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A Proportionately Increased Small and Equal Batch Delivery under Consignment Stock Agreement for a Single Vendor, Multiple Buyers Supply Chain System

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Abstract

Consignment stocks agreement had been very useful in inventory control. The benefit ranges from improved cash flow, reduced risk level, savings on investment, reduced ownership cost, low inventory carrying cost and regular restocking to mention few. Also, small batch delivery is an effective strategy for launching a product since it enables a business to assess the market and validate the product before committing to a large production run. In this paper, we combined small batch delivery and consignment stock policy by considering a supply chain setting where a vendor fulfilled the shipment requirement of each buyer sequentially in a single production set up. To achieve this, and as against the equal size shipments policy assumed in literature for different buyers, the vendor sends a smaller shipment first as early entry, followed by n equal shipments. These n shipments are proportionately increased according to the vendor rate of production to each buyer's demand rate. A mathematical cost function is developed to reduce the overall cost of the integrated supply chain system through the optimal cycle time and the optimal numbers of shipments to be delivered to each buyer. Numerical example is given using data from an existing literature, results were compared, and the new distribution policy gives better financial savings in terms of cost over the equal shipment policy assumed in literature. Sensitivity analyses were performed on key parameters to evaluate the robustness of the model.

Keywords: Consignment stock, Economic lot size, Distribution policy, Production rate, Financial savings, Demand rate.

1 | Introduction

Consignment inventory is an arrangement where the vendor stocks is kept in the consumer's repository without the seller transferring ownership to the buyer. It is carried out on a mutual agreement between both parties, who jointly benefit from the policy. Some of the benefits include good financial savings, regular restocking, flexible payment approach (upfront payment/profit sharing), reduced risk level and ease of launching new products. A small batch delivery on the other hand is a useful strategy for product entrance or introduction. It enables a business to test the market and validate the product before funds are invested into the large production run. A small batch delivery typically involves producing a limited quantity of the product and releasing it to a selected group of customers. It helps a company to avoid over production that can freeze up capital in inventory and create cash flow problems. By starting with a small quantity, a company can gauge

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customer demand and adjust production levels accordingly. Other benefits of small batch delivery include provision of platform to identify any production or quality issues before ramping up to full production. This can help ensure that the product meets customer expectations and avoids any potential recalls or quality issues.

Therefore, combining consignment stock agreement with small batch delivery can be a very profitable strategy in reducing the cost of the supply chain system. It allows for flexibility in batch size, while ensuring that the products are available for customers to purchase. This can help reduce the risk of overproduction, since the supplier can adjust production levels based on actual demand. It can also help increase sales and improve cash flow because the supplier can receive payment for the products as they are sold.

In view of the aforesaid, this research aimed at improving costs accruing from consignment stock agreement through a distribution policy that combines both small batch delivery and a proportionately increased equal batch size delivery for a single vendor, many buyers supply chain system. A review of past literature on consignment stock agreement shows that several policies involving equal and constant shipment size had been studied, but none as incorporated a flexible and proportionate shipment size and plan that allow supplier to spontaneously respond to the known buyers demand without shortages or any likely problem associated with bullwhip effect. To achieve this and like Zavanella and Zanoni [1], it is assumed that the vendor produces in a single set up and make delivery to the buyers sequentially. Each buyer consecutively received their total lot size per cycle over n+1 shipments, after which delivery is made to the next buyer. The sequence of customer arrangement is subjective and solely depends on the vendor discretion. The lots size to be deliver to each buyer consist of one (1) small shipment and n equal sized shipments. The equal shipments size is achieved by increasing the first smaller shipment proportionately by the vendor rate of production to the individual buyer's demand rate.

The rest of the paper is arranged as follows: Section 2 reviews literatures that are related to consignment stock policy and integrated modelling. Section 3 defines the problem formally and presents the proposed mathematical formulation. The mathematical formulation is solved, and the obtained results is compared with the equal shipment policy addressed by Zavanella and Zanoni [1] in Section 4. Section 5 presents sensitivity analysis on important parameters. Summary and conclusions are presented in the last Section 6.

2 | Literature Review

Different lot sizing problems had been addressed in literature. This ranges from single-vendor, single-buyer problems to single vendor, many buyers supply chain system. Some models that are applicable to this research work are discussed below.

2.1 | Single Vendor, Single Buyer

Goyal [2], [3] proposed a joint economic lot-size model that is aimed at minimising the overall costs incurred by both the buyer and vendor. Generalisation of this model was done later by Banerjee [4] and [5], Goyal [6] and Goyal et al. [7]. There models assumed equal power between the buyer and the vendor through a contractual deal. Lu [8] developed a model to reduce vendor overall cost at the expense of the highest cost the buyer might incurred.

Later, Hill [9], [10] considered minimising the total costs per year when buyer's demand and frequency of ordering is known to the vendor. This model is suitable when partnership exist between the two parties.

Goyal [11] improved the approach used in finding the best policy for the joint economic lot size problem when capacity constraint through transport equipment is considered. Valentini and Zavanella [12] discussed an industrial setting and a performance review of the consignment stocks agreement. Performance appraisal of the consignment stock agreement using analytical means was proposed by Braglia and Zavanella [13]. Zanoni and Grubbstrom [14] proposed a detailed analytical solution.

Ben-Daya and Hariga [15] assumed stochastic demand and equally defined the lead time as a linear function of the ordered lots and a fixed delay time. A heuristic approach for minimising the holding, lead-time reduction and ordering cost for an integrated system under adjustable lead time assumption between a vendor and a buyer was proposed by Hoque and Goyal [16]. Hill and Omar [17] reviewed literature works on the single vendor, single buyer inventory problem, and through different batch sizes within a replenishment cycle, they suggested a solution methodology that improved the cost attributed to the consignment stock case.

Sarmah et al. [18] identified models addressing buyer–vendor synchronization under deterministic settings. The models were classified based on the issues they addressed and the future line of research. Zhou and Wang [19] proposed a model that requires no numerical difference between buyer's and vendor's unit holding cost. Also, no shipment policy is assumed. The model, however, considered shortages and product deterioration from the buyer's inventory. Islam [20] studied consignment stock policy for a seasonal product. The authors, however, considered the selling period average inventory cost in formulating their profit function. Jaber et al. [21] showed the impact of collection and repairable rate of used items on cost and batch size under consignment stock agreements. The authors modified the remanufacturing, waste disposal and production model through additional purchaser to the vendor's system. Zanoni et al. [22] showed that the consignment stock agreement with Vendor managed Inventory (VMI) policy performed well under the emission trading plan than the traditional joint economic lot size model through the GHG emissions tax and penalty cost that was added to the buyer and vendor cost function. Giri et al. [23] proposed a joint economic lot size model where decision on customer's demand is influenced by the on-hand stock available with the buyer. The model is later generalised to include the space capacity constraint at the buyer's side. The impact of buyer's limited space on the optimal number of shipments, batch size and total cost were studied.

Zahran et al. [24] studied payments delay under consignment stock agreement settings. As a base case situation, an equal payment, equal interval scheme was considered against two possible delay payment plans, with or without interest rate. The results showed that accepting slight delay in payment on the side of the buyer is more advantageous in the system than making payments to the vendor at the time of invoicing. Khan et al. [25] developed a model to investigate the impact of product screening on defectives items and storage cost under both consignment stock agreement and VMI policy. Giri et al. [26], considered equal and unequal sized delivery from a single vendor to a single buyer under VMI and consignment stock policy. The authors considered warranty cost on the side of the vendor whose production system may produce defective items that are discovered during the screening phase by the buyer. The average expected profit is modelled mathematically, and a solution technique is proposed to determine the best possible number of delivery shipment from the vendor. An integrated model that studied consignment stock policy and variable production rate under random demand settings was developed by Aldurgam et al. [27]. The vendor's products that are delivered in full truck load are stored in the manufacturer's warehouse, where they serve as raw material for the manufacturer's product. The model was solved to determine the most economic production lot size, production rate, re-order level and number of full trucks. Islam et al. [28] developed a profit maximizing consignment model that considers realistic factors like shipping time inventory, work in process inventory, selling period inventory of sold products, transhipment cost and several other factors. Hariga et al. [29] investigated the effects of two carbon reduction strategies (carbon tax and carbon cap) on supply chain costs and carbon emissions under vendor managed consignment inventory arrangement. In order to have a more sustainable production process, Zavanella et al. [30] took into consideration energy-related objectives in lot sizing. Gharaei et al. [31] investigated quality control and green policies in a supply chain that was subject to sanctions with vendor-managed inventory and a consignment stock agreement. Giri and Masanta [32] investigated a learning-and-forgetting manufacturer production system with a random return rate for used goods and an inspection method to identify components that qualify for remanufacturing. Early, and late delivery when coordinating a two-level supply chain system were considered by Çömez-Dolgan et al. [33]. Hemmati et al. [34] evaluated bundling and separate sales for two related items under VMI with consignment stock policy. Sen et al. [35] took into account the channel members' (i.e., seller and buyer) warehouse space restrictions under both n-shipment and consignment stock policies in cases when the goods degrade. Marchi et al. [36] revisited consignment stock and delay payment. A methodology for setting appropriate lower and upper inventory limits for the buyer, taking into account the heterogeneity in the supplier base, was developed by Bogaert and Jaarsveld [37]. Zhang et al. [38] considered cap-and-trade regulation in a closed loop supply chain, under consignment stock agreement. Asadkhani et al. [39] integrate coordination, quality requirements and environmental issues in a vendor-buyer supply chain system, under Vendor-Managed Inventory with Consignment Stock (VMI-CS) agreement. The interaction between existing and new products in a diffusion process was studied by Keshavarz and Hamid [40]. Hemmati et al. [41] studied the effect of two points deterioration at the buyer's end under for VMI-CS agreement. The benefit of just-in-time, delay and advance for buyer's delivery under vendor and buyer space limitation were studied by Ambroszkiewicz and Bylka [42].

2.2 | Single Vendor, Multiple Buyer

Like the single-vendor, single-buyer integrated model, the single-vendor, many-buyers model has been of interest to many scholars presently and past. Lai and Staelin [43] proposed a quantity discount model for a seller dealing with many identical buyers. Joglekar [44] improved the work of Lai and Staelin [43] by showing that in many-purchase situation, purchasers' order quantity affects both the revenue stream of the vendor and the cost stream of the manufacturer. Joglekar and Tharthare [43] proposed an independent and logical decision approach for determining the economic lot sizes for a vendor and several buyers. The authors argued that collaboration recommended in literature negate the free enterprise system, and thus support the autonomy of each party to adopt its own independently derived optimal replenishment policy.

Banerjee et al. [45] later proposed a model that coordinates inventory between multiple buyers and one vendor, when trading with a single product, under stochastic demands and lead times via a common cycle approach. The authors centred more on Electronic Data Interchange (EDI) system as a means of having instantaneous communication between the vendor and multiple buyers under a pre-determined agreement and a pre-arranged decision system. Lu [8] object previous assumption that the buyer's ordering and holding costs must be known to the vendor, which are sometimes difficult to estimate. Lu thus proposed, a model that minimises the total cost of the vendor per year, subject to the highest cost the buyer might incurred.

Viswanathan and Piplani [46] developed a model to explore benefits of coordinating supply chain inventories under joint replenishment time for all buyers. The authors, however, failed to include the vendor inventory cost in the model. Woo et al. [47] continued the work of Banerjee [4] by considering a joint investment by the vendor and the buyers to reduce the ordering cost.

Boyaci and Gallego [48] examined the pricing and inventory policies that commonly improve the channel profit of a supply chain system under deterministic price-sensitive customer demand. The authors' showed through consignment stocks agreement how an optimal policy can be implemented jointly. Furthermore, Siajadi et al. [49] showed that for joint economic lot sizing problem, the single shipment policy is less advantageous to the multiple shipment policy. Kim et al. [50] examined a three stage multi echelon supply chain system. The last stage consists of numerous retailers that are interacting with only one buyer, who makes raw material purchase at the first and single-resource level. Zavanella and Zanoni [1] formulated a model that determines the optimal replenishment decision for a single-vendor, multiple-buyers under consignment stock agreement. There results proved that the consignment stock agreement is of higher benefit than the uncoordinated optimization. Controllable lead time under consignment stock agreement was investigated by Srinivas and Rao [51] as a strategy for reducing the expected overall cost of a supply chain system through the optimization of variables like quantities transported, delay deliveries and numbers of transportation. Hariga et al. [52] considered scheduling problems for multiple delivery under single vendor, multiple buyers' consignment stock arrangement. The problem was formulated using nonlinear mixed integer programming and a heuristic procedure was proposed to give a near optimal schedule. The solution obtained gives a significant savings that increases with the number of buyers. Bendaya et al. [53]

studied different vendor, buyer partnership under consignment stock agreement and the result obtained from their formulation showed that CS agreement is more advantageous when the vendor operates a flexible system, and the frequency of shipments is inversely related to the lot size. Mandal and Giri [54] considered both imperfection in the vendor production system and an adjustable lead time through a crashing cost paid by the buyer. Fauza et al. [55] proposed an integrated approach to address food inventory policy and supply chain under different quality characteristics. Variable production rate and imperfect quality of products under consignment stock agreement was considered by Sarker et al. [56]. Omar and Zulkipli [57] assumed the demand rate is positively dependent on the level of items displayed. Vendor imperfect production process as induced by dependent demand was considered by Guchhait et al. [58]. Chan et al. [59] synchronized the lengths of the buyers' ordering cycles and the vendors' production cycles under stochastic demand settings. Bendaya et al. [60] considered the remanufacturing of commercially viable used products recovered from the end consumer in a two-stage single vendor and multiple purchasers closed loop supply chain under a centralized consignment stock agreement. Delivery route and cost of emission of greenhouse gases were considered by Castellano et al. [61]. Agustiandi et al. [62] took into account warehousing, capital, and service level constraints. Castellano et al. [63] study the impact of controllable lead time and backorders-lost sales mixture. Charkraboty et al. [64] study a VMI contract wherein buyers charge the supplier a penalty endogenously on the excess of inventory supplied each time the provider exceeds some predetermined inventory level. Recently, Adegbola [65] studied a single vendor, multiple buyers supply chain problem under stochastic demand, full truck load assumption using simulation optimization. The author evaluated two distribution policies (JRP and VMI) and further showed that the coefficient of variation should be considered as a judgment criterion of when to embrace simulation modeling ahead of other modeling techniques.

Therefore, from the survey, it is obvious that the closest research work is the work of Zavanella and Zanoni [1]. The aim of this research work is to evaluate another coordinated product delivery policy in which a small batch delivery is combined with consignment stock agreement. Each buyer independently received one (1) small size shipment to launch an entry into the market, and n equal shipments that is increased proportionately according to the ratio of the vendor production rate to each manufacturer demand rate. A mathematical model is formulated for the proposed policy and a numerical example is given using the same data from Zavanella and Zanoni [1].

3 | Notations and Assumptions

Like Zavanella and Zanoni [1], the following notations and assumptions are employed in developing the analytical model.

3.1 | Model Assumptions

- *A cycle refers to the period used by the vendor to produce the total lot size required by Y buyers in a one set up.*
- *The cycle is repeated homogeneously within all period considered.*
- *The Vendor production rate outweigh the joint demand rate of the buyers i.e.,* $p > D$ *where* $D = \sum_i^Y d_i$.
- *Each buyer received a smaller lot on first shipment after which they receive an increased equal sized shipment (the numbers of which may differ for different buyers as shown in Fig. 1). Production continues and the final shipment is made immediately production finishes.*
- *The equal shipments received by each buyer after the first smaller shipment is this smaller shipment size increased by the ratio of the vendor production rate to individual buyer's demand rate i.e.,* $k = \frac{P}{d_i}$.

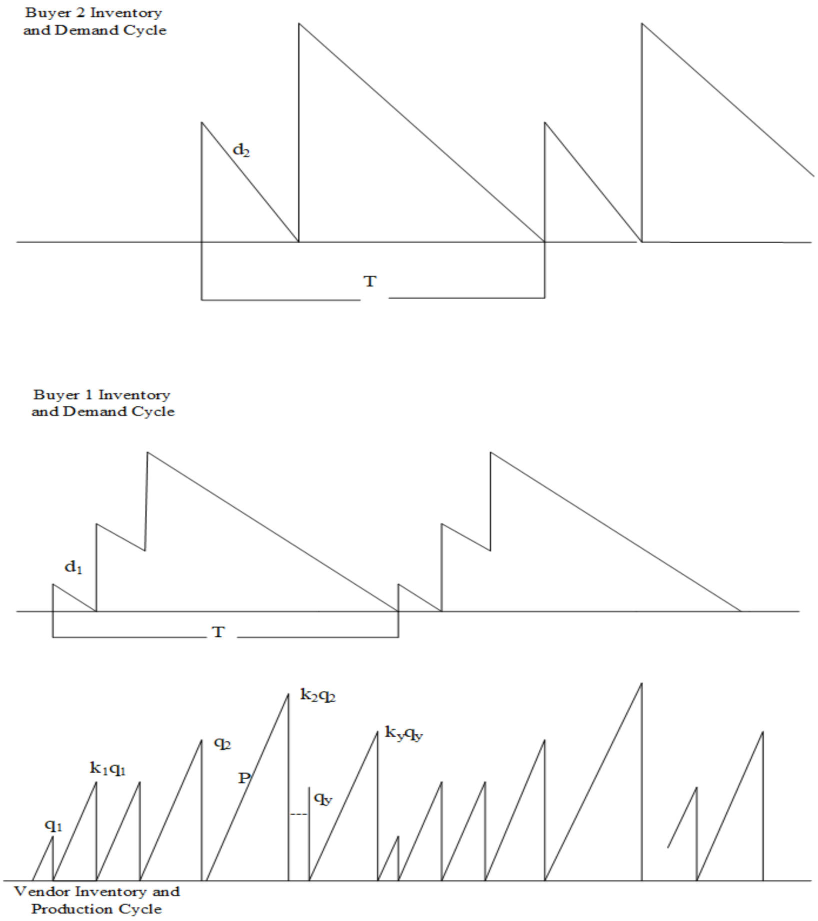

Fig. 1. The proposed inventory profile for the single vendor, multiple buyers' consignment stock policy.

3.2 | Notations

A_1	Vendor set up cost (€/set up).
$A_{2,i}$	Buyer ith ordering/emission cost (€/order).
h_1	Holding cost of vendor per item and per time unit (€/item time unit).
$h_{2,i}$	Holding cost of buyer ith per item and per unit time (€/item time unit).
p	Vendor rate of production (item/time unit).
d_i	Demand rate of buyer i_{th} buyer (continuous rate) (item/time unit).
Y	Number of buyers.
T	Cycle time (time unit).
n_i	Numbers of shipment delivery transported to buyer i per cycle time.
q_i	Buyer i_{th} batch size received per delivery from the vendor.
$T.C$	The average total cost per unit time.

3.3 | Mathematical Model Development

In this part, we derive the total cost function for the proposed single vendor, multiple buyers integrated inventory model in line with the assumptions stated above. For simplicity of modelling, we first consider single-buyer, single-vendor case, a situation where i=1, after which the model is generalized to multiple-buyers case i.e., i=Y. All cost identified in the proposed model are enumerated below.

I. Vendor set up cost: The vendor produces the lot sizes sent to all buyers in a single set up per cycle.

$$\frac{A_1}{T}. \tag{1}$$

II. Vendor holding cost: This is the cost of holding a given level of inventory in stock during each production runs. Since different lot sizes are shipped to each buyer, it consists of one (1) small lot size and n increased lot size to be deliver to each independent buyer.

$$\frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \quad (2)$$

The cost incurred by the vendor is the summation of both his set up cost and holding cost.

$$T.C_{vendor} = \frac{A_1}{T} + \frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \quad (3)$$

For all Y Buyers, the total cost per unit time incurred by the vendor is.

$$T.C_{\ vendor)} = \frac{A_1}{T} + \sum_{i=1}^{Y} \frac{h_1 T di^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right). \quad (4)$$

III. Ordering emission cost of each buyer:

$$n_i + 1)\frac{A_{2,i}}{T}. \quad (5)$$

IV. Holding cost of each buyer: To compute each buyer average inventory holding cost, the buyers' end of the profile shown in *Fig. 1* was analysed based on the inventory accumulation from the $q_i(1 + n_i k_i)$ lot size received from the vendor. The buyers' profile in *Fig. 1* was partitioned into one (1) small triangle, $\frac{n(n-1)}{2}$ rectangles, (n-1) triangle and one big triangle.

$$\frac{T d_i^2 h_{2,i}}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right). \quad (6)$$

For all Y Buyers, the total cost incurred is the summation of the emission cost and the inventory holding cost for each buyer.

$$TC_{(buyer)} = \sum_{i=1}^{Y}(n_i + 1)\frac{A_{2,i}}{T} + T h_{2,i} \sum_{i=1}^{Y} \frac{d_i^2}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right) \quad (7)$$

The total cost of the integrated supply chain system under the proposed distribution policy is the combined cost of the buyers and the vendor,

$$T.C_{system} = TC_{(vendor)} + TC_{(buyer)}.$$

$$\begin{aligned} TC_{system} = {} & \frac{A_1}{T} + \sum_{i=1}^{Y} \frac{h_1 T d_i^4}{2(n_i p + di)^2}\left(\frac{n_i p}{di^2} + \frac{1}{P}\right) + \sum_{i=1}^{Y}(n_i + 1)\frac{A_{2,i}}{T} \\ & + T h_{2,i} \sum_{i=1}^{Y} \frac{d_i^2}{(n_i p + d_i)^2}\left(\frac{n_i p}{2}\left(\frac{n_i p}{d_i} + 1 - n_i\right) + \frac{d_i}{2}\right). \end{aligned} \quad (8)$$

Furthermore, the objective here is to minimize the cost function, subject to having a complete and sequential delivery to all buyers i.e., the first buyer received his shipments completely before other buyers are considered per cycle. Precedence is however not given to the buyer's arrangement or chronological sequence, as this is subjective and solely depend on the discretion of the vendor. The two decision variables are the cycle time (T) and the number of delivery shipments $(1 + n_i)$ that minimizes the total cost of the integrated supply chain system. A joint optimum solution technique is proposed to find the best possible values for the decision variables, and the results obtained are compared with that from Zavanella and Zanoni [1] using the same data, so as to estimate the financial savings.

4 | Solution Method, Numerical Example and Sensitivity Analysis

4.1 | Joint Optimum Solution Approach

This approach optimizes the cost of the integrated supply chain system centrally. The cost is minimized by taking the first derivative of $TC_{system}$with respect to T, we have,

$$\frac{\partial T.C}{\partial T} = 0. \tag{9}$$

$$\frac{\partial T.C}{\partial T} = \frac{-1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right) + \frac{h_1}{2}\sum_{i=1}^{Y}\left(\frac{n_i P d_i^2}{(n_i p + d_i)^2} + \frac{di^4}{P(n_i p + d_i)^2}\right)$$
$$+\frac{1}{2}\sum_{i=1}^{Y} h_{2,i}\left(\left(\frac{n_i^2 p^2 d_i}{(n_i p + d_i)^2} + \frac{n_i p di^2}{(n_i p + d_i)^2} - \frac{n_i^2 d_i^2 p}{(n_i p + d_i)^2}\right.\right. \tag{10}$$
$$+\frac{d_i^3}{(n_i p + d_i)^2}) = 0\,.$$

$$= \frac{-1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right) + \sum_{i=1}^{Y}\frac{h_1}{2(n_i p + d_i)^2}\left(n_i P d_i^2 + \frac{di^4}{P}\right)$$
$$+\sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_i p + d_i)^2}\left(n_i^2 p^2 d_i + n_i p d_i^2 - n_i^2 d_i^2 p + d_i^3\right) = 0. \tag{11}$$

$$\frac{1}{T^2}\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)$$
$$= \sum_{i=1}^{Y}\frac{h_1}{2(n_i p + d_i)^2}\left(n_i P d_i^2 + \frac{di^4}{P}\right) \tag{12}$$
$$+\sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_i p + d_i)^2}\left(n_i^2 p^2 d_i + n_i p d_i^2 - n_i^2 d_i^2 p + d_i^3\right).$$

$$\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)$$
$$= T^2\frac{\left[h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2} + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 p^2\right)}{n_i P + d_i)^2}\right)\right]}{2P} \tag{13}$$

$$T^2 = \frac{\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)}{\frac{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 P^2\right)}{n_i P + d_i)^2}\right).}{2P}} \tag{14}$$

$$T^* = \sqrt{\frac{2P\left(A_1 + \sum_{i=1}^{Y} A_{2,i}\, n_i + 1)\right)}{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i P^2 + d_i^2)}{n_i P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i\left(d_i^2 - Pn_i\; n_i - 1)d_i + n_i^2 P^2\right)}{n_i P + d_i)^2}\right)}} \tag{15}$$

If

$$x=\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right),y$$

$$=\frac{\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)}{2P}$$

$$\begin{aligned}T.C_{system}&=\frac{\sqrt{y}}{\sqrt{x}}\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\\&+\frac{\sqrt{x}}{\sqrt{y}}\left(\sum\nolimits_{i=1}^{Y}\frac{h_1}{2(n_ip+d_i)^2}\left(n_iPd_i^2+\frac{di^4}{P}\right)\right.\\&\left.+\sum_{i=1}^{Y}\frac{h_{2,i}}{2(n_ip+d_i)^2}\left(n_i^2p^2d_i+n_ipd_i^2-n_i^2d_i^2p+d_i^3\right)\right).\end{aligned}\tag{16}$$

$$\begin{aligned}&T.C_{system}\\&=\frac{\sqrt{y}}{\sqrt{x}}\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\\&+\frac{\sqrt{x}}{\sqrt{y}}\left(\frac{\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)}{2P}\right).\end{aligned}\tag{17}$$

Since,

$$=\frac{\sqrt{m}}{\sqrt{x}}x+\frac{\sqrt{x}}{\sqrt{m}}m=2\sqrt{xm}.$$

$$T.C_{system}=$$

$$2\sqrt{\frac{1}{2P}\left(\left(A_1+\sum_{i=1}^{Y}A_{2,i}\ n_i+1)\right)\right)\left(\left(h_1\sum_{i=1}^{Y}d_i^2\frac{(n_iP^2+d_i^2)}{n_iP+d_i)^2}\right)+P\left(\frac{\sum_{i=1}^{Y}h_{2,i}\,d_i\left(d_i^2-Pn_i\ n_i-1)d_i+n_i^2P^2\right)}{n_iP+d_i)^2}\right)\right)}\tag{18}$$

To find both the optimal number of shipments n_i received by each buyer, and the cycle time T that minimizes the total cost function, the pseudocode below is used. n_i is an integer that is determined by performing a line search over a reasonable range to minimize the cost function of the integrated supply chain system. The optimal cycle time T is then determined by substituting n_i^* back into *Eq. (15)*.

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Algorithm 1. A pseudocode algorithm proposed to solve the small batch, equal shipment size model.

$T.C^*_{system} = big\ J$

$For\ n_i = 1: step\ size\ of\ 1: n_{max} \quad \forall\ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$Compute\ T.C_{system}$

$$= 2\sqrt{\frac{1}{2P}\left((A_1 + \sum_{i=1}^{Y} A_{2,i}(n_i+1))\right)\left(\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_iP^2 + d_i^2)}{(n_iP + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i \left(d_i^2 - Pn_i(n_i-1)d_i + n_i^2P^2\right)}{(n_iP + d_i)^2}\right)\right)}$$

$if\ \ T.C_{system} \le\ T.C^*_{system}$

$T.C^*_{system} = T.C_{system}\ (n_i^*) \qquad for\ all\ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$n_i^* = n_i \qquad for\ all\ \ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

$End\ if$

$$Compute\ T^* = \sqrt{\frac{2P\left(A_1 + \sum_{i=1}^{Y} A_{2,i}(n_i^*+1)\right)}{\left(h_1 \sum_{i=1}^{Y} d_i^2 \frac{(n_i^*P^2 + d_i^2)}{(n_i^*P + d_i)^2}\right) + P\left(\frac{\sum_{i=1}^{Y} h_{2,i}\, d_i \left(d_i^2 - Pn_i^*(n_i^*-1)d_i + n_i^{*2}P^2\right)}{(n_i^*P + d_i)^2}\right)}}$$

$End\ for$

$n_i^*, T^* = argmin\ T.C_{system}$

$min\ \ T.C_{system} = min\{T.C_{system}[T^*, n_i^*]\}\ \ for\ all\ \ i = 1,2,3, \dots \dots \dots \dots \dots, Y$

4.2 | Numerical Example

The model was evaluated using data in *Table 1* as extracted from Zavanella and Zanoni [1].

Table 1. Model data acquired from Zavanella and Zanoni [1].

P	3200 units/year
D	1500 units/ year
d_1	500 unit/ year
d_2	1000 unit / year
A_1	€400 / setup
A_{21}	€75 /order
A_{22}	€25 / order
h_1	€5/item/year
h_{21}	€4/item/year
h_{22}	€4 /item /year

The results obtained were then compared with that of Zavanella and Zanoni [1] as the base case and it was observed that the proposed model perform better in terms of financial savings.

Table 2. Comparison of results of the equal size shipment policy with those obtained from the proposed small batch, relatively increased equal shipment size policy.

Policy Type	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer2)	TC (System)
Equal size shipment policy	1	3	0.424	1134.1	601.7	849.9	2585.7
	$1 + n_1$	$1 + n_2$	T	TC (vendor)	TC (Buyer 1)	TC (Buyer2)	TC(System)
Small batch and relatively increased equal shipment policy	1	3	0.443	1125.0	612.4	744.8	2482.2

Furthermore, *Table 3* shows that the new policy gives better financial savings in terms of cost, and this is about 4% for the whole system, 1% for the vendor and 13% for buyer 2. Buyer 1 however had 2% increment in cost due to high holding cost resulting from his low demand rate, as the cycle time for all participants is relatively constant.

Table 3. Financial savings in percentage as obtained by comparing the proposed gradually increased shipment policy with the equal sized shipment proposed by Zavanella and Zanoni [1].

TC (Saving)	Vendor (Saving)	Buyer 1 (Increment)	Buyer 2 (Saving)
4%	1%	2%	13%

5 | Sensitivity Analysis

Using the previous example as the base case scenario, we change some model parameters to examine their effects on the decision variables and also, to evaluate the robustness of the proposed model.

Effect of vendor holding cost h_1

Table 4 illustrates how the vendor holding cost has an impact on the proposed policy. It is obvious that by raising the vendor unit holding cost, the cost at the buyer decreases, while the cost incurred by the vendor together with the overall cost of the integrated supply chain system increases. The reason for this is that any increase in the unit holding cost of the vendor reduces the joint cycle time. This compelled the vendor to reduce the shipment size delivered to each buyer, so as to prevent a high surge in costs that could amass from holding and sending a bigger constant lot size to the individual buyer. Meanwhile, if we compared the result obtained from the small batch with proportionate increase equal shipment policy with those obtained from the equal and constant sized shipment policy found in literature; the former gives better financial savings in term of cost. The gain ranges from 3.7% to 4.3% depending on the choice of other parameters.

Table 4. Effect of vendor holding cost h_1.

Parameter	Small Batch With Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
h_1	$1+n_1$	$1+n_2$	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
4.5	1	3	0.447	1096	643.97	748.5	2459.8	1	3	0.428	1109.0	603.6	853.7	2566.3	4.30
5	1	3	0.443	1125	641.2	744.8	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.7	4.20
5.5	1	3	0.439	1153	638.5	741.2	2504.3	1	3	0.422	1159.0	599.9	846.2	2605.0	4.00
6	1	3	0.435	1180	635.9	737.8	2526.5	1	3	0.419	1183.5	598.1	842.6	2624.2	3.90
6.5	1	3	0.432	1208	633.5	734.4	2548.0	1	3	0.416	1208.0	596.4	839.1	2643.2	3.70

Effect of vendor set-up cost A_1

The effect of the vendor set up cost (A_1) was studied in *Table 5*, the model responded through an increase in the joint cycle time, which is an indication that the shipment lot size or the number of shipment delivery increases. From *Table 5*, an increase in the vendor set up cost from 350 to 400 as example, retained the number of shipment deliveries, but it increases the size of shipments lots to be delivered at each buyer. This resulted in higher costs at the vendor and buyer, which combined to increase the total cost of the supply chain system. A proportionate increment in the quantities delivered is necessary to balance out the increment in the set-up cost so as to have an economy of scale. *Table 5* further shows that when the vendor set up cost changes, the small batch policy with comparatively increased equal shipment size gives better savings than equal sized shipment.

Table 5. Effect of vendor set-up cost A_1.

Parameter	Small batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
A_1	$1+n_1$	$1+n_2$	T	TC (vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
350	1	3	0.423	1040.3	627.5	726.3	2366.0	1	3	0.406	1047.7	590.5	827.1	2465.4	4.20
400	1	3	0.443	1125.0	641.2	744.8	2482.0	1	3	0.425	1134.1	601.7	849.8	2585.7	4.16
450	1	3	0.463	1204.5	655.0	763.2	2592.5	1	3	0.444	1012.8	613.1	872.3	2700.7	4.17
500	1	3	0.482	1279.6	668.8	781.4	2698.4	1	4	0.481	1228.0	636.7	944.0	2808.7	4.09
550	2	3	0.554	1244.8	871.2	855.4	2795.6	1	4	0.498	1104.1	648.7	963.5	2910.9	4.12

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Table 6. Effect of buyer holding cost $(h_{2,i})$.

Parameter	Small Batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
$h_{2,i}$	$1+n_1$	$1+n_2$	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
3.0	1	3	0.497	1054.1	547.9	635.1	2212.9	1	4	0.563	993.9	525.5	775.1	2294.5	3.69
3.5	1	3	0.468	1089.8	596.3	691.9	2351.0	1	3	0.528	1094.0	560.4	790.1	2444.5	3.98
4.0	1	3	0.443	1125.0	641.2	744.7	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.7	4.12
4.5	1	2	0.422	1159.5	683.3	794.4	2606.4	1	3	0.404	1173.2	640.5	905.9	2719.6	4.34
5.0	1	2	0.385	1305.1	753.8	743.5	2725.0	1	3	0.386	1211.4	677.1	958.8	2847.2	4.48
5.5	1	2	0.371	1336.0	794.1	784.5	2832.5	1	3	0.370	1248.6	711.8	1009.0	2969.4	4.83

Effect of buyer holding cost $h_{2,i}$

The buyer holding cost was equally varied to study how the model responded to a change in this parameter. *Table 6* showed clearly that the joint cycle time and the numbers of delivery.

Shipments reduce when the holding cost at the buyers increases because the vendor tried to save costs by reducing the goods available at the buyers through a reduced shipment lot size or numbers. The total cost of the supply chain and the vendor cost, however, keep growing because of the fixed set up cost, which remains unchanged regardless of the quantities of goods produced by the vendor. Also, from *Table 6*, the small batch with relatively increased equal shipment policy gives better financial savings over equal sized shipment when the holding cost at the buyer is changing.

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Table 7. Effect of the vendor production rate (P).

Parameter	Small Batch with Relatively Increased Equal Shipment Policy							Equal Size Shipment Policy							
P	1 + n_1	1 + n_2	T	TC (Vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	n_1	n_2	T	TC (vendor)	TC (Buyer 1)	TC (Buyer 2)	TC (sys)	% Savings
1600	2	5	0.575	1022.8	536.2	695.6	2345.8	2	6	0.557	972.1	739.2	803.0	2514.3	7.1
2000	2	4	0.538	1036.1	651.7	734.9	2415.7	1	4	0.447	1120.4	626.9	827.4	2571.4	6.4
2400	2	4	0.528	1004.9	724.6	793.5	2459.9	1	4	0.445	1129.7	613.9	837.1	2580.8	4.9
2800	1	4	0.444	1152.0	600.1	714.2	2479.2	1	4	0.444	1098.3	613.2	875.8	2587.4	4.4
3200	1	3	0.443	1125.0	641.2	744.8	2482.2	1	3	0.425	1134.1	601.7	849.8	2585.8	4.2

Effect of production rate

Finally, we studied the production rate effect in *Table 7*. For the small batch, relatively increased equal shipment policy, an increase in the production rate reduces the joint cycle time, and equally increase the shipment lot size to be deliver at the buyer. This is responsible for the reduction in the vendor cost due to the fixed set up cost that is charged for the gradually increased quantities of items produced, which also increase the cost at the buyers through the holding cost that varied with the quantities shipped. Comparing the results from the policy discussed above with those obtained from the equal shipment policy shows that the former policy performed better in terms of savings, which ranges between 7.1% and 4.2%.

6 | Summary and Conclusion

In this research, we further explore the single vendor, multiple buyers' consignment stock inventory problem by considering small batch delivery under known production and demand conditions. As compared to previous work, we proposed a new policy wherein the vendor coordinate shipments by sending smaller lots to the buyer first after which the buyer received an equal shipment that is increased at constant rate $(\frac{P}{d_i})$. The vendor ensures that each buyer is fully served before the next buyer is considered, and the buyer's sequence/arrangement is very flexible and subjective depending on the discretion of the vendor. An integrated mathematical model that described the cost associated with this policy from the vendor and buyer's perspectives was formulated, and like Zavanella and Zanoni [1], a joint optimal solution technique was adopted in solving the problem being the better of the two solution techniques evaluated by the previous authors.

Furthermore, the proposed policy was evaluated using the same data from Zavanella and Zanoni [1], and the results obtained was compared with that from equal shipment, equal interval policy proposed by Zavanella and Zanoni [1] at different holding cost, set up cost and production rate of the vendor and buyers. The percentage difference was computed and the small batch with relatively increased equal shipment policy gives better financial reward than the equal shipment, equal interval policy found in literature. Finally, we studied how the model responded to changes in key parameters through a sensitivity analysis performed on the holding cost, set-up cost and production rate. The impact of these parameters on the number of shipments delivered to each buyer, shipment size, vendors and buyers' cost were investigated and discussed.

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