Int. J. Research in Industrial Engineering, pp. 11-25

Volume 3, Number 1, 2014

International Journal of Research in Industrial Engineering

journal homepage: www.nvlscience.com/index.php/ijrie

Comprehensive Method to Determine Real Option Utilizing Probability Distribution

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A R T I C L E I N F O

Article history :
Received:
March 14, 2014
Revised:
April 15, 2014
Accepted:
May 03, 2014

Keywords :

Real option, optimization, Option pricing model

A B S T R A C T

Data envelopment analysis (DEA) is a non-parametric analytical methodology widely used in efficiency measurement of decision making units (DMUs). Conventionally, after identifying the efficient frontier, each DMU is compared to this frontier and classified as efficient or inefficient. This thesis introduces the most productive scale size (MPSS), and anti- most productive scale size (AMPSS), and proposes several models to calculate various distances between DMUs and both frontiers. Specifically, the distances considered in this paper include: (1) both the distance to MPSS and the distance to AMPSS, where the former reveals a unit's potential opportunity to become a best performer while the latter reveals its potential risk to become a worst performer, and (2) both the closest distance and the farthest distance to frontiers, which may proved different valuable benchmarking information for units. Subsequently, based on these distances, eight efficiency indices are introduced to rank DMUs. Due to different distances adopted in these indices, the efficiency of units can be evaluated from diverse perspectives with different indices employed. In addition, all units can be fully ranked by these indices.

1. Introduction

In these days, importance of making decisions in projects in order to manage budgets and direct actions towards predetermined goals is in growing. Decision making, according to its nature, often gets very complicated even though many methods are developed to contribute in convoluted situations. One of these methods increasingly used in recent years, is real options. Determination of proper template for project portfolio selection using real option valuation method is an increasingly used technique in financial engineering and project management.
Main characteristic of real option that makes it a useful tool is considering risks of projects in financial valuations. This feature becomes more important in high risk projects like R&D and huge oil projects since there is no data or cognition over the market or lots of stochastic factors affect valuation of projects.

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Project portfolio selection can be performed in different ways such as zero one programing including project value, switching model and real option valuation, developed NPV (Net Present Value) methods, to name but a few. In these techniques NPV of projects are being reported by a single number and only the information existing in expected value is used to valuate project portfolios

While as mentioned, the real options approach takes risks into account, so considering distribution of gathered data might provide some other useful information about real option value of projects in portfolio selection.

Distribution of real option value can provide several advantages. Distribution will show dispersal of variable, so optimum decisions can be made in different levels of certainty and according to the nature of the problem, different certainty levels might be defined for parameters of the problem. Another advantage in determining distribution of real option value is simplifying sensitivity analysis of the decision making optimization problem.

Due to the fact that the NPV of the projects are made up of different stochastic factors, a simplifying assumption can be made that the NPV distribution of project are considered as normal or lognormal distribution which has certain expected value and standard deviation.

By considering the NPV of the project as a stochastic variable with a specified distribution, the common methods of real option valuation can be developed in order to determine the distribution of the real option and if it is not possible, at least expected value and variance of real option value can be determined.

In section 2, a review of the performed studies in real option area are surveyed, in section 3, present equations for calculating real option are introduced and expanded, and the new methodology is developed. In section 4, a simple numerical example is presented and objective parameters are calculated using new methodology. In last section, conclusions are made.

2. Literature review

Real option is defined based on the similar principles of option in financial engineering. Real option, unlike option, includes real assets. First articles of applying real option to solve an investment problem were published by [1] and [2]. Afterwards, growing numbers of studies in the field of real option are published.

One of the most important fields in which real option is applied, is R&D and portfolio selection for new products. As an example of this, [3] presented a model to optimize making the decisions to select the best R&D portfolio for the risk-averse decision maker while considering Unreliability of R&D environment. This model is based on fuzzy logic. [4], following the elementary R&D management method, introduced a support tool to determine R&D occasions for the following purposes: (1) to find out the shadow options that are not yet clear in the format of cash flows, and (2) to consider these options as a choice in R&D portfolios. [5] provided a report of all the risks that are present R&D portfolios and investigated effects of existence of conditional decision options on correlations among options. [6] also developed an easily applicable model for evaluating the real option investments in R&D area.

Real option is also applied in other areas. [7] attempted to utilize real option in information systems and developed a model for this purpose. [8] utilized real data in aerospace industry and introduced a real option model in order to assess various licensing scenarios.

In recent years, applying real options in energy area is in growth. [9] presented a framework based on binomial tree to value existing policies defined for renewable energies and [10] incorporated two models from literature and studied effects of several parameters such as market risk on infrastructure investments in hydrogen section using the real options. [11] analyzed real option of abandoning investment in the corporations installing energy saving equipment.

Some of the studies have tried to create new frameworks to assess managerial decisions through combining real option and other concepts. [12] and [13] incorporated the concepts of real options and game theory. [14] applied probabilistic dynamic programing to real option for determining the optimal seller choice strategies in such multi-phase, multi-seller competitions. [15] determined value of the project by using the real option, while the cash flow and cost's mean were estimated through fuzzy numbers and Lee et al, (2005) employed Bayes' rule to determine the real options parameters' degree of fuzziness and used a four dimensional fuzzy decision space to describe decision space of investor in order to make the utilization of black-Scholes option pricing model possible.

Main characteristic of real option as a tool for analyzing the plans financially is considering the existing risk in the plans through volatility. But when the value of real option is reported by a single quantity, some of the vital information is ignored. The aim of this study is to initiate a new method to calculate the value of the real option and value of the project while probability distribution of the real option value is determined.

3. Methodology

Several methods are developed in order to determine the real option portfolios that are based on the value of the real option. The goal of this study is to expand these methods so the distribution of the real option value or at least the expected value and variance of the real option is used to select a portfolio with minimum risk.

A. Expected value and standard deviation of the real option in the binomial tree method:

1. Calculating parameters of the real option value in one period

In the real option valuation method, at first step the value of the project must be determined and after that the real option's value is designated based on future movement of project's value (Fig 1).

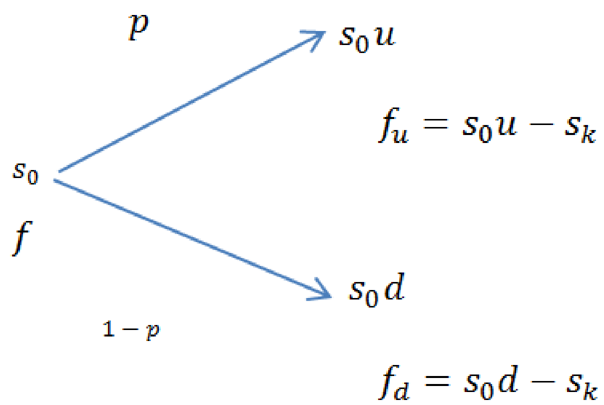

Figure 1: Binomial Tree

According to uncertainties present in the NPV of projects, it is argumentative to assume that the value of project has normal distribution along with a certain expected value and standard deviation. Together with the project's NPV distribution and its probabilistic change, value of the real option is calculated as follows.

- $s_0 \sim N(\mu, \sigma^2)$
- $s_0 u \sim N(u\mu, (\sigma)^2)$
- $s_0 d \sim N(d\mu, (\sigma)^2)$
- $s_k \sim N(e^{r\Delta t}\mu, (\sigma)^2)$
- $f_u = s_0 u - s_k \sim N(\mu(u - e^{r\Delta t}), 2\sigma^2)^+$
- $f_d = s_0 d - s_k \sim N(\mu(d - e^{r\Delta t}), 2\sigma^2)^+$
- $f = e^{-rT} * (pf_d + (1 - p)f_u)$
- $\mu_{fd} = \int_0^\infty xf_d$
- $\mu_{fu} = \int_0^\infty xf_u$
- $p = \frac{e^{r\Delta t} - d}{u - d}$

In these equations, S0 is the value of the project at time zero, Sk is the value of project at time period k, μ is the mean of the project value, σ2 is the variance of the project value, u is the ratio that shows the amount of growth in the project value and d is the ratio that shows amount of decline in the project value, r is annualized continuously compounded rate. It must be noted that in most the of real option cases, quantity of u and d are determined by adding a quantity to one and subtracting the same quantity from one respectively in order to form the ratios that display probable value of project in the next period of time. P is the probability of growth in the value of project in next period of time, and based on this, 1-p shows the probability of decline in value of the project.

$$\mu_f = e^{-r\Delta t} \times [p \times \int_0^{\infty} xN\left(\mu\left(d - e^{r\Delta t}\right), 2\sigma^2\right)$$
$$+(1-p) \times \int_0^{\infty} xN\left(\mu\left(u - e^{r\Delta t}\right), 2\sigma^2\right)] \quad (1)$$

μf is the expected value of real option.

$$\sigma_f{}^2 = E[var(x \mid y)] + var[E(x \mid y)] \quad (2)$$

$$\sigma_f = p \times [\int_0^{\infty} x^2 f_u - [\int_0^{\infty} x f_u]^2] +$$
$$(1-p) \times [\int_0^{\infty} x^2 f_d - [\int_0^{\infty} x^2 f_d]^2] +$$
$$[p \times u_f d^2 + (1-p) \times u_f d^2] -$$
$$[p \times u_f d + (1-p) \times u_f d]^2 \quad (3)$$

1. Calculating the real option's probability distribution parameters in several stages

Presented formulas are used to calculate the parameters for one step and have to be extended to several steps. In each step, the value of the real option has normal distribution with certain mean and standard deviation and only takes positive values.

At this point, by expanding pervious calculations used in the binomial tree, the value of the real option is computed according to the probability of growth or decline in the value of the project.

In previous section, only one interval was considered. In this section n intervals with equal length of Δt are assumed.

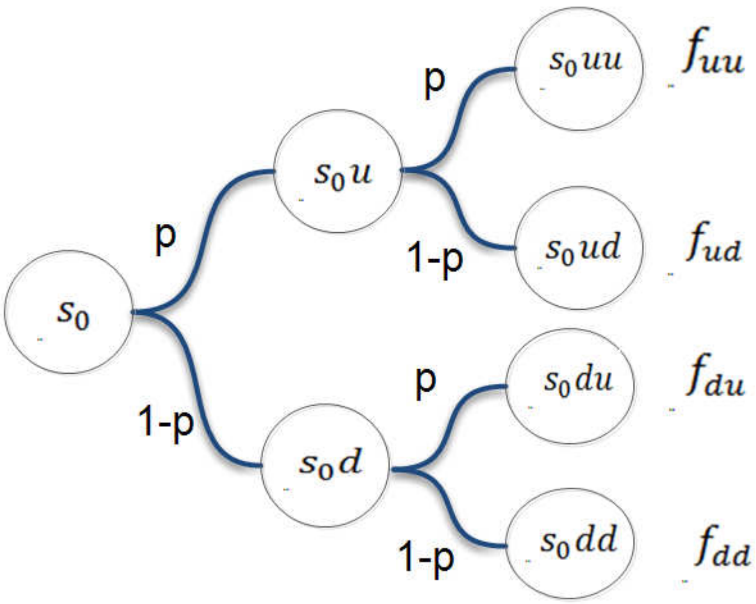

Figure 2: probability distribution of project value in Binomial tree

Presented network is the expansion of the following node (Fig. 2). Calculations for finding the value of the real option are performed from last layer of the nodes, so each node is calculated based on the value of its right hand node.

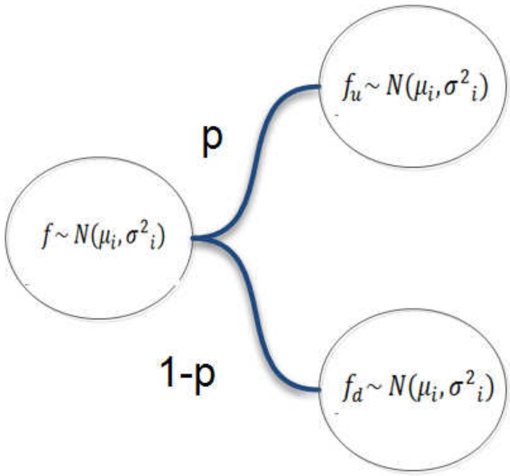

Figure 3: Value probability distribution for a single Node

In this unique stage, the parameters of the tree (mean and standard deviation) are specified for fu and fd and by using them, required parameters for $f\_\sim N(\mu_3, \sigma_3^2)^+$ are determined (Fig. 3).
In valuing the European real option, this procedure is repeated to starting point of the tree. At the end, mean and standard deviation of the real option value is determined. The only difference in valuing American real option is that in each node the quantity of $f_\sim N(\mu_3, \sigma_3^2)^+$is calculated using normal distribution of real option value, compared with ${s_{k_{node}}}^+$ and the one with higher quantity is selected. Rest of the procedure is the same as European procedure.

4. Numerical illustration

There is an oil project which is going to be operational within one year. According to the stochastic factors present in valuing the project, the NPV of project follows normal distribution N (100,302). It has to be noticed variance 30 against mean of 100 shows volatility of 30%.
To use the flexibility of binomial tree in valuation, project period is required to be broken into smaller periods with capability of exercising real option. In this example the project is broken into three periods each with length of four month.
Based on the Figure 1 growth or decline in the value of the project is calculated as follows.
$d = \frac{1}{u} = 0.84 \quad u = e^{\sigma\sqrt{\Delta t}} = e^{0.30*\sqrt{0.33}} = 1.19$
As mentioned above the probability of growth will be

$$p^* = \frac{e^{\mu\Delta t} - d}{u - d} = \frac{e^{0.15\times 0.33} - 0.82}{1.21 - 0.82} = 0.60$$

So the probability of decline would be 1-P.

Project value at final nodes follows normal distribution. Parameters of the distribution are represented in Table 1.

Table 1: Distribution parameters for third level of project value

	Mean	**Variance**
S_{uuu}	167.7007	30
S_{uud}	118.8078	30
S_{udu}	118.8078	30
S_{udd}	84.169558	30
S_{duu}	118.8078	30
S_{dud}	84.169558	30
S_{ddu}	84.169558	30
S_{ddd}	59.630044	30

These parameters are calculated as explained in previous section. For example

$$S_{uuu} = N\left(\mu \times u^3, \sigma^2\right) = N\left(100 \times 1.21^3, 30^2\right) \quad (5)$$

By assuming r = 15%, the value of project at the end of the tree is

$$S_{kkk} = N\left(\mu \times e^{r\times 1}, \sigma^2\right) = N\left(100 \times 1.16, 30^2\right) = N\left(116.18, 30^2\right) \quad (6)$$

And the distribution of real option value would be

$$f_{uuu} = s_{uuu} - s_{kkk} \sim N\left(\mu\left(u^3 - e^{r\times 1}\right), 2\sigma^2\right)^+ = N\left(167.7 - 116.18, 2\times(30)^2\right)^+ \quad (7)$$

$$f_{uud} = s_{uud} - s_{kkk} \sim N\left(\mu\left(u^2 d - e^{r\times 1}\right), 2\sigma^2\right)^+ \quad (8)$$

Distribution parameters of real option value and project value are presented in Table 2.

Table 2: Distribution parameters for third level of project value

	Mean	**Variance**
f_{uuu}	51.52	$2 * 30^2$
f_{uud}	2.62	$2 * 30^2$
f_{udu}	2.62	$2 * 30^2$
f_{udd}	-32.01	$2 * 30^2$
f_{duu}	2.62	$2 * 30^2$
f_{dud}	-32.01	$2 * 30^2$
f_{ddu}	-32.01	$2 * 30^2$
f_{ddd}	-56.55	$2 * 30^2$

Now distribution parameters of the call real option are calculated for pre final nodes.

$$\mu_{f_{uu}} = e^{-r\Delta t} \times [p \times \int_0^{\infty} xN\left(\mu\left(uuu - e^{r\times 1}\right), 2\sigma^2\right)$$
$$+(1-p) \times \int_0^{\infty} xN\left(\mu\left(uud - e^{r\times t}\right), 2\sigma^2\right)] \quad (9)$$

$$= e^{-0.15\times 0.4} \times [0.62 \times \int_0^{\infty} xN\left(118.8 - 116.2, 2\times 30^2\right)$$
$$+(1-0.62) \times \int_0^{\infty} xN\left(167.7 - 116.2, 2\times 30^2\right)] = 45.95 \quad (10)$$

Based on equation (1),

$$\sigma_{(f_u u)}{}^2 = p \times [\int_0^{\infty} x^2 f_{uuu} - u_{(f_{uuu})}{}^2] + (1-p)$$
$$\times [\int_0^{\infty} x^2 f_{uud} - u_{(f_{uud})}{}^2] + [p \times u_{(f_{uuu})}{}^2 + (1-p) \quad (11)$$
$$\times u_{(f_{uud})}{}^2] - [p \times u_{(f_{uuu})} + (1-p) \times u_{(f_{uud})}]^2$$

$$\mu_{f_{uuu}} = \int_0^{\infty} xf_{uuu} \quad (12)$$

$$\mu_{f_{uud}} = \int_0^{\infty} xf_{uud} \quad (13)$$

By replacing calculated values in equations:

$$f_{uuu} \sim N\left(167.8 - 116.2, 2\times 30^2\right) \quad (14)$$

$$f_{uud} \sim N\left(118.8 - 116.2, 2\times 30^2\right) \quad (15)$$

$$\sigma_{f_{uu}} = 56.5$$

Mathematica 7.1 is used to calculate the real option value parameters. The results are represented in Table 3 and Table 4.

Table 3: Distribution parameters for second level of project value

Node	**Mean of real option value**	**Standard deviation of real option value**
Uu	45.95	56.5
Ud	17.3	30
Du	17.3	30
Dd	6.02	16.15

At this point, according to the probability of growth for each node in binomial tree the value of the real option is determined through following equations. An example is represented for node u.

$$\mu_{(f_u)} = e^{(-r\Delta t)} \times [p \times \int_0^\infty xN(\mu(uu), \sigma_{uu}{}^2) + (1-p) \times \int_0^\infty xN(\mu(ud - e^{(r\times t)}), \sigma_u d\,?^2)] \tag{16}$$

$$= e^{(-0.15\times 0.33)} \times [0.60 \times \int_0^\infty xN(45.95, 56.5^2) + (1-0.60) \times \int_0^\infty xN(17.3, 30^2)] = 34.6 \tag{17}$$

By using equation 1-2 again,

$$u_{(f_u)}{}^2 = p \times [\int_0^\infty x^2 f_{uu} - u_{(f_{uu})}{}^2] + (1-p) \times [\int_0^\infty x^2 f_{ud} - u_{(f_{ud})}{}^2] + [p \times u_{(f_{uu})}{}^2 + (1-p) \times u_{(f_{ud})}{}^2] - [p \times u_{(f_{uu})} + (1-p) \times u_{(f_{ud})}]^2 \tag{18}$$

$$\mu_{f_u} = \int_0^\infty x f_{uu} \tag{19}$$

$$\mu_{f_d} = \int_0^\infty x f_{ud} \tag{20}$$

$\sigma_{fuu} = 56.5$

Value of other nodes can be determined in the same way.

Table 4: Distribution parameters for first level of project value

Node	Mean of real option value	Standard deviation of real option value
U	38.5	53.7
D	16.6	24.9

So distribution parameters of the real option value in the first node can be determined like other nodes.

These parameters are represented in Table 5.

Table 5: Distribution parameters for project value

Node	Mean of real option value	Standard deviation of real option value
-	33	46.7

A. Determining the expected value and standard deviation of the real option value in continuous mode

In this case, it is possible to assume that the value of the project follows a lognormal distribution with certain expected value and standard deviation. Expected value can be determined as follows.

$$E\left[\max(V-K), 0\right] = \int_K^\infty (V-K)\, g(V)\, dV \tag{21}$$

Based on the characteristics of lognormal distribution, variable lnV with standard deviation w, has a mean value equal to m, in which m is:

$$m = \ln\left[E(V)\right] - w^2/2 \tag{22}$$

Now a new variable is defined as:

$$Q = \frac{lnV - m}{w} \tag{23}$$

This variable has a standard normal distribution with the mean value equal to zero and the standard deviation equal to one.

$$h(Q) = \frac{1}{\sqrt{2\pi}} e^{\frac{-Q^2}{2}} \tag{24}$$

By using variable 19, equation 16 changes to equation 20.

$$\left[\max(V-K),0\right] = \int_{(lnK-m)/w}^{\infty} \left(e^{Qw+m} - K\right) h(Q)\,dQ \tag{25}$$

$$E\left[\max(V-K),0\right] = \int_{(lnK-m)/w}^{\infty} e^{Qw+m} h(Q)\,dQ$$
$$-k \times \int_{(lnK-m)/w}^{\infty} h(Q)\,dQ \tag{26}$$

$$\mathrm{E}\left[\max(\mathrm{V}-\mathrm{K}),0\right] = \int_{(\ln\mathrm{K}-\mathrm{m})/\mathrm{w}}^{\infty} \mathrm{e}^{\mathrm{Qw+m}} \mathrm{h}(\mathrm{Q})\,\mathrm{dQ}$$
$$-\mathrm{k} \times \int_{(\ln\mathrm{K}-\mathrm{m})/\mathrm{w}}^{\infty} \mathrm{h}(\mathrm{Q})\,\mathrm{dQ} \tag{27}$$

$$\mathrm{E}\left[\max(\mathrm{V}-\mathrm{K}),0\right] =$$
$$\int_{(\ln\mathrm{K}-\mathrm{m})/\mathrm{w}}^{\infty} \mathrm{e}^{\mathrm{Qw+m}} \mathrm{h}(\mathrm{Q})\,\mathrm{dQ} - \mathrm{k} \times \int_{(\ln\mathrm{K}-\mathrm{m})/\mathrm{w}}^{\infty} \mathrm{h}(\mathrm{Q})\,\mathrm{dQ} \tag{28}$$

While

$$e^{Qw+m} h(Q) = \frac{1}{\sqrt{2\pi}} e^{\frac{(-Q^2+2Qw+2m)}{2}} = 1/\sqrt{2\Pi} e^{(-(Q-w)^2+w^2+2m))/2)} = \frac{e^{m+w^2/2}}{\sqrt{2\pi}} e^{\frac{(-(Q-w)^2)}{2}}$$

$$= e^{(m+w^2 ?2)} \times h(Q-w) \tag{29}$$

$$E\left[\max\left(V-K\right),0\right]=e^{m+w^2/2}\times \int_{(\ln K-m)/w}^{\infty} h\left(Q-w\right)dQ-k\times \int_{(\ln K-m)/w}^{\infty} h\left(Q\right)dQ \tag{30}$$

By using characteristics of normal distribution, equation 20 would be equal to

$$1-N[\left(\ln K-m\right)/w-w] \tag{31}$$

Or

$$N[\left(-\ln K+m\right)/w+w] \tag{32}$$

By replacing m it will become

$$N\left[\frac{\left(-\ln[E\left(V\right)/K]+{w^2}/{2}\right)}{w}\right]=N\left(d_1\right) \tag{33}$$

For equation 21, going through the same route will result in:

$$E\left[\max\left(V-K\right),0\right]= e^{m+w^2/2}N\left(d_1\right)-KN\left(d_2\right) \tag{34}$$

In which

$$d_1=\frac{\ln[E(\left(V\right)/K]+w^2/2}{w} \tag{35}$$

and

$$d_2=\frac{\ln[E(\left(V\right)/K]-w^2/2}{w} \tag{36}$$

Calculation results for Black-Scholes model

For a call option with no dividend, exercise date T, exercise price k, risk free rate of return r, current price S0 and standard deviation σ, value of call option would be:

$$c=e^{-rT}\hat{E}\left(S_T-K,0\right) \tag{37}$$

While

$$\hat{E}(S_T)=S_0e^{(-rT)} \tag{38}$$

Standard deviation for lnST equals to $\sigma\sqrt[2]{T}$.

Following equations can be concluded from previous equations.

$$c=e^{-rT}\left[s_0e^{rT}N\left(d_1\right)-KN\left(d_2\right)\right] \tag{39}$$

And

$$c=\left[s_0N\left(d_1\right)-Ke^{-rT}N\left(d_2\right)\right] \tag{40}$$

1. Calculating variance

$$\operatorname{var}\left[\max\left(V-K\right),0\right]= E\left[\max\left(V-K\right)^2,0\right]-E\left[\max\left(V-K\right),0\right]^2 \tag{41}$$

$$E\left[\max\left(V-K\right)^2,0\right]=\int_0^\infty\left(V-K\right)^2 g\left(V\right)dV \tag{42}$$

$$=\int_0^\infty (V)^2 g(V)dV+\int_0^\infty (K)^2 g(V)dV -\int_0^\infty 2KVg(V)dV \tag{43}$$

While lnV follows a normal distribution with the expected value of m and variance w, it will be derived that:

$$E\left[\max\left(V-K\right)^2,0\right]=\left(e^{w^2}-1\right)e^{2m+w^2} +e^{2m+w^2}+k^2-2k\times e^{m+\frac{1}{2}w^2} \tag{44}$$

The details are presented in appedix A

According to equation (22)

$$\operatorname{var}\left[\max\left(V-K\right),0\right]=\left(e^{w^2}-1\right)e^{2m+w^2}+e^{2m+w^2} +k^2-2k\times e^{m+\frac{1}{2}w^2}-\left[e^{m+w^2/2}N\left(d_1\right)-KN\left(d_2\right)\right]^2 \tag{45}$$

$$d_1=\frac{ln[E(\left(V\right)/K]+w^2/2}{w} \tag{46}$$

And

$$d_2=\frac{ln[E(\left(V\right)/K]-w^2/2}{w} \tag{47}$$

As presented, Black-Scholes formula can be expanded, in order to calculate the expected value and the variance of the option value instead of a single quantity. This expansion results in more flexibility. The expected value and variance of a stochastic variable and -if it is possible to be determined- the distribution of it can provide useful information about degree and intensity of dispersion of the real option value. This will result in better understanding of problem and leads to making efficient decisions.

Calculating expected value and standard deviation of the real option value for continuous mode in general

In analogous cases in which, calculating value of real option is related to different parameters of distribution and determining the variance in closed form is difficult or impossible, it would be helpful to use the delta method.

In this method, the variance of a stochastic variable is estimated through following equations using first and second derivations of distribution function of the stochastic variable.

$$\operatorname{var}\left(f\left(x\right)\right) = (f'\left(E\left(x\right)\right)^2 \operatorname{var}\left(x\right) \tag{48}$$

$$f(x) = f(a) + f^{'}(a)(x-a) + f^{''}(a)(x-a)^2/2! \tag{49}$$

$$f\left(x\right) \approx f\left(a\right) + f'\left(a\right)\left(x-a\right) \tag{50}$$

Now, by assuming a=μx, it will become Taylor expansion of X around mean, which will be equal to

$$y = f\left(x\right) \approx f\left(\mu_x\right) + f'\left(\mu_x\right)\left(x-\mu_x\right) \tag{51}$$

$$\operatorname{var}\left(Y\right) = \operatorname{var}\left(f\left(x\right)\right) \approx \left[f'\left(\mu_x\right)\right]^2 \operatorname{Var}\left(X\right) \tag{52}$$

5. Conclusion

Many stochastic parameters are involved in the project portfolio selection problems that increase uncertainty and thus selecting the optimal project portfolio becomes a hard decision making problem.

One of useful methods in this area is the real option, which includes uncertainties in economic valuations and finds optimal solution based on these uncertainties. In this method, all the parameters and ambiguities are summarized in a solitary quantity and therefore a large part of information will be omitted.

Since the amount of these parameters are significant, based on big numbers' rule it can be assumed that the sum of these parameters follow normal or lognormal distribution. By this assumption, in this study, the real option value is considered as a stochastic variable and current methods of calculating the real option value are expanded in order to determine mean value and variance of this variable.

In the example presented here we determined mean and variance of real option value using binomial tree, while using classic method would lead only to the real option value of a project. Identifying distribution of the real option value will provide more vital information such as the dispersion rate of the variable and the reliability of the answer, and consequently it would be possible to analyze the sensitivity of the answer.

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Appendix A

$$Var(x) = E(x^2) - E(x)^2 \rightarrow$$
$$E(x^2) = Var(x) - E(x)^2 \tag{1}$$

$$E(x) = e^{(m+1/2w^2)} \tag{2}$$

$$\mathrm{Var}(\mathrm{x}) = \left(\mathrm{e}^{\mathrm{w}^2} - 1\right)\mathrm{e}^{2\mathrm{m}+\mathrm{w}^2} \tag{3}$$

$$\int_0^{\infty} (V)^2 g(V)\, dV = E(x^2)$$
$$= \left(e^{w^2} - 1\right) e^{2m+w^2} + e^{2m+w^2} \tag{4}$$

$$\int_0^{\infty} (k)^2 g(V)\, dV = k^2 \tag{5}$$

$$\int_0^{\infty} 2k \times v \times g(V)\, dV = 2k \times E(x) = 2k \times e^{m+\frac{1}{2}w^2} \tag{6}$$

$$E\left[\max(V-K)^2, 0\right] = \left(e^{w^2} - 1\right)e^{2m+w^2} + e^{2m+w^2} + k^2 - 2k * e^{m+\frac{1}{2}w^2} \tag{7}$$