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Some Results on $(c)-$Mapping

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Abstract

Compared with [1], in this paper, we will give first some sufficient conditions under which a $(c)-$mapping possesses an Approximate Fixed Point Sequence (AFPS). And then, we will prove that $(c)-$mapping has a fixed point. Finally, we will check some special properties of the fixed point sets of these mappings, such as closedness, convexity.

Keywords: $(c)-$mapping, Fixed point, Closedness, Convexity.

1 | Introduction

It is well known that various nonlinear generalizations of the contraction mapping are of great significance in the literature. Nonexpansive mappings, asymptotically nonexpansive mappings are some examples of such generalizations. We know that every nonexpansive mapping or asymptotically nonexpansive mapping on a non-empty closed, bounded, convex subset of a uniformly convex Banach space has at least one fixed point, see [2], [3] and [4]. Subsequently, many authors have introduced several kinds of nonlinear mappings generalizing the class of nonexpansive mappings such as asymptotically pseudocontractive mappings, uniformly asymptotically regular mappings, uniformly asymptotically regular mappings with sequence, uniformly $L-$Lipschitzian mappings, etc.

In proving the existence of fixed point of the above mentioned mappings, many authors took the help of Approximate Fixed Point Sequence (AFPS). Here, we mention a number of iterative AFPS.

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I. The Mann iteration [5]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)x_n + \lambda_n T x_n,$$

where (λ_n) is a sequence of real numbers satisfying $0 \leq \lambda_n < 1$ for all $n \in N$.

II. The Krasnoselskij iteration [6]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = \frac{1}{2}(x_n + Tx_n).$$

III. The Halpern iteration [7]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)Tx_n + \lambda_n u,$$

where (λ_n) is a sequence in $[0,1]$ and $u \in X$.

IV. The modified Mann iteration [8]: sequence $\{x_n\}$ is defined by

$$x_{n+1} = (1-\lambda_n)x_n + \lambda_n T^n x_n,$$

where (λ_n) is a sequence in $[0,1]$.

In this paper, we will extend these results to $(c)-$ mapping and we will first give some sufficient conditions under which a $(c)-$ mapping possesses an AFPS.

2 | Preliminaries

In this section, we collect some necessary definitions, which will be used in next section.

Definition 1. [1], [9]. Let X be a normed linear space, C a non-empty subset of X and $T: C \to C$ be a mapping. The mapping T is said to be a Reich type nonexpansive mapping if there exists non-negative real numbers a, b, c with $a+b+c=1$, such that the condition.

$$\|Tx - Ty\| \leq a\|x-y\| + b\|x-Tx\| + c\|y-Ty\|. \quad (1)$$

Holds for all $x, y \in C$. The mapping T is said to be a Chatterjea type nonexpansive mapping if there exists non-negative real numbers a, b, c with $a+b+c=1$, such that the condition

$$\|Tx - Ty\| \leq a\|x-y\| + b\|x-Ty\| + c\|y-Tx\|. \quad (2)$$

Holds for all $x, y \in C$. In both cases, we say that T is a Reich type nonexpansive (Chatterjea type nonexpansive) mapping with coefficients (a, b, c).

Definition 2. [10], [11]. Let (C,d) be a metric space. A self-mapping $T:C\to C$ is said to be a generalized nonexpansive mapping if there exist a,b and $c\in[0,1]$, such that $a+2b+2c\le 1$ and:

$$\begin{aligned} d(Tx_1,Tx_2) &\le ad(x_1,x_2)+b(d(x_1,Tx_1)+d(x_2,Tx_2)) \\ &+c(d(x_1,Tx_2)+d(x_2,Tx_1)), \end{aligned} \quad (3)$$

For all $x_1,x_2\in C$.

The most important case is when $a+2b+2c=1$, which covers in particular the following situations:

If $b=0$ and $c>0$, then T is called $(c)-$mapping [12] and [13].

Definition 3. [14], [15]. A self-mapping T of a metric space (X,d) is said to be asymptotically regular if $\lim_{n\to\infty} d(T^n x,T^{n+1}x)=0$ for all $x\in X$.

Definition 4. [1]. A sequence (x_n) in a normed linear space is said to be an AFPS if

$$\lim_{n\to\infty}\|x_n-Tx_n\|=0.$$

Proposition 1. [16]. Let (z_n) and (w_n) be two bounded sequences in a Banach space X and let $\lambda\in(0,1)$. Let $z_{n+1}=\lambda w_n+(1-\lambda)z_n$ and suppose $\|w_{n+1}-w_n\|\le\|z_{n+1}-z_n\|$ for all $n\in N$. Then $\lim_{n\to\infty}\|w_n-z_n\|=0$.

3 | Main Results

In this section, we compare Reich type nonexpansive, Chatterjea type nonexpansive mappings with $(c)-$mapping, therefore, we extend some results in [1] to $(c)-$mapping. We will give first some sufficient conditions under which a $(c)-$mapping possesses AFPS.

Theorem 1. Let X be a Banach space and C be a non-empty closed, convex, bounded subset of X. Let $T:C\to C$ be a $(c)-$mapping with coefficients (a, b, c), such that $a+2c=1, 0<c<1$. Also assume that for $x,y\in C$

$$\frac{1-c}{5}\|x-Ty\|\le\|x-y\|\Rightarrow\|Tx-Ty\|\le\|x-y\|.$$

Then T has an AFPS in C. Moreover, the AFPS is asymptotically regular.

Proof. Since T is a $(c)-$mapping with $a+2c=1, 0<c<1$ and

$$\|Tx-Ty\|\le a\|x-y\|+c\|x-Ty\|+c\|y-Tx\|. \quad (4)$$

For all $x,y\in C$. Let $x_0\in C$ be arbitrary but fixed. We consider the sequence (x_n) in X defined by

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$x_{n+1} = \lambda T x_n + (1-\lambda) x_n$ for all $n \geq 2$ where $\frac{1}{2} \leq \lambda < 1$. Since C is convex and bounded, it follows that (x_n) is a bounded sequence in C. Now putting $x = x_n, y = x_{n+1}$ in *Eq. (4)*, we get

$$\begin{aligned}\|Tx_n - Tx_{n+1}\| &\leq a\|x_n - x_{n+1}\| + c\|x_n - Tx_{n+1}\| + c\|x_{n+1} - Tx_n\| \\ &\leq a\|x_n - x_{n+1}\| + c\|x_n - x_{n+1}\| + c\|x_{n+1} - Tx_{n+1}\| \\ &\quad + c\|x_{n+1} - x_n\| + c\|x_n - Tx_n\| \\ &= \|x_n - x_{n+1}\| + c\|x_{n+1} - Tx_{n+1}\| + c\|x_n - Tx_n\|. \end{aligned} \tag{5}$$

Since $x_{n+1} = \lambda T x_n + (1-\lambda) x_n$ for all $n \geq 2$, we have $\|x_n - Tx_n\| = \frac{1}{\lambda}\|x_n - x_{n+1}\|$ and $\|x_{n+1} - Tx_{n+1}\| = \frac{1}{\lambda}\|x_{n+1} - x_{n+2}\|$. Using these in *Eq. (5)*, we get

$$\lambda\|Tx_n - Tx_{n+1}\| \leq \lambda\|x_n - x_{n+1}\| + c\|x_{n+1} - x_{n+2}\| + c\|x_n - x_{n+1}\|. \tag{6}$$

Now

$$\begin{aligned} x_{n+1} - x_{n+2} &= \lambda T x_n + (1-\lambda) x_n - \left[\lambda T x_{n+1} + (1-\lambda) x_{n+1}\right] \\ &= \lambda(Tx_n - Tx_{n+1}) + (1-\lambda)(x_n - x_{n+1}). \end{aligned}$$

$$\begin{aligned} \|x_{n+1} - x_{n+2}\| &\leq \lambda\|Tx_n - Tx_{n+1}\| + (1-\lambda)\|x_n - x_{n+1}\| \\ &\leq \lambda\|x_n - x_{n+1}\| + c\|x_{n+1} - x_{n+2}\| + c\|x_n - x_{n+1}\| \\ &\quad + (1-\lambda)\|x_n - x_{n+1}\|. \end{aligned}$$

$$\Rightarrow \|x_{n+1} - x_{n+2}\| \leq \frac{1+c}{1-c}\|x_n - x_{n+1}\|. \tag{7}$$

Again, we have

$$\begin{aligned} &\lambda(Tx_n - Tx_{n+1}) = (x_{n+1} - x_{n+2}) + (\lambda - 1)(x_n - x_{n+1}) \\ &\Rightarrow \lambda\|Tx_n - Tx_{n+1}\| = \|x_{n+1} - x_{n+2}\| + (1-\lambda)\|x_n - x_{n+1}\|. \end{aligned} \tag{8}$$

Therefore,

$$\begin{aligned} \|x_n - Tx_{n+1}\| &\leq \|x_n - Tx_n\| + \|Tx_n - Tx_{n+1}\| \\ &= \frac{1}{\lambda}\|x_n - x_{n+1}\| + \|Tx_n - Tx_{n+1}\|, \end{aligned}$$

$$\begin{aligned} \lambda\|x_n - Tx_{n+1}\| &\leq \|x_n - x_{n+1}\| + \lambda\|Tx_n - Tx_{n+1}\| \\ &\leq \|x_n - x_{n+1}\| + \|x_{n+1} - x_{n+2}\| + (1-\lambda)\|x_n - x_{n+1}\| \\ &\leq (2-\lambda)\|x_n - x_{n+1}\| + \frac{1+c}{1-c}\|x_n - x_{n+1}\| \\ &< \left(\frac{3}{2} + \frac{1+c}{1-c}\right)\|x_n - x_{n+1}\|. \end{aligned}$$

Thus

$$\frac{1}{2}\|x_n - Tx_{n+1}\| \le \lambda\|x_n - Tx_{n+1}\|$$
$$\le \left(\frac{3}{2} + \frac{1+c}{1-c}\right)\|x_n - x_{n+1}\|.$$

Hence,

$$\frac{1-c}{5}\|x_n - Tx_{n+1}\| < \|x_n - x_{n+1}\|.$$

Therefore, by given hypothesis, we get

$$\|Tx_n - Tx_{n+1}\| \le \|x_n - x_{n+1}\|.$$

Thus, by Proposition 1, we have $\|x_n - Tx_n\| \to 0$ as $n \to \infty$. So (x_n) is an AFPS of T. Further, we have $n \to \infty$,

$$\|x_n - x_{n+1}\| = \lambda\|x_n - Tx_n\| \to 0.$$

Therefore, the AFPS (x_n) is asymptotically regular also.

Next, we prove a result concerning the existence of fixed points of such mappings using *Theorem 1*.

Theorem 2. Suppose that all the conditions of *Theorem 1* are satisfied. Further, assume that for any $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\|x - y\| + \|x - Ty\| + \|y - Tx\| < 3\varepsilon + \delta \Rightarrow \|Tx - Ty\| \le \frac{\varepsilon}{2}. \quad (9)$$

Then T has a fixed point in C.

Proof. By *Theorem 1*, T has an AFPS (x_n), where $x_{n+1} = \lambda Tx_n + (1-\lambda)x_n$ and $\frac{1}{2} \le \lambda < 1$. Here we take $\lambda = \frac{1}{2}$. So we get an AFPS (x_n) given by $x_{n+1} = \frac{1}{2}(Tx_n + x_n)$, and this sequence is asymptotically regular also. Next, we show that (x_n) is a Cauchy sequence. Let $\varepsilon > 0$ be arbitrary. So there exists $\delta > 0$ such that *Eq. (9)* holds. Without loss of generality, we take $\delta < \varepsilon$. Since, (x_n) is asymptotically regular, there exists $N \in N$ such that

$$\|x_n - x_{n+1}\| < \frac{\delta}{4}.$$

For all $n \ge N$. Next, we show by induction on p that

$$\|x_N - x_{N+p}\| < \varepsilon \text{ for all } p \in N. \quad (10)$$

Clearly *Eq. (10)* is true for $p = 1$. Let *Eq. (10)* be true for some $p \in N$.

Therefore,

$$\begin{aligned}&\|x_N - x_{N+p}\| + \|x_N - Tx_{N+p}\| + \|x_{N+p} - Tx_N\| \\ &\le \|x_N - x_{N+p}\| + \|x_N - x_{N+p}\| + \|x_{N+p} - Tx_{N+p}\| + \|x_{N+p} - x_N\| + \|x_N - Tx_N\| \\ &= 3\|x_N - x_{N+p}\| + 2\|x_{N+p} - x_{N+p+1}\| + 2\|x_N - x_{N+1}\| \\ &< 3\varepsilon + \delta.\end{aligned}$$

$$\|Tx_N - Tx_{N+p}\| \le \frac{\varepsilon}{2}.$$

From *Eq. (9)*, we get

Again by the formation of (x_n) , we get

$$\|x_{N+p+1} - x_{N+1}\| \le \frac{1}{2}\|Tx_{N+p} - Tx_N\| + \frac{1}{2}\|x_{N+p} - x_N\| < \frac{3\varepsilon}{4}.$$

Thus

$$\|x_N - x_{N+p+1}\| \le \|x_N - x_{N+1}\| + \|x_{N+1} - x_{N+p+1}\| < \frac{\delta}{4} + \frac{3\varepsilon}{4} < \varepsilon.$$

Therefore, *Eq. (10)* is true for $p+1$. So *Eq. (10)* is true for all p. Continuing in a similar manner, we can show that

$$\|x_n - x_{n+p}\| < \varepsilon. \text{ for all } n \in N \text{ and for all } p \in N.$$

Therefore, (x_n) is a Cauchy sequence and hence convergent to some $z \in C$.

Again,

$$\begin{aligned}\|Tx_n - Tx_m\| &\le a\|x_n - x_m\| + c\|x_n - Tx_m\| + c\|x_m - Tx_n\| \\ &\le a\|x_n - x_m\| + c\|x_n - x_m\| + c\|x_m - Tx_m\| \\ &\quad + c\|x_m - x_n\| + c\|x_n - Tx_n\| \\ &= \|x_n - x_m\| + c\|x_m - Tx_m\| + c\|x_n - Tx_n\| \to 0 \text{ as } n, m \to \infty.\end{aligned}$$

Therefore, (Tx_n) is a Cauchy sequence in C. Also, since $x_{n+1} = \frac{1}{2}(Tx_n + x_n)$, we have that $Tx_n = 2x_{n+1} - x_n \to z \text{ as } n \to \infty.$ Again,

$$\begin{aligned}\|z - Tz\| &\le \|z - x_n\| + \|x_n - Tx_n\| + \|Tx_n - Tz\| \\ &\le \|z - x_n\| + \|x_n - Tx_n\| \\ &\quad + \left(a\|x_n - z\| + c\|x_n - Tz\| + c\|z - Tx_n\|\right).\end{aligned}$$

Letting $n \to \infty$ in above inequality, we get

$$\|z - Tz\| \le c\|z - Tz\|.$$

Since $1 > c > 0$, which gives $z = Tz$, i.e., z is a fixed point of T.

The following theorem characterizes the fixed point set of $(c)-$ mapping.

Theorem 3. Let X be a Banach space and C be a non-empty subset of X. Let $T: C \to C$ be a $(c)-$ mapping with coefficients (a, b, c), such that $a + 2c = 1, 0 < c < 1$, then $Fix(T)$ is a closed subset of C.

Proof. Since T is $(c)-$ mapping with coefficients (a, b, c), we have $a + 2c = 1, 0 < c < 1$ and

$$\|Tx - Ty\| \le a\|x - y\| + c\|x - Ty\| + c\|y - Tx\|.$$

For all $x, y \in C$. Let (z_n) be a sequence in $Fix(T)$ converging to some $z \in C$.

Then, we have

$$\|Tz_n - Tz\| \le a\|z_n - z\| + c\|z_n - Tz\| + c\|z - Tz_n\|$$
$$\Rightarrow \|z_n - Tz\| \le a\|z_n - z\| + c\|z_n - Tz\| + c\|z - z_n\|.$$

Taking limit as $n \to \infty$ in above inequality, we get $\|z - Tz\| \le c\|z - Tz\|$. Since $0 < c < 1$, so $z = Tz$, i.e., $z \in Fix(T)$ and hence $Fix(T)$ is a closed set.

In the next theorem, we give another characterization of the fixed point set of $(c)-$ mapping by taking the underlying space as a Hilbert space in place of Banach space.

Theorem 4. Let X be a Hilbert space and C be a non-empty subset of X. Let $T: C \to C$ be a (c)-mapping with coefficients (a, b, c), such that $a + 2c = 1, 0 < c < 1$, then $Fix(T)$ is a convex subset of C

Proof. Let $x, y \in Fix(T)$ be any two points and take $z = \lambda x + (1 - \lambda) y$, where λ is a scalar with $0 \le \lambda \le 1$. Then we have

$$\|Tz - Tx\| \le a\|z - x\| + c\|z - Tx\| + c\|x - Tz\|$$
$$\Rightarrow \|Tz - x\| \le a\|z - x\| + c\|z - x\| + c\|x - Tz\|$$

$$\Rightarrow (1 - c)\|Tz - x\| \le (a + c)\|z - x\|$$

$$\Rightarrow \|Tz - x\| \le \|z - x\|.$$

Next, using parallelogram law we have

$$\left\|\frac{z - x}{2} + \frac{Tz - x}{2}\right\|^2 + \left\|\frac{z - x}{2} - \frac{Tz - x}{2}\right\|^2 = 2\left(\left\|\frac{z - x}{2}\right\|^2 + \left\|\frac{Tz - x}{2}\right\|^2\right)$$

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$$\left\|\frac{z-x}{2}+\frac{Tz-x}{2}\right\|^2+\frac{1}{4}\|z-Tz\|^2=\frac{1}{2}\|z-x\|^2+\frac{1}{2}\|Tz-x\|^2 \\ \leq\frac{1}{2}\|z-x\|^2+\frac{1}{2}\|z-x\|^2$$

$$\Rightarrow\left\|\frac{z-x}{2}+\frac{Tz-x}{2}\right\|^2\leq\|z-x\|^2-\frac{1}{4}\|z-Tz\|^2$$

$$\Rightarrow\left\|\frac{z+Tz}{2}-x\right\|^2\leq(1-\lambda)^2\|x-y\|^2-\frac{1}{4}\|z-Tz\|^2.$$

$$\left\|\frac{z+Tz}{2}-y\right\|^2\leq\lambda^2\|x-y\|^2-\frac{1}{4}\|z-Tz\|^2.$$

Similarly, we have

Now if $z\neq Tz$, then we have

$$\left\|\frac{z+Tz}{2}-x\right\|<(1-t)\|x-y\|, \text{ and}$$

$$\left\|\frac{z+Tz}{2}-y\right\|<t\|x-y\|.$$

Then, we get

$$\|x-y\|\leq\left\|\frac{z+Tz}{2}-x\right\|+\left\|\frac{z+Tz}{2}-y\right\| \\ <(1-t)\|x-y\|+t\|x-y\| \\ =\|x-y\|,$$

which gives a contradiction. So we must have $z=Tz$, i.e., $z\in Fix(T)$. Therefore, $Fix(T)$ is a convex set.

4 | Applications

In this section, we will give a concrete example to illustrate the theorem 2 and show the rationality of the obtained theorems.

Example 1. Let us consider the Banach space R equipped with the ususal norm, take $C=\left[0,\frac{3}{2}\right]$ and define a mapping $T:C\to C$ by

$$Tx=\begin{cases}\frac{3}{2} & \text{if } x<\frac{1}{3};\\ \frac{5}{4} & \text{if } x\geq\frac{1}{3}.\end{cases}$$

Choose $a = c = \frac{1}{3}$. Then for any $x, y \in C$, if $x, y < \frac{1}{3}$ or if $x, y \geq \frac{1}{3}$, it is obvious to check that

$$\|Tx - Ty\| \leq a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|).$$

Next, suppose that $x < \frac{1}{3}$ and $y \geq \frac{1}{3}$. Then $\|Tx - Ty\| = \frac{1}{4}$ and

$$a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|) = \frac{1}{3}\left(y - x + \frac{5}{4} - x + \frac{3}{2} - y\right) \geq \frac{1}{4}.$$

So

$$\|Tx - Ty\| \leq a\|x - y\| + c(\|x - Ty\| + \|y - Tx\|).$$

Therefore, T is a $(c)-$mapping with coefficients $a = c = \frac{1}{3}$. Also, T has an AFPS $\left(\frac{5}{4} + \frac{1}{n+2}\right)_{n \in N}$ and $Fix(T) = \left\{\frac{5}{4}\right\}$, which is obviously closed and convex.

Remark 1. *Example 1* shows that the class of $(c)-$mapping is larger than that of nonexpansive mapping.

5 | Conclusion

In this article, we notice the differences between Reich type nonexpansive, Chatterjea type nonexpansive mappings and $(c)-$mapping in reference [1] and [9]. We first proved a new theorem that some sufficient conditions under which a $(c)-$mapping possesses an AFPS. And then, we also proved that $(c)-$mapping has a fixed point. Finally, we checked some special properties of the fixed point sets of these mappings, such as closedness, convexity. In the end, we gave a concrete real example to show the theorem 2. This shows that our work is meaningful. Through this study, we advanced the work of researching fixed point theory in $(c)-$mapping.

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Conflicts of Interest

The authors declare no conflict of interest.

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