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Performance Evaluation Function for Benchmarking Process Using Context-Dependent Data Envelopment Analysis

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ABSTRACT

Data envelopment analysis (DEA) is a methodology for identifying efficient frontier of decision making units (DMUs) with multiple outputs and inputs. Context-dependent DEA refers to a DEA approach where a set of DMUs are evaluated against a particular evaluation context. Each evaluation context represents an efficient frontier composed by DMUs in a specific performance level. Context-dependent DEA measures the attractiveness and the progress for each DMU. Current paper extends the context-dependent DEA by ranking all units on the basis of attractiveness and progress measures. The method is applied to measure the attractiveness and progress of 49 bank branches, and ranking them with Context-dependent DEA.

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes, Cooper and Rhodes (CCR) [1], is a mathematical programming method for measuring the relative efficiency of decision making units (DMUs) with multiple outputs and inputs.

The most models of DEA, the best performers have efficiency score unity, and, from experience, we know that usually there are plural DMUs which have this "efficient status".

Differentiating efficient DMUs is an interesting research area. The original DEA method evaluates each DMU against a set of efficient DMUs and cannot identify which efficient DMU is a better option with respect to the inefficient DMU. This is because all efficient DMUs have an efficiency score of one. Authors have proposed methods for ranking the best performers, for instance using super-efficiency DEA model.

In this paper, in order to rank DMUs, we use the evaluation contexts that are obtained by partitioning the set of DMUs into several levels of efficiency, and rank all DMUs with two criteria: the attractiveness and the progress. The influence of all DMUs, both efficient and inefficient, in ranking is this method’s preference.

2. Data envelopment analysis

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Consider n decision making units $(DMU_j; j = 1, \dots, n)$ in which each DMU consumes input levels $x_{ij}, i = 1, \dots, m$ to produce output levels $y_{rj}, r = 1, \dots, s$. Suppose that $X_j = (x_{1j}, \dots, x_{mj})^T$ and $Y_j = (y_{1j}, \dots, y_{sj})^T$ are the vectors of inputs and outputs values respectively, for DMU_j, in which it is assumed that $X_j \geq 0, X_j \neq 0$ and $Y_j \geq 0, Y_j \neq 0$. The relative efficiency score of the $DMU_o, o \in \{1, \dots, n\}$, is obtained from the following model which is called output-oriented CCR envelopment model [2].

$$
\begin{aligned}
& Max \quad \varphi \\
& \text{s.t} \\
& \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} && i = 1, \dots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \geq \varphi y_{ro} && r = 1, \dots, s \\
& \lambda_j \geq 0, && j = 1, \dots, n
\end{aligned} \tag{1}
$$

The dual of (1) is the following model which is called output-oriented CCR multiplier model.

$$
\begin{aligned}
& \text{Min} \quad \sum_{i=1}^{m} v_i x_{io} \\
& \text{s.t} \\
& \sum_{r=1}^{s} u_r y_{ro} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, && j = 1, \dots, n \\
& v_i \geq 0, && i = 1, \dots, m \\
& u_r \geq 0, && r = 1, \dots, s
\end{aligned} \tag{2}
$$

where, $u_r (r = 1, \dots, s)$and $v_i (i = 1, \dots, m)$are the weights on output r and input i, respectively.

It can be proven that $\varphi^* \geq 1$ and DMU_o is efficient (technical) in the CCR model if and only if $\varphi^* = 1$. Otherwise, the DMU_o is inefficient.

3. Context-dependent DEA

It is known that adding or deleting an inefficient DMU does not alter the efficiency of the existing DMUs and the efficient frontier .The efficiency scores change only if the efficient frontier is altered, and efficient frontier depends on the efficient DMUs. The performance of efficient DMUs is not influenced by the presence of inefficient DMUs. However, the evaluation is often influenced by the context. A DMU's performance will appear more

attractive against a background of less attractive alternatives and less attractive when compared to more attractive alternatives.
We define the set of all DMUs index as J^1, that is, $J^1 = \{1, \dots, n\}$ and the set of efficient DMU index in J^1 with model (1) as E^1. Then the sequences of J^l are defined interactively as $J^{l+1} = J^l + E^l$, and the set E^l can be found as efficient DMUs index in J^l with the following linear programming problem:

$$\varphi_o^l = Max \quad \varphi$$
s.t
$$\sum_{j \in J^l} \lambda_j x_{ij} \le x_{io} \qquad i = 1, \dots, m$$
$$\sum_{j \in J^l} \lambda_j y_{rj} \ge \varphi y_{ro} \qquad r = 1, \dots, s$$
$$\lambda_j \ge 0, \qquad j = 1, \dots, n \qquad (3)$$

We call E^l the l[st]-evalution context, when l=1, model (3) become the model (1). The DMUs in setE^1 define the first-level efficient frontier. When l=2, model (3) gives the second-level efficient DMUs, and so on. In this matter, several levels of efficient frontiers are identified.
The DMUs in set E^ldefine 1[st]-level efficient frontier that is efficient frontier of J^l.
Model (3) yields a stratification of the whole set of DMUs, which partitions into different subgroups of efficiency levels characterized byE^l. It is easy to present

i) $J^1 = \cup E^l \;\; and \;\; E^l \cap E^{l'} = \emptyset \;\; for \;\; l \neq l'$
ii) The DMUs in $E^{l'}$ are dominated by the DMUs in E^l if$l' > l$.

Then each DMU in set E^l is efficient with respect to the DMUs in set $J^{l'}$ for all$l' > l$.
Now, based on evaluation contexts, $E^l(l = 1, \dots, L)$, consider a specific DMU_o from a specific level $E^{l_o}(l_o \in \{1, \dots, L-1\})$,we can obtain the relative attractiveness measure of DMU_o with respect to $(l_o + d)^{th}$- evaluation context, $E^{l_o+d}(d = 1, \dots, L - l_o)$ by the following context-dependent DEA:

$$\Omega_o^d = Max \quad \varphi \qquad d = 1, \dots, L - l_o$$
s.t
$$\sum_{j \in E^{l_o+d}} \lambda_j x_{ij} \le x_{io} \qquad i = 1, \dots, m$$
$$\sum_{j \in E^{l_o+d}} \lambda_j y_{rj} \ge \varphi y_{ro} \qquad r = 1, \dots, s$$
$$\lambda_j \ge 0, \qquad j \in E^{l_o+d} \qquad (4)$$

It can be proven that

i) $0 < \Omega_o^d < 1 \;\; for \;\; each \;\; d = 1, \dots, L - l_o$
ii) $\Omega_o^{d+1} < \Omega_o^d$.

Definition 1. $A_o^d = \frac{1}{\Omega_o^d}$is called d-degree attractiveness of DMU_o from specific level E^{l_o}.
In model (4), each efficient frontier of E^{l_o+d} represents an evaluation context for measuring the relative attractiveness of DMUs in E^{l_o}. Note that A_o^d is the reciprocal of the optimal value to (4), therefore $A_o^d > 1$.
The smaller the value ofA_o^d , the more attractive, because DMU_o makes itself more farther from the evaluation contextE^{l_o+d}.
When we evaluate $DMU_o\ , o \in E^{l_o}$, with other DMUs in$E^{l_o} \neq \{0\}$, super-efficiency score is obtained as follows:

$$\Omega_o^0 = Max \qquad \varphi$$

s.t

$$\sum_{j \in E^{l_o - \{0\}}} \lambda_j x_{ij} \le x_{io} \qquad i = 1, \dots, m$$

$$\sum_{j \in E^{l_o - \{0\}}} \lambda_j y_{rj} \ge \varphi y_{ro} \qquad r = 1, \dots, s$$

$$\lambda_j \ge 0, \qquad j \in j \in E^{l_o+d} \qquad (5)$$

Definition 2. $A_o^0 = \frac{1}{\Omega_o^0}$is called 0-degree attractiveness of DMU_o from specific level E^{l_o} if $E^{l_o} \neq \{0\}$ and otherwise $A_o^0 = 1$.
It can be proven that $\Omega_o^0 \le 1$therefore $A_o^0 \ge 1$. The progress measure is the amount unattractive $DMU_o$when compared to more attractive (better) alternatives DMUs. To obtain progress measure for a specific $DMU_o \in E^{l_o}, l_o \in \{2, \dots, L\}$, we use the following context-dependent DEA:

$$P_o^g = Max \qquad \varphi \qquad\qquad g = 1, \dots, l_o - 1$$

s.t

$$\sum_{j \in E^{l_o - g}} \lambda_j x_{ij} \le x_{io} \qquad i = 1, \dots, m$$

$$\sum_{j \in E^{l_o - g}} \lambda_j y_{rj} \ge \varphi y_{ro} \qquad r = 1, \dots, s$$

$$\lambda_j \ge 0, \qquad j \in E^{l_o - g} \qquad (6)$$

We have

i) $P_o^g > 1 \quad for \quad each \ g = 1, \dots, l_o - 1$

ii) $P_o^{g+1} > P_o^g$.

Definition 3. P_o^g is called g-degree progress of DMU_o from specific level E^{l_o}.

Each efficient frontier, E^{l_o-g} , contains a possible target for a specific DMU in E^{l_o} to improve its performance. The progress here is a level-by-level improvement. For a smaller P_o^g, more progress is expected for DMU_o . Thus, a larger value of P_o^g is preferred.

Definition 4. For $DMU_o \in E^{l_o}, l_o \in \{1, \dots, L-1\}$, the mean attractiveness is defined as:

$$\overline{A_o} = \sum_{d=0}^{L-l_o} \frac{A_o^d}{L-l_o+1}$$

Definition 5. For, $DMU_o \in E^{l_o}, l_o \in \{2, \dots, L\}$ the mean progress is defined as:

$$\overline{P_o} = \sum_{g=1}^{l_o-1} \frac{P_o^g}{l_o-1}$$

And $\overline{P_o} = 1$, if $DMU_o \in E^1$ [3-5].

4. Ranking

In this section, using mean attractiveness and mean progress scores, we present a method for DMUs ranking, A method has been developed which is able to rank all (extreme and non extreme efficient and inefficient) DMUs.

The DMUs in $E^{l'}$ are dominated by the DMUs in E^l if, $l' > l$ then each DMU in set E^l have a more rank order with respect to the DMUs in set $J^{l'}$ for all $l' > l$.

In order to rank DMUs, we firstly sort them by levels, and then each DMU is compared with all DMUs in its level.

Assume $DMU_o \in E^{l_o}, l_o \in \{1, \dots, L-1\}$, for comparison $DMU_o$with DMUs in same level, E^{l_o} we apply following two factors

i) Mean attractiveness.
ii) Mean progress.

The more mean attractiveness and less mean progress, the more rank order is given. Therefore, the more $\overline{A_o}$ and the less $\overline{P_o}$, the more rank order.

Noting that in evaluation of mean attractiveness, we proposed super-efficiency factor, which is a very important factor in most of the ranking methods.

Based on mentioned factors, we present following measure for ranking DMU_o:

$$r_o = \frac{\overline{P_o}}{\overline{A_o} + \overline{P_o}} \qquad (7)$$

We have $r_o \le 1$ and the less r_o, the more rank order, that is , Measure (7) guarantee the influence mean attractiveness and mean progress in ranking.

5. An application

In current section, we employ above-mentioned method to rank 49 bank branches with three inputs and four outputs. Their information is given in Table 1.

Table 1: Data for 49 bank branches

DMUs	Input1	Input2	Input3	Output1	Output2	Output3	Output4
1	100.39	415467.80	435093.30	128313.00	66186.00	2661355.00	21059.00
2	137.04	14136.80	3861.90	16911.00	16238.00	157437.00	425.00
3	147.92	10660.00	2348.80	4471.00	5839.80	117608.00	587.00
4	73.15	4251.00	1248.90	5565.90	2541.00	52544.00	280.00
5	21.94	5204.00	1419.80	6199.00	2402.00	74335.00	252.00
6	137.11	6772.30	1635.00	2435.50	3228.00	49222.00	233.00
7	40.24	5272.50	1431.20	2870.60	669.00	75697.00	391.00
8	11.10	3501.00	1509.70	4852.20	1084.40	35609.00	521.00
9	82.99	5183.90	3903.80	5799.40	5201.00	42794.00	412.00
10	41.00	3297.00	972.40	1818.50	493.00	27730.00	458.00
11	261.19	4018.80	1124.00	2031.00	356.00	30729.00	312.00
12	2.67	3123.90	1117.90	983.70	457.00	21282.00	88.00
13	19.32	2893.30	858.00	1329.90	1663.60	16964.00	122.00
14	1.05	2461.40	645.90	737.60	94.50	6312.00	300.00
15	1.19	2050.70	818.90	447.00	15.80	9297.00	97.00
16	0.50	2290.60	966.60	238.10	13.60	6478.00	31.00
17	0.50	2036.70	5189.50	275.10	0.80	4483.00	27.00
18	24.33	4351.90	1053.30	2577.20	331.20	13708.00	182.00
19	0.50	2454.30	678.30	468.80	90.40	16784.00	57.00
20	29.02	2024.00	720.50	1052.20	72.40	4307.00	43.00
21	0.50	2442.00	853.90	333.18	15.80	7397.00	22.00
22	0.50	1956.80	1580.10	478.10	25.78	6763.00	90.00
23	0.50	2523.80	2968.80	827.80	16.80	7516.00	59.00
24	0.50	2017.40	975.00	798.80	42.94	7010.00	269.00
25	57.21	4200.80	1405.30	2392.90	402.90	60583.00	582.00
26	1.89	2556.30	1022.10	598.30	84.00	15432.00	250.00
27	0.50	2246.60	2162.10	434.50	14.90	6369.00	160.00
28	34.75	3333.80	1307.90	726.80	214.60	30746.00	283.00
29	5.84	2269.10	1424.00	221.90	56.30	7575.00	95.00
30	26.64	2779.10	882.60	354.70	140.60	21508.00	121.00
31	33.28	2562.10	1148.30	397.00	133.40	13843.00	446.00
32	2.30	1880.00	1383.00	63.79	10.50	2476.00	30.00
33	46.26	11132.00	3146.00	7794.40	4008.00	198162.00	1274.00
34	113.84	4602.20	1521.40	1580.70	667.30	59439.00	338.00
35	8.77	2426.00	1113.20	352.40	150.00	16165.00	577.00
36	39.21	5128.80	1203.90	1750.80	1103.40	108084.00	120.00
37	4.23	2191.40	1206.00	110.40	33.80	4685.00	44.00
38	46.97	2850.00	1392.70	321.00	473.00	22694.00	563.00
39	3.30	2181.00	617.70	186.83	79.30	6076.00	287.00
40	4.84	3701.30	967.50	2205.10	240.90	29661.00	768.00
41	3.54	2244.10	830.10	196.60	144.00	12994.00	209.00
42	8.77	4689.00	1070.70	2947.30	701.00	39461.00	294.00
43	0.50	2321.50	1072.70	520.50	1.30	5016.00	36.00
44	16.50	4645.60	1247.00	1746.50	930.50	66144.00	400.00
45	19.89	3183.60	1372.00	439.80	365.00	26229.00	624.00
46	0.61	1397.70	2385.40	105.80	8.00	4301.00	16.00
47	96.97	4871.80	1283.50	4673.60	549.00	106176.00	314.00
48	80.10	6347.80	2433.40	1784.50	2972.60	29252.00	240.00
49	29.32	2456.10	833.90	349.00	255.40	13923.00	419.00

Using the DEA model (1), we obtain first level of efficient frontiers. That is $E^1 = \{DMU_j | j = 1,2,4,5,8,9,12,14,19,23,24,33,35,36,40,47\}$.
It can be seen from the original DEA (CCR) model, sixteen bank branches in $E^1$are efficient. We remove these efficient branches from $J^1 = \{1, \dots, 49\}$and again by using above model, we obtain four other levels of efficient frontiers. They are
$E^2 = \{DMU_j | j = 3,7,10,13,21,22,25,26,27,38,39,42,43,44,45,49\}$,
$E^3 = \{DMU_j | j = 6,11,15,16,17,18,28,31,34,41,48\}$,
$E^4 = \{DMU_j | j| = 20,29,30,46\}$,
$E^5 = \{DMU_j | j = 32,37\}$.
Now, using (4) we consider the attractiveness and with (6) consider the progress for each DMU's in any level when different efficient frontiers are chosen as evaluation contexts. The results forE^1 are given in Table 2.

Table 2: Attractiveness and progress for DMU's in first- level efficient frontiers

No.	DMU No.	A_0	A_1	A_2	A_3	A_4	P
1	1	2.272	2.420	3.808	2.385	149.254	1
2	2	3.683	8.085	16.474	50.000	81.967	1
3	4	1.979	2.830	6.601	25.707	57.471	1
4	5	1.082	2.248	2.604	12.771	72.464	1
5	8	1.096	2.205	2.694	19.841	64.935	1
6	9	1.347	1.998	4.093	18.450	60.241	1
7	12	1.043	1.639	3.238	9.542	59.524	1
8	14	1.270	1.521	3.724	14.430	45.455	1
9	19	2.704	3.255	4.990	16.892	34.247	1
10	23	1.418	2.217	4.241	14.286	35.088	1
11	24	1.242	1.693	2.566	4.655	39.841	1
12	33	1.250	2.120	3.891	16.529	25.316	1
13	35	1.134	2.109	4.308	17.452	21.413	1
14	36	1.731	2.267	4.072	12.077	24.876	1
15	40	1.305	1.693	3.569	5.754	32.680	1
16	47	1.146	1.485	2.211	6.739	14.205	1

In Table 2, A_0 is super-efficiency and A_1 is 1-degree attractiveness of DMU_o from a specific level E^1 and A_2 is 2 –degree attractiveness and so forth. P is progress of DMU_o and basedefinition 5, P=1 if $DMU_o \in E^1$.
Now for ranking DMU's, we compute r_o for all DMU's in all efficient frontier levels. We should sort them by levels and then each DMU will be compared with all DMU's in its level. Table 3 shows the ranking DMU's in each specific level and total ranking.

Table 3: The DMU's ranking in each level and total ranking

DMUs	Stratification	r_o	Rank in E^1	Rank in E^2	Rank in E^3	Rank in E^4	Rank in E^5	Total ranking
1	E^1	0.0303	2					2
2	E^1	0.0264	1					1
3	E^2	0.0379		1				17
4	E^1	0.0520	4					4
5	E^1	0.0548	6					6
6	E^3	0.0465			1			33
7	E^2	0.1518		9				25
8	E^1	0.0803	10					10
9	E^1	0.0522	5					5
10	E^2	0.1586		10				26
11	E^3	0.1587			5			37
12	E^1	0.0972	13					13
13	E^2	0.0584		2				18
14	E^1	0.0924	12					12
15	E^3	0.1524			4			36
16	E^3	0.1767			7			39
17	E^3	0.1996			9			41
18	E^3	0.1088			3			35
19	E^1	0.0745	9					9
20	E^4	0.1411				1		44
21	E^2	0.1738		11				27
22	E^2	0.1127		3				23
23	E^1	0.0625	7					7
24	E^1	0.0502	3					3
25	E^2	0.1431		8				24
26	E^2	0.1742		12				28
27	E^2	0.1029		5				21
28	E^3	0.3434			11			43
29	E^4	0.5237				4		47
30	E^4	0.2512				3		46
31	E^3	0.1806			8			40
32	E^5	0.7802					2	49
33	E^1	0.0700	8					8
34	E^3	0.1598			6			38
35	E^1	0.1624	16					16
36	E^1	0.1000	15					15
37	E^5	0.4781					1	48
38	E^2	0.1810		14				30
39	E^2	0.2196		16				32
40	E^1	0.0999	14					14
41	E^3	0.2397			10			42
42	E^2	0.0790		3				19
43	E^2	0.1066		6				22
44	E^2	0.0997		4				20
45	E^2	0.1768		13				29
46	E^4	0.2231				2		45
47	E^1	0.0909	11					11
48	E^3	0.0810			2			34
49	E^2	0.1958		15				31

We use Andersen-Petersen method (A-P) [6], to reach certain result for ranking too. Table 4 shows comparison result of our method and Andersen-Petersen method for all DMU's.

Table 4: comparison context-dependent and Anderson-Peterson method for ranking DMU's

DMUs	Rank with context-dependent	Rank with A-P method	DMUs	Rank with context-dependent	Rank with A-P method
1	**2**	2	26	**28**	23
2	**1**	3	27	**21**	29
3	**17**	20	28	**43**	38
4	**4**	5	29	**47**	47
5	**6**	6	30	**46**	43
6	**33**	34	31	**40**	27
7	**25**	26	32	**49**	49
8	**10**	9	33	**8**	11
9	**5**	10	34	**38**	30
10	**26**	25	35	**16**	8
11	**37**	39	36	**15**	7
12	**13**	13	37	**48**	48
13	**18**	28	38	**30**	19
14	**12**	15	39	**32**	35
15	**36**	33	40	**14**	4
16	**39**	42	41	**42**	36
17	**41**	45	42	**19**	17
18	**35**	37	43	**22**	32
19	**9**	16	44	**20**	22
20	**44**	46	45	**29**	21
21	**27**	40	46	**45**	44
22	**23**	31	47	**11**	1
23	**7**	14	48	**34**	41
24	**3**	12	49	**31**	24
25	**24**	18			

6. Conclusion

Context-dependent DEA is developed to measure the attractiveness and progress of DMU's with respect to a given evaluation context. Different strata of efficient frontiers instead of the traditional first-level efficient frontier are used as evaluation contexts. In the original DEA, adding or deleting inefficient DMU's does not alter the efficiencies of the existing DMU's and the efficient frontier. While under the context-dependent DEA, such action changes the performance of both efficient and inefficient DMU's.

The efficient DMUs ranking has always been considered, in which most methods use super-efficiency concept in some way. Here, we presented a criterion for ranking all DMUs (not only efficient DMUs) via using two concepts namely; attractiveness and progress in context-dependent DEA, and super-efficiency score.

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