E-ISSN: 2717-2937 | P-ISSN: 2783-1337

International Journal of Research in Industrial Engineering

www.riejournal.com

Int. J. Res. Ind. Eng. Vol. 12, No. 3 (2023) 321–336.

Paper Type: Research Paper

Improving Classic Hungarian Algorithm Considering Uncertainty by Applying for Grey Numbers

Shahram Ariafar[1,*], Seyed Hamed Moosavirad[1], Ali Soltanpour[2]

[1] Department of Industrial Engineering, Faculty of Engineering, Shahid Bahonar University of Kerman, Kerman, Iran; ariafar@gmail.com; s.h.moosavirad@uk.ac.ir.

[2] Organization and Jobs Classification, National Iranian Copper Industries Company, Iran; soltanpourali1@gmail.com.

Citation:

Ariafar, Sh., Moosavirad, S. H., & Soltanpour, A. (2023). Improving classic hungarian algorithm considering uncertainty by applying for grey numbers. *International journal of research in industrial engineering, 12*(3), 321-336.

Received: 05/11/2022 Reviewed: 07/12/2022 Revised: 19/01/2023 Accepted: 24/02/2023

Abstract

The Hungarian Algorithm is the most famous method for solving Linear Assignment Problems (LAP). Linear Assignment Method (LAM), as an application of LAP, is among the most popular approaches for solving Multi Criteria Decision Making (MCDM) problems. LAM assigns a priority to each alternative based on a Decision Matrix (DM). The elements of the DM are often deterministic in MCDM. However, in the real world, the value of the elements of the DM might not be specified precisely. Hence, using interval grey numbers as the value of the DM to consider the uncertainty is reasonable. In this research, for providing a real circumstance, the classic Hungarian algorithm has been extended by using the concept of grey preference degree as the Grey Hungarian Algorithm (GHA) to solve LAM under uncertainty. To verify the proposed GHA, a real case for ranking several items of mining machinery warehouse from Sarcheshmeh Copper Complex has been solved by the GHA. Also, the same case study has been prioritized by two other methods: Grey TOPSIS and Grey VIKOR. The results of all mentioned approaches are identical, showing the validity of the proposed GHA developed in this research.

Keywords: Grey interval number, Hungarian algorithm, Grey VIKOR, Grey TOPSIS, Preference degree.

1 | Introduction

In a real environment, when the aim is to select alternatives by Multi Criteria Decision Making (MCDM) methods based on their maximum impacts, using the Linear Assignment Method (LAM) is reasonable. The main specification of this approach is that each alternative shall be assigned just to one rank, and each ranking shall be occupied just by one alternative. This kind of problem has been well studied and solved by the Hungarian algorithm developed by Kuhn [1].

Researchers have done various studies to extend the context of the Linear Assignment Problem (LAP) and Hungarian Algorithm. In early research done by Das and Deo [2], they developed a parallel Hungarian Algorithm by using an Exclusive Read and Exclusive Write (EREW) and a Parallel Random Access Machine (PRAM) [2]. In another study by Ishibuchi and Tanaka [3], they developed a binary linear assignment mathematical model that used interval numbers as the coefficients of the

Corresponding Author: ariafar@gmail.com

https://doi.org/10.22105/riej.2023.345845.1344

Ariafar et al.| Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

objective function. Aldous [4] used a usual probabilistic model to address a random assignment problem. Li et al. [5] developed a serial-parallel system to improve the Hungarian Algorithm. Rajabi-Alni and Bagheri [6] used the Hungarian Algorithm to solve the many-to-many matching problems considering demands and capacity. In the Many-to-many matching problems, each element of one set matches at least one item of another set.

Bai [7] introduced the grey LAM by providing two solution approaches; one of the solution methods was using the average of the grey numbers to transform the model into a deterministic problem, and another approach was applying the grey forecasting method. Majumdar [8] has extended the Hungarian method for solving assignment models with interval parameters. However, the proposed method is not completely accurate because in dividing two interval parameters in the proposed method, the lower endpoint of the first interval number has been divided by the lower endpoint of the second interval number as the lower endpoint of the generated interval number.

Similarly, the upper endpoint of the interval number has been calculated by dividing the upper endpoint of the first number by the upper endpoint of the second number that is not correct. For instance, if the interval number is [2, 4] that is divided by [1, 5] according to the dividing method that has been considered by Majumdar [8], the answer will be equal to $\left[\frac{2}{1}, \frac{4}{5}\right]$, which is not correct; because the lower endpoint of the generated interval is greater than the upper endpoint of this number.

Several studies applied the Hungarian Algorithm as a part of their solution approach; for instance, the application of the Hungarian Algorithm for solving error-tolerant matching [9], [10]. Lan et al. [11] studied the scheduling of physicians and medical staff in the outpatient ward of a large hospital. They proposed a meta-heuristic algorithm based on the Iterated Hungarian Algorithm named IHA. Yadav et al. [12] applied the Hungarian Algorithm to LAPs for solving resource allocation in wireless communication systems. Khan et al. [13] presented an algorithm based on the Hungarian Algorithm and fractional programming for beamforming and scheduling strategy in a cloud radio access network.

Yang et al. [14] proposed a multi-objective MIP model for internal truck allocation when the internal trucks are shared among several nearby container terminals and solved the model by applying the Hungarian Algorithm. Kumarnnath and Batri [15] developed a modified PSO-based iterative Hungarian Algorithm to provide maximum throughput traffic with the least blocking probability in the transmission of data between nodes of an optical network. MacLean et al. [16] developed a method for maximizing the satisfaction of medical students regarding their clinical preference rotation at the University of Texas Southwestern Medical Center (UTSW). They used the Hungarian Algorithm to solve the problem. Stevens and Sciacchitano [17] used the Hungarian Algorithm and hierarchical clustering method for vortex detection and tracking in the complex and turbulent flow study.

Katariya et al. [15] improved the Hungarian Algorithm to solve the unbalanced assignment problem. In this problem, some fictitious machines in the system cannot be assigned to any tasks. In this situation, all the tasks must be allocated to a given number of machines [15]. Zhu et al. [18] developed a model for infrared target trajectories based on Gaussian distribution. They used a Munkres version of the Hungarian Algorithm to solve the model. Zhang et al. [19] applied the Hungarian Algorithm for task assignment optimization in a remote sensing big data workflow.

However, investigating the requirements of the real world shows that in actual circumstances, precise, sufficient, or fully covered information cannot be provided often. For instance, when the inventory items should be ranked or prioritized based on two criteria ("Scarcity" and "Availability of the Technical Specifications" in this research) by the experts of the warehouse department, the experts might not be confident about the priorities of each inventory item based on the mentioned criteria. Hence, the uncertainty of the environment should be considered in the problem to make more reasonable and realistic decisions in such a situation.

On the other hand, reviewing the literature showed that several methods have been developed to consider the imprecise experts' opinions to make realistic decisions: fuzzy, stochastic, and grey methods [20]. The stochastic method needs a probability distribution function of the events and facts.

If there were enough data regarding the facts, then the experts' opinions regarding those facts would not be required. On the other hand, another method for considering the ambiguity of judgment is applying fuzzy numbers. The membership function of a fuzzy number should be available if the fuzzy number would like to be used. Hence, the interval grey numbers application to consider the uncertainty of the environment is more reasonable.

To the best of the researchers' knowledge of this study, no study has extended the context of grey numbers in the Hungarian Algorithm. Hence, this study develops a method for solving the Hungarian Algorithm with interval grey numbers, which is the main contribution of this study. The remainder of the paper is as follows. In Section 2, grey theory and interval grey numbers will be discussed. Section 3 will be dedicated to the Grey Hungarian Algorithm (GHA) development and an explanation of Grey VIKOR and Grey TOPSIS, followed by an illustration of the proposed methods by solving a sample problem, sensitivity analysis, and discussion in Section 4. Finally, Section 5 concludes the paper.

2 | Grey Theory and Interval Grey Numbers

At first, Deng [21] proposed the basic thinking of Grey Systems Theory. It is one of the new effective mathematical theories to solve uncertain problems with incomplete information. The theory is categorized into five major parts: 1) grey forecasting, 2) grey relational analysis, 3) grey decision-making, 4) grey programming, and 5) grey control.

In grey systems theory, the darkness of the color is usually used to show the degree of accuracy of information. In the grey theory, the entire system is divided into three categories: white, black, and grey sections. White represents the completion, certainty, and transparency of information in a system, and black shows unknown and incomplete information in a system [22], [23]. While the grey section is placed between two white and black sections, showing insufficient information between two clear boundaries. The concept of grey systems is shown in *Fig. 1*. So, a grey number $\otimes G$ in a grey system may be shown with a closed interval with upper and lower endpoint, i.e. $[\underline{\mu}_G(x), \overline{\mu}_G(x)]$ [24].

Each grey system is described with grey numbers, grey equations, grey matrices, etc., while grey numbers are considered minor parts of a grey system, like atoms or cells. In other words, a grey number's exact value is unclear, but its interval endpoints are clear. In a practical situation, a grey number will be stated by an interval or a whole set of numbers [23]. For example, ranking the alternatives based on a criterion in decision-making will be expressed by linguistic variables, that could be stated by numerical intervals that include uncertain information [21].

Ariafar et al. | Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

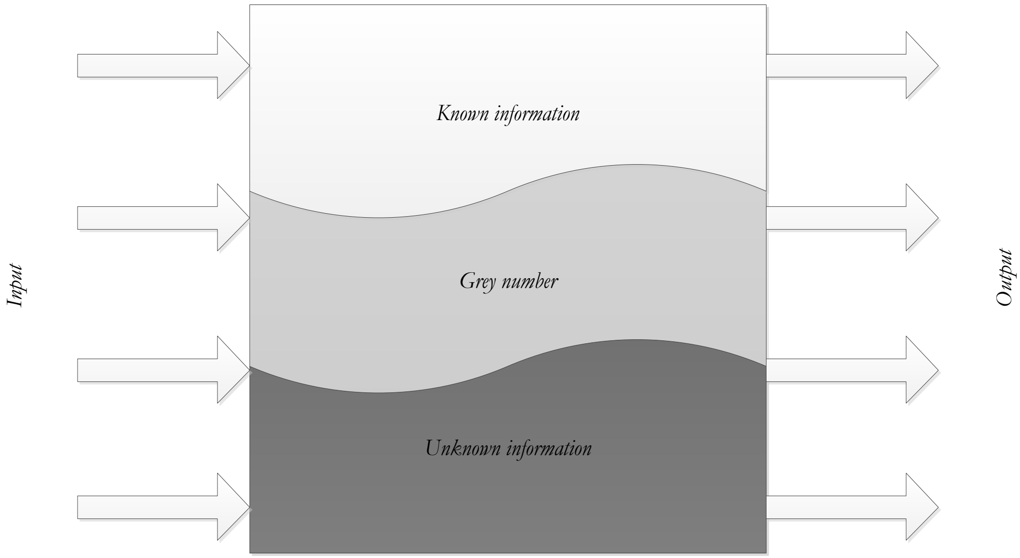

Fig. 1. The concept of the grey system.

Generally, the arithmetic operators defined for real numbers can also be extended for the set of grey intervals [23]. These interval operators were developed by Moore [25]. The basic operation laws of two grey numbers $\otimes G_1=[\underline{G_1},\overline{G_1}]$, and $\otimes G_2=[\underline{G_2},\overline{G_2}]$ are defined as follows [21]:

$$\otimes G_1 + \otimes G_2 = \left[\underline{G_1} + \underline{G_2}, \quad \overline{G_1} + \overline{G_2}\right]. \tag{1}$$

$$\otimes G_1 - \otimes G_2 = \left[\underline{G_1} - \overline{G_2}, \quad \overline{G_1} + \underline{G_2}\right]. \tag{2}$$

$$\otimes G_1 \times \otimes G_2 = \left[\begin{array}{c} \min(\underline{G_1}\times\underline{G_2}, \underline{G_1}\times\overline{G_2}, \overline{G_1}\times\underline{G_2}, \overline{G_1}\times\overline{G_2}) \\ \max(\underline{G_1}\times\underline{G_2}, \underline{G_1}\times\overline{G_2}, \overline{G_1}\times\underline{G_2}, \overline{G_1}\times\overline{G_2}) \end{array}\right]. \tag{3}$$

$$\begin{gathered}\otimes G_1 \div \otimes G_2 = \left[\underline{G_1}, \overline{G_1}\right] \times \left[\frac{1}{\overline{G_2}}, \frac{1}{\underline{G_2}}\right]. \\ \overline{G_2} \neq 0, \quad \underline{G_2} \neq 0.\end{gathered} \tag{4}$$

3 | Research Methodology

The developed solution approach of the study will be expressed in this section. Hence, at first, the GHA will be explained, followed by the explanation for the Grey VIKOR and Grey TOPSIS that have been applied to verify the developed method of this study.

3.1 | Developed Grey Hungarian Algorithm

The proposed GHA of this study is based on the classic Hungarian algorithm [1], [26] that has been extended to consider Grey interval numbers. The flowchart of the GHA method is shown in *Fig. 2*.

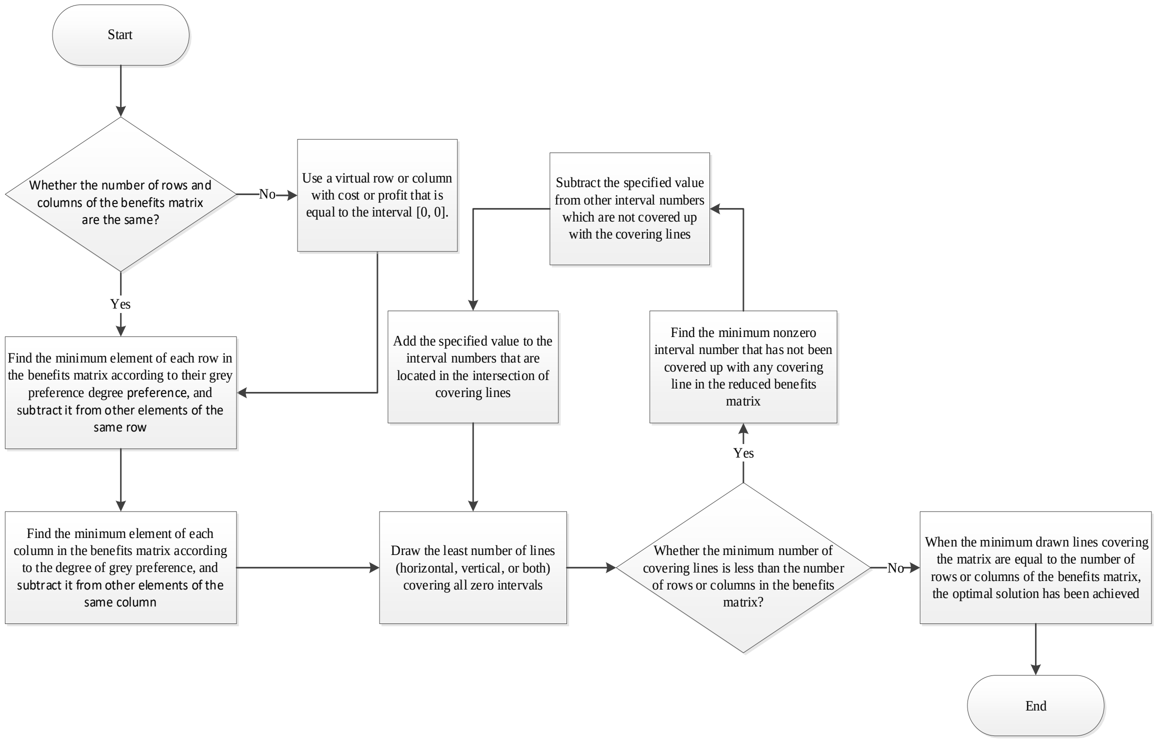

Fig. 2. The flowchart of the proposed GHA.

The steps of the proposed GHA are as follows:

Step 1. Find the minimum element of each row in the benefits matrix (objective function coefficients for the assignment problem) according to their grey preference degree relation and subtract it from other elements of the same row. Then, go to the *Step 2*.

Step 2. Find the minimum element of each column in the benefits matrix obtained from *Step 1* according to the degree of grey preference and subtract it from other elements of the same column. Then go to the *Step 3*.

Step 3. Draw the least number of lines (horizontal, vertical, or both) covering all zero intervals (zero intervals are those where their average interval points are equal to zero). Then go to the *Step 4*.

Step 4. The optimal solution will be achieved if the number of minimum drawn lines covering the matrix equals the number of rows or columns of the benefits matrix. It is available among the covered zero intervals. If the minimum number of covered lines is less than the number of rows or columns in the benefits matrix, go to *Step 5*.

Step 5. Find the minimum nonzero interval number that has not been covered up with any covering line in the reduced benefits matrix. Subtract the specified value from other interval numbers that are not covered up with the coverage lines; then, add it to the numbers that have been covered by two covering lines simultaneously. Then go to the *Step 3*.

3.2 | Grey VIKOR Algorithm

VIKOR method is an MCDM approach. This method is used when there are conflicting criteria, and this method finds a solution close to the ideal solution by using a compromised ranking list. The use of the VIKOR method for decision problems in the literature is abundant. It has been extended to solving interval numbers by Sayadi et al. [27], but the lack of normalization for the Decision Matrix (DM) can be sensed. This study has improved the Grey VIKOR method by normalizing the DM. The steps of Grey VIKOR of this study are as follows:

Table 1. DM in the Grey VIKOR method.

	C_1	C_2	$\dots$	C_n
A_1	$\left[g^l_{11}, g^u_{11}\right]$	$\left[g^l_{12}, g^u_{12}\right]$	$\dots$	$\left[g^l_{1n}, g^u_{1n}\right]$
A_2	$\left[g^l_{21}, g^u_{21}\right]$	$\left[g^l_{22}, g^u_{22}\right]$	$\dots$	$\left[g^l_{2n}, g^u_{2n}\right]$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
A_m	$\left[g^l_{m1}, g^u_{m1}\right]$	$\left[g^l_{m2}, g^u_{m12}\right]$	$\dots$	$\left[g^l_{mn}, g^u_{mn}\right]$

$$\otimes W = [\otimes w_1, \otimes w_2, \dots, \otimes w_n]$$

As can be seen from *Table 1*, which is the DM, $A_1, A_2, \dots, A_m$ are the alternatives, while $C_1, C_2, \dots, C_n$ are the required criteria for selecting alternatives. g_{ij} is the rank of alternative A_i according to the C_j^{th} criterion whose value is not crisp, $gij \in \left[g^l_{ij}, g^u_{ij}\right]$ while $\otimes W$ is the weight of j^{th} criteria.

Step 1. In the development of the Grey VIKOR Method, at first, the DM should be normalized. In the following, the normalization process for the cost (minimizing) and profit (maximizing) criterion has been provided.

$$\otimes f_{ij} = \left[\frac{\otimes G_j^{max} - \otimes \overline{G_{ij}}}{\otimes G_j^{max}}, \quad \frac{\otimes G_j^{max} - \otimes \underline{G_{ij}}}{\otimes G_j^{max}}\right], \quad \otimes G_j^{max} = \max_{1<i<m}\left\{\otimes \overline{G_{ij}}\right\}, \quad \text{for Cost} \tag{5}$$

Criterion.

$$\otimes f_{ij} = \left[\frac{\otimes \underline{G_{ij}}}{\otimes G_j^{max}}, \quad \frac{\otimes \overline{G_{ij}}}{\otimes G_j^{max}}\right], \qquad \otimes G_j^{max} = \max_{1<i<m}\left\{\otimes \overline{G_{ij}}\right\}, \text{ for Benefit Criterion.} \tag{6}$$

Step 2. Determining the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) according to the *Eqs. (7)* and *(8)*.

$$A^+ = \left\{f_1^+, f_2^+, \dots, f_n^+\right\} = \left\{\left(\max_i f_{ij}^u \middle| j \in B\right) \quad \text{or} \quad \left(\min_i f_{ij}^l \middle| j \in C\right)\right\}. \tag{7}$$

$$A^- = \left\{f_1^-, f_2^-, \dots, f_n^-\right\} = \left\{\left(\min_i f_{ij}^l \middle| j \in B\right) \quad \text{or} \quad \left(\max_i f_{ij}^u \middle| j \in C\right)\right\}, \tag{8}$$

where B is the set of benefit criteria, while C is the set of cost criteria.

Step 3. Computing the intervals $\left[S_i^L, S_i^U\right]$ (Satisfaction Index), and $\left[R_i^L, R_i^U\right]$ (Regret Index) by using *Eqs. (9)* and *(10)* by considering grey uncertainty for every W_j.

$$\begin{aligned} S_i^l &= \sum_{j \in B} W_j \left(\frac{f_j^+ - f_{ij}^u}{f_j^+ - f_j^-}\right) + \sum_{j \in C} W_j \left(\frac{f_{ij}^l - f_{ij}^+}{f_j^- - f_j^+}\right) \quad i = 1,2,\dots,m, \\ S_i^u &= \sum_{j \in B} W_j \left(\frac{f_j^+ - f_{ij}^l}{f_j^+ - f_j^-}\right) + \sum_{j \in C} W_j \left(\frac{f_{ij}^u - f_{ij}^+}{f_j^- - f_j^+}\right) \quad i = 1,2,\dots,m. \end{aligned} \tag{9}$$

$$\begin{aligned} R_i^l &= \max\left\{W_j \left(\frac{f_j^+ - f_{ij}^u}{f_j^+ - f_j^-}\right) j \in B \quad , \quad W_j \left(\frac{f_{ij}^l - f_{ij}^+}{f_j^- - f_j^+}\right) j \in C\right\} \quad i = 1,2,\dots,m, \\ R_i^l &= \max\left\{W_j \left(\frac{f_j^+ - f_{ij}^l}{f_j^+ - f_j^-}\right) j \in B \quad , \quad W_j \left(\frac{f_{ij}^u - f_{ij}^+}{f_j^- - f_j^+}\right) j \in C\right\} \quad i = 1,2,\dots,m. \end{aligned} \tag{10}$$

Step 4. Computing the VIKOR Index: $Q_i = \left[Q_i^L,\ Q_i^U\right]; i = 1, \dots, m$ according to the *Eq. (11)*.

$$
\begin{aligned}
Q_i^l &= v\frac{\left(S_i^l - S^+\right)}{\left(S^- - S^+\right)} + \left(1-v\right)\frac{\left(R_i^l - R^+\right)}{\left(R^- - R^+\right)}, \\
& \qquad\qquad\qquad\qquad\qquad\qquad i = 1,2,\dots,m. \\
Q_i^u &= v\frac{\left(S_i^u - S^+\right)}{\left(S^- - S^+\right)} + \left(1-v\right)\frac{\left(R_i^u - R^+\right)}{\left(R^- - R^+\right)},
\end{aligned}
\tag{11}
$$

where, $S^+ = \min_i S_i^l, \quad S^- = \max_i S_i^u, \quad R^+ = \min_i R_i^l, \quad R^- = \max_i R_i^u$.

While v is defined as the strategic weight of "the majority of criteria," which is considered to be 0.5, in this study.

Step 5. According to the VIKOR method, the best alternative is an alternative with the lowest value of S_i, R_i, and Q_i, while their value is an interval Grey number. The preference degree concept is used in this study to compare the alternatives and select the best one with the minimum value.

The preference degree between two numbers $\otimes Q_1$ and $\otimes Q_2$ can be defined as follows [28]:

$$
\begin{aligned}
P\left(\otimes Q_1 > \otimes Q_2\right) &= \frac{\max\left(0, \overline{Q}_1 - \underline{Q}_2\right) - \max\left(0, \underline{Q}_1 - \overline{Q}_2\right)}{\left(\overline{Q}_1 - \underline{Q}_1\right) + \left(\overline{Q}_2 - \underline{Q}_2\right)}, \\
P\left(\otimes Q_2 > \otimes Q_1\right) &= \frac{\max\left(0, \overline{Q}_2 - \underline{Q}_1\right) - \max\left(0, \underline{Q}_2 - \overline{Q}_1\right)}{\left(\overline{Q}_1 - \underline{Q}_1\right) + \left(\overline{Q}_2 - \underline{Q}_2\right)}.
\end{aligned}
\tag{12}
$$

Also, *Eq. (13)* also exists for the degree of preference between two grey numbers:

$$
\begin{aligned}
& P\left(\otimes Q_1 > \otimes Q_2\right) + P\left(\otimes Q_2 > \otimes Q_1\right) = 1, \\
& \text{If} \quad \otimes Q_1 = \otimes Q_2 \quad \text{Then} \quad P\left(\otimes Q_1 > \otimes Q_2\right) = P\left(\otimes Q_2 > \otimes Q_1\right) \equiv 0.5.
\end{aligned}
\tag{13}
$$

If the midpoint of Q1 is greater than the midpoint of Q2, then,

$$
P\left(\otimes Q_1 > \otimes Q_2\right) > P\left(\otimes Q_2 > \otimes Q_1\right).
$$

As mentioned before, the best alternative would be the one with the minimum value of the VIKOR index if the two following conditions are met [29]:

Condition 1: If alternatives A_1 and A_2 hold the first and the second ranks among the m alternatives, the following equation, *Eq. (14)* shall be met:

$$
Q(A_2) - Q(A_1) \geq \left[\frac{1}{m-1} \quad , \quad \frac{1}{m-1}\right].
\tag{14}
$$

Condition 2: Alternative A_1 shall be recognized as the superior rank, at least in one of the R or S groups.

In the case that the first condition is not met, all the alternatives are considered to be the best alternatives. Moreover, if the second condition is not met, both Alternatives will be selected as the best alternatives.

3.3 | Grey TOPSIS Method

Step 1. To start with the Grey TOPSIS Method, the DM should be normalized. The process of normalizing is as follows [30]:

$$\otimes f_{ij} = \left[\frac{-\otimes \overline{G}_{ij}}{\otimes G_j^{min}} + 2, \quad \frac{-\otimes \underline{G}_{ij}}{\otimes G_j^{min}} + 2 \right], \qquad \otimes G_j^{min} = \min_{1<i<m}\left\{\otimes \underline{G}_{ij}\right\}, \text{ for Cost Criterion.} \tag{15}$$

$$\otimes f_{ij} = \left[\frac{\otimes \underline{G}_{ij}}{\otimes G_j^{max}}, \quad \frac{\otimes \overline{G}_{ij}}{\otimes G_j^{max}} \right], \qquad \otimes G_j^{max} = \max_{1<i<m}\left\{\otimes \overline{G}_{ij}\right\}, \text{ for Benefit Criterion.} \tag{16}$$

$$A^{+} = \left\{f_1^{+}, f_2^{+}, \ldots, f_n^{+}\right\} = \left\{\left(\max_i f_{ij}^{u} \middle| j \in B\right) \text{ or } \left(\min_i f_{ij}^{l} \middle| j \in C\right)\right\}. \tag{17}$$

$$A^{-} = \left\{f_1^{-}, f_2^{-}, \ldots, f_n^{-}\right\} = \left\{\left(\min_i f_{ij}^{l} \middle| j \in B\right) \text{ or } \left(\max_i f_{ij}^{u} \middle| j \in C\right)\right\}. \tag{18}$$

Step 2. The next step is determining the NIS and the PIS based on the following equations.

Where B is the set of benefit criteria, while C is the set of cost criteria.

Step 3. Calculate the Euclidean distance from the PIS as D_i^{+}, and the Euclidean distance from the NIS as D_i^{-}.

$$D_i^{+} = \sqrt{\frac{1}{2}\left(\sum_{j=1}^{m} W_j\left[\left(f_j^{+} - f_{ij}^{l}\right)^2 + \left(f_j^{+} - f_{ij}^{u}\right)^2\right]\right)}. \tag{19}$$

$$D_i^{-} = \sqrt{\frac{1}{2}\left(\sum_{j=1}^{m} W_j\left[\left(f_j^{-} - f_{ij}^{l}\right)^2 + \left(f_j^{-} - f_{ij}^{u}\right)^2\right]\right)}. \tag{20}$$

Step 4. Calculate the closeness grade of TOPSIS based on the D_i^{+} and D_i^{-} that have been calculated in the previous step.

$$C_i = \frac{\left(D_i^{-}\right)^2}{\left(D_i^{+}\right)^2 + \left(D_i^{-}\right)^2}. \tag{21}$$

4 | Numerical Example

In this section, a real case for multi-criteria inventory classification of items has been done from the General Mechanic and Standardization (GMS) Ordering Group of Sarcheshmeh Copper Complex to verify the proposed Grey Hungarian. Sarcheshmeh Copper Complex is one of the world's largest open deposit copper mines that produces copper products such as 8 mm Copper Rods, Copper Slabs, and non-copper materials such as Sulfuric Acid and Slime. In this research, four items based on the opinion of an expert from the GMS Ordering Group have been introduced to be ranked by the two criteria: "Scarcity" and "Availability of the Technical Specifications". According to the two qualitative criteria that the mentioned expert has considered, the information for these four items has been converted to quantitative grey numbers based on [31] and summarized in *Table 2*.

Ariafar et al.| Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

Table 2. Initial information and grey DM.

	Name of Items	Scarcity (C1)		Availability of the Technical Specifications (C2)	
A_1	LINK, CRAWLER 36" for DRILL MODEL: BE 45 _R	High	[6,8]	High	[6,8]
A_2	C-CLAMPS PIN	Medium	[4,6]	Medium	[4,6]
A_3	CORE-VALVE TIRE WHEEL GP. for SLAG POT 621B	Low	[2,4]	Low	[2,4]
A_4	SCREW BRASS -FLAT HEAD 3/8"X15/16" UNC TPI=16	Very Low	[1,2]	Very Low	[1,2]

Since the policy of GMS Ordering Group is to order items for Sarcheshmeh Copper Complex, then based on the mentioned criteria, the supplying of an item will be highly ranked from the point of view of "Scarcity" the item has been ranked high, so "Scarcity" is a Benefit criterion. On the other hand, based on another criterion, which is "Availability of Technical Specification," whether the specification of an item is unavailable, then the supplying process of the item because of the unavailability of the specification of the product is difficult, so the "Availability of Technical Specifications" will be taken into account as a cost criterion. Moreover, the weights of these two criteria have been evaluated by linguistic variables according to the expert's opinion as "Extremely Important," which have been converted to a grey number as [0.8, 1], based on previous research that has been done by [32]. Hence, the ranking of inventory items based on these two criteria will be summarized in *Table 3*. In the following, based on the acquired data from the GMS Ordering Group of Sarcheshmeh Copper Complex, two solution methods, the GHA and the Grey VIKOR method, have been developed for the real case for multi-criteria inventory classification of items of the GMS group.

As can be seen from *Table 2*, the available data in this table are cardinal. Hence, for using the data by the LAM, the data has been converted into ordinal data based on two previously mentioned criteria, "Scarcity," a Benefit criterion, and "availability of the technical specifications," a cost criterion.

Table 3. Ranking of alternatives according to each criterion.

	Scarcity (C1) (Benefit Criteria)	Availability of the Technical Specifications (C2) (Cost Criteria)
A1	1^{st} Rank	4^{th} Rank
A2	2^{nd} Rank	3^{rd} Rank
A3	3^{rd} Rank	2^{nd} Rank
A4	4^{th} Rank	1^{st} Rank

4.1 | Results

After identifying the real case in the following, the case will be solved by GHA, Grey VIKOR, and Grey TOPSIS to show the validity of the proposed GHA.

4.1.1 | Grey Hungarian Algorithm

In this section, based on *Table 3*, *Table 4* has been constructed by considering each criterion's weight. As seen from *Table 3*, A1 has been ranked as the first rank according to the C1 criterion, while it has been considered the fourth rank based on the C2 criterion. Hence, in *Table 4* in the A1 row, under First rank and Fourth rank, there will be amounts in the Table that for zero values are [0, 0] and for one value are [0.8, 1]. *Table 4* is the DM of the LAM, which contains the objective function coefficients of the LAM Problem.

Table 4. The benefits matrix (objective function coefficients for the assignment problem).

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[0.8,1]	[0,0]	[0,0]	[0.8,1]
A_2	[0,0]	[0.8,1]	[0.8,1]	[0,0]
A_3	[0,0]	[0.8,1]	[0.8,1]	[0,0]
A_4	[0.8,1]	[0,0]	[0,0]	[0.8,1]

After obtaining the DM, to start solving the problem by GHA, the linear assignment model should be checked to determine whether it is maximization or minimization. If the model is in the form of maximization, then all the elements of *Table 4* will be multiplied by (-1). In this way, the problem will be transformed into a minimization problem, as seen in *Table 5*.

Then, by considering the existence of entirely negative grey numbers among the elements of *Table 5*, the grey number [0.8, 1], the highest existing symmetric number among all the grey numbers, will be added to all matrix elements. The results are shown in *Table 6*. Otherwise, there will be no need for any change at this step.

Table 5. Converting the problem from maximization to minimization.

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[-1,-0.8]	[0,0]	[0,0]	[-1,-0.8]
A_2	[0,0]	[-1,-0.8]	[-1,-0.8]	[0,0]
A_3	[0,0]	[-1,-0.8]	[-1,-0.8]	[0,0]
A_4	[-1,-0.8]	[0,0]	[0,0]	[-1,-0.8]

For a maximization problem, by considering the existence of fully negative interval values among the elements of *Table 5*, the interval [0.8, 1], the highest existing symmetric value among the interval values, will be added to all elements of this matrix. The results are shown in *Table 6*.

Table 6. Adding the highest existing symmetric grey number.

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]
A_2	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_3	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_4	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]

Subtracting row minima

In this step, the minimum grey number of each row will be subtracted from the entire grey numbers of that row that will change every grey number. However, because each row contains at least one zero interval, subtracting row minima does not affect the generated matrix.

Subtracting column minima

After subtracting row minima, the minimum grey number of every column will be subtracted from other grey numbers of that column. In this problem, subtracting column minima does not affect the matrix because each column has at least one zero interval.

Covering the generated matrix with minimal coverage lines

In this step, coverage lines with the minimum number of lines are drawn, according to the GHA, which has been explained previously. The results are shown in *Table 7*.

Ariafar et al.| Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

Table 7. The drawn coverage lines.

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]
A_2	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_3	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_4	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]

The optimal solution has been obtained since all rows and columns of the matrix were drawn with at least a coverage line. However, this problem has multiple optimal solutions, as has been shown in *Table 8* and *Table 9*.

Solution 1:

- A_1 = 1^{st} *Rank,*
- A_2 = 2^{nd} *Rank,*
- A_3= 3^{rd} *Rank,*
- A_4 = 4^{th} *Rank.*

Solution 2:

- A_4 = 1^{st} *Rank,*
- A_3 = 2^{nd} *Rank.*
- A_2= 3^{rd} *Rank,*
- A_1 = 4^{th} *Rank.*

Table 8. First solution.

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]
A_2	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_3	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_4	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]

Table 9. Second solution.

	1st Rank	2nd Rank	3rd Rank	4th Rank
A_1	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]
A_2	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_3	[0.8,1]	[-0.2,0.2]	[-0.2,0.2]	[0.8,1]
A_4	[-0.2,0.2]	[0.8,1]	[0.8,1]	[-0.2,0.2]

4.1.2 | Grey VIKOR

In this section, the Grey VIKOR method is based on [27], except for the normalization of the DM that is proposed in this research.

Normalizing the elements of DM

If the criterion type is a Cost criterion, *Eq. (5)* will be applied for normalization to normalize the elements of the DM. For the benefit criterion, *Eq. (6)* will be used [21], [33]. The results can be seen in *Table 10*.

Table 10. Normalized DM elements.

	C_1	C_2
A_1	[0.75,1]	[0,0.25]
A_2	[0.5,0.75]	[0.25,0.5]
A_3	[0.25,0.5]	[0.5,0.75]
A_4	[0.125,0.25]	[0.75,0.875]

Determining the positive and NISs

In this step, the Positive and NISs are calculated based on *Eqs. (7)* and *(8)*, and are shown in *Table 11*.

Table 11. Positive and NISs.

	C_1	C_2
f_j^+	1	0
f_j^-	0.125	0.875

Calculating satisfaction and regret index

Satisfaction and Regret Index will be calculated based on *Eqs. (9)* and *(10),* and will be shown in the following Table *(Table 12)*:

Table 12. The satisfaction and regret index.

	$[S_i^L, S_i^U]$	$[R_i^L, R_i^U]$
A_1	[[0,0], [0.457,0.571]]	[[0,0], [0.229,0.286]]
A_2	[[0.457,0.571], [0.914,1.143]]	[[0.229,0.286], [0.457,0.571]]
A_3	[[0.914,1.143], [1.371,1.714]]	[[0.457,0.571], [0.686,0.857]]
A_4	[[1.371,1.714], [1.6,2]]	[[0.686,0.857], [0.8,1]]

To simplify the computation of the VIKOR index, a whitening method derived from [23] is used according to *Eq. (22)*. The results are presented in *Table 13*.

$$\otimes m = \left[a, b\right] \quad \Rightarrow \quad \otimes m = \alpha a + (1-\alpha) b. \tag{22}$$

Table 13. Whitening values for the previous Table.

	$[S_i^L, S_i^U]$	$[R_i^L, R_i^U]$
A_1	[0,0.514]	[0,0.257]
A_2	[0.514,1.029]	[0.257,0.514]
A_3	[1.029,1.543]	[0.514,0.771]
A_4	[1.543,1.8]	[0.771,0.9]

Calculating the VIKOR index

In this step, the VIKOR index, based on *Eq. (11)* will be calculated as shown in *Table 14*.

Table 14. VIKOR index.

	$[Q_i^L, Q_i^U]$
A_1	[0,0.286]
A_2	[0.286,0.571]
A_3	[0.571,0.857]
A_4	[0.857,1]

Ranking the alternatives and finding the best alternative

Considering the VIKOR algorithm, the alternative with the least value $[Q_i^L, Q_i^U]$ is the best.

Ariafar et al. | Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

$$P\left(\left[0,0.286\right] > \left[0.286,0.571\right]\right)=\frac{\max\left(0,(0.286-0.286)\right) - \max\left(0,(0-0.571)\right)}{\left(0.286-0\right) + \left(0.571-0.286\right)}=0<0.5,$$

$$P\left(\left[0.286,0.571\right] > \left[0.571,0.857\right]\right)=\frac{\max\left(0,(0.571-0.571)\right) - \max\left(0,(0.286-0.857)\right)}{\left(0.571-0.286\right) + \left(0.857-0.571\right)}=0<0.5,$$

$$P\left(\left[0.571,0.857\right] > \left[0.857,1\right]\right)=\frac{\max\left(0,(0.857-0.857)\right) - \max\left(0,(0.571-1)\right)}{\left(0.857-0.571\right) + \left(1-0.857\right)}=0<0.5.$$

So, according to the VIKOR index and Grey Preference Degree Relation, the ranking for the GMS inventory items is as follows:

- $A_1 = 1^{st}$ *Rank,*
- $A_2 = 2^{nd}$ *Rank,*
- $A_3 = 3^{rd}$ *Rank,*
- $A_4 = 4^{th}$ *Rank.*

Now, two previously mentioned conditions will be checked to find the best alternative in the Grey VIKOR algorithm.

Condition 1:

$$\begin{gathered} Q(A_2)-Q(A_1) \geq \left[\frac{1}{5-1},\frac{1}{5-1}\right], \\ \left[0.286,0.571\right] - \left[0,0.286\right] \geq \left[0.25,0.25\right]. \end{gathered} \tag{23}$$

Condition 2: Alternative A_1 in both R and S interval values ranks first among other alternatives. So, both A_1 and A_2 are the best alternatives.

4.1.3 | Grey TOPSIS

The same problem has been solved by the Grey TOPSIS method derived from [30]. The results of Grey TOPSIS are as follows:

- $A_1 = 1^{st}$ *Rank,*
- $A_2 = 2^{nd}$ *Rank,*
- $A_3 = 3^{rd}$ *Rank,*
- $A_4 = 4^{th}$ *Rank.*

4.2 | Sensitivity Analysis

In this section, it is assumed that the opinions of the GMS ordering group have been changed. For this case, the DM is as what is in *Table 15*.

Ariafar et al.| Int. J. Res. Ind. Eng. 12(3) (2023) 321-336

Table 15. Change in the information of DM.

	Name of Items	Scarcity (C1)		Availability of the Technical Specifications (C2)	
A_1	LINK, CRAWLER 36" FOR DRILL MODEL: BE 45 _R	Medium	[4,6]	Medium	[4,6]
A_2	C-CLAMPS PIN	High	[6,8]	High	[6,8]
A_3	CORE-VALVE TIRE WHEEL GP. FOR SLAG POT 621B	Very low	[1,2]	Very Low	[1,2]
A_4	SCREW BRASS -FLAT HEAD 3/8"X15/16" UNC TPI=16	Low	[2,4]	Low	[2,4]

For this case, the problem has been solved by GHA, Grey VIKR, and Grey TOPSIS. The results for this case will be changed as follows:

- $A_1 = 1^{st}$ *Rank,*
- $A_2 = 2^{nd}$ *Rank,*
- $A_3 = 3^{rd}$ *Rank,*
- $A_4 = 4^{th}$ *Rank.*

As can be seen from the results, all the priorities for the parts have been changed according to the change in the DM, but all the methods have been prioritized in the same order.

4.3 | Discussion

The GHA was developed in this study to solve the Grey LAPs. Then, a real case from the GMS Ordering Group of Sarcheshmeh Copper Complex was solved by GHA and two other methods: Grey VIKOR and Grey TOPSIS. Based on the obtained ranks that have been provided by GHA, Grey VIKOR, and Grey TOPSIS for the initial case, and also the case for sensitivity analysis, the Spearman's correlation coefficient rank has been calculated according to *Eq. (24)* that is derived from [33].

$$\rho = 1 - \frac{6\sum_{a=1}^{A} D_a^2}{A(A^2-1)}. \tag{24}$$

In *Eq. (24)*, A is the total number of alternatives, and D_a indicates the difference between the ranks obtained from different MADM methods for alternative a. The results for Spearman's Correlation Coefficient Rank have been summarized in *Table 16*.

Table 16. Results of Spearman's correlation coefficient rank.

	Grey TOPSIS	Grey VIKOR	Grey Hungarian 1st Solution	Grey Hungarian 2nd Solution
Grey TOPSIS	-----	1.000**	1.000**	-1.000**
Grey VIKOR	1.000**	----	1.000**	-1.000**
Grey Hungarian 1st Solution	1.000**	1.000**	-----	-1.000**
Grey Hungarian 2nd Solution	-1.000**	-1.000**	-1.000**	----

The results show the validity of the two solution methods proposed in this study, the GHA and the Grey VIKOR method. Compared to other approaches, the advantage of using the GHA is that it finds multiple optimal solutions where more than one optimal solution is available.

5 | Conclusion

In this study, because of the necessity of considering uncertainty in making decisions, the GHA was developed based on the grey numbers operation rules and the concept of the grey preference degree. To show the validity of the proposed GHA approach, a real case from the Sarcheshmeh Copper

Complex warehouse has been solved and compared to the results of Grey VIKOR and grey TOPSIS. The identical results for all the solution approaches in the initial case and after the sensitivity analysis imply the validity of the proposed solution method. When there is poor information, and the data distribution or membership function is not available, using the GHA and other grey approaches is reasonable. If there is a lack of experience or a small size of data sampling, the grey methods application is not recommended. As a suggestion for future research, it is recommended to modify the method to overcome this limitation. The use of uncertainty concepts, which, in addition to grey numbers, leads to a combination of grey numbers and fuzzy sets, is recommended.

Acknowledgments

The Sarcheshmeh Copper Complex partially supported this study. Hence, the authors express sincere thanks for their partial support. Also, the authors intend to thank the anonymous reviewers for their valuable comments that improved this paper.

References

[1] Kuhn, H. W. (1955). The Hungarian method for the assignment problem. *Naval research logistics quarterly*, *2*((1-2)), 83–97. DOI:10.1007/978-3-540-68279-0_2

[2] Das, S. K., & Deo, N. (1990). Parallel hungarian algorithm. *Computer systems science and engineering*, *5*(3), 131-136. https://stars.library.ucf.edu/scopus1990/1508/

[3] Ishibuchi, H., & Tanaka, H. (1990). Multiobjective programming in optimization of the interval objective function. *European journal of operational research*, *48*(2), 219–225. DOI:10.1016/0377-2217(90)90375-L

[4] Aldous, D. (1992). Asymptotics in the random assignment problem. *Probability theory and related fields*, *93*(4), 507–534. DOI:10.1007/BF01192719

[5] Li, T., Li, Y., & Qian, Y. (2016). Improved Hungarian algorithm for assignment problems of serial-parallel systems. *Journal of systems engineering and electronics*, *27*(4), 858–870.

[6] Rajabi-Alni, F., & Bagheri, A. (2022). Computing a many-to-many matching with demands and capacities between two sets using the Hungarian algorithm. *Journal of mathematics*, *2023*. https://doi.org/10.1155/2023/7761902

[7] Bai, G. Z. (2009). Grey assignment problems. In *Fuzzy information and engineering* (pp. 245-250). Springer Berlin Heidelberg. https://link.springer.com/chapter/10.1007/978-3-540-88914-4_31

[8] Majumdar, S. (2013). Interval linear assignment problems. *Journal of applied mathematics*, *1*(6), 14–16.

[9] Serratosa, F. (2015). Computation of graph edit distance: Reasoning about optimality and speed-up. *Image and vision computing*, *40*, 38–48.

[10] Serratosa, F., & Cortés, X. (2015). Graph edit distance: Moving from global to local structure to solve the graph-matching problem. *Pattern recognition letters*, *65*, 204–210.

[11] Lan, S., Fan, W., Liu, T., & Yang, S. (2019). A hybrid SCA--VNS meta-heuristic based on Iterated Hungarian algorithm for physicians and medical staff scheduling problem in outpatient department of large hospitals with multiple branches. *Applied soft computing*, *85*, 105813. https://www.sciencedirect.com/science/article/abs/pii/S1568494619305940

[12] Yadav, S. S., Lopes, P. A. C., Ilic, A., & Patra, S. K. (2019). Hungarian algorithm for subcarrier assignment problem using GPU and CUDA. *International journal of communication systems*, *32*(4), e3884. https://doi.org/10.1002/dac.3884

[13] Khan, A. A., Adve, R. S., & Yu, W. (2020). Optimizing downlink resource allocation in multiuser MIMO networks via fractional programming and the hungarian algorithm. *IEEE transactions on wireless communications*, *19*(8), 5162–5175.

[14] Yang, X., Zhao, N., & Yu, S. (2020). Combined internal trucks allocation of multiple container terminals with hungarian algorithm. *Journal of coastal research*, *103*(SI), 923–927.

[15] Kumarnath, J., & Batri, K. (2021). An optimized traffic grooming through modified pso based iterative hungarian algorithm in optical networks. *Information technology and control*, *50*(3), 546–557.

[16] MacLean, M. T., Lysikowski, J. R., Rege, R. V, Sendelbach, D. M., & Mihalic, A. P. (2021). Optimizing medical student clerkship schedules using a novel application of the Hungarian algorithm. *Academic medicine*, *96*(6), 864–868.

[17] Stevens, P., & Sciacchitano, A. (2021). Application of clustering and the Hungarian algorithm to the problem of consistent vortex tracking in incompressible flowfields. *Experiments in fluids*, *62*, 1–11.

[18] Zhu, Z., Lou, K., Ge, H., Xu, Q., & Wu, X. (2022). Infrared target detection based on Gaussian model and Hungarian algorithm. *Enterprise information systems*, *16*(10–11), 1573–1586.

[19] Zhang, S., Xue, Y., Zhang, H., Zhou, X., Li, K., & Liu, R. (2023). Improved Hungarian algorithm--based task scheduling optimization strategy for remote sensing big data processing. *Geo-spatial information science*, 1–14. https://doi.org/10.1080/10095020.2023.2178339

[20] Xie, N., & Liu, S. (2010). Novel methods on comparing grey numbers. *Applied mathematical modelling*, *34*(2), 415–423.

[21] Li, G. D., Yamaguchi, D., & Nagai, M. (2007). A grey-based decision-making approach to the supplier selection problem. *Mathematical and computer modelling*, *46*(3–4), 573–581.

[22] Tseng, M. L. (2009). A causal and effect decision making model of service quality expectation using grey-fuzzy DEMATEL approach. *Expert systems with applications*, *36*(4), 7738–7748.

[23] Liu, S., & Lin, Y. (2006). Grey clusters and grey statistical evaluations. In *Grey information: theory and practical applications* (pp. 139–189). Springer. https://link.springer.com/chapter/10.1007/1-84628-342-6_6

[24] Sadeghieh, A., Dehghanbaghi, M., Dabbaghi, A., & Barak, S. (2012). A genetic algorithm based grey goal programming (G3) approach for parts supplier evaluation and selection. *International journal of production research*, *50*(16), 4612–4630.

[25] Moore, R. E. (1979). *Methods and applications of interval analysis*. SIAM. https://epubs.siam.org/doi/pdf/10.1137/1.9781611970906.bm

[26] Winston, W. L. (2004). *Operations research: applications and algorithm*. Thomson Learning, Inc. https://www.academia.edu/download/58159784/Winston_4th_ed.pdf

[27] Sayadi, M. K., Heydari, M., & Shahanaghi, K. (2009). Extension of VIKOR method for decision making problem with interval numbers. *Applied mathematical modelling*, *33*(5), 2257–2262.

[28] Sevastianov, P. (2007). Numerical methods for interval and fuzzy number comparison based on the probabilistic approach and Dempster--Shafer theory. *Information sciences*, *177*(21), 4645–4661.

[29] Parkouhi, S. V., & Ghadikolaei, A. S. (2017). A resilience approach for supplier selection: Using Fuzzy Analytic Network Process and grey VIKOR techniques. *Journal of cleaner production*, *161*, 431–451.

[30] Lin, Y. H., Lee, P. C., & Ting, H.-I. (2008). Dynamic multi-attribute decision making model with grey number evaluations. *Expert systems with applications*, *35*(4), 1638–1644.

[31] Nguyen, H. T., Dawal, S. Z. M., Nukman, Y., & Aoyama, H. (2014). A hybrid approach for fuzzy multi-attribute decision making in machine tool selection with consideration of the interactions of attributes. *Expert systems with applications*, *41*(6), 3078–3090.

[32] Esangbedo, M. O., & Che, A. (2016). Grey weighted sum model for evaluating business environment in West Africa. *Mathematical problems in engineering*, *2016*. https://www.hindawi.com/journals/mpe/2016/3824350/abs/

[33] Baykasouglu, A., Subulan, K., & Karaslan, F. S. (2016). A new fuzzy linear assignment method for multi-attribute decision making with an application to spare parts inventory classification. *Applied soft computing*, *42*, 1–17.