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Presenting an Analytic Hierarchy Process- Assurance Region- Joint Multiple Layer Data Envelopment Analysis Model for Evaluating the Performance of an Educational System: A Case Study

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A B S T R A C T

One of the major problems in organizations is resource limitation, so the main goal of each union is to maximize the usage of their recourses and improve its efficiency. Universities as a main body of each countries educational system have an important role in developing a country. Therefore, assessing the efficiency of universities and improving the quality of them are important goals. The method of the data envelopment analysis (DEA) can rank the efficiency while the number of indicators does not exceed the specific amount. But in measurement of the universities' efficiency, main intention is toward considering a comprehensive set of indicators and in this case discrimination power of the DEA method decreases and its results are unacceptable. In this study, we present a combined model of the joint multiple layer DEA (MLDEA) model and weight restrictions method in order to try to eliminate the weakness of the mentioned method. Educational units of ShahidBahonar University of Kerman are evaluated and ranked as a case study. Empirical results shows the efficiency of presented model based on discrimination power, weight allocating and possibility of implementing this model in evaluating the function of activities, which have many indicators along with hierarchical structure.

1. Introduction

The data envelopment analysis (DEA) is a non-parametric optimization technique. This technique is used for measuring the relative efficiency of a homogeneous set of decision making units (DMUs) on the basis of multiple inputs and multiple outputs which for the first time was presented by Charnes, Cooper and Rhodes (CCR model) [1]. DEA is a powerful tool for evaluating Performance the function. Some of the usage of it is shown in [2-4]. Management is

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becoming more complex nowadays leads to reveal the structural weakness of putting the DEA model in action. There is a large amount of activities for evaluating the performance which needed to be in the group of functional indicators and they may belong to different categories or have hierarchical structure. In this situation, assuming all the input-output indicators in one layer- ignoring the hierarchical structure- leads to weak discrimination power among DMUs and allocating inappropriate weights to indicators.

Therefore for the first time, Meng et al. [5] introduced a layered hierarchy in the DEA model. The main idea of their study was making one the weights of indicators among categories, using the DEA method while the weights within categories (or internal weights) were determined by the weighted sum approach. Because of nonlinear nature of the model while all the weights are deducted from mathematical model, Kao [6] showed that non-linear model of Meng et al. [5] can become a linear model using variable substitutions, which it’s limited to two-layer hierarchy. While Shen et al. [7] proposed a generalized multiple layer DEA (MLDEA) model. They incorporated different types of possible weight restrictions for each category of each layer to their model. Although they used large numbers of indicators, fortunately their model has a high discrimination power.

In addition to structural weakness of DEA model, we can mention to distribution of unreal weights in this model. It happens when we evaluate the DMUs as efficient just because of their input-output weights which are so big or zero. Various techniques of weights restriction were presented to overcome this problem [8 and 9].

In this study, the focal point is on the educational system, it is also known the DEA methodology is especially appropriate to evaluate the efficiency of non-profitable entities including academic departments [10]. In recent years, several studies have been undertaken to analyze the efficiency of academic departments in universities. Some of these main studies utilized DEA model include [11-13]. One of these previous studies on DEA applications which are related to university departments is the Beasley’s essay. He presented Joint DEA model and succeeded in determining the teaching and research efficiencies of DMUs at the same time [14]. Kuah and Wong [15] used the joint DEA model in their studies. The results of their studies showed that although Joint DEA model is good in determining teaching and research efficiencies, it couldn’t discriminate DMUs effectively and reach to appropriate ranking of DMUs.

So far there is no model which can cover all the mentioned cases (weak discriminating power among DMUs, unrealistic weight allocations, investigating educational and research activities simultaneously) at the same time in the DEA model, after doing research and studies, Joint DEA model for evaluating the performance of academic departments in determining teaching and research efficiencies is used. To improve the Joint DEA model in discrimination power and considering hierarchical structure of input-output indicators, the concept of MLDEA model is putted into the joint DEA model and a combined model of the joint MLDEA model is presented. Finally to make the allocated weights of this model to input-output indicators more real, the authors use weight restrictions using expert’s opinions and add to the joint MLDEA model and finally present a novel analytic hierarchy process - assurance region - joint MLDEA (AHP-AR-Joint MLDEA) model.

The rest of paper is arranged in following order; after a brief and general review of DEA models including the basic DEA, joint DEA, and multiple layers DEA models in Section 2, the way to make the joint MLDEA model and a hybrid AHP-AR-joint MLDEA model are presented in Section 3. In Section 4, the application of this hybrid model to evaluate the academic departments' performance is demonstrated and the results from different models are subsequently provided and compared in Section 5. The paper ends with conclusions and topics for further research in Section 6.

2. Review of DEA Models

In this section, we review the DEA models used in the proposed hybrid model. They include the basic DEA, joint DEA, and multiple layer DEA models.

2.1. Basic DEA Model

Nowadays, the DEA is one of the most widely accepted methods to measure the relative efficiency of homogenous group of DMUs. Consider an n-DMUs set: DMU1, DMU2,…, and DMU*n*. Each DMU*j*, (*j*=1,2,…,*n*) uses m inputs x_{ij} (*i*=1,2,…,*m*) and produces s outputs yrj (r=1,2,…, s). Let the input weights vi (*i*=1,2,…,*m*) and the output weights *ur* (*r*=1,2,…,*s*) as variables. ε is a small non-Archimedean number for preventing the DMU to assign a weight of zero to unfavorable factors [16]. Let DMU*j* to be evaluated on any trail be designated as DMUo (*o*=1,2,…,*n*). The efficiency score of each DMU*o*, *Zo*, is obtained by solving the following linear programming:

$$\begin{aligned} \max Z_0 &= \sum_{r=1}^{s} u_r * y_{ro} \\ \text{s.t.} & \\ &\sum_{i=1}^{m} v_i * x_{io} = 1 \\ &\sum_{r=1}^{s} u_r * y_{rj} - \sum_{i=1}^{m} v_i * x_{ij} \le 0, \quad j = 1,\ldots,n \\ &u_r, v_i \ge \varepsilon, \quad r = 1,\ldots.,s \quad i = 1,\ldots.,m \end{aligned} \tag{1}$$

This linear programming model for all the DMUs separately runs ways to identify the relative optimal efficiency scores by selecting the best possible input and output weights. In general an efficient DMU obtains a score of 1 and an inefficient DMU obtains a score of less than 1.

2.2. Joint DEA Model

Lots of studies have been done related to performance assessment in academic departments, based on teaching and research activities [14 and 15]. The basic DEA model gives the value for the overall efficiency of each department. However, it couldn't determine how efficient each department is at each of its two basic activities, teaching and research. So a type of DEA model called Joint DEA was presented by Beasley [14]. For more information we refer to [14].

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2.3. Multiple Layer DEA Model

Nowadays, performance evaluation became routine action in performance management. It is clear that a single indicator may not to be sufficient for effective performance management in this domain, especially when the performance evaluation is for academic departments and also these indicators might belong to different categories and to be linked to one another constituting a multilayer hierarchical structure [5]. Shen et al. [7] proposed MLDEA model by incorporating different types of possible weight restrictions for each category in each layers. This model is able to discriminate DMUs even with relatively large numbers of indicators. For more information we refer to [7].

3. Proposed Hybrid Model

In this section at first, the joint MLDEA model then the AHP – AR- joint MLDEA model is presented.

3.1. Joint MLDEA Model

In this subsection, the combined joint MLDEA model for evaluating academic departments is presented. The main idea of MLDEA model is to aggregate the values of the input and output factors within a particular category of a particular layer by the weighted sum approach in which the sum of the (internal) weights equals to 1 and about measuring final layers' weights are determined by using the basic DEA [7]. The researcher used this idea to create the combined model. The notations and variables are used in combined model are as follow:

L	the number of inputs layer	L= 1, …, L
K	the number of outputs layer	K= 1, …, K
m^l	the number of input categories in the l^{th} layer (l=1, …, L) $g_l=1, \dots, m^l$	
s^k	the number of output categories in the k^{th} layer (k=1, …, K) $f_k= 1, \dots, s^k$	
$B_{g_l}^{(l)}$	the set of input factors of g^{th} category in the l^{th} layer	
$A_{f_k}^{(k)}$	the set of output factors of f^{th} category in the k^{th} layer	
$x_{g_l j}^{(l)}$	the aggregated performance of DMU_j related to the sets of input layers	
$y_{f_k j}^{(k)}$	the aggregated performance of DMU_j related to the sets of output layers	
v_{g_l}	the weight given to the g^{th} input in the l^{th} layer	
u_{f_k}	the weight given to the f^{th} output in the k^{th} layer	
$p_{g_l}^{(l)}$	the internal weights associated with the factors of the g^{th} category in the l^{th} input layer	
$q_{f_k}^{(k)}$	the internal weights associated with the factors of the f^{th} category in the k^{th} output layer	

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For making new mathematical model, the following notations and decision variables are introduced:

Notations

B_{E_L}	the set of input factors of L[th] layer associated with teaching
$B_{E\&R_L}$	the set of input factors of L[th] layer associated with teaching and research
A_{E_K}	the set of output factors of K[th] layer associated with teaching
A_{R_K}	the set of output factors of K[th] layer associated with research

Decision variables

M_{g_1}	the proportion of g_1 th shared input factor for both teaching and research activities associated with teaching
$1-M_{g_1}$	the proportion of g_1 th shared input factor for both teaching and research activities associated with research
To	the teaching efficiency of under evaluation DMU (DMU_o)
Ro	the research efficiency of under evaluation DMU (DMU_o)

Formulation

$$\max z=\sum_{f_k=1}^{s^{(k)}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1o}}\right)\right)\right)\right) \tag{2}$$

s.t.

$$\sum_{g_L=1}^{m^{(L)}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} p^{(1)}_{g_1} * x^{(1)}_{g_{1o}}\right)\right)\right)\right)=1 \tag{3}$$

$$\begin{aligned}&\sum_{f_k=1}^{s^{(k)}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1j}}\right)\right)\right)\right)-\\ &\sum_{g_L=1}^{m^{(L)}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right)\le 0 \qquad j=1,\ldots,n\end{aligned} \tag{4}$$

$$\begin{aligned}&\sum_{f_k\in A_{E_K}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1j}}\right)\right)\right)\right)-\\ &\sum_{g_L\in B_{E_K}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right)-\\ &\sum_{g_L\in B_{E\&R_K}}^{m^{(L)}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} M_{g_1} * p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right)\le 0 \qquad j=1,\ldots,n\end{aligned} \tag{5}$$

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$$\sum_{f_k\in A_{RK}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1j}}\right)\right)\right)\right) -$$
$$\sum_{g_L\in B_{E\&RK}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} \left(1-M_{g_1}\right) * p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right) \le 0 \qquad j=1,\dots,n \tag{6}$$

$$T_o = \frac{\sum_{f_k\in A_{EK}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1j}}\right)\right)\right)\right)}{\begin{array}{c}\sum_{g_L\in B_{EK}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right) + \\ \sum_{g_L\in B_{E\&RK}}^{m^{(L)}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_l\in B^{L+1}_{g_{L+1}}} p^{(l)}_{g_l}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} M_{g_1} * p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)\right)\end{array}} \tag{7}$$

$$R_o = \frac{\sum_{f_k\in A_{RK}} u_{f_k}\left(\sum_{f_{K-1}\in A^{(K)}_{f_K}} q^{(K-1)}_{f_{K-1}}\left(\cdots\sum_{f_k\in A^{(K+1)}_{f_{k+1}}} q^{(k)}_{f_k}\left(\cdots\sum_{f_2\in A^{(3)}_{f_3}} q^{(2)}_{f_2}\left(\sum_{f_1\in A^{(2)}_{f_2}} q^{(1)}_{f_1} * y^{(1)}_{f_{1j}}\right)\right)\right)\right)}{\sum_{g_L\in B_{E\&RK}} v_{g_L}\left(\sum_{g_{L-1}\in B^{(L)}_{g_L}} p^{(L-1)}_{g_{L-1}}\left(\cdots\sum_{g_2\in B^{(3)}_{g_3}} p^{(2)}_{g_2}\left(\sum_{g_1\in B^{(2)}_{g_2}} \left(1-M_{g_1}\right) * p^{(1)}_{g_1} * x^{(1)}_{g_{1j}}\right)\right)\right)} \tag{8}$$

$$\sum_{g_l\in B^{(l+1)}_{g_{l+1}}} p^{(l)}_{g_l} = 1, \quad p^{(l)}_{g_l} \ge \xi, \qquad f_k = 1,\dots,s^{(k)},, \quad l = 1,\dots,L-1 \tag{9}$$

$$\sum_{f_k\in A^{(k+1)}_{f_{k+1}}} q^{(k)}_{f_k} = 1, \quad q^{(k)}_{f_k} \ge \xi, \qquad f_k = 1,\dots,s^{(k)}, \quad k = 1,\dots,K-1 \tag{10}$$

$$v_{g_L}, u_{f_K} \ge \varepsilon, \qquad f_k = 1,\dots,s^{(k)}, \quad f_k = 1,\dots,s^{(k)} \tag{11}$$

$$\eta \le M_{g_1} \le 1-\eta \qquad g_1 \in B_{E\&R} \tag{12}$$

Equation (2) representing the objective function regarding hierarchical structure of input and output factors, finds the optimum set of weights that give the maximum relative overall efficiency (Zo) to under evaluation DMU (DMUo) and this mentioned equation is subjected to the constraints (3) to (12). Constraint (3) forces the weighted sum of the inputs for DMUo to1. Constraints (4) - (6) are to limit the relative efficiencies (Zo, To, Ro) of all DMUs to be less than 1. Equations (7) and (8) define teaching and research efficiency of DMUo respectively, which are determined with the optimum set of weights obtained for DMUo. Constraints (9) and (10) show the sum of the internal weights is required to be equal to 1. Equation (11) is the weights restriction of the weights, because in this model all weights were required to be strictly positive. In addition; constraint (12) is to prevent zero proportion of the g_1^{th} input in the 1^{th} layer on either function.

It is clear; the efficiency score of each DMU calculated from this Joint MLDEA model will not exceed the efficiency score of the one layer Joint DEA model. Because this Joint MLDEA model is less flexible than the Joint DEA model since the sum of the internal weights in each category of each layer is required to be equal to 1. As a result, it will improve the discriminating power of Joint DEA to a certain extent.

Now, For Simplification this nonlinear joint MLDEA model, it could be transformed to another model by using variable substitution ($\hat{u}_{f_1} = \prod_{k=1}^{K-1} p_{f_k}^{(k)} * u_{f_K},\ f_k \in A_{f_{k+1}}^{(k+1)}, \hat{v}_{g_1} = \prod_{l=1}^{L-1} q_{g_l}^{(l)} * v_{g_L},\ g_l \in B_{g_{l+1}}^{(l+1)}$) has been described in [7].

$$\max z = \sum_{f_1=1}^{s^{(1)}} \hat{u}_{f_1} * y_{f_1 o}^{(1)} \tag{13}$$

$s.t:$

$$\sum_{g_1=1}^{m^{(1)}} \hat{v}_{g_1} * x_{g_1 o}^{(1)} = 1 \tag{14}$$

$$\sum_{f_1=1}^{s^{(1)}} \hat{u}_{f_1} * y_{f_1 o}^{(1)} - \sum_{g_1=1}^{m^{(1)}} \hat{v}_{g_1} * x_{g_1 o}^{(1)} \le 0 \qquad \forall j = 1,\dots,n \tag{15}$$

$$\sum_{f_1 \in A_{EK}^{(K)}} \hat{u}_{f_1} * y_{f_1 j}^{(1)} - \sum_{g_1 \in B_{EL}^{(L)}} \hat{v}_{g_1} * x_{g_1 j}^{(1)} - \sum_{g_1 \in B_{E\&RL}^{(L)}} M_{g_1} * \hat{v}_{g_1} * x_{g_1 j}^{(1)} \le 0 \qquad \forall j = 1,\dots,n \tag{16}$$

$$\sum_{f_1 \in A_{RK}^{(K)}} \hat{u}_{f_1} * y_{f_1 j}^{(1)} - \sum_{g_1 \in B_{E\&RL}^{(L)}} (1 - M_{g_1}) * \hat{v}_{g_1} * x_{g_1 j}^{(1)} \le 0 \qquad \forall j = 1,\dots,n \tag{17}$$

$$T_o = \frac{\sum_{f_1 \in A_{EK}^{(K)}} \hat{u}_{f_1} * y_{f_1 o}^{(1)}}{\sum_{g_1 \in B_{EL}^{(L)}} \hat{v}_{g_1} * x_{g_1 o}^{(1)} + \sum_{g_1 \in B_{E\&RL}^{(L)}} M_{g_1} * \hat{v}_{g_1} * x_{g_1 o}^{(1)}} \tag{18}$$

$$R_o = \frac{\sum_{f_1 \in A_{RK}^{(K)}} \hat{u}_{f_1} * y_{f_1 o}^{(1)}}{\sum_{g_1 \in B_{E\&RL}^{(L)}} (1 - M_{g_1}) * \hat{v}_{g_1} * x_{g_1 o}^{(1)}} \tag{19}$$

$$V_{g_L} = \sum_{g_1 \in B_{g_L}^{(L)}} \hat{v}_{g_1} \qquad g_1 = 1,\dots,m^{(1)} \qquad L = 1,\dots,L-1 \tag{20}$$

$$U_{f_K} = \sum_{f_1 \in A_{f_K}^{(K)}} \hat{u}_{f_1} \qquad f_1 = 1,\dots,s^{(1)} \qquad K = 1,\dots,K-1 \tag{21}$$

$$p_{g_L}^{(L)} = \frac{\sum_{g_1 \in B_{g_L}^{(L)}} \hat{v}_{g_1}}{\sum_{g_1 \in B_{g_{(L+1)}}^{(L+1)}} \hat{v}_{g_1}} \qquad g_L = 1,\dots,m^{(L)} \qquad L = 1,\dots,L-1 \tag{22}$$

$$q_{f_K}^{(K)} = \frac{\sum_{f_1 \in A_{f_K}^{(K)}} \hat{u}_{f_1}}{\sum_{f_1 \in A_{f_{K+1}}^{(K+1)}} \hat{u}_{f_1}} \qquad f_K = 1,\dots,s^{(K)} \qquad K = 1,\dots,K-1 \tag{23}$$

$$\eta \le M_{g_1} \le 1 - \eta \qquad g_1 \in B_{E\&R} \tag{24}$$

$$\hat{u}_{f_1} \ge \varepsilon * \xi^{(K-1)} \qquad f_1 = 1,\dots,s^{(1)} \tag{25}$$

$$\hat{v}_{g_1} \ge \varepsilon * \xi^{(L-1)} \qquad g_1 = 1,\dots,m^{(1)} \tag{26}$$

$$V_{g_L} \ge \varepsilon \qquad g_L = 1,\dots,m^{(L)} \tag{27}$$

$$U_{f_K} \ge \varepsilon \qquad f_K = 1,\dots,s^{(K)} \tag{28}$$

$$p_{g_l}^{(l)} \ge \xi \qquad g_l = 1,\dots,m^{(L)} \qquad l = 1,\dots,L-1 \tag{29}$$

$$q_{f_k}^{(k)} \ge \xi \qquad f_k = 1,\dots,s^{(K)} \qquad k = 1,\dots,K-1 \tag{30}$$

This model completely reflects the layered hierarchy of indicators by specifying the weights of the factors in each category of each layer, and also determines the relative teaching and research efficiencies of academic departments jointly. The other advantage of this model is determining the proportion of shared resource between teaching and research

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activities. Constraints (20) and (21), with regard to summing up the weights of the factors in each category of each layer (i.e., $p^{(l)}_{g_l}$, and $q^{(k)}_{f_k}$), whose sum is equal to 1, are obtained. Constraints (22) and (23) demonstrate internal weights in input and output layers deduced from the best possible input and output weights, (i. e., $\hat{u}_{f_1}$ and $\hat{v}_{g_1}$). This model can be solved with a software package, such as LINGO 8.0.

3.2. AHP-AR-Joint MLDEA Model

To improve the amount of weights in Section 3.1, the researcher used experts' opinions. Therefore with restricting the weight flexibility in each category of layers adaption, realistic and acceptance of final weights using experts' opinions will be guaranteed. So we utilize a version of the assurance region (AR) model proposed by Thompson et al. [17]. For every pair (i.e., i1, i2) of input (output) factor, the ratio $\frac{v_{i_1}}{v_{i_2}}$ ($\frac{u_{r_1}}{u_{r_2}}$) must be bounded by L_{i_1,i_2} (L_{r_1,r_2}) and U_{i_1,i_2} (U_{r_1,r_2}) as Equation (31):

$$L_{i_1,i_2} \le \frac{v_{i_1}}{v_{i_2}} \le U_{i_1,i_2} \tag{31}$$

where the bounds are calculated by using the experts' weights W_{ki} as Equation (32):

$$L_{i_1,i_2} = \min \frac{w_{ki_1}}{w_{ki_2}}, \qquad U_{i_1,i_2} = \max \frac{w_{ki_1}}{w_{ki_2}} \quad k = 1,\dots,k \tag{32}$$

The analytic hierarchy process (AHP) proposed by Saaty [18] are used to quantify their subjective judgments. The results derived from AHP survey then served as a guideline for setting the lower and upper bounds of weight restrictions to be used in the AR-DEA model. The model combining the AHP and AR-DEA (i.e., AHP-AR-DEA model) had been previously applied to some different fields before [19]. In this paper this type of weight restriction is used. The model combining AHP-AR-DEA model and Joint MLDEA model, the hybrid AHP-AR-joint MLDEA model is presented as follows:

$$\max z = \sum_{f_1=1}^{s^{(1)}} \hat{u}_{f_1} * y^{(1)}_{f_{1o}} \tag{33}$$

s.t.

$$\sum_{g_1=1}^{m^{(1)}} \hat{v}_{g_1} * x^{(1)}_{g_{1o}} = 1 \tag{34}$$

$$\sum_{f_1=1}^{s^{(1)}} \hat{u}_{f_1} * y^{(1)}_{f_{1o}} - \sum_{g_1=1}^{m^{(1)}} \hat{v}_{g_1} * x^{(1)}_{g_{1o}} \le 0 \qquad \forall j = 1,\dots,n \tag{35}$$

$$\sum_{f_1 \in A^{(K)}_{EK}} \hat{u}_{f_1} * y^{(1)}_{f_{1j}} - \sum_{g_1 \in B^{(L)}_{EL}} \hat{v}_{g_1} * x^{(1)}_{g_{1j}} - \sum_{g_1 \in B^{(L)}_{E\&RL}} M_{g_1} * \hat{v}_{g_1} * x^{(1)}_{g_{1j}} \le 0 \qquad \forall j = 1,\dots,n \tag{36}$$

$$\sum_{f_1 \in A^{(K)}_{RK}} \hat{u}_{f_1} * y^{(1)}_{f_{1j}} - \sum_{g_1 \in B^{(L)}_{E\&RL}} (1 - M_{g_1}) * \hat{v}_{g_1} * x^{(1)}_{g_{1j}} \le 0 \qquad \forall j = 1,\dots,n \tag{37}$$

$$T_o = \frac{\sum_{f_1 \in A_{E_K}^{(K)}} \hat{u}_{f_1} * y_{f_{1o}}^{(1)}}{\sum_{g_1 \in B_{E_L}^{(L)}} \hat{v}_{g_1} * x_{g_{1o}}^{(1)} + \sum_{g_1 \in B_{E\&R_L}^{(L)}} M_{g_1} * \hat{v}_{g_1} * x_{g_{1o}}^{(1)}} \tag{38}$$

$$R_o = \frac{\sum_{f_1 \in A_{R_K}^{(K)}} \hat{u}_{f_1} * y_{f_{1o}}^{(1)}}{\sum_{g_1 \in B_{E\&R_L}^{(L)}} (1 - M_{g_1}) * \hat{v}_{g_1} * x_{g_{1o}}^{(1)}} \tag{39}$$

$$V_{g_L} = \sum_{g_1 \in B_{g_L}^{(L)}} \hat{v}_{g_1} \qquad g_1 = 1,\dots,m^{(1)} \qquad L = 1,\dots,L-1 \tag{40}$$

$$U_{f_K} = \sum_{f_1 \in A_{f_K}^{(K)}} \hat{u}_{f_1} \qquad f_1 = 1,\dots,s^{(1)} \qquad K = 1,\dots,K-1 \tag{41}$$

$$p_{g_L}^{(L)} = \frac{\sum_{g_1 \in B_{g_L}^{(L)}} \hat{v}_{g_1}}{\sum_{g_1 \in B_{g_{(L+1)}}^{(L+1)}} \hat{v}_{g_1}} \in \Phi \qquad g_L = 1,\dots,m^{(L)} \qquad L = 1,\dots.,L-1 \tag{42}$$

$$q_{f_K}^{(K)} = \frac{\sum_{f_1 \in A_{f_K}^{(K)}} \hat{u}_{f_1}}{\sum_{f_1 \in A_{f_{K+1}}^{(K+1)}} \hat{u}_{f_1}} \in \psi \qquad f_K = 1,\dots,s^{(K)} \qquad K = 1,\dots,K-1 \tag{43}$$

$$\eta \le M_{g_1} \le 1 - \eta \qquad g_1 \in B_{E\&R} \tag{44}$$

$$\hat{u}_{f_1} \ge \varepsilon * \xi^{(K-1)} \qquad f_1 = 1,\dots,s^{(1)} \tag{45}$$

$$\hat{v}_{g_1} \ge \varepsilon * \xi^{(L-1)} \qquad g_1 = 1,\dots,m^{(1)} \tag{46}$$

$$V_{g_L} \ge \varepsilon \qquad g_L = 1,\dots,m^{(L)} \tag{47}$$

$$U_{f_K} \ge \varepsilon \qquad f_K = 1,\dots,s^{(K)} \tag{48}$$

$$p_{g_l}^{(l)} \ge \xi \qquad g_l = 1,\dots,m^{(L)} \qquad l = 1,\dots.,L-1 \tag{49}$$

$$q_{f_k}^{(k)} \ge \xi \qquad f_k = 1,\dots,s^{(K)} \qquad k = 1,\dots,K-1 \tag{50}$$

In this model, by restricting the weight flexibility in each category of a layer, denoted as Φ and Ψ in equations (42) and (43), consistency of weights in each layer with experts' opinions is guaranteed, which cannot be realized in the one layer model.

4. A Case Study: Performance Evaluation of Academic Departments

There is no doubt that academic education has an important role in countries' progress because of training experts and producing new knowledge and technology. Therefore choosing practical and effective strategies for improving the quality along with quantity is necessary. So the role of quality evaluation in academic education management is undeniable. Generally universities are looking for different strategies to improve the quality of their curriculum and teaching and research system. As it mentioned before, in the field of performance assessment in academic departments, there are lots of studies have been done, based on teaching and research activities. The basic DEA model only gives the value for the overall efficiency of each department. And it couldn't determine how efficient each department is in its two basic activities, teaching and research. Also often the decision makers (DMs) wish to select comprehensive indicators to reduce the risk of excluding any important

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measure which could eventually affect the performance of the model, on the other hand it is well-known that the discrimination power of DEA models will be much decreased if too many indicators are used. So we apply the presented model which is described in the section 3.2, to study the performance of academic departments based on teaching and research performance with considering a large numbers of indicators that the standard DEA models have not been able to be applied.

4.1. Academic Department's Indicators Based on Teaching and Research Performance and Their Hierarchy

In this study, by considering universities two teaching and research goals the input-output indicators along with hierarchy structure are considered. The researcher applied the AHP-AR- Joint MLDEA model to evaluate the overall efficiency, teaching efficiency, and research efficiency of all academic departments.

Consider there are 30 academic departments, each department j, DMUj (j=1, 2,…, 30) has 2 layers of inputs and 3 layers of outputs, in 2^{th} layer of inputs, there are 2 inputs that are decomposed by different types in the first layer. Analogously, in 3th layer of outputs, there are 2 outputs which are decomposed too. These indicators are described in detail, in next section.

4.1.1. Academic Department's Indicators and Their Hierarchy

Indicators that are used in evaluation of academic departments' efficiency can be divided in two broad categories: research and teaching activities.

Inputs and outputs indicators for teaching efficiency include: Number of students, this input is decomposed by different degrees (bachelors and masters, respectively). Percentage of the graduates, this output is decomposed by different degrees (bachelors and masters, respectively). Grade Point Average (GPA) of graduates, this output is decomposed by different degrees (bachelors and masters, respectively). Average graduates semester, this output is decomposed by different degrees (bachelors and masters, respectively). Number of students who have reached high levels, this output is decomposed by different degrees (master accepted and PHD accepted, respectively). Employment rate, this output is decomposed by different degrees (bachelors and masters, respectively). Teaching outputs are numbered from 1 to 10 as they are listed above.

Inputs and outputs for research efficiency include: Number of papers, this output is decomposed by paper types (journal papers and conference papers). Number of edited books, this output is decomposed by book types (written books and translated books). Number of projects, this output is decomposed by project types (internal projects and external projects). Two other considered outputs are the number of explorations and inventions and the number of awards. Research outputs are also numbered from 11 to 18.

It should be noted that the number of academic staffs, which is decomposed by different Academic qualifications (e.g., Lecturer, Assistant Professor, Associate Professor and Professor) is a shared input for both teaching and research activities and therefore in evaluating teaching and research efficiencies, the proportion of the academic staff for both functions needs to be determined that presented model in Section 3.2 is capable of doing this.

Totally, the hierarchical structure of 6 inputs and 18 outputs are considered in this study, are shown in the Figure 1.

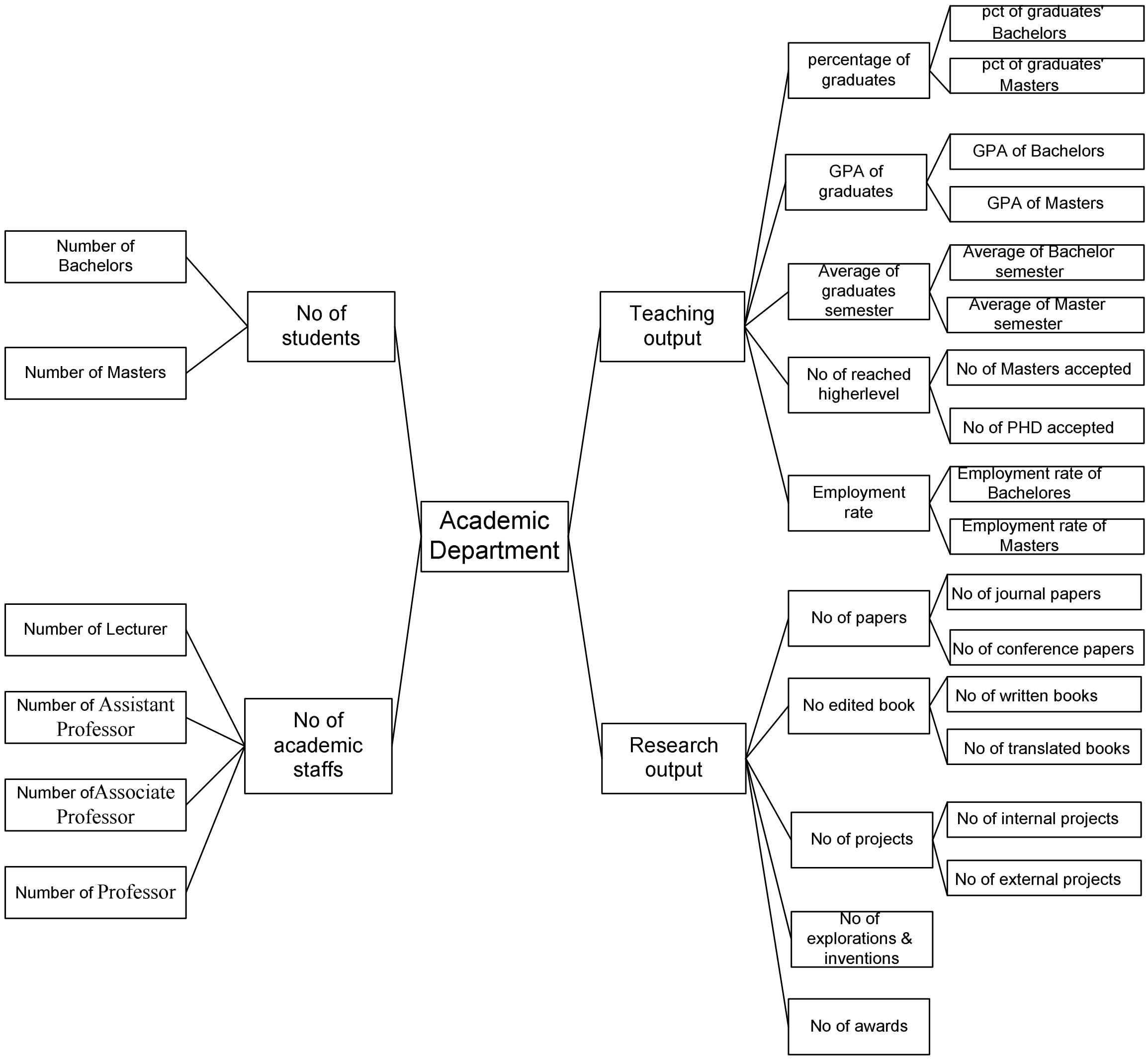

FIGURE 1. The hierarchical structure of academic department indicators in terms of teaching and research performance.

4.2. Data Collection

All the data needed for this study are collected from wide range of data sources in the 30 academic departments of the ShahidBahonar University of Kerman for the year 2011–2012. Before applying the AHP- AR- Joint MLDEA model, the raw data should be normalized so as to eliminate the scale differences of the indicators and the effects of the measurement unit. Therefore in this study, the Nardo approach is used. In this approach the maximum value for each indicator in the data set is selected as the reference, and the other indicator values are divided by this reference [20]. The resulting normalized data are presented in Tables 1 to 3.

TABLE 1. Data of normalized input parameters

Academic Department	No. of Bachelors	No. of Masters	No of Lecturer	No. of Assistant Professors	No. of Associated Professors	No. of Professors
1	0.475	0.397	0.091	0.350	0.286	0.800
2	0.275	0.147	0.182	0.350	0	0
3	0.825	0.044	0.909	0.300	0	0
4	0.513	0.221	0.182	0.250	0.143	0.200
5	0.500	0.132	0	0.300	0	0
6	0.850	0.294	0.455	0.350	0	0
7	0.388	0.103	0.091	1	1	0.200
8	0.475	0.088	0.364	0.250	0	0.200
9	0.550	1	0.182	0.900	0.714	1
10	0.513	0.147	0.273	0.250	0	0
11	0.550	0.206	0.091	0.400	0.286	0.800
12	0.500	0.338	0.182	0.850	0.286	0.200
13	0.563	0.206	0.182	0.400	1	0.800
14	0.438	0.221	0.091	0.700	0.286	0.600
15	0.763	0.647	0.364	0.600	0.571	0
16	1	0.603	0	0.550	0.429	0
17	0.900	0.103	0.364	0.200	0	0
18	0.438	0.294	0.091	0.250	1	0.600
19	0.338	0.088	1	0.200	0	0
20	0.388	0.250	0.273	0.200	0.714	0
21	0.838	0.485	0	0.550	0.429	0.400
22	0.563	0.441	0.182	0.450	0.571	0.600
23	0.375	0.176	0.091	0.350	0.143	0
24	0.300	0.118	0.364	0.100	0.143	0.200
25	0.550	0.103	0	0.250	0.143	0
26	0.438	0.147	0.455	0.150	0.143	0
27	0.413	0.147	0.182	0.500	0	0
28	0.450	0.235	0.364	0.350	0.571	0
29	0.350	0.132	0	0.450	0.143	0.200
30	0.513	0.132	0.182	0.200	0.286	0

TABLE 2. The data of normalized teaching outputs parameters

Teaching outputs / Academic Department	1	2	3	4	5	6	7	8	9	10
1	1	1	0.934	0.861۰	0.885	0.815	0.171	0.636	0	0.530
2	0.816	0.800	1	0.875	0.847	0.847	0.132	0	0.443	1
3	1	1	0.977	0.873	0.846	0.826	0.395	0	0.243	0.170
4	0.818	1	0.984	0.942	0.919	0.912	0.158	0.182	0.571	0.720
5	0.976	0.778	0.977	0.944	1	0.998	0.132	0.091	0.571	0.720
6	0.950	1	0.858	0.830	0.869	0.731	0.237	0	0.886	1
7	0.809	0.571	0.965	0.875	0.871	0.869	0.039	0.273	0.814	0.570
8	0.968	1	0.843	0.882	0.850	0.884	0.158	0	0.414	0.750
9	0.553	1	0.918	0.869	0.852	0.930	1	1	0.643	0.550
10	0.727	1	0.894	0.776	0.835	0.794	0.132	0	0.471	1
11	0.805	0.929	0.877	0.833	0.831	0.672	0.368	0.636	0.143	1
12	0.659	1	0.877	0.833	0.856	1	0.197	0	0.179	0.600
13	1	1	0.877	0.833	0.834	0.838	0.368	0.727	0.214	0.500

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TABLE 2. Continued

14	0.822	1	0.831	0.819	0.874	0.805	0.171	0.455	1	0.540
15	1	1	0.931	0.847	0.796	0.761	0.671	0.091	0.671	0.830
16	1	1	0.902	0.870	0.883	0.730	0.684	0	0.300	0.670
17	0.800	1	0.884	0.874	0.862	0.850	0.145	0	0.600	0.430
18	1	1	0.829	0.807	0.815	0.738	0.263	0.364	0.671	0.940
19	1	1	0.975	0.856	0.807	0.918	0.158	0	0.600	0.420
20	0.963	1	0.894	0.804	0.864	0.857	0.250	0	0.514	0.800
21	1	1	0.800	0.815	0.868	0.981	0.513	0.545	0.457	0.600
22	1	1	0.875	0.839	0.815	0.843	0.658	0.273	0.643	1
23	1	0.583	0.828	0.769	0.811	0.761	0.197	0	0.900	0.570
24	1	0.750	0.916	0.931	0.872	0.799	0.132	0.273	0.157	0.710
25	0.708	1	0.831	0.856	0.818	0.781	0.066	0	0.429	0.300
26	0.909	1	0.880	0.855	0.798	0.810	0.263	0	0.429	0.800
27	0.914	1	0.928	0.883	0.858	0.712	0.132	0	0.243	0.500
28	0.727	0.875	0.853	0.840	0.815	0.815	0.197	0.364	0.714	0.500
29	0.806	0.778	0.853	0.884	0.876	0.705	0.105	0.455	0.571	0.670
30	0.893	1	0.897	1	0.894	0.864	0.105	0.182	0.714	1

TABLE 3. Data of normalized research outputs parameters

Research outputs / Academic Department	11	12	13	14	15	16	17	18
1	0.178	0.053	0.333	0.286	0	0	0	0
2	0.025	0.008	1	0	0.400	0	0	0
3	0.102	0.084	0.667	0.143	0	0	0	0
4	0.119	0.038	0.500	0.143	0.800	0.143	0	0
5	0.034	0.023	0.333	0	0	0	0	0
6	0.025	0.046	0	0	0.400	0.143	0	0
7	1	0.756	0.167	0.571	0	0.143	0	0
8	0.178	0.198	0	0.143	0	0.143	0	0
9	0.458	0.183	0.333	0.143	0	0.143	0	0
10	0.102	0.214	0	0	0.200	0	0	0
11	0.203	1	0.167	0	0	0	0	1
12	0.314	0.374	0	0.286	0.200	0.286	0	0
13	0.534	0.588	0.167	0	0	0	0	0
14	0.263	0.198	0	0.286	0	0.143	0	0
15	0.432	0.947	0.167	0	0.100	0	0	0
16	0.322	0.481	0	0	0.800	1	0	0
17	0.008	0.046	0	0	0	0	0	0
18	0.356	0.878	0	0	0.200	0	0	0
19	0.076	0.107	0	0	0	0.143	1	1
20	0.356	0.824	0	0.429	0.100	0.143	0	0
21	0.212	0.695	0.333	0	1	0.143	1	0
22	0.475	0.466	0.167	0	0.200	0	0	0
23	0.068	0.290	0	0	0	0.429	0	0
24	0.254	0.145	0	0	0	0.143	0	0
25	0.076	0.115	0	0	0	0	0	0
26	0.042	0.176	0	0	0	0	0	0
27	0.161	0.626	0.333	1	0.200	0	0	0
28	0.508	1	0.667	0.571	0.100	0.571	0	0
29	0.297	0.496	0	0	0.200	0.286	0	0
30	0.059	0.061	0	0	0.100	0	0	0

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4.3. Imposed Weight Restrictions

Weight restrictions should be determined at each layer in hierarchical structure of academic department's indicators before applying the proposed AHP-AR-Joint MLDEA model to obtain realistic and acceptable weights. In this case, to avoid such an unreasonable distribution of weights the AHP-AR-DEA model is used [19]. For setting the lower and upper bounds of weight restrictions based on expert opinions (heads of the academic departments), questionnaires are used and by applying AHP approach subjective judgments of experts turned to quantitative judgments and finally with regard to Takamura and Ton [21] approach, lower and upper bounds of weight restrictions are obtained.

For estimation of absolute weight restriction, Roll et al. [22] and Roll and Golany [23], suggested a two-phase Process which relies on relative information is obtained from the DMUs included in the analysis. Described approach is used and for equation (44), 0.3 and 0.5 are lower and upper bounds respectively. In all models $\xi = 0.1$ and $\varepsilon = 0.01$.

5. Experimental Results

So far, the hybrid AHP-AR-Joint MLDEA model can be applied to evaluate the relative efficiency of 30 academic departments, and the indicator weights allocated in each layer of the hierarchy can be deduced. In the following sections, comparisons of the results among the AHP-AR-Joint MLDEA model, Joint MLDEA model, Joint DEA model and DEA model are illustrated.

5.1. Efficiency Score

In this section, the performance of the proposed AHP-AR-Joint MLDEA model is examined by comparing with the Joint MLDEA model, Joint DEA model and DEA model.

Due to the large number of indicators relative to the number of DMUs (or academic departments), we find that CCR model only results in one inefficient DMU (i.e., 12thDMU), which implies its weak capability of discriminating among DMUs' efficiency. By applying the Joint DEA model, in term of their overall efficiency, the result shows that out of 30 DMUs, only 11 DMUs are inefficient. All the remaining DMUs are efficient, so the result shows stronger discriminatory power than CCR model. Therefore, by considering the hierarchy structure of indicators and specifying the weights of the factors in each category of each layer, the Joint MLDEA model is applied which results in 12 more underperforming DMUs and relatively lower efficiency scores. In this case we obtain the better ranking of DMUs' efficiency. For further improvement of the discrimination power and to adjust the weights in each layer with prior knowledge, we adopt the AHP- AR- Joint MLDEA model. As a result, 2, 5 and 19th DMUs are the only three best-performing academic departments since they obtain the optimal efficiency score of 1, whereas the remaining 27 academic departments with a value less than 1 are considered to be under performing, and can be ranked by their scores directly. Therefore, the discriminating power is obviously improved by using the AHP-AR-Joint MLDEA model.

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Another useful application of the AHP-AR-Joint MLDEA model is it can determine which department has overall efficiency, teaching efficiency and research efficiency. Some departments (e.g., 17th DMU) are high in teaching efficiency but low in research efficiency. This could imply that they are more competent and productive in teaching but less capable in conducting research and they should focus more on research activities than teaching activities. Results of teaching and research efficiency for 30 DMUs based on AHP-AR-Joint MLDEA model are shown in Table 5.

TABLE 4. Efficiency scores for the 30 academic departments based on the four models (AHP- AR- Joint MLDEA, Joint MLDEA, Joint DEA, and DEA).

	AHP- AR- Joint MLDEA		Joint MLDEA		Joint DEA		DEA	
Academic Department	Efficiency Score	Ranking	Efficiency Score	Ranking	Efficiency Score	Ranking	Efficiency Score	Ranking
1	0.57661	26	0.88409	25	0.98896	22	1	1
2	1	1	1	1	1	1	1	1
3	0.85975	15	0.99929	11	1	1	1	1
4	0.90755	11	0.97644	15	1	1	1	1
5	1	1	1	1	1	1	1	1
6	0.72619	21	0.86016	26	1	1	1	1
7	0.84573	16	1	1	1	1	1	1
8	0.88521	12	0.99965	10	1	1	1	1
9	0.52098	28	0.95178	19	0.95178	28	1	1
10	0.92335	7	0.95678	18	1	1	1	1
11	0.88201	13	0.99438	12	1	1	1	1
12	0.47204	30	0.57007	30	0.63805	30	0.822	2
13	0.50731	29	0.77981	28	0.99127	21	1	1
14	0.65126	23	0.97242	16	0.97242	27	1	1
15	0.58995	25	0.75613	29	0.97381	26	1	1
16	0.74323	19	0.94248	20	1	1	1	1
17	0.71090	22	0.80021	27	0.93491	29	1	1
18	0.57194	27	0.91811	22	1	1	1	1
19	1	1	1	1	1	1	1	1
20	0.72863	20	1	1	1	1	1	1
21	0.79178	18	0.91966	21	1	1	1	1
22	0.63094	24	0.90527	24	0.98428	25	1	1
23	0.97653	5	0.99998	8	1	1	1	1
24	0.92230	8	0.99994	9	1	1	1	1
25	0.94586	6	0.99184	13	0.99227	20	1	1
26	0.81875	17	0.90641	23	0.98602	23	1	1
27	0.91769	9	0.97145	17	1	1	1	1
28	0.91095	10	1	1	1	1	1	1
29	0.98403	4	1	1	1	1	1	1
30	0.87615	14	0.98457	14	0.98457	24	1	1

TABLE 5. Teaching and research efficiency scores for 30 academic departments based onthe AHP-AR-Joint MLDEA

Academic department	Teaching efficiency	Research efficiency
1	0.66438	0.29008
2	1	1
3	0.94027	0.68525
4	0.83244	1
5	1	1
6	0.77464	0.62411
7	0.864998	0.82293
8	0.98950	0.61213
9	0.60234	0.29874
10	1	0.73673
11	0.71626	1
12	0.48272	0.43774
13	0.57116	0.41719
14	0.79262	0.33004
15	0.53343	0.66254
16	0.52284	1
17	0.91490	0.10877
18	0.63396	0.47215
19	1	1
20	0.72539	0.73652
21	0.55755	1
22	0.67078	0.53211
23	1	0.85831
24	1	0.67853
25	1	0.45055
26	0.96229	0.23545
27	0.89034	1
28	0.84248	1
29	1	0.93749
30	1	0.22511

5.2. Weight Allocation

For better understanding of the computational process leading to the final optimal efficiency scores in Table 4, we further explore the indicator weights allocated in each layer of the hierarchy for a specific academic department. Taking the 1^{th} DMU as an example, which obtains the efficiency score of 1 in the basic CCR model, while an inefficiency value (0.57661) in the AHP-AR-Joint MLDEA model, the assigned weights from these two models are presented in Figures 2 and 3, respectively.

In the basic CCR model, all indicators are treated to be in the same layer and no layer related weight restrictions can be imposed. Therefore, weights will be allocated with the only purpose of maximizing the efficiency score regardless of the indicators' position in the hierarchical structure. Figure 2 shows that the 1^{th} DMU obtains the optimal efficiency score of 1, while only 4 inputs and 5 outputs are allocated non-negligible weights larger than $\varepsilon = 0.01$.

On the contrary, the AHP-AR-Joint MLDEA model not only pursues the optimal efficiency scores, but also guarantees its consistency with prior knowledge and the obtainment of realistic and acceptable weights by restricting the weight flexibility in each category of each layer. Moreover, insight can be gained into the relative importance of the different indicators. Figure 3 shows the accordance of the weights from the AHP- AR- Joint MLDEA model with the imposed restrictions.

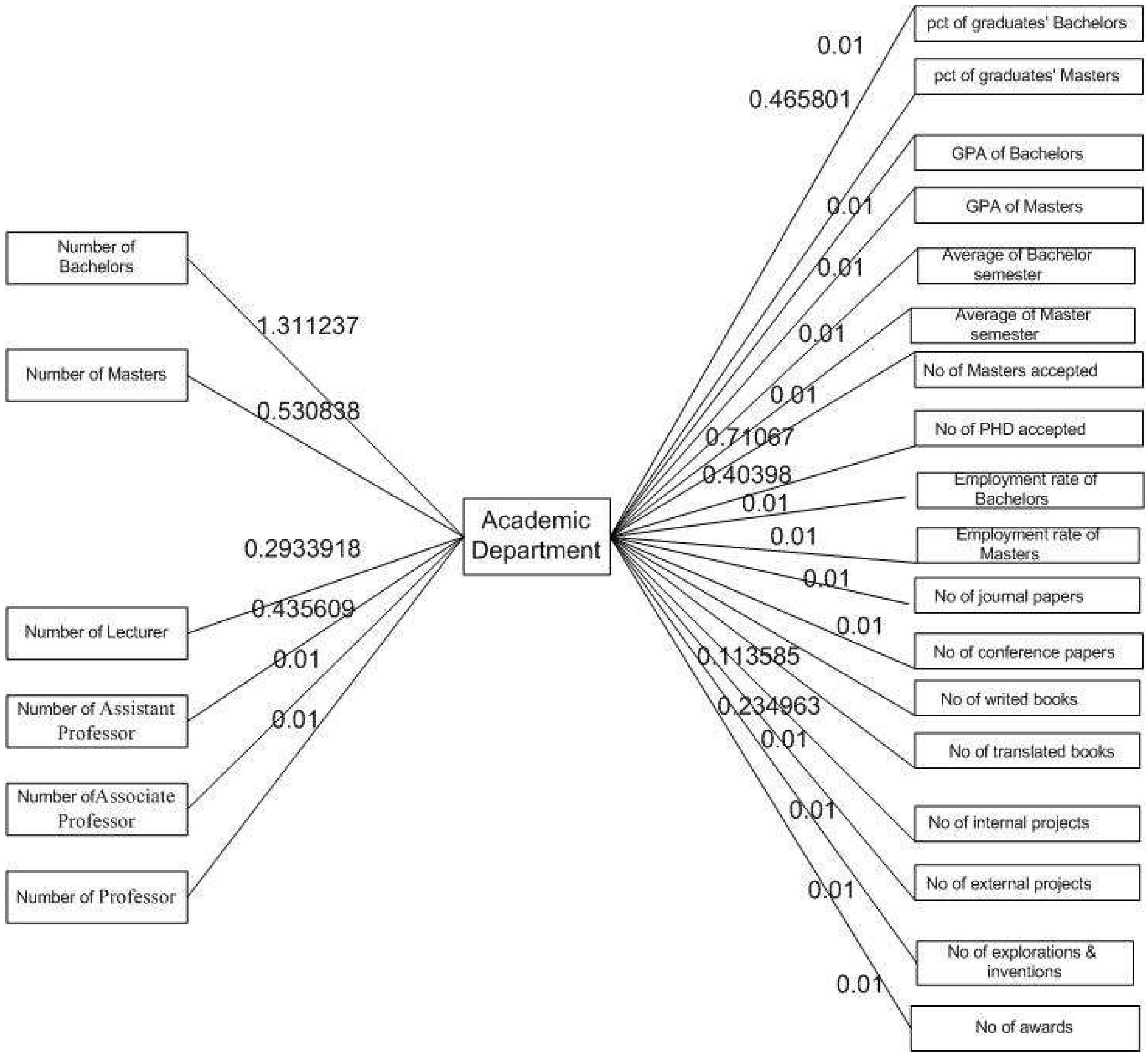

FIGURE 2. Assigned weights for 1th academic department based on the DEA model

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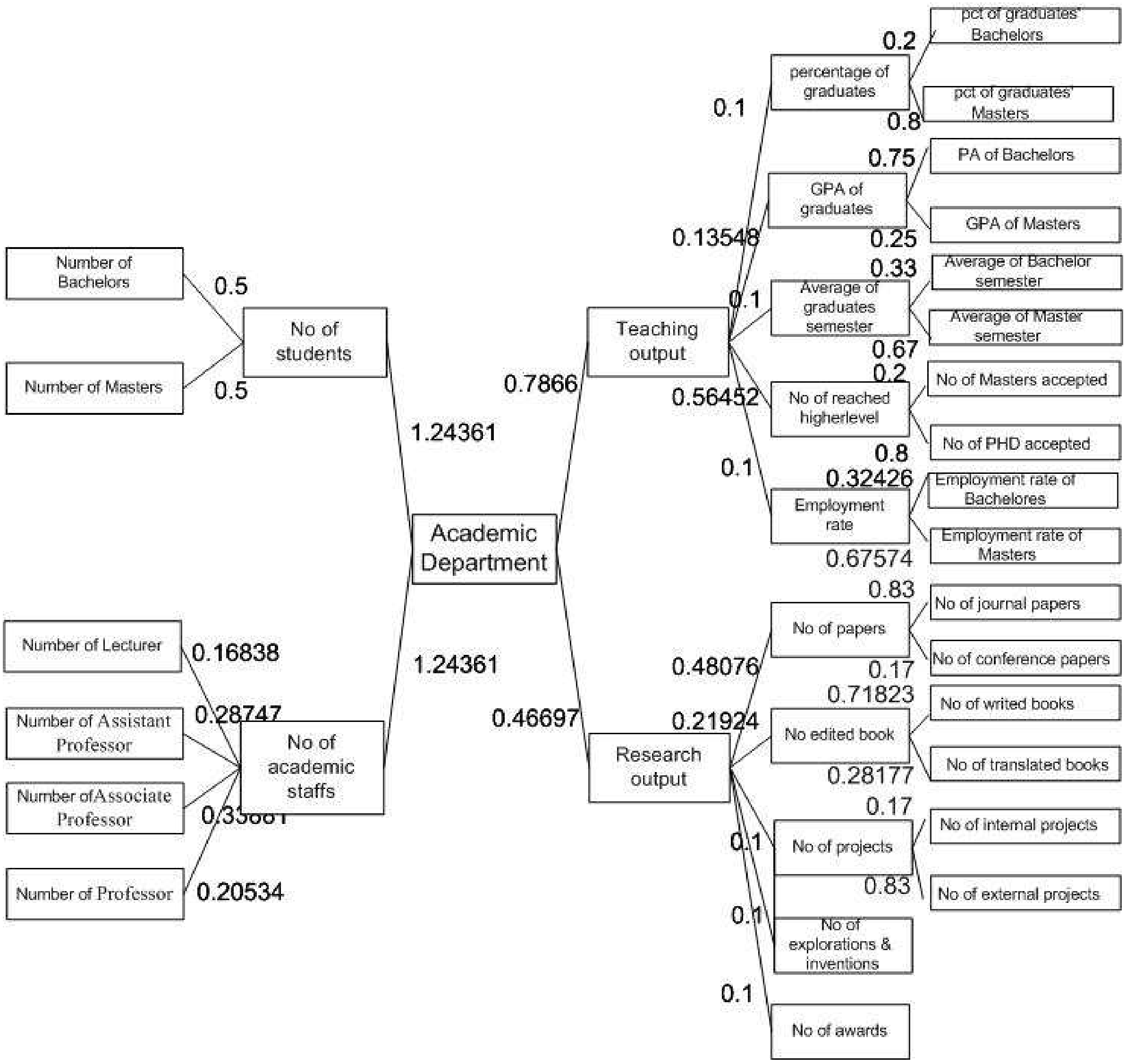

FIGURE 3. Assigned weights in each layer of the hierarchy for the 1stacademic department based on the AHP-AR-Joint MLDEA model.

6. Conclusions and Future Research

This paper addressed the incorporation of a layered hierarchy in the Joint DEA framework, and proposed the joint MLDEA model. This new model utilized hierarchy structure of indicators to measure the efficiencies of academic departments based on their teaching and research activities. Also for obtaining realistic and acceptable weights, we imposed weight restrictions at each category of each layer in hierarchical structure of academic department indicators and presented the hybrid AHP-AR-Joint MLDEA model. It is notable that, So far this combination of models for improving discrimination power of the DEA model has not been considered in literature review.

Moreover, for examining the recommended model, this model was applied for evaluating the performance of the academic departments at ShahidBahonar University of Kerman. Using

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the 6 hierarchical indicators as the model's input and the 18 layered final outcomes as the output based on their teaching and research activities, we computed the most optimal efficiency score for the 30 academic departments, and analyzed the weights assigned to each layer of the hierarchy. A comparison of the results with the one layer CCR model, Joint DEA model, and Joint MLDEA model indicated the effectiveness of the proposed AHP- AR- Joint MLDEA model in terms of discrimination power, weight allocating, and possibility of implementing this model in evaluating the function of activities which have many indicators along with hierarchical structure.

In this study, we incorporated the concept of layered hierarchy into other Joint DEA model; it could be incorporated into other DEA models such as additive model, slacks-based measure of efficiency (SBM), and free disposal hull (FDH). Also the proposed AHP- AR- Joint MLDEA model could be easily applied for assessment of other academic departments in upcoming studies.

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