



Fuzzy Big-M Method for Solving Fuzzy Linear Programs with Trapezoidal Fuzzy Numbers

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ABSTRACT

The fuzzy primal simplex method [15] and the fuzzy dual simplex method [17] have been proposed to solve a kind of fuzzy linear programming (FLP) problems involving symmetric trapezoidal fuzzy numbers. The fuzzy simplex method starts with a primal fuzzy basic feasible solution (FBFS) for FLP problem and moves to an optimal basis by walking truth sequence of exception of the optimal basis obtained in fuzzy primal simplex method don't satisfy the optimality criteria for FLP problem. Also this method has no efficient when a primal fuzzy basic FBFS is not at hand. The fuzzy dual simplex method needs to an initial dual FBFS. Furthermore, there exists a shortcoming in the fuzzy dual simplex method when the dual feasibility or equivalently the primal optimality is not at hand and in this case, the fuzzy dual simplex method can't be used for solving FLP problem. In this paper, a fuzzy Big-M method is proposed to solve these problems in which the primal FBFS is not readily available. A numerical example is given to illustrate the proposed method.

1. Introduction

The basic concepts of fuzzy decision making were first proposed by Bellman and Zadeh [1]. Tanaka et al. [2] adopted these concepts for solving mathematical programming problems. Zimmermann [3] initially proposed FLP formulation by using of both the minimum operator and the product operator. Afterwards, several authors considered different kinds of the FLP problems and proposed several approaches for solving these problems. Maleki et al. [4] used the concept of comparison of fuzzy numbers and proposed a new method for solving linear programming problems with fuzzy variables using an auxiliary problem. Maleki [5] proposed a new method for solving linear programming problems with vagueness in constraints by using a certain ranking function. Mishmast Nehi et al. [6] used the lexicographic ranking function to solve fuzzy number linear programming problems. Nasseri et al. [7, 8] developed the fuzzy primal simplex algorithms for solving both fuzzy number linear programming and FLP with fuzzy variables problems. Safi et al. [9] introduced a geometric approach for solving FLP problems with fuzzy goal and fuzzy constraints in two-dimensional space. Allahviranloo et al. [10] proposed a new method for solving fully FLP problems by applying the concept of comparison of fuzzy numbers. Hosseinzadeh Lotfi et al. [11] discussed fully

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FLP problems with triangular fuzzy numbers. Ebrahimnejad and Nasseri [12] developed the complementary slackness theorem for solving FLP with fuzzy parameters. Kumar et al. [13] proposed a generalized simplex algorithm for solving a FLP problem with ranking of generalized fuzzy numbers. Nasseri et al. [14] proposed a fuzzy two-phase method for solving FLP with fuzzy variables problems.

In addition, Ganesan and Veeramani [15] introduced a fuzzy primal simplex algorithm for solving FLP problem with symmetric trapezoidal fuzzy numbers. Ebrahimnejad et al. [16] generalized their method for situations in which some or all variables are restricted to lie within fuzzy lower and fuzzy upper bounds. Ebrahimnejad and Nasseri [17] developed a new fuzzy dual simplex algorithm by using the duality which has been proposed by Nasseri et al. [18, 19]. Kheirfam and Verdegay [20] studied sensitivity analysis for these problems when the data are perturbed, while the fuzzy optimal solution remains invariant. Fuzzy primal and dual simplex algorithms have been developed with the assumption that an initial FBFS is at hand. In many cases, finding such a FBFS is not readily available and some works may be needed to get the fuzzy primal simplex algorithm started. In this paper, a fuzzy Big-M method is proposed to solve these problems in which the initial FBFS is not readily available. This paper is organized as follows: In section 2 some basic definitions and arithmetics between two symmetric trapezoidal fuzzy numbers are reviewed. A review of formulation of FLP problem and the method proposed by Ganesan and Veeramani [15] for solving this problem are given in section 3. In section 4 a fuzzy Big-M method is proposed for FLP problems with the assumption that an initial FBFS is not readily available. A numerical example is solved in section 5. Finally, conclusions are discussed in section 6.

2. Preliminaries

Here, some necessary definitions and arithmetic operations of fuzzy numbers are presented.

Definition 2.1 A fuzzy number \tilde{a} on real \mathbb{R} numbers is said to be a symmetric trapezoidal fuzzy number if there exist real numbers, a^L and a^U , $a^L \leq a^U$ and $h > 0$ such that

$$\tilde{a}(x) = \begin{cases} \frac{x-(a^L-h)}{h} & a^L - h \leq x \leq a^L \\ 1 & a^L \leq x \leq a^U \\ \frac{-x+(a^U+h)}{h} & a^U \leq x \leq a^U + h \\ 0 & \text{else} \end{cases} \quad (\text{I})$$

A symmetric trapezoidal fuzzy number is denoted as $\tilde{a} = (a^L, a^U, h)$ when $h=0$; $\tilde{a} = (a^L, a^U)$ the set of all symmetric trapezoidal fuzzy numbers on \mathbb{R} by $F(\mathbb{R})$. The symmetric trapezoidal fuzzy number is shown in Figure 1.

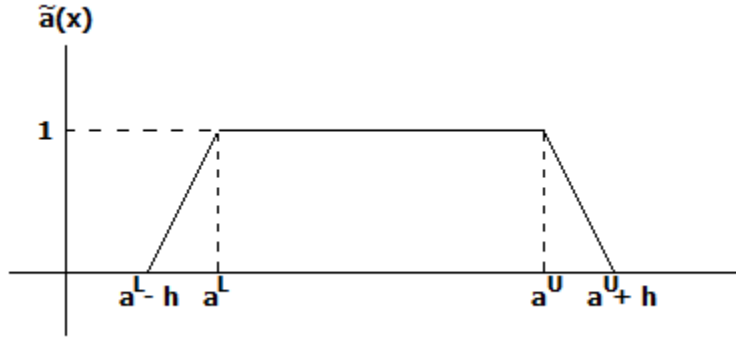


Figure 1. A symmetric trapezoidal fuzzy number

Remark 2.1 In this paper, a large fuzzy number is considered as $\tilde{M} = (M, M)$ and M is mathematically $M \rightarrow +\infty$.

Let $\tilde{a} = (a^L, a^U, h)$ and $\tilde{b} = (b^L, b^U, k)$ be two symmetric trapezoidal fuzzy numbers.

The arithmetic operations on these fuzzy numbers as follows:

Addition: $\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, h + k)$

Subtraction: $\tilde{a} - \tilde{b} = (a^L - b^U, a^U - b^L, h + k)$

Scalar multiplication: $\lambda \in \mathbf{R}, \lambda \tilde{a} = (\lambda a^L, \lambda a^U, |\lambda| h)$

Multiplication: $\tilde{a}\tilde{b} = ((\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) - w, (\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) + w, |a^U k + b^U h|)$

Where $w = \frac{\beta - \alpha}{2}$, $\alpha = \min\{a^L b^L, a^L b^U, a^U b^L, a^U b^U\}$ and $\beta = \max\{a^L b^L, a^L b^U, a^U b^L, a^U b^U\}$

Definition 2.2 Let $\tilde{a} = (a^L, a^U, h)$ and $\tilde{b} = (b^L, b^U, k)$ be two symmetric trapezoidal fuzzy numbers. Define the relations \preceq and \approx as

$\tilde{a} \preceq \tilde{b}$ if and only if

$$\frac{(a^L - h) + (a^U + h)}{2} < \frac{(b^L - k) + (b^U + k)}{2}$$

in this case can be writing $\tilde{a} \prec \tilde{b}$

Or $\frac{a^L + a^U}{2} = \frac{b^L + b^U}{2}, b^L < a^L$ and $a^U < b^U$

Or $\frac{a^L + a^U}{2} = \frac{b^L + b^U}{2}, a^L = b^L, a^U = b^U$ and $h \leq k$

In these cases can be saying that \tilde{a} and \tilde{b} are equivalent. ($\tilde{a} \approx \tilde{b}$)

3. Fuzzy Linear Programming

Let $A = (a_{ij})_{m \times n}, \tilde{c} = (c_j)_{1 \times n}, \tilde{b} = (b_i)_{m \times 1}, \tilde{x} = (x_j)_{1 \times n}$ where $a_{ij} \in \mathbf{R} (i = 1, 2, \dots, m, j = 1, 2, \dots, n)$ and $b_i, c_j, x_j \in F(\mathbf{R}), \tilde{b}_i, \tilde{x}_j \succcurlyeq \tilde{0}$ Then

$$\min \tilde{z} \approx \tilde{c}\tilde{x}$$

$$\text{s. t. } A\tilde{x} \succcurlyeq \tilde{b}$$

(II)

$\tilde{x} \succcurlyeq \tilde{0}$

is said to be a FLP.

Definition 3.1 Any fuzzy vector $\tilde{x} = (F(R))^n$ is called a fuzzy feasible solution to (II) if \tilde{x} non-negativity restrictions and satisfies the constraints of the problem.

Definition 3.2 A fuzzy feasible solution \tilde{x}_0 is said to be a fuzzy optimal solution for (II) if $\tilde{c}\tilde{x}_0 \preccurlyeq \tilde{c}\tilde{x}$ for all fuzzy feasible solution \tilde{x} for (II).

Definition 3.3 Let the i-th fuzzy constraint of (II) be $\sum_{j=1}^n a_{ij}\tilde{x}_j \preccurlyeq \tilde{b}_i$ where $\tilde{b}_i \succcurlyeq \tilde{0}$ then a fuzzy variable \tilde{S}_i such that $\tilde{S}_i \succcurlyeq \tilde{0}$ and $\sum_{j=1}^n a_{ij}\tilde{x}_j + \tilde{S}_i \approx \tilde{b}_i$ is said to be a fuzzy slack variable.

Definition 3.4 Let the i-th fuzzy constraint of (II) be $\sum_{j=1}^n a_{ij}\tilde{x}_j \succcurlyeq \tilde{b}_i$ where $\tilde{b}_i \succcurlyeq \tilde{0}$ then a fuzzy variable \tilde{S}_i such that $\tilde{S}_i \succcurlyeq \tilde{0}$ and $\sum_{j=1}^n a_{ij}\tilde{x}_j - \tilde{S}_i \approx \tilde{b}_i$ is said to be a fuzzy surplus variable.

The standard form of FLP problem (II) as follows

$$\begin{aligned} \min \tilde{z} &\approx \tilde{c}\tilde{x} & (III) \\ \text{s. t. } A\tilde{x} &\approx \tilde{b} \\ \tilde{x} &\succcurlyeq \tilde{0} \end{aligned}$$

Definitions 3.5 Given a system of m fuzzy linear equation involving symmetric trapezoidal fuzzy number in n unknowns, $A\tilde{x} \approx \tilde{b}$ where $\text{rank}(A)=m$. Let B any matrix $m \times m$ the basis matrix formed by m linearly independent columns of A. Then vector $\tilde{x} = (\tilde{x}_B^T, \tilde{x}_N^T)^T = (B^{-1}\tilde{b}, \tilde{0})$, is a FBFS also can be saying \tilde{x}_B is a fuzzy basic solution. $\tilde{x}_B \succcurlyeq \tilde{0}$

Suppose a FBFS of (III) with basis B is at hand. Let y_j be the solution to $By_j = a_j$. And fuzzy objective value is $\tilde{z}_j = \tilde{c}_B y_j = \tilde{c}_B B^{-1} a_j$.

Theorem 3.1 If there exists a FBFS with fuzzy objective value \tilde{z} such that $\tilde{z}_k - \tilde{c}_k > \tilde{0}$ for some non-basic fuzzy variable \tilde{x}_k , and $y_k > 0$, then it is possible to obtain a new FBFS with new fuzzy objective value \tilde{z} . That satisfies $\tilde{z} \preccurlyeq \tilde{z}$. See in [15].

Theorem 3.2 If there exists a FBFS $\tilde{z}_k - \tilde{c}_k > \tilde{0}$ for some non-basic fuzzy variables \tilde{x}_k , and $y_k \leq 0$, then the FLP problem (III) has an unbounded optimal solution. See in [7].

Theorem 3.3 If a fuzzy basic solution $\tilde{x}_B = B^{-1}\tilde{b}$, $\tilde{x}_N = \tilde{0}$ is feasible (III) and $\tilde{z}_j - \tilde{c}_j \preccurlyeq \tilde{0}$ for all $j, 1 \leq j \leq n$, then the fuzzy basic solution is a fuzzy optimal solution to (III). See in [19].

Ganesan and Veeramani [15] based on these theorems proposed a new algorithm for solving FLP problems in which the initial FBFS is at hand. Here, a summary of their method is given:

Algorithm 3.1 A fuzzy primal simplex method for FLP

Initialization step

Choose a starting FBFS with Basis B. Form the initial tableau similar to Table 1.

Main step

- Step1. Calculate $\tilde{z}_j - \tilde{c}_j$ for all nonbasic variables. Suppose $\tilde{z}_j - \tilde{c}_j = (d_j^L, d_j^U, d_j)$. Let $d_k^L + d_k^U = \max_{j \in T} \{d_j^L + d_j^U\}$ where T is the index set of the current nonbasic variables. If $d_k^L + d_k^U \leq 0$ then stop; the current solution is optimal. Otherwise, go to step 2 with \tilde{x}_k as entering variable.

Table 1. The current fuzzy basic feasible solution

Basis	\tilde{x}_1	...	\tilde{x}_r	...	\tilde{x}_m	...	\tilde{x}_j	...	\tilde{x}_k	...	RHS
\tilde{z}	$\tilde{0}$...	$\tilde{0}$...	$\tilde{0}$...	$\tilde{z}_j - \tilde{c}_j$...	$\tilde{z}_k - \tilde{c}_k$...	-
\tilde{x}_1	1	...	0	...	0	...	y_{1j}	...	y_{1k}	...	\tilde{b}_1
\vdots	\vdots		\vdots		\vdots		\vdots		\vdots		\vdots
\tilde{x}_r	0	...	1	...	0	...	y_{rj}	...	y_{rk}	...	\tilde{b}_r
\vdots	\vdots		\vdots		\vdots		\vdots		\vdots		\vdots
\tilde{x}_m	0	...	0	...	1	...	y_{mj}	...	y_{mk}	...	\tilde{b}_m

- Step2. Let $y_k = B^{-1}a_k$. If $y_k \leq 0$, then stop; the problem is unbounded. Otherwise, suppose $\tilde{b}_i = (\bar{b}_i^L, \bar{b}_i^U, p_i)$ and determine the index of the variable \tilde{x}_{B_r} leaving the basis as follows:

$$\frac{\frac{\bar{b}_r^L + \bar{b}_r^U}{2}}{y_{rk}} = \min_{1 \leq i \leq m} \left\{ \frac{\frac{\bar{b}_i^L + \bar{b}_i^U}{2}}{y_{ik}} \mid y_{ik} > 0 \right\}$$

- Step3. Update the basic B where a_k replace a_{B_r} , and go to step 1.

4. The Fuzzy Big-M Method for Solving FLP Problems

4.1. Fuzzy Artificial Variables

All the inequalities of the constraints are converted into equation by introducing fuzzy slack and fuzzy surplus variables. The coefficients of these fuzzy slack and fuzzy surplus variables are put equal to zero in the objective function.

After introducing fuzzy slack and fuzzy surplus variables, the constraints are put in the format $A\tilde{x} \approx \tilde{b}, \tilde{x} \geq \tilde{0}$ where A is an $m \times n$ matrix and $\tilde{b} \geq \tilde{0}$ is an m vector. Further suppose that A does not have identity sub-matrix (if A has an identity sub-matrix then there is an obvious starting FBFS). In this case, a fuzzy artificial variable is added to form a starting solution and then the fuzzy primal simplex method is used to get rid of these fuzzy artificial variables.

To illustrate, suppose that restrictions are changed by adding a fuzzy artificial vector \tilde{R} leading to the system $A\tilde{x} + \tilde{R} \approx \tilde{b}, \tilde{x}, \tilde{R} \geq \tilde{0}$. This forces an identity sub-matrix corresponding to the fuzzy artificial vector and gives an immediate FBFS of the new system, namely $\tilde{R} \approx \tilde{b}$ and $\tilde{x} \approx \tilde{0}$.

Now there is a starting FBFS and the fuzzy primal simplex method can be applied. In original problem, these fuzzy artificial variables must be forced to zero, because $A\tilde{x} \approx \tilde{b}$ if and only if $A\tilde{x} + \tilde{R} \approx \tilde{b}$ with $\tilde{R} \approx \tilde{0}$.

4.2. The Fuzzy Big-M Method

In fuzzy Big-M method the fuzzy artificial variables are not part of the original FLP model, they are assigned a very high penalty in the objective function, thus forcing them (eventually) to equal zero in the optimum solution. This will always be the case if the problem has a

FBFS. In the Big-M method, using of the penalty \tilde{M} , which by definition must be large relative to the actual objective coefficients of the model.

To illustrate, a FLP Problem (III) is solved if no convenient basis is known, the fuzzy artificial vector \tilde{R} is introduced this leads to the following system:

$$A\tilde{x} + \tilde{R} \approx \tilde{b}, \tilde{x}, \tilde{R} \succcurlyeq \tilde{0} \quad (\text{IV})$$

The Starting FBFS is given by $\tilde{R} \approx \tilde{b}$, $\tilde{x} \approx \tilde{0}$. In order to reflect the undesirability of a non-zero fuzzy artificial vector, the objective function is modified such that a large penalty is paid for any such solution. The following problem is solved:

$$\begin{aligned} \min \tilde{z} &\approx \tilde{c}\tilde{x} + \tilde{M}\tilde{R} \\ \text{s. t. } A\tilde{x} + \tilde{R} &\approx \tilde{b} \\ \tilde{x}, \tilde{R} &\succcurlyeq \tilde{0} \end{aligned} \quad (\text{V})$$

where $\tilde{M} = (M, M)$ is a very large fuzzy number. The term $\tilde{M}\tilde{R}$ can be interpreted as a penalty to be paid by any solution with $\tilde{R} \neq \tilde{0}$. Therefore the fuzzy primal simplex method itself will try to get the fuzzy artificial variables out of the basis, and then continue to find the fuzzy optimal solution of the original problem.

Now, the possible cases that are presented may arise while solving the fuzzy Big-M problem: since problem (V) has a FBFS (say $\tilde{R} \approx \tilde{b}, \tilde{x} \approx \tilde{0}$), then while solving it by the fuzzy primal simplex method one of the following two cases may arise:

1. Conclude that (V) has a fuzzy optimal solution.
2. Conclude that (V) has an unbounded optimal solution.

Case1. Finite fuzzy optimal solution of (V)

Under this case, there are two possibilities:

- (i): $(\tilde{x}_0, \tilde{0})$ is a fuzzy optimal solution of (V)
- (ii): $(\tilde{x}_0, \tilde{R}_0)$ is a fuzzy optimal solution of (V) and $\tilde{R}_0 \neq \tilde{0}$

Case2. FLP problem (V) has an unbounded

Under this case, there are two possibilities:

- (i): $\tilde{z}_k - \tilde{c}_k = \max(\tilde{z}_i - \tilde{c}_i) > \tilde{0}, y_k \leq 0$, and all fuzzy artificial variables are equal to zero.
- (ii): $\tilde{z}_k - \tilde{c}_k \approx \max(\tilde{z}_i - \tilde{c}_i) > \tilde{0}, y_k \leq 0$, and not all fuzzy artificial variables are equal to zero.

5. A Numerical Example

For an illustration of the above method a FLP problem is solved. Consider the following problem:

$$\begin{aligned} \min \tilde{z} &\approx (8,12,6)\tilde{x}_1 + (4,8,6)\tilde{x}_2 \\ \text{s. t. } \tilde{x}_1 + 6\tilde{x}_2 &\succcurlyeq (46,52,2) \\ 4\tilde{x}_1 + 2\tilde{x}_2 &\succcurlyeq (42,48,4) \\ \tilde{x}_1, \tilde{x}_2 &\succcurlyeq \tilde{0} \end{aligned}$$

The standard form of the FLP problem becomes

$$\begin{aligned} \min \tilde{z} &\approx (8,12,6)\tilde{x}_1 + (4,8,6)\tilde{x}_2 \\ \text{s. t. } \tilde{x}_1 + 6\tilde{x}_2 - \tilde{S}_1 &\approx (46,52,2) \\ 4\tilde{x}_1 + 2\tilde{x}_2 - \tilde{S}_2 &\approx (42,48,4) \end{aligned}$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{S}_1, \tilde{S}_2 \geq \tilde{0}$$

As it's seen no initial fuzzy basic variables is available and hence we may use the fuzzy Big-M method to solve the problem. Therefore, the problem changes into the following form:

$$\min \tilde{z} \approx (8,12,6)\tilde{x}_1 + (4,8,6)\tilde{x}_2 + (M, M)\tilde{R}_1 + (M, M)\tilde{R}_2$$

$$s. t. \tilde{x}_1 + 6\tilde{x}_2 - \tilde{S}_1 + \tilde{R}_1 \approx (46,52,2)$$

$$4\tilde{x}_1 + 2\tilde{x}_2 - \tilde{S}_2 + \tilde{R}_2 \approx (42,48,4)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{S}_1, \tilde{S}_2, \tilde{R}_1, \tilde{R}_2 \geq \tilde{0}$$

The problem is written in the tableau form as:

Table 2. The initial fuzzy simplex tableau

Basis	\tilde{x}_1	\tilde{x}_2	\tilde{S}_1	\tilde{S}_2	\tilde{R}_1	\tilde{R}_2	RHS
\tilde{z}	(-12,-8,6)	(-8,-4,6)	$\tilde{0}$	$\tilde{0}$	(-M,-M)	(-M,-M)	-
\tilde{R}_1	1	6	-1	0	1	0	(46,52,2)
\tilde{R}_2	4	2	0	-1	0	1	(42,48,4)

Because \tilde{R}_1, \tilde{R}_2 are fuzzy variables, so the cost row must be equal to zero. Hence, the next tableau will be as following:

Table 3. The change step

Basis	\tilde{x}_1	\tilde{x}_2	\tilde{S}_1	\tilde{S}_2	\tilde{R}_1	\tilde{R}_2	RHS
\tilde{z}	(5M-12,5M-8,6)	(8M-8,8M-4,6)	(-M,-M)	(-M,-M)	$\tilde{0}$	$\tilde{0}$	-
\tilde{R}_1	1	6	-1	0	1	0	(46,52,2)
\tilde{R}_2	4	2	0	-1	0	1	(42,48,4)

From the above tableau $\tilde{z}_2 - \tilde{c}_2: \frac{(8M-8)+(8M-4)}{2}$ is greater than others, so \tilde{x}_2 is an entering fuzzy variable and $-\frac{2}{6} < \frac{42+48}{2}$, so \tilde{R}_1 is a leaving fuzzy variable. Then after pivoting, the next tableau is given as:

Table 4. The first iteration

Basis	\tilde{x}_1	\tilde{x}_2	\tilde{S}_1	\tilde{S}_2	\tilde{R}_1	\tilde{R}_2	RHS
\tilde{z}	$(\frac{11M-34}{3}, \frac{11M-20}{3}, 7)$	$\tilde{0}$	$(\frac{M-4}{3}, \frac{M-2}{3}, 1)$	(-M,-M)	$(\frac{2-4M}{3}, \frac{4-4M}{3}, 1)$	$\tilde{0}$	-
\tilde{x}_2	$\frac{1}{6}$	1	$-\frac{1}{6}$	0	$\frac{1}{6}$	0	$(\frac{46}{6}, \frac{52}{6}, \frac{2}{6})$
\tilde{R}_2	$\frac{11}{3}$	0	$\frac{1}{3}$	-1	$-\frac{1}{3}$	1	$(\frac{74}{3}, \frac{98}{3}, \frac{14}{3})$

According to the above tableau $\tilde{z}_1 - \tilde{c}_1: \frac{(\frac{11M-34}{3})+(\frac{11M-20}{3})}{2}$, so \tilde{x}_1 is an entering fuzzy variable

and $\frac{\frac{74}{3} + \frac{98}{3}}{\frac{2}{11}} < \frac{\frac{46}{6} + \frac{52}{6}}{\frac{2}{11}}$, so \tilde{R}_2 is a leaving fuzzy variable. The last tableau is given in the below

(Now all fuzzy artificial variables leave the basis):

Table 5. The optimal solution

Basis	\tilde{x}_1	\tilde{x}_2	\tilde{S}_1	\tilde{S}_2	\tilde{R}_1	\tilde{R}_2	RHS
\tilde{z}	$\tilde{0}$	$\tilde{0}$	$(\frac{-8}{11}, \frac{4}{11}, \frac{18}{11})$	$(\frac{-34}{11}, \frac{20}{11}, \frac{21}{11})$	$(\frac{-11M-4}{11}, \frac{-11M+8}{11}, \frac{18}{11})$	$(\frac{-11M+20}{11}, \frac{-11M+37}{11}, \frac{21}{11})$	-
\tilde{x}_2	0	1	$-\frac{2}{11}$	$\frac{1}{22}$	$\frac{2}{11}$	$\frac{1}{22}$	$(\frac{68}{11}, \frac{83}{11}, \frac{6}{11})$
\tilde{x}_1	1	0	$\frac{1}{11}$	$-\frac{3}{11}$	$-\frac{1}{11}$	$\frac{3}{11}$	$(\frac{74}{11}, \frac{98}{11}, \frac{14}{11})$

Therefore the fuzzy optimal solution of the FLP problem which is obtained by the fuzzy Big-M method is $\tilde{x}_1 = (\frac{74}{11}, \frac{98}{11}, \frac{14}{11})$, $\tilde{x}_2 = (\frac{68}{11}, \frac{83}{11}, \frac{6}{11})$ and the fuzzy optimal value of its objective function is $\tilde{z} = (\frac{825}{11}, \frac{1801}{11}, \frac{1302}{11})$.

6. Conclusions

In this paper, a fuzzy Big-M method proposes for solving fuzzy linear programming problem in which the element of coefficient matrix of the constraints are represented by real number and rest of the parameters are represented by symmetric trapezoidal fuzzy number and this method proposed for solving this problem in which the primal FBFS is not readily available without converting to the classical linear programming problem.

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