Volume 3, Number 1, 2014

International Journal of Research in Industrial Engineering

journal homepage: www.nvlscience.com/index.php/ijrie

A New Estimator Based on Likelihood Function for Drift Time of Change in Poisson Rate Parameter

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A R T I C L E I N F O

Article history :
Received:
March 14, 2014
Revised:
April 14, 2014
Accepted:
May 03, 2014

Keywords :

Quality control; Statistical process control; Change point; Poisson process; c chart; CUSUM charts

A B S T R A C T

Although a control chart can signal an out-of-control state in a process, but it does not always indicate when the process change has begun. Identifying the real time of the change in the process, called the change point, is very important for eliminating the source(s) of the change and assists process engineers in identifying the responsible special cause and ultimately in improving the process. In this paper, we first introduce an estimator for a change point with linear trend in the Poisson process, based on the likelihood function using a slope parameter. Then we apply Monte Carlo simulation to evaluate the accuracy and the precision performance of the proposed change point estimator. Finally we compare, the proposed estimator with the MLE of the Poisson process change point derived under linear trend disturbance on the basis of cumulative sum (CUSUM) and Shewhart C control charts. The results show that the proposed procedure outperforms the MLE designed for drift time with regard to variance and is more effective in detecting drift time when the magnitude of change is relatively large.

1. Introduction

Statistical process control (SPC) has played an important role in industry for many years. After examining the process, SPC uses statistical tools to find the sources of variation in the process parameter(s). Control charts are used to monitor for changes in a process by distinguishing between special causes and common causes of variation. When a control chart signals an out-of-control alarm, process engineers initiate a search for the assignable cause disturbing the process. By signaling, control charts do not provide specific information regarding the cause nor when the process changed; rather, they only suggest that a change has occurred.

Cumulative sum (CUSUM) control charts suggested by Page were among the primary tools for detecting change in the process [1]. Though they were not as simple to operate as Shewhart control charts but they have been shown to be more efficient in detecting small

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shifts in the process mean. Exponentially weighted moving average (EWMA) control charts also offered a procedure for estimating the process change point [2]. Despite their ability in detecting small shifts, they were not quite effective tools for finding large shifts in the process mean. Knowing the exact point of change in a process would help to search and identify special causes, resulting in time saving to find the causes. Therefore, it is useful to identify the difference between the change point and the time when an out of control signal is generated by control charts.

Poisson count processes are often used to model the number of occurrences over some interval unit. The interval unit can be time, distance, area, volume or some similar unit. Often in an industrial quality control setting, the Poisson distribution is used to model the number of defects or nonconformities per unit of product. That is, the probability that a randomly selected unit of product contains x nonconformities is given by

$$P(X = x) = \frac{\lambda^x}{x!} \cdot e^{-\lambda}$$

(1)

Where $x \geq 0$, and $\lambda \geq 0$ denotes the mean rate of nonconformities. The on-line monitoring of λ is typically accomplished through the use of a SPC chart. For monitoring Poisson rates, the c, CUSUM and EWMA control charting procedures are most commonly used.

2. Literature review

The presence of a linear trend change in the mean of a sequence of independent exponential random variables was investigated by Gupta and Ramanayake [3]. They also studied analogues testing procedures to detect a linear trend change followed by an abrupt change in a random sequence of exponentially distributed random variables.

Samuel and Pignatiello [4] analyzed a step change in the rate parameter for a Poisson process. Samuel et al. [5] & [6] considered step change in a normal process mean and normal process variance. Samuel and Pignatiello [7] proposed an MLE for the process fraction non-conforming change point by applying the step change likelihood function. They evaluated the performances of their proposed estimator when an np chart signals and concluded that their estimator provides good accuracy and precision performances. Moreover, Perry et al. [8] developed a change-point estimator from the change likelihood function for a binomial random variable without assuming the previous information of the exact change type. The only assumption in their research is that the predicted change type is should belong to a family of monotonic change type. Further, Perry et al. compared the performances between their estimator and the one suggested by Samuel and Pignatiello [7].

The maximum likelihood estimator has been widely used to identify the time of the drift in processes. In this context, Perry and Pignatiello [9] proposed the MLE for the change point of a normal process mean when a linear trend disturbance is present. The proposed estimator performance was studied and compared with the performance of MLE designed for step changes. The MLE provides good overall performance in comparison with the estimators given by the CUSUM and EWMA control charts [10] & [11]. Perry et al. [12] compared the performance of the MLE for the time of drift in a Poisson rate parameter designed for linear trends to the MLE of the process change point designed for step changes when a linear trend disturbance is present. Performance comparisons showed that the MLE of the process change point designed for linear trends outperforms the MLE designed for step changes and CUSUM control chart estimator.

Noorossana and Shademan [13] proposed MLE for the change point of a normal process mean that does not require the knowledge of the exact change type showed by the process. The only required assumption is that the change type present should belong to a family of monotonic change, either isotonic or antitonic. Furthermore, they compared performances between their estimator and those suggested by Samuel et al. [5] and Perry and Pignatiello [9] following a genuine signal from the Shewhart $\bar{X}$ control chart. Zandi et al. [14] introduced MLE for the change point of process fraction nonconforming when the process was subjected to a linear trend disturbance. Atashgar and Noorossana [15] applied artificial neural networks to identify the change point in a bivariate environment when the process mean vector shifted linearly.

3. Poisson Process Linear Trend Change point model

Consider a linear trend change model for the behavior of Poisson process rate parameter λ. It is assumed the process is initially in control for the first τ subgroups and independent observations come from a Poisson distribution with in-control parameter $\lambda = \lambda_0$. The first disturbance in the rate parameter occurs at an unknown point in time τ (the process change point). After this point, the process changes from $\lambda = \lambda_0$ to an out of control state $\lambda = \lambda_i$ for $i = \tau + 1, \dots, T$, where T denotes the time when a control chart generates a signal. Assuming the signal is not a false alarm, the change model of λ is given by Equation (1), where β is the slope of the linear trend disturbance or the magnitude of process change.

$$\lambda_i = \lambda_0 + \beta(i - \tau) \qquad i = \tau + 1, \dots . T . \tag{2}$$

The above model has two unknown parameters τ and β. The parameter τ represents the last subgroup taken from the in-control process, and β is the slope parameter of the linear trend model. $\beta > 0$ means a linear change with an additive trend in λ has occurred and $\beta < 0$ represents a descending trend in the Poisson process. Based on these assumptions, we derived an estimation for the process change point τ with non-decreasing change type $\beta > 0$.

The likelihood function is:

$$L(\tau, \lambda_1 | C) = \prod_{i=1}^{\tau} ({\lambda_0}^{c_i}/c_i!) \cdot \exp(-\lambda_0) \times \prod_{i=\tau+1}^{T} \left(\left(\lambda_0 + \beta(i-\tau)\right)^{c_i}/c_i!\right) \cdot \exp\left(-\left(\lambda_0 + \beta(i-\tau)\right)\right), \tag{3}$$

Where c_i is the count corresponding to the ith subgroup. The MLE of τ is the value of τ that maximizes the likelihood function (Equation (2)), or equivalently, its logarithm. The logarithm of the likelihood function is

$$\ln L(\tau, \beta|D) = K - \frac{\beta}{2}(T-\tau)(T+1-\tau) + \log(\lambda_0)\sum_{i=1}^{\tau} c_i + \sum_{i=\tau+1}^{T} c_i \ln(\lambda_0 + \beta(i-\tau)). \quad (4)$$

Where K is constant. Perry [9] used the Newton method for estimate value of β. But this method needs many calculations. We propose a new method for finding β and use it in the likelihood function to introduce an estimator for the real time of change. An instance of the drift change in the Poisson process is depicted in Figure 1.

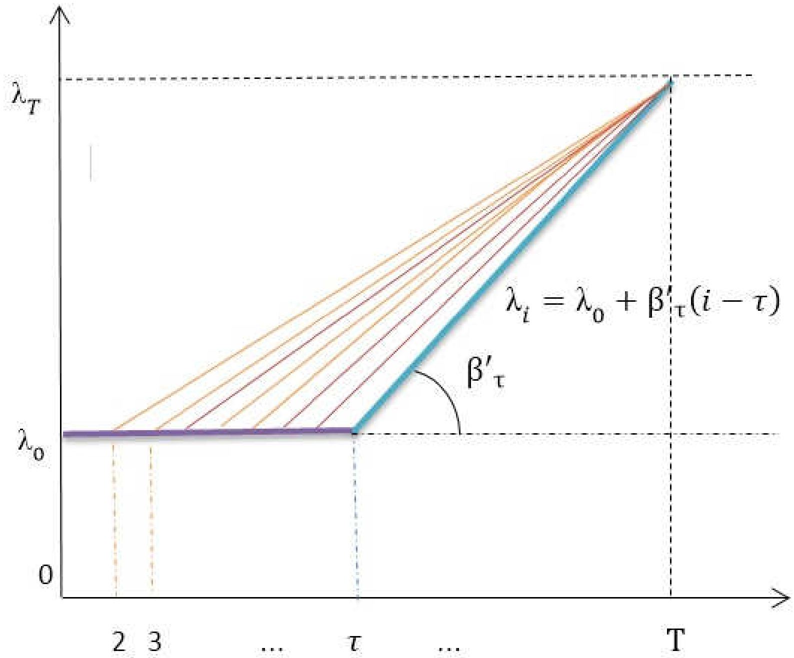

Figure 1. Drift change in the Poisson process for some possible change point τ.

We see that $\beta'_\tau = (\lambda_T - \lambda_0)/(T-\tau)$. Since c_T is an estimate of λ_T, hence $\beta'_\tau = (c_T - \lambda_0)/(T-\tau)$ may be used as an initial value for β'_τ. Substituting such an estimate, β'_τ for β in (3) and evaluating over all possible change points τ to find the maximum value leads to:

$$\hat{\tau} = \arg\max_{0 \le t < T} \{ -\frac{\beta'_t}{2}(T-t)(T+1-t) + \log(\lambda_0)\sum_{i=1}^{t} c_i + \sum_{i=t+1}^{T} c_i \ln(\lambda_0 + \beta'_t(i-t))\}. \quad (5)$$

Therefore, we propose $\hat{\tau}$ as an estimator of the change point.

4. CUSUM control chart estimator

Although developed by Page for normal process means, a CUSUM chart to monitor Poisson count data was suggested by Brook and Evans. This procedure cumulates the difference between an observed value X_i and a reference value k. If this sum exceeds a decision interval h, the chart signals a disturbance is present. The CUSUM control statistic for detecting increases and in the mean count rate are given as $S_i^+ = \max\{0, X_i - k^+ + S_{i-1}^+\}$ where $S_0^+ = 0$ and

$$k^+ = (\lambda_a^+ - \lambda_0)/(\ln(\lambda_a^+) - \ln(\lambda_0))$$

The value for $\lambda_a$is the out of control process rate for which to design the CUSUM. If S_i^+ exceeds a specified decision interval h^+, then the chart signals that an increase in the mean count rate has occurred. Lucas provided a comprehensive study on Poisson CUSUM control charts. Hawkins and Olwell provided extensive detail pertaining to the theoretical foundation and construction of CUSUM control charts in general, including the Poisson CUSUM [12].

5. False alarms

This section addresses the handling of false alarms in the simulation model. When $\tau > 0$ and a control chart issues a signal at subgroup T where $T \leq \tau$, the signal is a false alarm since the signal was given before the simulated process change occurred. Otherwise, however, if the signal time is greater than the real process change point, i.e. $T > \tau$, we call it a genuine signal and use it for searching for the change point.

In the simulation runs, the false alarm signal is not considered for the performance analysis. Whenever a signal is a false alarm, the process is assumed in control and, therefore, the control chart continues its action to monitor the process. In other words, when a false alarm happens in a simulation run at subgroup T, the control chart resumes at subgroup T+1 while not altering the change-point estimation process [12].

6. Comparison of Change Point Estimator

In this section the performances of the proposed estimator, $\hat{\tau}$, are compared with the MLE derived for linear change disturbance proposed by Perry et al. (2006) when a linear trend disturbance is present, and the out-of-control signal comes from a Poisson CUSUM and c control charts. This is referred to $\hat{\tau}_m$. We investigate in-control rate parameter value of $\lambda_0 = 5, 10$.

- Using a Poisson CUSUM control chart

In this section we use simulation to compare accuracy of $\hat{\tau}$ and $\hat{\tau}_m$ following a signal from a Poisson CUSUM control chart. The process change point was simulated to occur at $\tau = 100$. Independent observations were simulated from a Poisson process with rate parameter λ_0 for subgroup $i = 1,2, \dots, 100$. Following subgroup 100, observations were simulated from a Poisson process with rate parameter $\lambda_i = \lambda_0 + \beta(i - \tau)$, where $\beta > 0$, until the CUSUM chart produced a signal. Then two estimates of the process change point, $\hat{\tau}$ and $\hat{\tau}_m$, were computed. This procedure was repeated a total of N = 5000 times for each β value investigated. Averages of the change point estimates obtained from the 5000 runs, $\hat{\tau}$ and $\hat{\tau}_m$, were computed along with their corresponding estimated standard errors.

Table I shows accuracy performances for the two estimators, as well as the estimated ARL of the CUSUM procedure, over a range of β values. The mean squared errors, MSE, and the expected time of the change-point estimates were calculated as shown in Table 1. Except for$\beta = 0.05$, and $\beta = 0.15$, $MSE(\tau)$is smaller than $MSE(\hat{\tau}_m)$ for all other considered values of β. And for $\beta \geq 0.5$ $\hat{\tau}$ is closer to $\tau = 100$ than is $\hat{\tau}_m$. It means that for these values of β the proposed estimator performs better than the other estimator.

Table I. Estimated accuracy performances for new estimator and MLE of the change point for different $\boldsymbol{\beta}$ values following a genuine signal from an CUSUM control chart when a linear trend change is present. ($\boldsymbol{\lambda_0 = 5}$, $\mathbf{h = 7, k = 6.38, \tau = 100}$, N=5000)

β	ARL	$\hat{\tau}$	$\hat{\tau}_m$	$MSE(\tau)$	$MSE(\hat{\tau}_m)$
0.05	56.603	124.43	97.34	19.96	17.92
0.15	20.95	104.7	101.3	7.51	6.97
0.35	10.89	102.01	99.1	4.21	6.10
0.5	9.1	100	98.4	3.05	5.83
0.8	6.22	101.70	97.8	1.49	4.79
1	5.20	99.8	97.8	2.88	3.61
2	3.38	99.4	98.05	1.89	3.05
3	2.634	99.9	100.44	0.73	2.50

Note that, in this table, as the magnitude of the slope parameter, β, increases to 3, the mean squared error for the two estimators decreases. However, more accurate estimates are obtained using the proposed method in almost all cases. Thus, it can be concluded from Table I that the proposed estimator outperforms the other estimator especially for large value of β.

Furthermore we use simulation to study the precision of the proposed estimator following a signal from a CUSUM control chart. To evaluate and compare the precision of $\hat{\tau}$ and $\hat{\tau}_m$, we recorded (for each estimator) the proportion of the N = 5000 simulation runs that the estimator was within a specified tolerance of the simulated change point value. Doing this for a range of β yields the results shown in Tables II and III.

Table II. Precision of the new estimator and MLE of the change point for different $\boldsymbol{\beta}$ values ($\boldsymbol{\beta} < 0.8$). ($\boldsymbol{\lambda_0 = 5}$, $\mathbf{h = 7}$, $\mathbf{k = 6.38}$ $\boldsymbol{\tau = 100}$, N=5000)

Precision	**β**			
	0.05	***0.15***	***0.35***	***0.5***
$\hat{p}(\hat{\tau}_m = \tau)$ $\hat{p}(\hat{\tau} = \tau)$	0.045 0.017	0.070 0.039	0.135 0.101	0.150 0.121
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 1)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 1)$	0.045 0.017	0.221 0.119	0.324 0.242	0.397 0.330

Precision	β			
	0.05	*0.15*	*0.35*	*0.5*
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 2)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 2)$	0.211 0.079	0.329 0.179	0.473 0.368	0.550 0.517
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 3)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 3)$	0.261 0.096	0.435 0.243	0.600 0.496	0.666 0.651
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 4)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 4)$	0.323 0.113	0.530 0.307	0.682 0.609	0.732 0.752
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 5)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 5)$	0.37 0.127	0.595 0.371	0.755 0.696	0.773 0.834
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 6)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 6)$	0.045 0.017	0.666 0.437	0.797 0.781	0.810 0.892

Table III. Precision of the new estimator and MLE of the change point for different $\boldsymbol{\beta}$ values ($\boldsymbol{\beta \geq 0.8}$). ($\boldsymbol{\lambda_0 = 5}$, $\mathbf{h = 7}$, $\mathbf{k = 6.38}$ $\boldsymbol{\tau = 100}$, N=5000)

Precision	β			
	0.8	*1*	*2*	*3*
$\hat{p}(\hat{\tau}_m = \tau)$ $\hat{p}(\hat{\tau} = \tau)$	0.168 0.187	0.200 0.244	0.267 0.357	0.286 0.412
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 1)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 1)$	0.445 0.447	0.483 0.510	0.513 0.724	0.481 0.770
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 2)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 2)$	0.608 0.632	0.619 0.730	0.641 0.877	0.614 0.890
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 3)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 3)$	0.704 0.772	0.703 0.839	0.715 0.933	0.695 0.940
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 4)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 4)$	0.755 0.869	0.761 0.919	0.769 0.954	0.767 0.972
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 5)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 5)$	0.785 0.918	0.801 0.951	0.798 0.965	0.801 0.98
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 6)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 6)$	0.814 0.956	0.828 0.975	0.826 0.977	0.827 0.985

From Table II and III it can be seen that the $\hat{\tau}$ is more precise for $\beta > 0.5$. As the magnitude of β increases, the precision of the two estimators improves. The precision of the two estimates is plotted in Figures 2-4. These Figures confirms the above results.

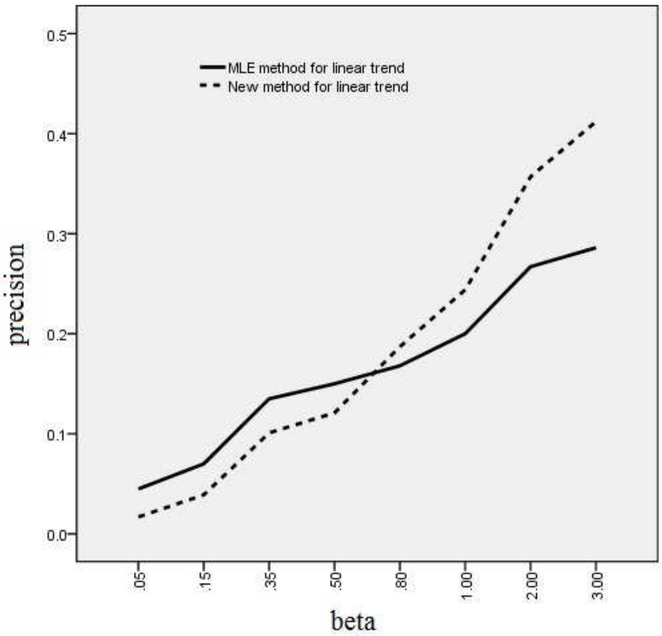

Figure 2. Precision of estimators for the estimated accurate change point $\hat{\mathbf{p}}(\hat{\boldsymbol{\tau}} = \boldsymbol{\tau})$.

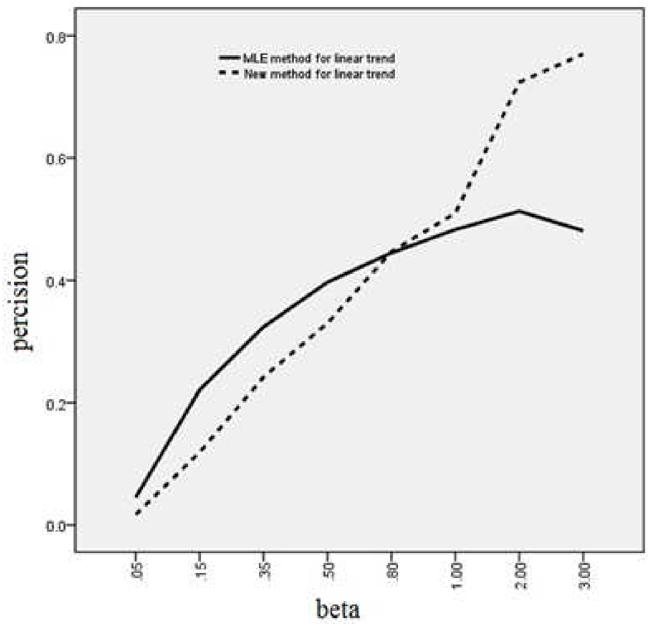

Figure 3. Precision of estimators for tolerance 1 subgroup $\hat{p}(|\hat{\tau}-\tau|\leq 1)$.

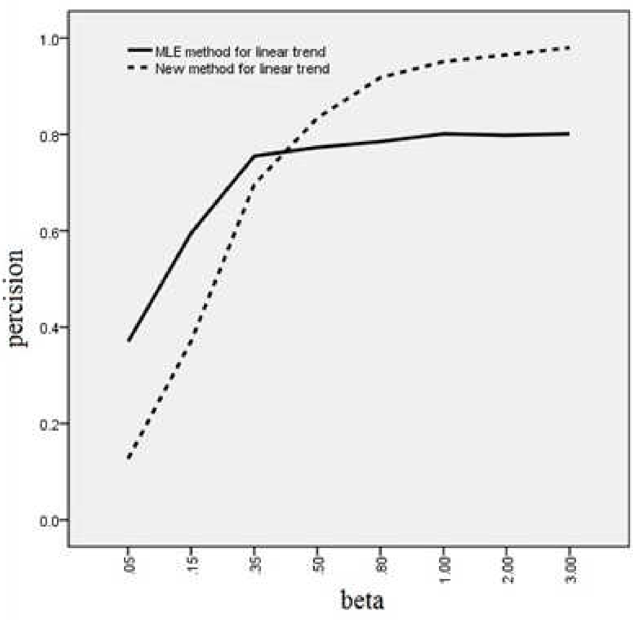

Figure **4**. Precision of estimators for tolerance 1 subgroup $p(|\hat{\tau}-\tau|\leq 2)$.

- Using a *c* control charts

In this section we consider the performance of our proposed change point estimator when using a c chart and $\lambda_0 = 10$. In this case, the upper control limit is $\mathrm{UCL} = 10 + 3\sqrt{10} = 19.49$ and the lower control limit is $\mathrm{LCL} = 10 - 3\sqrt{10} = 0.51$. A Monte Carlo simulation study was conducted to examine the performance of our estimator and compare it whit the other estimator.

The change point was simulated at time 100. Starting at time 101, observations were simulated from the changed process until the c chart signaled that the process was out of control. The expected time of the first genuine alarm, $\mathrm{E(T)}$, is the expected time at which the control chart first signals a change in the Poisson rate parameter. Since no false alarms were observed, the expected time of the first genuine alarm is equal to the average run length (ARL) plus τ. Thus, $(\mathrm{T}) = \mathrm{ARL} + \tau$. Based on the simulation data, the two aforementioned estimators of the Poisson process, i.e. $\hat{\tau}$ and $\hat{\tau}_m$, were then obtained for various values of β. This procedure was repeated $\mathrm{N} = 5000$ times over a range of β values for each estimator. The mean squared errors, MSE, and the expected time of the change-point estimates were calculated as shown in Table IV.

From Table IV, it can be seen for all values of β, $\mathrm{MSE}(\hat{\tau})$ is smaller than$\mathrm{MSE}(\hat{\tau}_m)$ and for $\beta \geq 0.5$, our proposed estimator performs well in estimating the time of the process change. It can also be seen from Table IV that the performance of the two estimators improves considerably with increases in magnitude of the change. We next consider the frequency with which the change point estimation is within a distance of m the true change point, for $= 1,2,..,10,15$. The results are reported in Table V and VI for different β values. Table V show that for small values of β the precision provided by the $\hat{\tau}_m$, in most cases is better than the proposed estimator. However, it is not absolutely better than it. For example for $\beta = 0.5$, the precision performance of $\hat{\tau}$ that was within 9,10 and 15 subgroups of the true process change point, is slightly better than the precision provided by the $\hat{\tau}_m$.

Table IV. Estimated accuracy performances for new estimator and MLE of the change point for different $\boldsymbol{\beta}$ values from an **c** control chart when a linear trend change is present. ($\boldsymbol{\lambda_0 = 10}$, $\boldsymbol{\tau = 100}$, N=5000)

$\boldsymbol{\beta}$	$\boldsymbol{E(T)}$	$\hat{\boldsymbol{\tau}}$	$\hat{\boldsymbol{\tau}}_{\boldsymbol{m}}$	$\boldsymbol{MSE(\tau)}$	$\boldsymbol{MSE(\hat{\tau}_m)}$
0.05	158.516	121.98	105.852	20.18	23.63
0.15	125.128	114.236	103.103	8.04	14.908
0.35	114.06	107.16	97.66	4.833	10.219
0.5	112.04	105.28	96.24	3.81	8.60
0.8	107.18	102.28	97.54	2.347	6.719
1	107.04	101.88	98.66	2.00	4.817
2	104.22	100.12	98	1.90	3.07
3	103.36	100.01	99.18	0.904	2.70

Table V. Precision of the new estimator and the MLE of the change point for different $\boldsymbol{\beta}$ values ($\boldsymbol{\beta} < 0.8$). ($\boldsymbol{\lambda_0 = 10}$, $\boldsymbol{\tau = 100}$, N=5000)

Precision	$\boldsymbol{\beta}$			
	0.05	***0.15***	***0.35***	***0.5***
$\hat{p}(\hat{\tau}_m = \tau)$ $\hat{p}(\hat{\tau} = \tau)$	0.025 0.009	0.045 0.015	0.087 0.023	0.102 0.055
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 1)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 1)$	0.055 0.024	0.135 0.113	0.26 0.085	0.304 0.150
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 2)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 2)$	0.099 0.046	0.212 0.125	0.383 0.138	0.457 0.247
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 3)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 3)$	0.137 0.052	0.295 0.137	0.476 0.216	0.577 0.360
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 4)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 4)$	0.166 0.074	0.366 0.157	0.582 0.284	0.682 0.477
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 5)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 5)$	0.203 0.087	0.429 0.169	0.663 0.354	0.767 0.598
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 6)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 6)$	0.244 0.095	0.485 0.184	0.739 0.449	0.819 0.686
$\hat{p}(\lvert\hat{\tau}_m - \tau\rvert \leq 7)$ $\hat{p}(\lvert\hat{\tau} - \tau\rvert \leq 7)$	0.273 0.184	0.553 0.190	0.801 0.547	0.851 0.774

Precision	β			
	0.05	***0.15***	***0.35***	***0.5***
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{8})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{8})$	0.303 0.297	0.602 0.339	0.839 0.637	0.886 0.859
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{9})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{9})$	0.334 0.304	0.645 0.473	0.866 0.711	0.905 0.922
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{10})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{10})$	0.364 0.439	0.684 0.521	0.880 0.769	0.917 0.956
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{15})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{15})$	0.845 0.614	0.893 0.729	0.948 0.966	0.955 1

Table VI. Precision of the new estimator and the MLE of the change point for different β values ($\beta \geq 0.8$). ($\lambda_0 = 10, \tau = 100$, N=5000)

Precision	β			
	0.8	***1***	***2***	***3***
$\hat{\boldsymbol{p}}(\hat{\boldsymbol{\tau}}_m = \boldsymbol{\tau})$ $\hat{\boldsymbol{p}}(\hat{\boldsymbol{\tau}} = \boldsymbol{\tau})$	0.096 0.137	0.144 0.173	0.257 0.291	0.296 0.673
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{1})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{1})$	0.305 0.378	0.387 0.456	0.560 0.692	0.608 0.816
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{2})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{2})$	0.462 0.574	0.589 0.655	0.700 0.882	0.749 0.959
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{3})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{3})$	0.629 0.697	0.757 0.786	0.784 0.966	0.822 0.99
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{4})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{4})$	0.777 0.747	0.829 0.877	0.823 0.992	0.857 0.995
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{5})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{5})$	0.836 0.853	0.864 0.945	0.882 0.996	0.960 1
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{6})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}\rvert \leq \mathbf{6})$	0.871 0.911	0.887 0.976	0.887 0.999	0.968 1
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m - \boldsymbol{\tau}\rvert \leq \mathbf{7})$	0.888 0.952	0.908 0.995	0.974 1	0.987 1

Precision	**β**			
	0.8	***1***	***2***	***3***
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}-\boldsymbol{\tau}\rvert \le \mathbf{7})$				
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m-\boldsymbol{\tau}\rvert \le \mathbf{8})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}-\boldsymbol{\tau}\rvert \le \mathbf{8})$	0.906 0.981	0.925 1	0.993 1	1 1
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m-\boldsymbol{\tau}\rvert \le \mathbf{9})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}-\boldsymbol{\tau}\rvert \le \mathbf{9})$	0.921 0.992	0.932 1	0.950 1	1 1
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m-\boldsymbol{\tau}\rvert \le \mathbf{10})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}-\boldsymbol{\tau}\rvert \le \mathbf{10})$	0.93 0.997	0.94 1	1 1	1 1
$\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}_m-\boldsymbol{\tau}\rvert \le \mathbf{15})$ $\hat{\mathbf{p}}(\lvert\hat{\boldsymbol{\tau}}-\boldsymbol{\tau}\rvert \le \mathbf{15})$	0.957 1	0.964 1	1 1	1 1

It can be seen from Table VI that our proposed change point estimator has a very good precision in estimating the time of a change in the Poisson rate parameter when the magnitude of β is equal or bigger than 0.8. for $\beta = 0.8$, our estimator identified the process change point in 13.7% of the simulation trials. Our estimator was within two (six) subgroups of the true process change point in 57.4% (91.1%) of the simulations. The proposed estimator performs very well for large values of β. For $\beta = 3$, the c chart has an ARL of 3.36. In this case, our proposed estimator identified the change point correctly in 67.3% of the simulations and was within one subgroup of the time of the process change in 81.6% of the simulation trials.

7. Conclusion

Knowing the time when the process change began would simplify the search for the special cause. If process engineers knew when the change in the process began, the search would simply reduce to discovering which process variables or procedures changed during that time. Thus, process engineers would increase their chances of correctly identifying the special cause quickly. This would allow them to take appropriate actions to improve quality sooner.

In this paper, we have proposed a new estimator based on the likelihood function that is useful for identifying the time of a drift change in the Poisson process. The performance of the proposed estimation method was evaluated and compared with the MLE methods developed by Perry et al. [12] for linear trend in Poisson rate parameter. For this purpose, we consider the out-of-control signal comes from a Poisson CUSUM and c control charts. When the out-of-control signal is detected, the diagnostic is started. Results showed that the proposed approach has a smaller variance almost in all cases and for large values of β it outperforms the other estimator when a linear trend disturbance is present.

The proposed method provides an estimate to the drift change in Poisson process. Employing such an estimator for other kind of processes such as binomial, normal and etc. may be a subject for future research.

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