^L_k, \quad \forall k \in K,
\end{align*}
\]
\[
\theta_i(d_{k}^U - d_{k}^L) \geq -2\text{cong}_{R_k} + \left( \sum_{v_j \in V_j} \sum_{l \in k} 2(z_{-CC_{klv_j}}) \cdot \text{cong}_{F_l} \right) + (d_{k}^L + d_{k}^U),
\]
\forall k \in K, \tag{94}

As Constraint (41) is an interval equation, we turn it into the following equations,

Equation (41) →
\[
\text{cong}_{F_k} - \left( \sum_{v_j \in V_j} \sum_{l \in k} z_{-CC_{klv_j}} \cdot \text{cong}_{F_l} \right) \geq d_{k}^L \cdot p_{k}^L, \quad \forall k \in K, \tag{95}
\]

\[
\theta_i\left((d_{k}^L \cdot p_{k}^U) - (d_{k}^L \cdot p_{k}^L)\right) \geq -2\text{cong}_{F_k} + \left( \sum_{v_j \in V_j} \sum_{l \in k} 2(z_{-CC_{klv_j}}) \cdot \text{cong}_{F_l} \right)
+ \left( (d_{k}^L \cdot p_{k}^L) + (d_{k}^U \cdot p_{k}^L) \right), \quad \forall k \in K, \tag{96}
\]

As Constraint (44) is an interval equation, we turn it into the following equations,

Equation (44) →
\[
y_{-CC_{klv_j}} - \left( \sum_{j \in J} y_{-DC_{jlv_j}} + \sum_{h \in K} y_{-CC_{hlv_j}} \right) + M\left(1 - z_{-CC_{klv_j}}\right) \geq 0,
\quad \forall k, l \in K, \forall v_j \in V_j
\]
\[
\left(p_{k}^L \cdot d_{k}^L\right) - d_{k}^L, \tag{97}
\]

\[
\theta_i\left((p_{k}^U \cdot d_{k}^L) - d_{k}^U\right) - \left((p_{k}^L \cdot d_{k}^L) - d_{k}^L\right) \geq -2y_{-CC_{klv_j}}
+ \left( \sum_{j \in J} 2(y_{-DC_{jlv_j}}) + \sum_{h \in K} 2(y_{-CC_{hlv_j}}) \right) - M\left(1 - 2(z_{-CC_{klv_j}})\right)
+ \left((p_{k}^L \cdot d_{k}^L) - d_{U}\right) \quad \forall k, l \in K, \forall v_j \in V_j, \tag{98}
\]

As Constraint (59) is an interval equation, we turn it into the following equations,

Equation (59) →
\[
\left(1 - p_{l}^U\right) \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{-DD_{jmv_j}} - \sum_{v_m \in V_M} \sum_{i \in I} y_{-DM_{miv_m}} \leq 5, \quad \forall m \in M, \tag{99}
\]
\[
\left( 1 - p_{l}^U \right) + \left( 1 - p_{l}^U \right) \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{-DD_{jmv_j}} - \sum_{v_m \in V_M} \sum_{i \in I} y_{-DM_{miv_m}} \leq
10\theta_2 + \theta_2 \cdot \left(1 - p_{l}^U\right) \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{-DD_{jmv_j}}, \quad \forall m \in M, \tag{100}
\]

As Constraint (60) is an interval equation, we turn it into the following equations,

Equation (60) →
\[
\sum_{v_m \in V_M} \sum_{i \in I} y_{-DM_{miv_m}} + p_{l}^U \cdot \sum_{v_j \in V_j} \sum_{j \in J} y_{-DD_{jmv_j}} - y_m \cdot Cd_m \leq 5, \quad \forall m \in M, \tag{101}
\]
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\[
\sum_{\nu_i \in V_i} \sum_{i \in I} 2y_{-DM_{miv}} + \left(p_{l_m}^U + p_{l_m}^L\right) \sum_{\nu_j \in V_j} \sum_{j \in J} y_{-DD_{jmiv}} - 2\gamma_m \cdot Cd_m \leq 10\theta_2 + \theta_2 \left[p_{l_m}^U - p_{l_m}^L\right] \sum_{\nu_j \in V_j} \sum_{j \in J} y_{-DD_{jmiv}}, \quad \forall m \in M.
\] (102)

4. Numerical illustration

To demonstrate the applicability of the proposed methodology, a numerical example fuzzified from the deterministic case given in [20] will be presented in the following with the parameter analysis.

Step-by-Step procedure

The procedure of analysis follows the following steps:

**Step 1: Specify the input and define the membership functions of the fuzzy data.**

In this example, there are three manufactories (i), four DCs (j), seven customers (k), and two dismantlers (m) with €2 unit landfilling cost (\(Cd_m\)). There are four types of transportation mode (air, rail, road, and water) used to transfer product from manufactories to DCs, one type of transportation mode (road) used to transfer product from DCs to customers and dismantlers, and two types of transportation mode (rail and road) used to transfer product from dismantlers to manufactories. The average carbon emissions of the entire system is set to be 1431(kg).

The input data with the fuzzified parameters \(\bar{p}_{r_k}\), demand \(\bar{d}_{c_k}\) for each customer \(k\), and \(\bar{P}_{l_m}\) for dismantler \(m\) are listed in Tables 2 and 3. These fuzzy inputs are assumed to be symmetric triangular and denoted by their respective mode and spread as (mode, spread).

| Table 2. The input fuzzy recovery percentage (\(\bar{p}_{r_k}\)) and demand (\(\bar{d}_{c_k}\)) (mode, spread) |
|---|---|---|---|---|---|---|---|
| \(\bar{p}_{r_k}\) | \(\bar{d}_{c_k}\) |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| (0.1, 0.1) | (0.3, 0.1) | (0.5, 0.1) | (0.2, 0.1) | (0.8, 0.1) | (0.1, 0.1) | (0.4, 0.1) |
| (20, 5) | (18, 5) | (10, 5) | (12, 5) | (20, 5) | (14, 5) | (10, 5) |

| Table 3. The input fuzzy landfilling rate (\(\bar{P}_{l_m}\)) (mode, spread) |
|---|---|---|
| \(\bar{P}_{l_m}\) |
| 1 | 2 |
| (0.3, 0.1) | (0.38, 0.1) |

**Step 2: Defuzzify the fuzzy number into a crisp interval form by the possibilistic mean.**

Using formula (2), the intervals of possibilistic mean value of fuzziness can be obtained as follows:

\[
M(\bar{p}_{r_1}) = [0.066, 0.133] \\
M(\bar{p}_{r_2}) = [0.266, 0.333] \\
M(\bar{p}_{r_3}) = [0.466, 0.533] \\
M(\bar{p}_{r_4}) = [0.166, 0.233] \\
M(\bar{d}_{c_1}) = [18.33, 21.66] \\
M(\bar{d}_{c_2}) = [16.33, 19.66] \\
M(\bar{d}_{c_3}) = [8.33, 11.66] \\
M(\bar{d}_{c_4}) = [10.33, 13.66] \\
M(\bar{d}_{c_5}) = [18.33, 21.66] \\
M(\bar{d}_{c_6}) = [12.33, 15.66] \\
M(\bar{d}_{c_7}) = [8.33, 11.66]
\]
\[ M(\tilde{p}_1) = [0.266, 0.333] \quad M(\tilde{p}_2) = [0.3466, 0.4133] \]

Assuming that the intervals of zeros for RHS with respect to constraints (59) and (60) are \( M(\tilde{0}) = [-5.5] \), then by calculating the possibilistic mean interval of uncertain parameters, the proposed preference model can be applied.

In addition, the standard deviations of the fuzzy numbers can be calculated from formula (3) as follows:

\[
\sqrt{\text{Var}(\tilde{p}_k)} = \sqrt{\text{Var}(\tilde{p}_m)} = \sqrt{\frac{0.1^2}{6}} = 0.04083, \quad \forall k \in K, \forall m \in M,
\]

\[
\sqrt{\text{Var}(\tilde{d}_c)} = \sqrt{\text{Var}(\tilde{d}_c)} = \sqrt{\text{Var}(\tilde{d}_c)} = \sqrt{\text{Var}(\tilde{d}_c)} = \sqrt{\text{Var}(\tilde{d}_c)} = \sqrt{\frac{5^2}{6}} = 2.04124.
\]

To facilitate the computations in our mixed integer programming (MIP) model, GAMS 22.9 software package is applied. Based on the input data given in [20], our MIP model is solved for the neutral case (with \( \theta_1 = 0 \) and \( \theta_2 = 0 \)). The total objective mean of the cost \( f \) with intervals \( \{M_\text{r}(\tilde{f}), M_\text{r}^*(\tilde{f})\} \), \( 2\sigma \), with 96.63% confidence level are listed in Table 4 as an example. Product flow rates and amount of CO2 (kg) emitted from journeys of selective paths are shown in this table. The visual graph of the solution is shown in Figure 2 as a logistic pattern of the case.

There are five types of connection links in the selective path column:

Links connected between the manufactory and DC is indicated by a-b: [n] format; where, a and b are numbers which indicate selective manufactory and DC, respectively. n is a number which indicates a selective path on the figure. [ ] is a symbol related to this kind of connection links.

- Links connected between the manufactory and DC is indicated by a-b: [n] format; where, a and b are numbers which indicate selective manufactory and DC, respectively. n is a number which indicates a selective path on the figure. [ ] is a symbol related to this kind of connection links.

- Links connected among the customers is indicated by e-f: [n] format; where, e and f are numbers which indicate selective customers. { } is a symbol related to this kind of connection links.

- Links connected between the DC and dismantler is indicated by g-h: [n] format; where, g and h are numbers which indicate selective DC and dismantler, respectively. ( ) is a symbol related to this kind of connection links.

- Links connected between the dismantler and manufactory is indicated by i-j: [n] format; where, i and j are numbers which indicate selective dismantler and manufactory, respectively.  is a symbol related to this kind of connection links.

The optimal closed-loop chain is shown in Figure 2. In this figure, we consider a particular color for each tour in which a salesman depart from selective DCs and arrive to the customers.
So, the selective path given in Table 4 is indicated by different colors. The suitable paths to deliver product to customers from manufactories and DCs in the forward flows, to deliver recovered product to dismantlers from DCs and customers, and to deliver reused product to manufactories from dismantlers in the reverse flows for our model is shown by this figure. As well as, the selected vehicles for carrying product and the corresponding amount of product are illustrated in it which also includes the amount of CO2 (kg) emitted from journeys and the amount of landfills.

Table 4. Optimal solution for a neutral DM of the example

<table>
<thead>
<tr>
<th>Emission-constraint model</th>
<th>Selective path</th>
<th>Amount of product flow</th>
<th>Amount of CO2 (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3-3: [1]</td>
<td>92.31</td>
<td>23.49</td>
</tr>
<tr>
<td></td>
<td>3-5: [1]</td>
<td>34.66</td>
<td>193.55</td>
</tr>
<tr>
<td></td>
<td>3-6: [2]</td>
<td>22.66</td>
<td>153.31</td>
</tr>
<tr>
<td></td>
<td>3-7: [3]</td>
<td>34.99</td>
<td>69.14</td>
</tr>
<tr>
<td></td>
<td>5-2: [1]</td>
<td>29.37</td>
<td>55.52</td>
</tr>
<tr>
<td></td>
<td>6-4: [2]</td>
<td>7.41</td>
<td>39.8</td>
</tr>
<tr>
<td></td>
<td>7-3: [3]</td>
<td>26.71</td>
<td>120.47</td>
</tr>
<tr>
<td></td>
<td>3-1: [4]</td>
<td>19.6</td>
<td>53.87</td>
</tr>
<tr>
<td></td>
<td>2-3: [4]</td>
<td>24.6</td>
<td>236.08</td>
</tr>
<tr>
<td></td>
<td>4-3: [5]</td>
<td>5.27</td>
<td>42.75</td>
</tr>
<tr>
<td></td>
<td>1-3: [6]</td>
<td>14.14</td>
<td>60.15</td>
</tr>
<tr>
<td></td>
<td>3-1: [1]</td>
<td>44</td>
<td>146.48</td>
</tr>
<tr>
<td></td>
<td>1-3:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Landfill</td>
<td>13.2</td>
<td>-</td>
</tr>
</tbody>
</table>

Expected objective value: 220288.12
Interval of the expected objective value: [219768.77, 220807.47]
Root mean square imprecision index (RMSII): 636.071
96.63% confidence interval: [219015.978, 221560.26]
Corresponding to the Table 4, apart from the expected objective value and the interval value, the standard deviation and confidence interval can help the DM to understand the scale of the uncertainty. In this example, where $2\sigma$ is adopted with 3.37% significance level, the DM has 96.63% confidence level to control the solution between $[220288.12 - 2\sigma, 220288.12 + 2\sigma]$ from the current solution.

5. Conclusion

The issue of green supply chain management has received more attention in the last decade. In this work, we introduced an uncertain situation with the corresponding analytical models and solutions. Based on the deterministic closed-loop model introduced by Mohajeri and Fallah [20], a more realistic model to cope with uncertain situation was introduced by adopting the fuzzy approach. To avoid the shortcoming of using level cuts in the conventional solutions, the interval programming approach from the concept of possibilistic mean was introduced so that all information of level cuts can be integrated towards providing an effective solution. Here, when transforming from the fuzzy program to an interval program, the DM's preference was taken into account in the model formulation. Therefore, the applicability and effectiveness of our proposed model was tested through numerical example.
References


