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A Hybridized Lagrangian Genetic Algorithm for Designing an Integrated Supply Chain Network: A Case Study Approach

A. Jafari[1], P. Chiniforooshan[2,*], F. Zabihi[3]

[1]Department of Industrial Engineering, Science and Culture University, Tehran, Iran
[2]Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[3]Technology Development Institute, Department of Industrial Engineering, Tehran, Iran

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ABSTRACT

This paper investigates the problem of designing an integrated production-distribution system which supports strategic and tactical decision levels in supply chain management. This overall optimization is achieved using mathematical programming for modeling the supply chain functions such as location, production, and distribution functions. Our model intends to minimize the total cost including production, location, transportation, and inventory holding costs. In view of the NP-hard nature of the problem, this paper provides a hybrid algorithm incorporates Genetic Algorithm into Lagrangian Relaxation method (namely HLRGA) to update the lagrangian multipliers and improve the performance of LR method. The effectiveness of HLRGA has been investigated by comparing its results with those obtained by CPLEX, hybrid genetic algorithm, and simulated annealing on a set of supply chain network problems with different sizes. Finally, an industrial case demonstrates the feasibility of applying the proposed model and algorithm to the real-world problem in a supply chain network.

1. Introduction

Supply chain management (SCM) is the systematic analysis and educated decision-making within the different business functions of an organization resulting in smooth and cost-effective flows of resources – material, information, and money. In other words, it is the coordination and synchronization of the flow of resources in the network of suppliers, manufacturing facilities, distribution centers (DCs) and customers. These network elements form the different echelons of the supply chain [1]. Decisions are made across the supply chain on three levels: strategic, tactical and operational. Strategic decisions are long term decisions where the time horizon may be anything from one year to several years i.e. it involves multiple planning horizons. Tactical decisions are taken over a shorter period of time, maybe a few months. These are more localized decisions taken to keep the organization on the track set at the strategic level. Operational decisions are similar to day-to-day decisions for planning a few days worth of operations. These take into consideration the most profitable way to carry out daily activities for satisfying immediate requirements.

[*]Corresponding author
E-mail address: p.chiniforooshan@yahoo.com

The strategic configuration of the supply chain is thus a key factor influencing the efficiency at tactical and operational level. Its long term impact on the efficiency of the supply chain, combined with the commitment of substantial capital resources, render this level crucial. One of the most important aspects of strategic level is location problem that leads the system to define the optimal number of facilities, location of facility in geographical manner, assigning the customers to facility and configuration of transportation network. Melo et al. [2] presented a comprehensive review based on SC features. According to [2], approximately 82% of literature deals with single period problems. Likewise, there is a gap in 3-layers, Multi periods and Multi products integrated SC and facility location models, as they have suggested researching in this case. So, in this paper, a 3-layers SC model includes location-allocation problem is presented. Also, the proposed model is extended in multi periods and multi products.

There is a vast amount of literature available on supply chain management research dealing with the different aspects of the subject. Numerous models in the literature, conceptual as well as quantitative, refer to the planning and quantitative aspects of the different business functions: location, production, inventory and transportation. A number of quantitative models use mixed-integer programming (MIP) to solve the supply chain optimization problems. One of the first attempts was done by Geoffrion and Graves [1], where a MIP model was formulated for the multiple commodity location problem. This seminal research involved the determination of distribution center (DC) locations, their capacities, customer zones and transportation flow patterns for all commodities. A solution to the location portion of the problem was presented, based on Bender's Decomposition (BD).

Cohen and Lee [3] develop an analytical model to establish materials requirements policy based on stochastic demand. They develop four different sub-models with a minimum-cost objective. A mathematical algorithm at the end decides the optimal ordering policies to minimize the costs. A MIP model for a production, transportation, and distribution problem has been developed by Pirkul and Jayaraman [4] to represent a multi-product tri-echelon capacitated plant and warehouse location problem. The model minimizes the sum of fixed costs of operating the plants and warehouses, and the variable costs of transporting multiple products from the plants to the warehouses and finally to the customers.

Schmidt and Wilhelm [5] present a review of the work done on different decisional levels in the supply chain with respect to time frames – strategic, tactical and operational. Modeling issues are discussed at each level and a prototype formulation is provided as an extension of the discussion. Cordeau et al. [6] propose a static model considering a multi-commodity, multi-facility and single-country network. The decision variables concern the number of locations, the capacity and technology of manufacturing in plants and warehouses, selection of suppliers, selection of distribution channels, transportation modes and material flows.

Vila et al. [7] propose a dynamic model in a much more specialized context. They consider an application in the lumber industry, but their model can be applied to other sectors. The authors consider an international network, with deterministic demands. They consider external suppliers, capacitated plants and warehouses, the choice between a set of available technologies, the possibility of adding capacity options to the facilities, and a list of substitute products to replace standard ones. Other studies which address the SC coordination issues at different decision levels have been developed [8-12]. These conventional methods generally consider an overall production strategy, inventory strategy and flow of products through a facility over a single period to minimize total costs or maximize profits [13].

Supply chains have been more or less integrated to some extent as a whole, or in parts. Integration, if done at all, has been mostly done in patches throughout the supply chain. In many cases, this has been driven more by the need to survive and improvise, than by the willingness to improve and advance further. Therefore, efforts must be made to integrate suppliers, manufacturers, distributors, and customers, so that they will collaborate effectively with each other in the entire network. During the past few years, there have been significant attempts for providing integrated supply chain problems, which includes suppliers, manufacturers, distributors and retailers. The primary objective of an integrated supply chain is to optimize all cost components from converting raw materials into final products delivered to end users [14-16].

The decisions made for network design determines the suppliers, manufacturing plants, and intermediate inventory warehouses, selects the distribution channel from suppliers to customers, and identifies the transportation volume among distributed facilities for multiple period horizon. From operational perspective, it is critical to have a coordinated plan for production and distribution activities of multiple level production factories and distribution centers in order to take full advantage of the supply network [17, 18]. In general, production and distribution planning involve raw material suppliers, manufacturing plants, intermediate warehouses, distribution centers and customers which are interconnected in terms of the interconnected in terms of supplier/customer relations.

The objective of this paper is to simultaneously optimize the decision variables of different functions that have been traditionally optimized sequentially [9]. Hence in this study, we will develop an integrated strategic and tactical supply chain model in a multi-echelon, multi-level, multi-period supply chain network. The problem, therefore, is modeled as a mixed integer linear programming formulation that seeks to optimize fixed charge DCs costs, fixed and variable production costs, transportation costs between plant to DCs and DCs to customer zones, inventory holding costs and backorder costs while satisfying all customer demands, plant and DCs capacity. After formulating the problem

The main contributions of my project can be summarized as follows:

- Introducing a novel integrated strategic and tactical SC planning model by integrating location and production-distribution planning activities into a multi-echelon, multi-level and multi-period SC network.
- Developing a hybrid algorithm which combined the lagrangian relaxation method and genetic algorithm to solve the candidate problem.
- Applying the model and algorithm to a real industrial case for implementing the feasibility of applying the proposed model to a real-world problem.

This paper is organized as follows: the mathematical model of the SCN design problem is given in section two. While third section includes an explanation about the proposed HLRGA, the brief description of the hybrid genetic algorithm and simulated annealing algorithms used in computational experiments are given in section four. Moreover, the fifth section gives computational results and presents an industrial case. This is followed by conclusions in the sixth section.

2. Mathematical Model

In this research we consider a multi-plant, and multi-customer location-production-distribution system. The system contains a set of manufacturing facilities with limited production capacities situated within a geographical area. Each of these facilities can produce one or all of the products in the company's portfolio. The customer demands for product are to be satisfied from this set of manufacturing facilities. There are fixed costs associated with each facility location which may include land costs, construction and fabrication costs etc. Although we assume that the customer allocation has to be done within the existing set of manufacturing facilities, sometimes it may be necessary to make changes or expansions in the current facilities to accommodate the production quantities which ultimately will prove to be beneficial. Costs for these changes would be included in the fixed costs. So the production capacities of each of these facilities effectively represent its current and potential capacities.
When the number of customers is large and the distances between them and the deot, or sources are long it is often beneficial to utilize distribution centers. This makes the model much more complicated as products can now be shipped straght from the sourcce to customer, or products may be shipped to the customer via a distribution center, as shown in Figure 1.

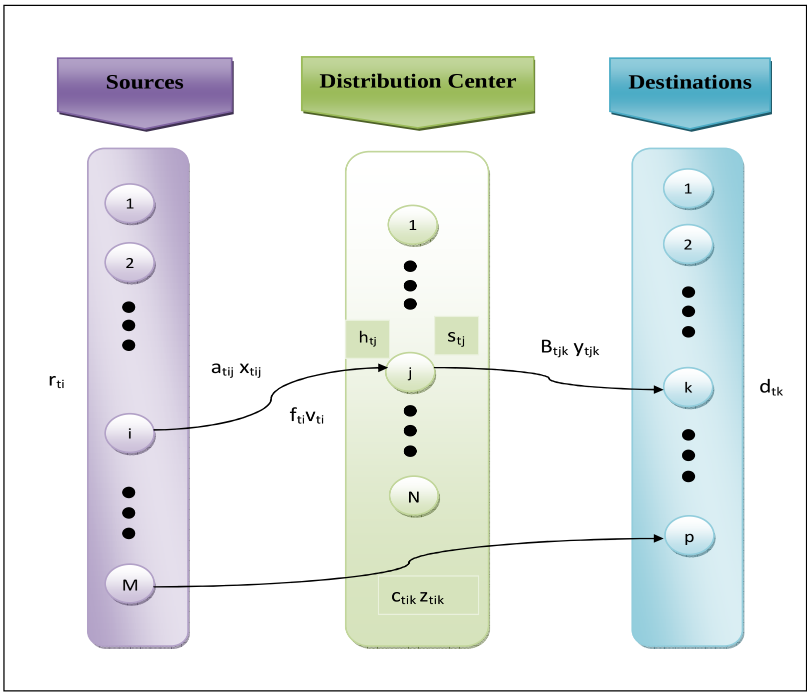

Figure 1. A real-world supply chain network

The addition of the distribution centers makes the system much more flexible. For example, transshipments can now occur at the distributions centers: product may be shipped from multiple sources to the distribution centers and then shipped from there to a single customer. It also allows for the possibility of inventory to be held at the distribution centers. Thus, inventory can be shipped to a distribution center during one time period and held there for delivery during another time period. To optimize this model, however, both fixed and variable costs now need to be taken into account. Variable costs include the costs of transporting products between the sources, the distribution centers, and the final customers, as well as the cost of holding inventory at the distribution centers.

Below is a proposed mathematical model to represent the distribution and inventory problem with set-up costs. This model takes into account possible inventory at the distribution centers, set-up costs associated with operating each source, and time periods. This model works on the assumption that all demands are met in the time period they occur, and that the starting and ending inventories at all distributions centers are zero. The following notations are used to define the mathematical model:

Parameters:

M= Number of sources (plants)
N= Number of distribution centers
H= Number of time periods in the planning horizon
P= Number of customers
BM= A big number
a_{tij}= Unitary transportation cost from source i to distribution center j
PC_{it}= Variable cost to produce a unit of product in source i during time period t
h_{tj}= Inventory cost in distribution center j at the end of period t
b_{tjk}= Unitary transportation cost from distribution center j to customer k during time period t
c_{tik}= Unitary cost of transportation units directly from source i to customer k during time period t
r_{ti}= Products available in source i during time period t
d_{tk}= Demand at customer k during time period t
f_{ti}= Setup cost associated with transportation products from source i

Variables:

x_{tij}= Number of products to be sent from source i to distribution center j during time period t
q_{it}= Quantity of products produced in source i during period t
s_{tj}= Inventory at distribution center j at the end of time period t
y_{tjk}= Number of products to be sent from distribution center j to customer k during time period t
z_{tik}= Number of products to be sent directly from source i to customer k during time period t
u_j= 1 if distribution center j is opened, 0 otherwise
v_{ti}= 1 if $x_{tij} > 0$, 0 otherwise

Mathematical model:

The production-distribution problem can be formulated as follows:

$$Min\ Z = \sum_{t=1}^{H}\sum_{i=1}^{M}\left(\sum_{j=1}^{N} a_{tij}x_{tij} + \sum_{k=1}^{P} c_{tik}z_{tik} + f_{ti}v_{ti}\right) + \sum_{t=1}^{H}\sum_{j=1}^{N}\left(\sum_{i=1}^{M} b_{tij}y_{tij} + h_{tj}s_{tj}\right) + \sum_{t=1}^{H}\sum_{i=1}^{M} PC_{it}q_{it} \qquad (1)$$

Subject to

$$\sum_{j=1}^{N} x_{tij} + \sum_{k=1}^{P} z_{tik} \le r_{ti} \qquad \forall t,i \qquad (2)$$

$$\sum_{j=1}^{N} y_{tjk} + \sum_{i=1}^{M} z_{tik} = d_{tk} \qquad \forall t,k \qquad (3)$$

$$\sum_{j=1}^{N} x_{tij} \le q_{it} \qquad \forall t,i \qquad (4)$$

$$\sum_{j=1}^{N} z_{tij} \le q_{it} \qquad \forall t,i \qquad (5)$$

$$\sum_{i=1}^{M} x_{tij} + s_{t-i,j} - s_{t,j} - \sum_{k=1}^{P} y_{tjk} = 0 \qquad \forall t,j \qquad (6)$$

$$\left(\sum_{j=1}^{N} x_{tij} + \sum_{k=1}^{P} z_{tik}\right) - Bv_{ti} \le 0 \qquad \forall t,j \qquad (7)$$

$$x_{tij} \le BMu_j \qquad \forall t,i,j \qquad (8)$$

$$y_{tij} \le BMu_j \qquad \forall t,i,j \qquad (9)$$

$$s_{0,j} = s_{Q,j} = 0 \qquad \forall j \qquad (10)$$

$$x_{tij} \ge 0, y_{tjk} \ge 0, z_{tik} \ge 0, s_{tj} \ge 0 \qquad \forall t,i,j,k \qquad (11)$$

In this model, namely PRDIS model, the goal of objective function (1) is to minimize the producing, shipping, inventory, and set up costs. The summations in the objective function represent these costs. The first set of summations represents the cost of shipping from the source to the distribution centers, the cost of shipping from a source straight to a customer, and the setup of cost of operating from those sources. The set-up cost is included with the use of the binary variable v_{ti}. The second set of summation represents the cost of shipping from the distribution centers to the customers, and the cost of holding any inventory at the distribution centers.

The next set of equations represents the constraints of the model. The first set of constraints (2) ensures the total amount shipped from each source (total to all distribution centers and to all customers) is less than that source's capacity in each time period. The second set of constraints (3) ensures the demands at each customer are met during each time period. Constraints (4) ensure that the total number of products delivered from the source *i* to distribution center *j* during time period *t* is equal to production number in a given period *t*. Constraints (4) ensure that the number of products to be sent directly from source *i* to customer *k* during time period *t* is equal to production number in a given period. The next set of constraints (6) is a conservation constraint. This set of constraints ensures that the products at the beginning of the time period plus the products entering the distribution center minus the products shipped from the distribution center is equal to the products counted as on-hand inventory at the start of the next time period. The next set of constraints (7) ensures that the set-ups costs are considered. If any products are shipped from a source (either to a distribution center or to a customer) during a time period, then the value of the binary variable v_{ti} will be 1, and the set-up costs for that time period will be included in the objective function. If no products are shipped, then this value is 0, and the set-up costs for that source and time period are disregarded. Constraints (8) and (9) guarantee the assignment of customers and transportation to open DCs. The next set of constraints (10) ensures that the inventory at the beginning and end of the planning horizon at each distribution center is zero. Finally, the last set of constraints (11) ensures that non-negativity and binary conditions hold

3. Proposed Hybrid Algorithm

Supply chain network design is to provide an optimal platform for efficient and effective SCM. This is an important and strategic operations management problem in SCM. The design task involves the choice of facilities (plants and distribution centers (DCs)) to be opened and the distribution network design to satisfy the customer demand with minimum cost. It belongs to a production-distribution and facility location-allocation problem. Solution approaches for these problems are optimization algorithms within the framework of Benders' decomposition [1, 19], heuristics based on branch-and-bound [20], and Lagrangian relaxation [21]. However, these techniques consume extensive amounts of time and effort in finding optimal solutions for realistically sized problems. The problem under consideration can be reduced to the well-known *p*-median problem which is known to be NP-hard [22]. Therefore, researchers have utilized heuristic and meta-heuristic approaches to solve this problem.

Syarif et al. [23] have developed a spanning tree-based GA approach for the multi-source, single-product, multi-stage SCN design problem. Jayaraman and Ross [24] have also proposed a heuristic approach based on simulated annealing for the designing of distribution network and management in supply chain environment. Yeh [25] has proposed a memetic algorithm (MA) which is a combination of GA, greedy heuristic, and local search methods for the same problem. The author has extensively investigated the performance of the MA on the randomly generated problems.

Due to the limitations involved in exhaustive enumeration, branch and bound and mathematical modeling for solving large sized problems, researchers started developing heuristics. Even though these heuristics did not guarantee optimal solutions, they gave feasible solutions within a reasonable computational time [26]. A heuristic method is a procedure that is likely to find a good feasible solution but leaves no guarantee of its quality or whether it is optimal or not [27]. All the possible solutions are not considered, since that would require an infinite amount of time, but rather a part of the solution space with solutions that might or might not be optimal. The solution space is searched smarter, discarding those parts that certainly not will contain good solutions and focusing more on those parts that

could include a good one. Nevertheless, a well-designed heuristic method can often provide a near-optimal solution, or indicate that no optimal solution exists. The method should also be efficient enough, so that it can deal with large problems within a reasonable time.
The problem with ordinary heuristic methods is that for every problem given, a procedure must be designed to fit and to solve the problem [27, 28]. However, in recent years another type of procedure has been developed, the meta-heuristic that consists of both a general structure and strategy guidelines to adjust to the specific problem given. This approach is very timesaving and meta-heuristics have become an important tool for solving a wide range of practical problems. Furthermore, ordinary heuristics often are local improvement procedures, i.e. they try to improve the current solution within the local neighborhood of that solution. This means that for every iteration, the method will find a solution near the current one and accept it if it's better, converging towards the local optimum within the neighborhood of the starting solution. The drawback of this approach is that if the given problem consists of multiple local optima, the procedure applied will converge to one local optimum and then stop.
The algorithm developed to solve the integrated production-distribution with set-up costs uses a combination of genetic algorithm and Lagrangian relaxation techniques. First, an algorithm using the method of GA is developed to solve an uncapacitated PRDIS model. Next, a relaxed model of PRDIS is developed using the method of Lagrangian relaxation. Finally, the GA algorithm is used in conjunction with subgradient optimization to solve the Lagrangian relaxation model, and produce a solution to the PRDIS model itself.

3.1.Proposed Genetic Algorithm

For larger and more complicated problems, meta-heuristic procedures are used, where the search method combines local improvement procedures with more advanced and intelligent strategies to create a process capable of escaping from local optima and performing a robust search of the solution space. An important characteristic of the meta-heuristic is hence the possibility to escape from a local optimum after reaching it, and depending on the method, there are different ways of doing so. One common way, besides searching locally for better solutions than the current one, is to also have the possibility to accept a neighbor solution if it is worse than the current one. Another way is to prohibit solutions, within the current solution neighborhood, to force the procedure to search in another direction away from a local optimum.
In this section, representation and genetic operators which were used in GA for multi-objective design of SCN will be explained.

3.1.1. Representation

The representation scheme for the decision variables is a key point before using GAs to solve an optimization problem. This representation decides how the problem will be shown in the GA. There are three ways of encoding tree: (1) edge-based encoding, (2) vertex-based encoding and (3) edge-and-vertex encoding [29].

In this study, to escape from these repair mechanisms in the search process of GA, we used priority based encoding developed by Gen and Cheng [29]. They had successfully applied this encoding to the shortest path problem and the project scheduling problem. The first application of this encoding structure to a single product transportation problem was carried out by [30]. In priority- based encoding, the position of a gene is used to represent a node (source in transportation network), and the value is used to represent the priority of corresponding node for constructing a tree among candidates.

3.1.2. Evaluation

The purpose of the evaluation function is to measure the fitness of candidate solutions in the population with respect to the objective functions and constraints of the model. The fitness values are used to select parent solutions to create the next generation of solutions. The fitness of an individual dictates the number of copies of that solution in the mating pool. In this paper, the evaluation is achieved by the sum of the objective function (Eq. (1)) and the penalty terms of constraint violation.

3.1.3. Crossover

The crossover operators produce offspring by exchanging information contained in the parents. The proposed GA uses a simple crossover operator in which a random crossover point is determined along the length of the chromosome and swamps the right hand side segments of the parents. The crossover operation is done with a probability called the crossover probability. A segment-based crossover operator which was based on uniform crossover is used in this paper. In this operator, each segment of offspring is randomly selected with equal chance among the corresponding segments of parents. As it is seen in Figure 2, crossover operator utilizes from a binary mask. Its length is equal to number of stage in SCN. While “0” means that the first parent will transfer its genetic materials to the offspring, “1” means that the offspring will take genetic materials from the second parent for the corresponding segment. This crossover operator tends to preserve good gene segments of both parents.

	1	2	3	1	2	3	4	1	2	3	1	2	3	4	5	1	2	3	4	5
Parent 1	3	7	4	2	6	5	1	4	5	6	8	3	2	7	1	1	1	3	3	3
Parent 2	1	4	5	6	7	3	2	8	1	3	5	7	6	4	2	1	3	2	2	3
Binary Mask	0							1							0					
Child	3	7	4	2	6	5	1	8	1	3	5	7	6	4	1	1	1	3	3	3

Figure 2. An illustration of crossover operator

3.1.4. Mutation

The mutation operator acts on a single chromosome to alter the information contained in the genes. In this operator, firstly, a decision about which segments will be mutated is given with probability of 0.5 (i.e. using a binary mask), and then selected segments are mutated. Swap operator is used for chromosome. The scheme of mutation operator is shown in Figure 3.

	1	2	3	1	2	3	4	1	2	3	1	2	3	4	5	1	2	3	4	5
Parent	3	7	4	2	6	5	1	8	1	2	5	7	6	4	1	1	1	3	3	3
Binary Mask	1							0							1					
	1	2	3	1	2	3	4	1	2	3	1	2	3	4	5	1	2	3	4	5
Child	3	2	4	7	6	5	1	8	1	2	5	7	6	4	1	1	2	3	3	3

Figure 3. An illustration of mutation operator

3.1.5. Selection strategy

After obtaining the fitness value of each chromosome, chromosomes will then be selected. This is the vital process in the algorithm since it selects the parents to produce offspring and optimal solution will be obtained among the new solutions. Chromosomes with higher fitness value will have a higher chance of being selected more often. This is achieved by assigning a probability value to each chromosome selected, so that better chromosomes will be assigned a higher probability. The roulette wheel selection strategy is used in this research.

3.2.Hybrid Lagrangian Genetic Algorithm

One method frequently used to solve linear programming problems is that of Lagrangian relaxation. The general approach of Lagrangian relaxation can be find in [31, 32]. This method can also be applied to the DIPS model. Using the same parameter and variable definitions as the proposed model, the relaxed the model is:

$$Min\ Z = \sum_{t=1}^{Q}\sum_{i=1}^{M}\left(\sum_{j=1}^{N} a_{tij}x_{tij} + \sum_{k=1}^{P} c_{tik}z_{tik} + f_{ti}v_{ti}\right) + \sum_{t=1}^{Q}\sum_{j=1}^{N}\left(\sum_{i=1}^{M} b_{tij}y_{tij} + h_{tj}s_{tj}\right) + \sum_{t=1}^{Q}\sum_{i=1}^{M} PC_{it}q_{it} + \sum_{t=1}^{Q}\sum_{i=1}^{M}\lambda_{ti}\left(\sum_{j=1}^{N} x_{tij} + \sum_{k=1}^{P} z_{tik} - r_{ti}\right) \quad (12)$$

Subject to

$$\sum_{j=1}^{N} y_{tjk} + \sum_{i=1}^{M} z_{tik} = d_{tk} \qquad \forall t,k \qquad (13)$$

$$\sum_{j=1}^{N} x_{tij} \le q_{it} \qquad \forall t,i \qquad (14)$$

$$\sum_{j=1}^{N} z_{tij} \leq q_{it} \qquad \forall t, i \qquad (15)$$

$$\sum_{i=1}^{M} x_{tij} + s_{t-i,j} - s_{t,j} - \sum_{k=1}^{P} y_{tjk} = 0 \qquad \forall t, j \qquad (16)$$

$$\left(\sum_{j=1}^{N} x_{tij} + \sum_{k=1}^{P} z_{tik}\right) - Bv_{ti} \leq 0 \qquad \forall t, j \qquad (17)$$

$$x_{tij} \leq BMu_j \qquad \forall t, i, j \qquad (18)$$

$$y_{tij} \leq BMu_j \qquad \forall t, i, j \qquad (19)$$

$$s_{0,j} = s_{Q,j} = 0 \qquad \forall j \qquad (20)$$

$$x_{tij} \geq 0, y_{tjk} \geq 0, z_{tik} \geq 0, s_{tj} \geq 0, \lambda_{ti} \geq 0 \qquad \forall t, i, j, k \qquad (21)$$

In this model, the objective function is updated to reflect the lagrangian multipliers and the first constraint from the first model as a penalty function. The remaining constraints ((13)-(21)) now directly mirror the constraints of the uncapacitated model ((3)-(11)).

For the resulting model it will be necessary to find an efficient scheme to update the Lagrangian multipliersλ_{ti}. The general approach to this updating is the *Subgradient Optimization* technique as described in [32, 33]. In the subgradient optimization method, the lagrangian multipliers, λ_{ti}, are updates using the gradient of the solution with respect to λ. Thus, the entire model can be solved by combining the methods of genetic algorithm with subgradient optimization. In this algorithm, the relaxed model is solved using the method of genetic algorithm as presented in section 3.1 by updating the objective function of the uncapacitated production-distribution model with the objective function of the Lagrangian relaxed model. This solution is then used to update the lagrangian multipliers by the method of subgradient optimization. This process is repeated until a stopping criteria is reached (either the lagrangian multipliers converge or a set number of iterations is performed). This method is outlined below in Figure 4.

The program first initializes the lagrangian multipliers, passes them to the Genetic Algorithm to update the uncapacitated production-distribution model objective function, solve the model, and projects the solution back to the feasible region. The program then passes that solution and the lagrangian multipliers to the subgradient optimization algorithm to update the lagrangian multipliers and evaluates the improvement of the solution, and repeats this process until a final solution is reached.

4. Methods for Comparisons

To investigate the effectiveness of the HLRGA, two heuristic approaches have utilized. These two approaches are based on GA which is hybridized by linear programming and called hybrid genetic algorithm (hGA), and simulated annealing (SA). The section presents a brief explanation about the hGA and SA developed for the problem by the authors of the paper.

Require: an upper bound $\bar{L}$, initial values $\lambda_{t,i}^{0} \geq 0$, a sequence θ_v, and N_{iter}.
Ensure: best solution $(\boldsymbol{X}^*, \boldsymbol{Y}^*, \boldsymbol{Z}^*, \boldsymbol{S}^*)$ for production-distribution model

1. Set iteration $\tau = 0$
2. **While** $\left|\lambda_{t,i}^{\tau+1} - \lambda_{t,i}^{\tau}\right| \leq \varepsilon$ for all *t*, *i*, and $\tau < N_{iter}$ **do**
3. Call GA to solve uncapacitated production-distribution model and obtain $\boldsymbol{L}(\lambda^\tau)$
4. **If** GA solution is an improved feasible solution to production-distribution model **then**
5. $\bar{\boldsymbol{L}}$ ←GA solution of uncapacitated problem {update $\bar{\boldsymbol{L}}$}
6. **End if**
7. Calculate
$$\gamma^\tau = \frac{\partial \boldsymbol{L}}{\partial \lambda}$$ {gradient of $\boldsymbol{L}(\boldsymbol{\lambda}_{t,i}^{\tau})$}
8. Calculate
$$k = \frac{\theta_v(\bar{\boldsymbol{L}} - \boldsymbol{L}(\lambda^\tau))}{\|\gamma^\tau\|^2}$$ {step size}
9. $\lambda_{t,i}^{\tau+1} = max\{0, \lambda_{t,i}^{\tau} + k\gamma_{t,i}^{\tau}\}$
10. **If** no progress in *l* iteration **then**
11. $\theta_{v+1} = \frac{\theta_v}{2}$
12. **End if**
13. $\tau = \tau + 1$
14. **End while**

Figure 4. Pseudo code for Proposed Hybrid Lagrangian Genetic Algorithm

4.1.Hybrid genetic algorithm

As is known, when the opened plants and DCs are known on the multi-stage SCN design problem, the problem is reduced to a capacitated transshipment problem (CTP), which is relatively easier to solve by commercial software packages such as LINGO, CPLEX, etc., since it is a linear programming formulation. Based on this property, a GA hybridized with CPLEX (hGA) is developed. In hGA, a chromosome consists of two segments having same encodings. First, a transportation tree between DCs and customers and demands of the DCs are obtained by decoding of the second segment of the chromosome, and the cost of the last stage is calculated. The GA procedure is the same as explained in section 3.1. After determining which plants will be opened using binary encoding of the first segment, the problem for the first two stages is reduced to CTP, and the CTP is solved by CPLEX. The fitness value of the chromosome is the summation of the cost of CTP and the cost of last stage which is obtained by decoding of the second segment of chromosome.

4.2.Simulated annealing

Simulated annealing (SA) is a effective optimization algorithm motivated from an analogy between the simulation of the annealing of solid and the strategy of solving combinatorial optimization problems. In this paper, in order to enhance the exploitation ability of the proposed algorithm, DE is hybridized with a simulated annealing (SA) algorithm. All current solution vectors are improved by using SA.

The applied SA could be briefly introduced as follows: It starts with an initial solution, each solution vector of the current generation, and for each vector a neighbor solution is generated. In the proposed SA, a neighbor vector $\boldsymbol{N}_i = [N_{1,i}, \dots, N_{D,i}]$ for each solution vector ($\boldsymbol{X}_i$) is generated according to Equation 22.

$$\boldsymbol{N}_i = \boldsymbol{X}_i + rand_j(X_j^U - X_j^L) \times \rho \qquad (22)$$

where ρ is used to ensure that parameter values lies inside their allowed ranges in neighbor vector. Let $F(\boldsymbol{X})$ and $F(\boldsymbol{N})$ denote the objective function values of the current solution and the neighbor solution, respectively and define Δ as the difference between these objectives; that is $\Delta = F(\boldsymbol{X}) - F(\boldsymbol{N})$. If $\Delta \leq 0$, the neighbor solution is accepted; otherwise it is accepted with probability equal to $e^{-\Delta/T}$. where T is the temperature parameter such that $T > 0$. At the beginning, the temperature is set at the initial temperature T_0. Then T is decreased after generations according to the formula$T = \propto T$, where $\propto$ is the coefficient controlling the cooling schedule ($0 < \alpha < 1$).

5. Experimental Results

This section gives numerical results on the performance of HLRGA. All algorithms are coded in MATLAB 7 and executed on an Intel® Core 2 DuoE4500 at 2.20 GHz with 2.0GB of RAM. Before the numerical results, information about the test problems and the parameter setting of each algorithm will be given.

5.1.Test problems

Two sets of the test problems are considered in this research. The first set consists of 10 classes of problems called small size problems, and each class contains 10 randomly generated problem instances. Therefore, 100 problem instances are considered for the small size problems. These problems can be solved optimally and they are used to provide a better sense of the performance of the HLRGA. The second set called large size problems includes six classes of problems. Each class of this set contains 20 randomly generated problems, and a total of 120 problem instances are considered as large size problems. This set of large size problems are used to provide an idea of the comparative performance of the heuristic approaches with respect to objective function value. The methods presented in [34] are used to generate some of the tests' data.

The parameter settings of HLRGA, hGA, and SA are as follows: since one offspring is generated by crossover and mutation operators at each generation of the HLRGA and hGA, the crossover rate is set to 0.9, and the mutation is applied to offspring with the probability of 0.15. Based on the preliminary runs, the population sizes of HLRGA and hGA are taken as 50. The initial temperature in SA is set to 1000 in which an inferior solution (inferior by 70% relative to current solution) is accepted with a probability of 0.90. The final temperature is taken as 0.15 such that a solution which is inferior by %1 relative to current solution is accepted with a probability of 0.1%. The best individual of the initial population of the

HLRGA is taken as the initial solution of SA. In order to make a fair comparison between heuristic algorithms, CPU time is chosen as a stopping criterion.

5.2.Computational Results

The parameters of the small size problems are given in Table 1. The problems are described by providing the number of customers (|P|), the number of potential DCs (|N|), the number of sources (|M|), and the number of time horizons (|H|). As mentioned in section 5.1, each class of problems contains 20 randomly generated problem instances. To investigate the solution quality of the heuristic approaches, the optimum solution of each problem instance is obtained by CPLEX. The CPU time of each problem instance, which is used as a stopping criterion for the HLRGA, hGA, and SA, is obtained by solving the problem instance with the LH. The last two columns of the Table 1 report the average CPU times for the CPLEX. As is seen in this table, the optimum solutions of the first six problems are obtained in a short time by CPLEX.

Table 1. Small size problems

Problem Class	Parameters				CPU Time (S)
	|P|	|N|	|M|	|H|	CPLEX
S1	10	5	2	2	3.25
S2	10	5	2	2	8.43
S3	20	5	2	2	7.46
S4	20	10	3	3	16.67
S5	30	5	3	3	87.93
S6	30	10	4	3	213.03
S7	40	10	4	4	723.90
S8	40	15	5	4	1305.11
S9	50	10	5	4	4988.30
S10	50	15	10	6	12895.83

Table 2. Comparison of meta-heuristics for small size problems

Problem Class	Optimality gaps (%)					
	HLRGA		hGA		SA	
	Average	Maximum	Average	Maximum	Average	Maximum
S1	0.00	0.43	0.00	0.32	0.03	0.45
S2	0.05	0.41	0.00	0.24	0.19	0.35
S3	0.13	0.65	0.09	0.44	0.32	0.87
S4	0.34	1.04	0.27	0.69	0.45	0.98
S5	0.86	1.54	0.95	1.43	1.43	1.90
S6	1.14	2.26	1.59	2.98	1.96	3.16
S7	1.88	2.59	2.13	3.22	2.29	3.35
S8	1.94	2.46	2.09	3.18	3.14	3.44
S9	2.62	3.11	3.62	4.26	3.80	4.19
S10	3.19	3.75	4.67	5.52	4.43	6.87
Average	1.215	1.82	1.54	2.23	1.80	2.56

In Table 2, we have reported the summary of results for 10 classes of the small size problems with 20 instances in each. As performance measures, we have used the average percentage gap and maximum percentage gap between heuristic solution and optimum solution. The gap is defined as 100 × (heuristic solution value - optimum solution value)/optimum solution value. When the heuristic approaches are compared with respect to average gap over all 10 classes of the problems, it is seen that the HLRGA exhibits the best performance with the average gap of 1.215%. As seen in Table 2, while the average gap between the optimum solution and the HLRGA is less than 2%, the average gaps for the hGA and SA are less than 2%. Also, as seen in Table 2, the average and maximum gaps of the hGA and SA are close to each other. With respect to maximum gap over all 10 classes of the problems, it is seen that the HLRGA is comparable to the hGA.

Table 3. Large size problems

Problem Class	Parameters				
	|I|	|J|	|K|	|L|	|H|
S1	100	20	2	5	3
S2	100	30	2	5	3
S3	200	30	4	10	6
S4	200	40	4	10	6
S5	300	40	6	20	12
S6	300	50	6	20	12

Table 3 reports the parameters of the six classes of large size problems with 20 instances in each. As seen in Table 3, the number of customers, the number of potential DCs and the number of sources vary from 100 to 300, from 20 to 50, and from 10 to 25, respectively. Since the problems are very large, it is not possible to obtain their optimum solutions by CPLEX. As observed in Table 1, the average CPU times in CPLEX for the problem classes, where the number of product is 3, are higher than for the problems having two products. This shows that one of the important factors affected on the solution time of the problem instances is the number of the product.

Table 4. Comparison of meta-heuristics for large size problems

Problem Class	Optimality gaps (%)					
	HLRGA		hGA		SA	
	Average	Maximum	Average	Maximum	Average	Maximum
S1	2.43	4.63	4.23	5.98	4.68	6.78
S2	2.25	4.12	4.59	6.32	4.56	6.96
S3	2.89	4.76	5.22	6.88	4.68	7.43
S4	3.23	5.14	5.89	7.33	5.37	7.82
S5	3.53	5.64	6.19	7.47	5.89	8.37
S6	3.41	5.60	6.48	7.68	5.84	8.63
Average	3.20	5.22	5.84	7.26	5.39	7.98

Table 4 shows the average and maximum gaps of the proposed meta-heuristic approaches. Since the optimal solutions for large size problems are not known, the gap is defined as 100 × (heuristic solution value - lower bound)/lower bound. The lower bound is the best value of the objective function found by any of algorithms (i.e., HLRGA, hGA, and SA). When the heuristic approaches are compared with respect to average and maximum gaps over all six classes of the problems, it is seen that the HLRGA outperforms the other heuristics. Its average and maximum gaps are between 2.25% and 3.53%, and 4.12% and 5.64%, respectively. While the average and maximum gaps of the SA over the six classes of the problems are 5.39 % and 7.98%, respectively, these values are 5.84% and 7.26%, respectively, for the hGA. When the problem size increases, it is seen that the performance of the SA drastically decreases.

Furthermore, the proposed model has been evaluated by using date from an industrial real-world case in Iran. The conventional production and distribution strategy used by case is to maintain a constant work force level over the planning horizon, and fluctuated demands can then be met by using some combination of inventories, overtime. However, the expected performance was unable to achieve because of drawbacks on the current experiential method focus on a single component of the overall system, for example procurement, production, transportation, warehouses, or scheduling, despite the integration of these components in a single supply chain, and evaluation comparisons can only be done for specific plans under specific conditions and indication is vague for the optimum plan.

Alternatively, the decision maker would apply a mathematical programming method to develop an integrated production-distribution planning decisions plan for the industrial case. The planning horizon is 12 months. The forecasted demand fluctuates a lot over the planning horizon because seasonal variations. The product model includes 27 types of products. Also it comprises 4 sources and 160 customers.

In order to compare the quality of heuristic solutions obtained by the HLRGA with that of optimal solutions if they are available or feasible solutions obtained after running CPLEX for 10 h (i.e. 36,000 s), we have conducted the additional computational experiments. It is seen that CPLEX could not reach to optimal solutions for the problems after running 10 h. The average and maximum gaps based on the feasible solution of the CPLEX for this real case which solve 10 times are 3.38% and 5.59%, respectively. These results support the fact that the HLRGA can be used as an effective and efficient tool for designing the supply chain networks in real world case.

6. Conclusion

This work develops an integrated production distribution system in real-world industrial case which is a multi-source, multi-stage supply chain network design problem in dynamic environment. In this paper, a hybrid lagrangian relaxation genetic algorithm (HLRGA) is proposed for the candidate problem, which is a NP-hard problem. In the proposed algorithm the genetic algorithm (GA) is incorporated into the lagrangian relaxation (LR) method to update the lagrangian multipliers and improve the performance of LR method. The effectiveness of the HLRGA was investigated comparing its results with those obtained by CPLEX, hybrid GA and simulated annealing on two sets of test problems consisting of a total of 320 instances. Experimental study showed that the HLRGA found better heuristic solutions than the other heuristic approaches and reached the good heuristic solutions with lower computation time when compared with CPLEX. This is particularly attractive in large-scale systems. Future research is achieved on consideration of this modeling under uncertainty conditions with other critical SC problems such as vehicle routing, carriers loading, etc.

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