Volume 2, Number 4, 2013

International Journal of Research in Industrial Engineering

journal homepage: www.nvlscience.com/index.php/ijrie

Data Envelopment Analysis Based on MPSS Efficient and Inefficient Frontiers

F. Piran[1], F. Hosseinzadeh Lotfi[2,*], M. Rostami-Malkhalifeh[2]

[1]Department of Mathematics, Islamic Azad University, Zahedan Branch, Zahedan, Iran.

[2]Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.

A R T I C L E   I N F O

Article history :
Received:
September 21, 2013
Revised:
November 5, 2013
Accepted:
December 14, 2013

Keywords :

Data Envelopment Analysis, Efficiency index, Most productive scale size.

A B S T R A C T

Data envelopment analysis (DEA) is a non-parametric analytical methodology widely used in efficiency measurement of decision making units (DMUs). Conventionally, after identifying the efficient frontier, each DMU is compared to this frontier and classified as efficient or inefficient. This thesis introduces the most productive scale size (MPSS), and anti- most productive scale size (AMPSS), and proposes several models to calculate various distances between DMUs and both frontiers. Specifically, the distances considered in this paper include: (1) both the distance to MPSS and the distance to AMPSS, where the former reveals a unit's potential opportunity to become a best performer while the latter reveals its potential risk to become a worst performer, and (2) both the closest distance and the farthest distance to frontiers, which may proved different valuable benchmarking information for units. Subsequently, based on these distances, eight efficiency indices are introduced to rank DMUs. Due to different distances adopted in these indices, the efficiency of units can be evaluated from diverse perspectives with different indices employed. In addition, all units can be fully ranked by these indices.

1. Introduction

Data envelopment analysis (DEA) is a methodology based on a linear programming model for evaluating relative efficiency of decision making units (DMUs) with multiple inputs and outputs. Over the last decade DEA has gained considerable attention as a managerial tool for measuring performance. It has been widely used in the public and private sectors, such as banks, airlines, hospitals, universities and manufacturers. For a comprehensive overview of DEA methodology, the reader is referred to the books of Cooper et al.

Most DEA models measure the distance between DMUs and the efficient frontier as their relative efficiency. By this measurement of efficiency, many DMUs are found to have the same efficiency score, e.g., all efficient DMUs, and thus there is no difference between the efficiency of these units. In order to break the tie, some extended tools are proposed to differentiate them and fully rank them, such as cross-efficiency methods and super-efficiency

[*]Corresponding author
E-mail address: farhad@hosseinzadeh.ir

methods [1]. However, the non-uniqueness of the DEA optimal weights possibly reduces the usefulness of the cross-efficiency method and for super-efficiency methods some problematic areas exist, for example, they can give an excessively high ranking score to some DMUs, and an infeasible issue may occur [2].

Evaluating each DMU according to its distance to the efficient frontier means it is compared against the best performers. Such comparison can provide some valuable insights into the performance of each unit, such as its potential opportunity to become a best performer and how to improve its performance. On the other hand, identifying worst performers and comparing units to those worst performers is also important, which is particularly relevant to crisis and weakness evaluation. A few reasons for such comparison are mentioned as follows. Firstly, based on this comparison we are able to reveal how far a unit is from the worst ones so as to find out its potential risk to become a worst performer and its superiority over others. Secondly, comparison of a unit from two aspects, i.e., comparing this unit against the worst performers and against the best performers, can help us fully understand its relative efficiency among all units. Thirdly, in general, all units can be differentiated by the distance to the worst performers and to the best performers and therefore they can be fully ranked. To realize the comparison against the worst performers, an inefficient frontier which is comprised of those worst ones is introduced, in a manner similar to the efficient frontier. Both the distance to the efficient frontier and the distance to the inefficient frontier are employed to evaluate each DMU's efficiency in this paper.

This paper simultaneously considers two sets: the conventional production possi- bility set (T_V) and the called anti-production possibility set (AT_V). Their bounded frontiers are then called the MPSS efficient frontier (MPSSF) and MPSS inefficient frontier (AMPSSF), respectively. The distances between each DMU and the two frontiers, including both the closest distances and farthest distances, are calculated by some proposed models. Subsequently, based on these distances eight alternative efficiency indices are suggested to rank units. These models and efficiency indices contribute to DEA methodology not only in the ranking of DMUs but also in providing some valuable benchmarking information. Specifically, the contributions include the following aspects. Firstly, both the distance to MPSSF and the distance to AMPSSF are employed to evaluate DMUs, where the former indicates a DMU's potential opportunity to become a best performer while the latter reveals its potential risk to become a worst performer. Secondly, both the closest distance and the farthest distance are suggested to measure how far a unit is from a frontier and consequently different benchmarking information can be obtained. Thirdly, the alternative efficiency indices based on these distances can help to differentiate all units and thus in general all units can be fully ranked with one index supplemented by some other index (indices).

The rest of the paper is organized as follows. Sections 2, 3 and 4 are the theoretical part of our proposed DEA method. Section 2 introduces the anti-production possibility set and MPSS inefficient frontier which are similar to PPS and MPSS efficient frontier in conventional DEA analysis. In Sec. 3, some models for measuring the farthest and closest distances to MPSS and AMPSS are proposed. Based on these distances, eight alternative efficiency indices are presented in Sec. 4. Finally, some conclusions are drawn in Sec. 5.

2. Efficiency and inefficiency

Introduced by Charnes et al. [3], the well-known CCR model assigns an efficiency score to each DMU. It identifies an efficient frontier, and DMUs that lie on the frontier are recognized as efficient, while those that do not are recognized as inefficient. CCR model has a basic assumption of constant returns-to-scale (CRS) for the inputs and outputs. To take into consideration variable returns-to-scale (VRS), a BCC model is introduced by Banker et al [4]. (1984).

First some related concepts are stated, which have been introduced in previous DEA studies. The definition of the production possibility set (T_V) is given as follows:

$$T_V = \left\{ (x, y) |\ x \geq \sum_{j=1}^{n} \lambda_j x_j, y \leq \sum_{j=1}^{n} \lambda_j y_j \ \& \ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \right\} \tag{1}$$

Definition 1: The set

$$MPSS = \{(x, y) \in T_v | \forall \alpha, \forall \beta \left(\alpha(x), \beta(y)\right) \in T_v \Rightarrow \beta < \alpha\} \tag{2}$$

is called the MPSS efficient frontier.

Now we are at the stage to introduce some similar concepts on inefficiency. First, the anti-production possibility set (AT_V) is defined as

$$AT_V = \left\{ (x, y) |\ x \leq \sum_{j=1}^{n} \lambda_j x_j, y \geq \sum_{j=1}^{n} \lambda_j y_j \ \& \ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \right\} \tag{3}$$

Definition 2: Based on the definition of AT_V, the MPSS inefficient frontier is defined as:

$$AMPSS = \{(x, y) \in AT_v | \forall \alpha, \forall \beta \left(\alpha(x), \beta(y)\right) \in AT_v \Rightarrow \beta > \alpha\} \tag{4}$$

3. Distance to MPSS efficient frontier and MPSS inefficient frontier

In this section four distances are discussed: farthest distance to MPSS, farthest distance to AMPSS, closest distance to MPSS and closest distance to AMPSS. These distances and their corresponding benchmarking points may be illustrated by a simple example depicted with Figure 1, where seven DMUs A, B, C, D, E are included and DMU A is considered. Each of these units comprises one input and one output. In this example, the MPSS efficient frontier and the MPSS inefficient frontier are DE and BC, respectively. Under l1 -distance, $D = (\hat{x}_D, \hat{y}_D)$ is the farthest point to DMU A in DE (throughout the paper farthest/closest point in DE to one unit means it is the farthest/closest point to this unit among all points in DE which dominate this unit) while $C = (\hat{x}_C, \hat{y}_C)$ is its farthest point in BC (similarly, throughout the paper farthest/closest point in BC to one unit means it is the farthest/closest point to this unit among all points in BCD which are dominated by this unit) and thus AD and AC are its farthest distances to DE and BC, respectively. Both D and C can be used as benchmarking points for DMU A. Note that AD and D are the distance and benchmarking point which are used to evaluate DMU A. G and F are the closest points to DMU A in DE and BC, respectively, and its closest distances to DE and BC are respectively AG and AF . Similar to D and C, G and F can also be chosen as benchmarking points for DMU A.

For DMU A, both AD and AG reveal its potential opportunity to become a best performer while the other two distances, AC and AF, indicate its potential risk to become a worst performer. It should be mentioned that such four distances are pro- posed from different perspectives. The conventional farthest distance to MPSS is based on a conservative perspective since the longest path to become MPSS efficient will be obtained. On the contrary the closest distance to MPSS is from an optimistic perspective because the shortest path for a DMU to be MPSS efficient is found. Similarly, the farthest distance and the closest distance to AMPSS are respectively optimistic and conservative since the former finds the longest path for a unit to be strongly inefficient while the latter reveals the shortest path to AMPSS. Table 1 summarizes different perspectives of these distances and the performance of each unit can be evaluated from different perspectives by choosing different distances.

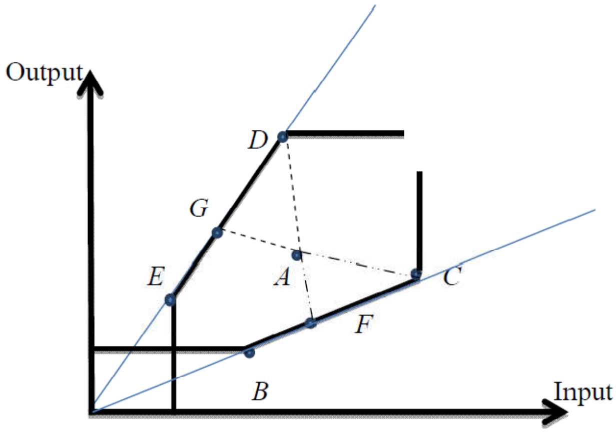

Figure 1. The farthest and closest distances and points for DMU A

DE: strongly efficient frontier
BC: strongly inefficient frontier
AD: farthest distance to SEF
AG: closest distance to SEF
AC: farthest distance to SIF
AF: closest distance to SEF
D: farthest point to SEF
G: closest point to SEF
C: farthest point to SIF
F: closest point to SEF

Table 1. Different perspectives of four distances

	MPSS	*AMPSS*
Closest distance	Optimistic	Conservative
Farthest distance	Conservative	Optimistic

3.1. Farthest distances to MPSS efficient frontiers

Suppose that E_C be a set of all units of T_c vertex efficient, Thus E_C is a set of all the T_v efficient MPSSs.

Farthest distance to MPSS efficient frontier (FDMPSS) can be calculated by the follow- ing model:

$$
\begin{aligned}
&PFDMPSS: max\ FDMPSS = \sum_{i=1}^{m} w_i^- s_i^- + \sum_{r=1}^{s} w_r^+ s_r^+ \\
&s.t. \quad s_i^- = \theta x_{io} - \sum_{j\in E_C} \lambda_j x_{ij} \qquad ,i = 1, \dots, m \\
&s_r^+ = \sum_{j\in E_C} \lambda_j y_{rj} - \theta y_{ro} \qquad ,r = 1, \dots, s \\
&\sum_{j\in E_C} \lambda_j = 1 \\
&s_i^- \geq 0 \qquad ,i = 1, \dots, m \\
&s_r^+ \geq 0 \qquad ,r = 1, \dots, s \\
&\lambda_j \geq 0, \qquad j \in E_C
\end{aligned} \tag{5}
$$

where w_i^- and w_r^+ represent the relative weights assigned to the slacks.
The chosen weights are as follows:

$$
w_i^- = 1/\left(\max_j(x_{ij}) - \min_j(x_{ij})\right), w_r^+ = 1/\left(\max_j(y_{rj}) - \min_j(y_{rj})\right) \tag{6}
$$

3.2. Farthest distances to MPSS inefficient frontiers

Suppose that AE_C is a set of all units of AT_c vertex efficient. Then, AE_C is the set of all AT_c vertex quite inefficient units. That Means, the set of all AMPSSs is in AT_v.
Similarly, farthest distance to MPSS inefficient frontier (FDAMPSS) can be calculated by the following linear programming, denoted by PFDAMPSS:

$$
\begin{aligned}
&PFDAMPSS: max\ FDAMPSS = \sum_{i=1}^{m} w_i^- s_i^- + \sum_{r=1}^{s} w_r^+ s_r^+ \\
&s.t. \quad s_i^- = \sum_{j\in AE_C} \lambda_j x_{ij} - \theta x_{io} \qquad ,i = 1, \dots, m \\
&s_r^+ = \theta y_{ro} - \sum_{j\in AE_C} \lambda_j y_{rj} \qquad ,r = 1, \dots, s \\
&\sum_{j\in AE_C} \lambda_j = 1 \\
&s_i^- \geq 0 \qquad ,i = 1, \dots, m \\
&s_r^+ \geq 0 \qquad ,r = 1, \dots, s \\
&\lambda_j \geq 0 \qquad ,j \in AE_C
\end{aligned} \tag{7}
$$

Its weights are also selected as (6). Note that the only difference between model PFDMPSS and PFDAMPSS is in their first two constraints, which mean (x_o, y_o) belongs to T_V in (5) while in (7) it belongs to AT_V.

3.3. Closest distance to MPSS inefficient frontier

For each evaluated $DMU(x_o, y_o)$, the closest point and corresponding closest distance to MPSS inefficient frontier (CDAMPSS) can be calculated by the following model PCDAMPSS:

$$
\begin{aligned}
&PCDAMPSS: min\ CDAMPSS = \sum_{i=1}^{m} w_i^- (x_i - x_{io}) + \sum_{r=1}^{s} w_r^+ (y_{ro} - y_r) \\
&s.t. \quad (x, y) \in AMPSS \\
&x_i \geq x_{io}, i = 1, \dots, m \\
&y_r \leq y_{ro}, r = 1, \dots, s
\end{aligned} \tag{8}
$$

where the weights w_i^- and w_r^+ are also selected as range adjusted ones given by (6). (x, y) is a point in MPSS inefficient frontier with $x = (x_1, \dots, x_m)^T$and $y = (y_1, \dots, y_s)^T$. Denote the optimal solution of (8) as (x^*, y^*). then this problem is to find a point (x^*, y^*) inMPSS inefficient frontier AMPSS which will minimize the total weighted l1 -distance between $(x, y)$and $DMU(x_o, y_o)$. The closest point $(x^*, y^*)$is used as the benchmarking point for evaluating the

efficiency of $(x_o, y_o)$and it would provide some valuable benchmarking information. For example, the closest distance CDMPSS indicates the inefficiency of $DMU(x_o, y_o)$: the smaller the optimal objective value is, the closer it is to the MPSS inefficient frontier and so it has the highest risk to become a worst performer. In some cases those units in critical situations deserve special attention and improving their performance is desirable. Model PCDAMPSS can help us to find them out by indicating that $(x^* - x_o, y_o - y^*)$ is the shortest path for a DMU to become MPSS inefficient.

To calculate the closest point and the closest distance to AMPSS, the following algorithm is suggested.

Step1. by using the below model, obtain the image of DMUP

$$
\begin{aligned}
& min \;\; \theta - \varepsilon\left(\sum_{i=1}^{m} w_i^- \, s_i^- + \sum_{r=1}^{s} w_r^+ \, s_r^+\right) \\
& s.t. \;\; \sum_{j \in AE_C} \lambda_j x_{ij} - s_i^- = \theta x_{io}\,, \qquad i = 1, \dots, m, \\
& \sum_{j \in AE_C} \lambda_j y_{rj} + s_r^+ = y_{ro}, \qquad r = 1, \dots, s, \\
& \lambda_j \geq 0, \qquad j \in AE_C, \\
& s_i^- \geq 0, \qquad i = 1, \dots, m \\
& s_r^+ \geq 0, \qquad r = 1, \dots, s
\end{aligned} \tag{9}
$$

and show as following;

$$
\left(\theta^* x_p + s^{-*}, \quad y_p - s^{+*}\right) = \left(\hat{x}_p, \hat{y}_p\right) \tag{10}
$$

Step2. Use the

$$
\begin{aligned}
& max = \alpha \\
& s.t. \, \alpha \hat{x}_{io} \leq \sum_{j=1}^{n} \lambda_j x_{ij} \qquad , i = 1, \dots, m \\
& \alpha \hat{y}_{ro} \geq \sum_{j=1}^{n} \lambda_j y_{rj} \qquad , r = 1, \dots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \alpha \geq 0
\end{aligned} \tag{11}
$$

model for obtained the point of image of the biggest AMPSS

$$
\begin{aligned}
& min = \alpha \\
& s.t. \, \alpha \hat{x}_{io} \leq \sum_{j=1}^{n} \lambda_j x_{ij} \qquad , i = 1, \dots, m \\
& \alpha \hat{y}_{ro} \geq \sum_{j=1}^{n} \lambda_j y_{rj} \qquad , r = 1, \dots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \alpha \geq 0
\end{aligned} \tag{12}
$$

and use the following model for obtained the Smallest AMPSS.
Step3. Calculate the distance between the two points and DMUP. Get the minimum distance as the closest distance to inefficiency boundary of MPSS.

3.4. Closest distance to MPSS efficient frontier

Similarly, the following linear programming problem, PCDMPSS, is solved to calculate the closest point and thus the closest distance to MPSS efficient frontier (CDMPSS) can be obtained:

$$PCDMPSS: min\ CDMPSS = \sum_{i=1}^{m} w_i^- (x_{io} - x_i) + \sum_{r=1}^{s} w_r^+ (y_r - y_{ro})$$
$$s.t.\ \ (x, y) \in MPSS$$
$$x_i \le x_{io}\ , i = 1, \dots, m$$
$$y_r \ge y_{ro}\ , r = 1, \dots, s \qquad (13)$$

Let (x^*, y^*) be the optimal solution of model PCDMPSS which is the closest point in MPSS efficient frontier MPSS to DMU(x_o, y_o). The objective is to minimize the total weighted l1 -distance between (x, y) and (x_o, y_o). It is clear that the smaller the optimal objective value of CDMPSS is, the more efficient the evaluated DMU will be. Particularly, one DMU(x_o, y_o) with its optimal objective value of 0 means it is identical to its benchmarking point $(x^*, y^*)$and thus it is MPSS efficient. This model also reveals $(x_o - x^*, y^* - y_o)$ is the shortest path to bring one DMU to MPSS efficient frontier.
To calculate the closest point and the closest distance to MPSS, the following algorithm is suggested.
Step1. by using the below model, obtain the image of DMUP

$$min\ \ \theta - \varepsilon\left(\sum_{i=1}^{m} w_i^-\, s_i^- + \sum_{r=1}^{s} w_r^+\, s_r^+\right)$$
$$s.t \sum_{j \in E_C} \lambda_j x_{ij} + s_i^- = \theta x_{io}, \qquad i = 1, \dots, m,$$
$$\sum_{j \in E_C} \lambda_j y_{rj} - s_r^+ = y_{ro}, \qquad r = 1, \dots, s,$$
$$\lambda_j \ge 0, \qquad j \in E_C,$$
$$s_i^- \ge 0, \qquad i = 1, \dots, m,$$
$$s_r^+ \ge 0, \qquad r = 1, \dots, s \qquad (14)$$

and show as following;

$$\left(\theta^* x_p - s^{-*}, \quad y_p + s^{+*}\right) = \left(\hat{x}_p, \hat{y}_p\right) \qquad (15)$$

Step2. Use the

$$max = \alpha$$
$$s.t. \alpha \hat{x}_{io} \ge \sum_{j=1}^{n} \lambda_j x_{ij} \qquad , i = 1, \dots, m,$$
$$\alpha \hat{y}_{ro} \le \sum_{j=1}^{n} \lambda_j y_{rj} \qquad , r = 1, \dots, s$$
$$\sum_{j=1}^{n} \lambda_j = 1$$
$$\alpha \ge 0 \qquad (16)$$

model for obtained the point of image of the biggest AMPSS

$$min = \alpha$$

$$s.t. \alpha \hat{x}_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij} \qquad , i = 1, \dots, m,$$

$$\alpha \hat{y}_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \qquad , r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\alpha \geq 0 \qquad (17)$$

and use the following model for obtained the Smallest AMPSS.

Step3. Calculate the distance between the two points and DMUP. Get the minimum distance as the closest distance to efficiency boundary of MPSS.

4. Some alternative efficiency indices

In this section eight alternative efficiency indices are proposed based on the four distances mentioned in the previous section and all DMUs are ranked according to these efficiency indices. Note that the FDMPSS as an efficiency index and then ranks units according to this index with the following rule: the smaller the value of FDMPSS, the more efficient the evaluated unit will be and thus it is ranked higher. However, as discussed earlier the distance to AMPSS is also as important as the distance to MPSS, which can reveal the potential risk of a unit to become a worst per- former and its superiority over others. In addition, besides the farthest distance which is often considered in the literature, the closest distance to a frontier can also be used to evaluate efficiency from a different perspective. So based on these observations, eight alternative efficiency indices are presented in Table 2. The column of Relation indicates the relationship between a unit's efficiency and the index's value. It is set as "+" or "−". "+" means the larger the index's value the better the corresponding unit will be, while "−" indicates the opposite: the larger the index's value the worse the corresponding unit.

Table 2. The proposed efficiency indices

No. of index	*Name of index*	*Definition of index*	*Range of value*	*Relation*
Index 1	*FDMPSS*	*Model PFDMPSS*	$[0, +\infty)$	−
Index 2	*FDAMPSS*	*Model PFDAMPSS*	$[0, +\infty)$	+
Index 3	*CDMPSS*	*Model PCDMPSS*	$[0, +\infty)$	−
Index 4	*CDAMPSS*	*Model PCDAMPSS*	$[0, +\infty)$	+
Index 5	*DFD*	$DFD = (FDAMPSS - FDMPSS)/(FDAMPSS + FDM$	$[-1,1]$	+
Index 6	*DCD*	$DCD = (CDAMPSS - CDMPSS)/(CDAMPSS + CDM$	$[-1,1]$	+
Index 7	*RFD*	$RFD = FDMPSS/FDAMPSS$	$[0, +\infty]$	−
Index 8	*RCD*	$RCD = CDMPSS/CDAMPSS$	$[0, +\infty]$	−

Among these indices the first four are the distances discussed in Sec. 3 while the remaining four are made up by combining two of these distances into one whole. Any of the eight efficiency indices can be chosen to evaluate efficiency according to one's requirement. The motivation and justification for proposing these indices are explained as follows. Firstly, the indices FDMPSS and CDMPSS, i.e., the farthest distance and the closest distance to MPSS, indicate how far a DMU is from the best performers. So in order to reveal a unit's potential opportunity to become a best performer and find out a way to improve its performance, FDMPSS and CDMPSS are appropriate. Secondly, the indices FDAMPSS and CDAMPSS, i.e., the closest distance and the farthest distance to AMPSS, reflect how far a unit is from the worst performers. As a result, the two indices can reveal a unit's potential risk to become a worst performer and its superiority over others. Thirdly, in order to fully understand each unit's situation, its potential opportunity to become a best performer and its potential risk to become a worst performer, i.e., its distances to both frontiers, can be combined for consideration. Clearly, for each DMU, it is desirable that its distance to MPSS is as small as possible, and meanwhile its distance to AMPSS is as large as possible. If the distance to MPSS is almost equal to the distance to AMPSS, this unit seems to be in an average situation. If the former distance is much smaller than the latter, it may be regarded as a good one even though its distance to MPSS is not equal to zero. Conversely, when the former is larger than the latter this unit tends to be a bad one. Therefore, the distance to MPSS combined with the distance to AMPSS can better position the evaluated unit and more accurately reveal its relative efficiency among all units. Index 5 to Index 8 in Table 2 are suggested based on such observations.

The index of DFD combines FDAMPSS and FDMPSS into one whole, which satisfies $-1 \leq \mathrm{DFD} \leq 1$. The efficiency of the evaluated DMU can be derived from its value of DFD, which will be compared to 0. If DFD is equal to 0, the evaluated DMU has an average performance. Those DMUs with DFD greater than 0 are good performers and among them some DMUs are in an extreme condition, i.e., their values of DFD are equal to 1. It is clear that a DMU with DFD equal to 1 means its farthest distance to the MPSS efficient frontier is 0. Thus it is MPSS efficient and has the best performance. Those DMUs with DFD less than 0 are poor performers and among them the extreme targets are those DMUs with DF D equal to −1. A DMU with DFD equal to −1 implies it is MPSS inefficient and has the worst performance. Generally speaking, the larger the value of DFD, the more efficient the corresponding unit will be.

More detailed explanation is required for DFD. This index considers the ratio of $\mathrm{FDAMPSS} - \mathrm{FDMPSS}$ to $\mathrm{FDAMPSS} + \mathrm{FDMPSS}$. As we have explained, by comparing to 0, the difference of the two distances reveals whether the performance of the evaluated DMU is good or bad. However, the question of whether $\mathrm{FDAMPSS} - \mathrm{FDMPSS}$ is sufficient for measuring the efficiency of a DMU may be asked. Suppose that there are two units with the same positive values of $\mathrm{FDAMPSS} - \mathrm{FDMPSS}$. This implies that the two units almost have the same distance to the average performers. However, their performances might be different due to their different locations, which can be reflected by the sum of FDAMPSS and FDMPSS. It is clear that the DMU with a lower sum is closer to the MPSS efficient frontier and it is likely to be more efficient than the other. Therefore, the sum of FDAMPSS and FDMPSS also needs to be taken into consideration when measuring a unit's efficiency. By considering the difference of FDMPSS and FDAMPSS as well as their sum, the ratio of DFD can accurately assess the relative efficiency of the evaluated unit.

There is a special case which can be revealed by Index 5. Note that it is possible for a DMU to have both its FDAMPSS and FDMPSS be equal to 0 and thus it is not only MPSS efficient but also MPSS inefficient. Since for such a DMU its distances to both frontiers are equal (though equal to 0), it can be regarded to have an average perfor- mance and thus its value of DFD is 0.

Index 6 is similar to Index 5 and the only difference between them is that the closest distance is adopted in DCD while DFD uses the farthest distance. The range of the value of DC D is also $[-1,1]$. If DCD is equal to 1 then the corresponding unit is MPSS efficient. If DCD is -1 then this unit is MPSS inefficient. Those DMUs with DCD less than 0 have poor performances while those with DCD greater than 0 have good performances.

Compared to DFD and DCD, the last two indices of RFD and RCD also combine the distance to AMPSS and the distance to MPSS but the ratio of the two distances are adopted. The range of the value of RFD is $[0, +\infty]$ and this value is compared to 1.

If it is equal to 1, the two distances are equal and the evaluated DMU is in an average situation. This case is the same as that of $\mathrm{DFD} = 0$. In the case of $\mathrm{RFD} < 1$, FDMPSS is smaller than FDAMPSS and so the evaluated unit has a good performance. Conversely, it has a poor performance for the case of $\mathrm{RFD} > 1$. All MPSS efficient DMUs have the same form of RFD as 0, while all MPSS inefficient ones have the form of ∞ . A DMU which is not only MPSS efficient but also MPSS inefficient has an average performance and thus its value of RCD is 1. Overall, the smaller the value of RFD, the more efficient the corresponding DMU will be. The index of RCD is quite similar to RF D, in which the closest distances are utilized instead of the farthest distances adopted in RFD.

Although there are eight efficiency indices suggested in total, it should be stated that we do not aim to tell which index performs better than others and should be used in applications. In fact, as mentioned in the motivation and justification for these indices, the choice of index depends on one's requirements. In addition, with different indices employed various efficiency results are obtained for all units. Based on these results each unit can be evaluated from different aspects, from which all units can comprehensively understand their efficiency and better position themselves so as to meet the challenges from their competitors.

Both the indices DCD and RCD are constructed by CDE and CDI. However, they are different at least in the following aspects. Firstly, their ranges are different: $\mathrm{DCD} \in [-1,1]$ while $\mathrm{RCD} \in [0, +\infty]$. Secondly, DCD is compared to 0 while RCD is compared to 1. Thirdly, DCD and RCD differ in their focus: DCD is more concerned about the relative difference of the closest distances while RCD pays more attention to the ratio of the closest distances. However, DCD and RCD are related to each other.

All eight efficiency indices are employed to evaluate efficiency of DMUs and thus eight ranking methods are obtained according to these indices, which are given by Table 3. It should be noted that for each efficiency index some DMUs are always found to have the same score and thus their rankings are identical in the corresponding ranking results. In order to break the tie some other indices can be used as a supplement to the corresponding index to rank DMUs. The last column of Table 3 gives a possible way to break the tie.

Table 3. Ranking DMUs according to the efficiency indices in Table 2

Ranking method	Index	An alternative to break the tie
R1	*FDMPSS*	*FDAMPSS*
R2	*FDAMPSS*	*FDMPSS*
R3	*CDMPSS*	*CDAMPSS*
R4	*CDAMPSS*	*CDMPSS*
R5	*DFD*	*FDMPSS&FDAMPSS*

Table 3. Continued

R6	*DCD*	*CDMPSS&CDAMPSS*
R7	*RFD*	*FDMPSS&FDAMPSS*
R8	*RCD*	*CDMPSS&CDAMPSS*

5. Conclusions

In this paper, after identifying MPSS efficient frontier and MPSS inefficient frontier some models are introduced to calculate the distances between DMU and frontiers. The optimal solutions of these models can be used as benchmarking points. Subsequently, some efficiency indices are proposed for efficiency measurement based on these distances, and these indices can be employed to rank DMUs. Since the distance to the MPSS efficient frontier and the MPSS inefficient frontier are adopted for DMU efficiency measurement, each DMU's potential opportunities and crises are revealed. As both the closest distance and farthest distance is used to measure a DMU's distance to frontiers, different valuable benchmarking information can be obtained. With different distances employed in efficiency indices, each DMU is evaluated from different perspectives and some new insights about their efficiency have been revealed. Furthermore, with one index supplemented by some other index (indices), it has been shown that all DMUs can be fully ranked by our proposed efficiency indices.

References

[1] Andersen, P. and Petersen, N.C. (1993), A pdrocedure for ranking efficient units in data envelopment analysis, *Managemente science*, Vol. 39, No. 10, pp.1261-1264.

[2] Adler, N., Friedman, L. and Sinuanny-Stern, Z. (2002), Review of ranking methods in the data envelopment analysis, *European jornal of operalional research*, Vol. 140, pp. 249-265.

[3] Charnes, A., Cooper, W.W. and Rhodes, E. (1978), Measuring the efficiencies of DMUs, *European Journal of Operational Research*, Vol. 2, pp. 429-444.

[4] Banker, R.D., Charnes, A. and Cooper, W.W. (1984), Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management science*, Vol. 30, pp. 1078-1092.