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Modeling of the Supply Chain of Cooperative Game between Two Tiers of Retailer and Manufacturer under Conditions of Uncertainty

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Abstract

The design of a repurchase agreement related to the amount of goods remaining in the two-echelon supply chain between the retailer and the manufacturer is examined. Two scenarios are considered quite separately; In the first scenario (decentralized) in which the retailer determines the price of the product and the optimal amount of the economic order and the producer is persuaded to follow this method. In the second (centralized) scenario, the goal is to maximize the profit of the whole chain, in which case the price of the product and the amount of the economic order are determined based on the profit of the whole chain. Then, a model of repurchase agreement related to the remaining goods was considered based on cooperative play and contract between two members of the supply chain, in which the goal is to maximize the profit of chain members. Due to the uncertainty of the competitive environment, the demand is considering under uncertain in modeling to determine the optimal level of cooperation in a competitive and cooperative market. The results of the implementation of this contract in a numerical example showed that the profit of the whole chain and the amount of economically optimal order in the centralized state increased compared to the decentralized state and the optimal price of the product decreased. Due to the fact that in the decentralized state the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized state and the producer's profit increases.

Keywords: Leader-follower game, Cooperative game, Stackelberg game .



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1 | Introduction

A supply chain consists of independent companies that include all the processes of preparation and conversion of raw materials into final products, their maintenance, distribution, and transfer to end customers as a chain. Today, competition along or across chains is of particular importance among researchers. The supply of products and services by changing the pattern of customer competition has led to a change in the type of market competition from the state of competition between



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independent companies to competition between supply chains. Supply chain design has an extensive literature, most of which is related to the single (exclusive) supply chain, and the existence of competing supply chains and their emergence in the future have not been considered. However, supply chains compete with each other for greater market share. Therefore, supply chains must be prepared for future competitiveness even if there is no competitor at the same time.

In the last two decades, members of scientific societies and business owners have paid more attention to supply chain management. The globalization of trade, increasing competition, and narrowing the gap between products in terms of quality and performance have led all scientists and craftsmen to re-examine how to make business activities more efficient and effective. In fact, they tend to use every means possible to increase their final profit and increase their market share. In particular, in order to achieve the above goals, they seek to establish coordination between members of the supply chain [1]. In supply chain management, the major issue is the management of separate but dependent members of a chain. To achieve an efficient supply chain, chain members must behave as a whole and coherently. But in reality, we often face decentralized supply chains in which each member makes decisions in order to optimize their goals in conflict with the goals of other members [2]. One of the most important issues in the supply chain is coordination and efforts to integrate the operations of the independent units to achieve the maximum possible benefit from the entire chain. Consequently, a key point in supply chain management is the development of mechanisms that align the goals of the independent components and create interaction between their activities and decisions to optimize the performance of the entire system [3].

Today, business units prefer to work with supply chain components to gain a competitive advantage, improve performance, and reduce costs. The supply chain can be divided into three main components, including the upstream part which is responsible for supplying raw materials, the middle part which receives the raw materials from the upstream part and produces the goods, and delivers them to the lower part of the supply chain, i.e., retailers to sell the product to the end customer [4]. There are conflicting goals and activities between parts of the supply chain that may increase the cost of each component of the supply chain. The goal of supply chain management is to coordinate, integrate, and establish mechanisms to build the dependent goals of the independent components of the supply chain [5].

The present study set to examine a two-echelon supply chain network between retailers and manufacturers that both members of the supply chain seek to maximize profits from the sale of their products. Therefore, two completely separate cases are considered. First, in the non-cooperative mode, a Stackelberg game takes place between two members of the supply chain, in which the retailer determines the price of the product and the optimal amount of the economic order, and the producer is persuaded to follow this method, which is called the decentralized scenario. Second, the goal is to maximize the profit of the whole chain, in which the price of the product and the amount of the economic order are determined based on the profit of the chain, which includes the profits of the members of the firms. This is known as a centralized scenario. In the second mode, a model is assumed based on cooperative game and signing a contract between two members of the supply chain, in which the goal is to maximize the profits of chain members by cooperating with each other in pricing and determining the optimal amount of economic order, i.e. the two members of the chain will increase their profits when they enter the contract. Therefore, modeling of such a system in which demand is considered uncertain is examined. The most important part of the proposed model is related to providing a cooperative model between two levels of supply chain network in conditions of demand uncertainty.

2 | Literature Review

In the operations management literature, the relationship between the implementation of single-commodity two-echelon and three-echelon supply chains, including the manufacturer and the retailer,

has been discussed. In the traditional supply chain, decision-making power was in the hands of manufacturers, and recently, with the introduction of the concept of supply chain, this decision-making power has been transferred to retailers [5]. Several models have been introduced for manufacturers and retailers. The relationship between producer and retailer is modeled using the Stackelberg game when there is no cooperation. To describe Stackelberg game in the supply chain, it is possible that the producer and retailer are the two actors in this game. Here, decision making is sequential. First, one of the players decides that in this game he is called the leader and the second player gives his best answer according to the behavior of the first player; the second player is called the follower. In several studies, the producer has been modeled as a leader and the retailer as a follower, and in some studies, the retailer has been the leader and the producer the follower [6]. In a cooperative game, the producer and retailer are introduced as the players of the game, and their decision making is such that the collective profit of both is maximized, and their desirability is the profit of each player due to decision making in the cooperative game. The decision making of both players is such that neither the producer nor the retailer, assuming that the other game is constant, has no incentive to change their decision and introduce this point as the Pareto optimal point in this game [7].

Noh et al. [8] played a Stackelberg game between two members of the chain, including retailer and manufacturer. The goal of this game was to increase the profit of the producer and retailer to determine the optimal price for the retailer. Mohagheghian et al. [23] examined a single-product supply chain between several retailers and a manufacturer in which demand is defined as a new nonlinear function. The structure of their model was Stackelberg, the purpose of which was to determine the optimal selling price of each retailer. Numerical examples and sensitivity analysis were performed on the parameters to examine the model. Leng and Parlar [9] examined Stackelberg's game in a chain with several suppliers and a leading producer under return-sell agreements and sharing of lost sales costs, in which the manufacturer first determines the parameters of the contract and then the suppliers decide on a Nash game. Esmailzadeh and Taleizadeh [10] determined the optimal pricing for two complementary products in a two-echelon supply chain under two different scenarios. The levels defined in this article included one retailer and two manufacturers. The ultimate goal of this paper was to investigate the effect of different market forces between producers and retailers and demand leakage on optimal wholesale and retail pricing as well as chain profits. In this modeling, manufacturers are considered as leaders and retailers are considered as followers. Hou et al. [11] considered a three-stage supply chain model consisting of one manufacturer, one distributor, and one retailer for a single sales cycle. In this paper, the coordination of a decentralized supply chain is analyzed with a simultaneous move game or a leader-follower game, based on a revenue sharing agreement. Zhang and Huang [12] reviewed a supply chain consisting of a manufacturer and several suppliers, forming a coalition. There is a common platform for building a family of products with different interchangeable modules. The manufacturer, as the customer and the leader of the chain, first decides on the selection of suppliers and the arrangement of the platform, and then the suppliers decide on the prices and the order size in order to maximize the profit of the whole coalition.

Ke et al. [13] examined a supply chain pricing decision problem in a two-echelon supply chain with one manufacturer and two competing retailers. Production costs, sales costs, and market rates are all considered uncertain parameters whose distribution is estimated by experienced experts. Considering the strength of different markets of channel members, three decentralized game models were used to detect equilibrium behaviors in appropriate decision-making conditions. Innovation of this article is how channel members should choose their most profitable pricing strategies in the face of uncertainties. Nematollahi et al. [14] considered a retailer corporate supplying goods. It is possible for the supplier to invest in social responsibility activities of corporates, which can increase the popularity of products and consequently market demand. In the first stage, the decisions of both members are examined under decentralized and centralized decision-making models. Then, a new cooperative model is proposed to determine the optimal order quantity and CSR investment. Mahdiraji et al. [15] modeled and solved a two-echelon supply chain model for retailers and manufacturers in non-cooperative conditions. In this article, social responsibilities play a role as a basic parameter in product pricing. Malekian and Rasti-Barzoki [16] considered increasing prices and national advertising of the producer in a producer-retailer supply chain assuming the effect of

consumer reference price. They examined the proposed model in two scenarios: centralized and decentralized. The results indicate that the reference price and customer memory coefficient have profound effects on the profit of supply chain members, the optimal depth of price promotion, and the level of advertising. Ghosh et al. [17] considered a two-echelon supply chain model with one manufacturer and one retailer. They assumed that demand for uncertain goods and part of the demand performance depend on the retailer unit price.

Yadav et al. [18] considered the shortage as a seller's decision variable and demand is receptive to selling price and marketing expenditure of the buyer. In this paper layer's interaction is reviewed and determined as non-cooperative Stackelberg game. Yang et al. [19] designed a two-layer game model to optimize the park integrated energy system internally and externally. Firstly, an upper-level Stackelberg game model of the superior energy network and park system designed to carry out external optimization of the park integrated energy system. Second, a cooperative game model presented for the park users, the gas supply system, and the park integrated energy system to undertake internal optimization of the park integrated energy system. Das et al. [20] suggested a discount mechanism by which companies can coordinate their ordering and pricing strategies throughout a supply chain model with a single manufacturer and single retailer. Also, the demand curve is iso-elastic price sensitive. Hong and Meng [21] used fuzzy set theory to examine the optimal decision of each member of a two-stage supply chain, which includes a manufacturer and a retailer. In this supply chain, the retailer takes the leading position and makes sales efforts. By considering the market demand function, the manufacturer's manufacturing cost and the retailer's operating cost as fuzzy variables, and by employing sequential game, expected value, and opportunity constraint models, the optimal decision-making solutions are resolved.

According to the research background, the articles have designed the supply chain cooperation network. However, very few articles discuss uncertainty in potential demand and pricing in uncertainty conditions, leading to research gaps. In this paper, a new model is presented for cooperation (signing a contract) between two chains of retailer (leader) and manufacturer (follower). Therefore, in general, the research gap can be raised in providing a collaborative model between the two levels of the supply chain to increase profits in the face of demand uncertainty. In this cooperative game, the level of the return product contract is also determined.

3 | Problem Definition and Modeling

A single-cycle supply chain consisting of two manufacturers and a retailer is examined. In this chain, the manufacturer produces the product and sells it to the retailer, and the retailer sells the product to the customer. The main purpose is to maximize the profit of the producer and retailer by determining the optimal amount of order and the price of the product. Also, customer demand is considered uncertain. In this model, the retailer decides on the order quantity Q and the product price p . The manufacturer also produces the product at cost c and sells it to the retailer at a wholesale price w . The main parameters and variables of the modeling are defined as follows:

- p : Total price.
- Q : Order quantity.
- w : Wholesale price.
- h : Maintenance costs.
- c : Production costs.
- π_r : Retailer profit.
- π_m : Producer profit.
- π_{sc} : The profit of the whole chain.
- $X(\epsilon, p)$: Demand function.
- $D(p)$: The certain part of the demand function.
- ϵ : Random demand variable.

– $[A,B]$: The lower and upper bound of the uniform distribution function.

According to the defined symbols, the demand function is displayed as the following model:

$$X(p,\varepsilon)=D(p)+\varepsilon. \tag{1}$$

In the demand function, $D(p)$ is the definite part of demand, which is linear and a function of the variable p . The value of ε is a random variable that has a uniform distribution in the $[A,B]$ range [22]. In the initial part of the model, two decentralized and centralized scenarios are presented, and the optimal value of the decision variable of the order quantity Q and the product price p are obtained. Demand function is a linear and descending function of the final price of the product, which is shown as follows:

$$D(p)=\alpha-\beta p \quad \alpha,\beta \geq 0. \tag{2}$$

Since ε is a random variable with a uniform distribution function, it has the mean μ as below:

$$\varepsilon \sim U(A,B), \quad \mu = \frac{A+B}{2}. \tag{3}$$

In the

following, the decentralized market scenario is first examined, and the optimal quantity of order Q and product price p in this market are determined.

3.1 | Decentralized Scenario

In this scenario, the members of the chain are considered as separate economic enterprises, each of which determines the decision variables based on their interests and profits. In this model, the retailer is responsible for determining the optimal order quantity Q and product price p . Therefore, in this scenario, the retailer determines the order quantity based on its profit, and the manufacturer follows the retailer's decisions. The profit function of the retailer, manufacturer, and the profit of the whole supply chain network are expressed as follows:

$$\pi_R = \begin{cases} p \cdot X(p,\varepsilon) - wQ - h[Q - X(p,\varepsilon)] & X(p,\varepsilon) \leq Q \\ pQ - wQ & X(p,\varepsilon) > Q \end{cases} \tag{4}$$

$$\pi_m = (w - c)Q, \tag{5}$$

$$\pi_{sc} = \pi_m + \pi_r. \tag{6}$$

Eq. (4) shows the retailer's profit function; Eq. (5) shows the manufacturer's profit function, and Eq. (6) shows the profit function of the whole supply chain network. In the following, the retailer's profit function is examined, and the optimal quantity of order and product price are determined. By placing the $X(p,\varepsilon)$ value of Eq. (1) in the retailer profit function, Eq. (4) is corrected as follows:

$$\pi_R = \begin{cases} (p-w)Q - (h+p)(Q - \alpha + \beta p - \varepsilon) & Q \geq \alpha - \beta p + \varepsilon \\ (p-w)Q & Q < \alpha - \beta p + \varepsilon \end{cases} \tag{7}$$

Therefore, the expected amount of the retailer's profit function is equal to the following:

$$E[\pi_r] = (p-w)Q - (h+p) \int_A^{Q-\alpha+\beta p} (Q-\alpha+\beta p-\varepsilon) f(\varepsilon) d\varepsilon, \tag{8}$$

$$\rightarrow E[\pi_r] = Q(p-w) + \frac{(h+p)(A-Q+\alpha-\beta p)^2}{2(A-B)}. \tag{9}$$

To determine the optimal quantity of economic order, first-order derivative should be obtained from the profit function of the retailer with respect to the economic order quantity variable, and the value of the obtained function should be zero.

$$\frac{\partial E[\pi_r]}{\partial Q} = p-w - \frac{(h+p)(A-Q+\alpha-p\beta)}{A-B} \tag{10}$$

Also, to determine the optimal quantity of the product price, first-order derivative should be obtained from the profit function of the retailer with respect to the product price variable, and the value of the obtained function should be zero.

$$\frac{\partial E[\pi_r]}{\partial p} = Q - \frac{(h+p)\beta(A-Q+\alpha-p\beta)}{A-B} + \frac{(A-Q+\alpha-p\beta)^2}{2(A-B)} \tag{11}$$

In order to determine the economic order quantity and the optimal amount of product price, the concavity of the retailer's profit function must be proved. Therefore, the Hessian function matrix should be defined as follows:

$$H = \begin{bmatrix} \frac{\partial^2 E[\pi_r]}{\partial Q^2} & \frac{\partial^2 E[\pi_r]}{\partial Q \partial p} \\ \frac{\partial^2 E[\pi_r]}{\partial p \partial Q} & \frac{\partial^2 E[\pi_r]}{\partial p^2} \end{bmatrix} = \begin{bmatrix} \frac{h+p}{A-B} & \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} \\ \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} & \frac{\beta(-2A+2Q-2\alpha+h\beta+3p\beta)}{A-B} \end{bmatrix} \tag{12}$$

Theorem 1. To calculate the optimal quantity of economic order and the price of the final product as follows, the $\frac{h+p}{A-B}$ value must be negative and the $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ value must be positive. Therefore, considering the positive value of price and maintenance cost, as well as the higher value of B compared to A , it can be shown that $\frac{h+p}{A-B} < 0$. Also, the value of $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ can be easily shown to be strictly positive. Therefore, the above function is concave, and the quantity of economic order and the optimal price can be determined by solving the following equations.

$$\frac{\partial E[\pi_r]}{\partial Q} = 0 \rightarrow Q_{dc}^* = \frac{B(p-w)+A(h+w)+(h+p)(\alpha-p\beta)}{h+p} \tag{13}$$

$$\frac{\partial E[\pi_r]}{\partial p} = 0 \rightarrow p_{dc}^* = \frac{2A\beta-2Q\beta+2\alpha\beta-h\beta^2 + \sqrt{\beta^2(A^2+6BQ+2A(-4Q+\alpha+h\beta) + (-Q+\alpha+h\beta)^2)}}{3\beta^2} \tag{14}$$

3.2 | Centralized Scenario

In the centralized scenario, the supply chain is considered by a central decision maker, and the goal is to maximize the profit of the whole chain. Therefore, the value of the decision variable of the economic order quantity is obtained from the total profit function:

$$\pi_{sc} = \begin{cases} p.X(p,\epsilon) - cQ - h[Q-X(p,\epsilon)] & X(p,\epsilon) \leq Q \\ pQ - cQ & X(p,\epsilon) > Q \end{cases} \tag{15}$$

$$\pi_m = (w-c)Q, \tag{16}$$

$$\pi_r = \pi_{sc} - \pi_m. \tag{17}$$

Eq. (15) shows the total profit of the supply chain network; Eq. (16) shows the total profit of the manufacturer, and Eq. (17) shows the profit of the retailer. By placing the value of $X(p, \epsilon)$ from Eq. (1) in the supply chain profit function, Eq. (15) is corrected as follows:

$$\pi_{sc} = \begin{cases} (p-c)Q - (h+p)(Q-\alpha+\beta p-\epsilon) & Q \geq \alpha - \beta p + \epsilon \\ (p-c)Q & Q < \alpha - \beta p + \epsilon \end{cases} \quad (18)$$

Therefore, the expected value of the whole chain profit function in this case is equal to:

$$E[\pi_{sc}] = (p-c)Q - (h+p) \int_A^{Q-\alpha+\beta p} (Q-\alpha+\beta p-\epsilon) f(\epsilon) d\epsilon, \quad (19)$$

$$\rightarrow E[\pi_{sc}] = (-c+p)Q + \frac{(h+p)(A-Q+\alpha-p\beta)^2}{2(A-B)}. \quad (20)$$

Hence, the profit of the whole chain becomes more than the decentralized scenario, but one of the members suffers loss. In this scenario, in order to determine the optimal quantity of economic order, the first-order derivative of the profit function of the whole supply chain network must be set equal to zero in relation to the quantity of economic order.

$$\frac{\partial E[\pi_{sc}]}{\partial Q} = p - \frac{(h+p)(A-Q+\alpha-p\beta)}{A-B} - c. \quad (21)$$

Also, to determine the optimal value of the product price, the first-order derivative of the retailer's profit function must be taken relative to the product price variable, and the value of the obtained function must be set equal to zero.

$$\frac{\partial E[\pi_{sc}]}{\partial p} = \frac{2(A-B)Q + (A-Q+\alpha-p\beta)^2 + 2(h+p)\beta(-A+Q-\alpha+p\beta)}{2(A-B)}. \quad (22)$$

In order to determine the quantity of economic order and the optimal amount of product price, the concavity of the retailer's profit function must be proved. Therefore, the Hessian function matrix should be defined as follows.

$$H = \begin{bmatrix} \frac{\partial^2 E[\pi_{sc}]}{\partial Q^2} & \frac{\partial E[\pi_{sc}]}{\partial Q \partial p} \\ \frac{\partial E[\pi_{sc}]}{\partial p \partial Q} & \frac{\partial^2 E[\pi_{sc}]}{\partial p^2} \end{bmatrix} = \begin{bmatrix} \frac{h+p}{A-B} & \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} \\ \frac{-B+Q-\alpha+h\beta+2p\beta}{A-B} & \frac{\beta(-2A+2Q-2\alpha+h\beta+3p\beta)}{A-B} \end{bmatrix}. \quad (23)$$

Theorem 2. To calculate the optimal quantity of economic order and the price of the final product as follows, the $\frac{h+p}{A-B}$ value must be negative and the $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ value must be positive.

Therefore, considering the positive value of price and maintenance cost, as well as the higher value of B compared to A , it can be shown that $\frac{h+p}{A-B} < 0$. Also, the value of $\frac{(-B+Q-\alpha+h\beta+2p\beta)^2}{(A-B)^2}$ can be easily shown to be strictly positive. Therefore, the above function is concave, and the quantity of economic order and the optimal price can be determined by solving the following equations.

$$\frac{\partial E[\pi_{sc}]}{\partial Q} = 0 \rightarrow Q_c^* = \frac{A(c+h) + B(-c+p) + (h+p)(\alpha-p\beta)}{h+p}, \quad (24)$$

$$\frac{\partial E[\pi_{sc}]}{\partial p} = 0 \rightarrow p_c^* = \frac{2A\beta - 2Q\beta + 2\alpha\beta - h\beta^2 + \sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)}}{3\beta^2} \quad (25)$$

3.3 | Contract Model

In this section, the contract related to the remaining products is evaluated. The value of the retailer's profit, taking into account the contract, is as follows:

$$\pi_r = \begin{cases} (p-w_b)Q - (p+h-b)(Q-X(p,\epsilon)) & Q-X(p,\epsilon) > \gamma \\ (p-w_b)Q - (p+h)(Q-X(p,\epsilon)) & 0 < Q-X(p,\epsilon) < \gamma \\ (p-w_b)Q & Q-X(p,\epsilon) < 0 \end{cases} \quad (26)$$

$$\pi_r = \begin{cases} (p-w_b)Q - (p+h-b)(Q-\alpha+\beta p-\epsilon) & Q-\alpha+\beta p-\gamma > \epsilon \\ (p-w_b)Q - (p+h)(Q-\alpha+\beta p-\epsilon) & Q-\alpha+\beta p-\gamma < \epsilon \\ (p-w_b)Q & \epsilon < Q-\alpha+\beta p \\ & Q-\alpha+\beta p < \epsilon \end{cases} \quad (27)$$

$$E[\pi_r] = (p-w_b)Q - (p+h) \int_A^{Q-\alpha+\beta p} (Q-\alpha+\beta p-\epsilon)f(\epsilon)d\epsilon + b \int_A^{Q-\alpha+\beta p-\gamma} (Q-\alpha+\beta p-\epsilon)f(\epsilon)d\epsilon \quad (28)$$

The value of producer's profit can also be calculated from the following equation.

$$\pi_m = \begin{cases} (w_b-c)Q - b(Q-X(p,\epsilon)) & Q-X(p,\epsilon) > \gamma \\ (p-w_b)Q & Q-X(p,\epsilon) < \gamma \end{cases} \quad (29)$$

$$\pi_m = \begin{cases} (w_b-c)Q - b(Q-\alpha+\beta p-\epsilon) & Q-\alpha+\beta p-\gamma > \epsilon \\ (w_b-c)Q & Q-\alpha+\beta p-\gamma < \epsilon \end{cases} \quad (30)$$

$$E[\pi_m] = (w_b-c)Q - b \int_A^{Q-\alpha+\beta p-\gamma} (Q-\alpha+\beta p-\epsilon)f(\epsilon)d\epsilon \quad (31)$$

As a result, the profit of the whole supply chain network is equal to:

$$\pi_{sc} = \pi_m + \pi_r \quad (32)$$

In the above relations, w_b , b , and γ are the contract parameters. By integrating the profit function of the retailer and production, the ultimate profit of these two functions is as follows:

$$\pi_r = - \frac{b(A-Q+\alpha-2p\beta-p\beta\gamma)(A-Q+\alpha+p\beta\gamma) + (A-Q+\alpha-p\beta)^2(-h+p(-1+Q)-Qw_b)}{2(A-B)} \quad (33)$$

$$\pi_m = \frac{b(A-Q+\alpha-2p\beta-p\beta\gamma)(A-Q+\alpha+p\beta\gamma)}{2(A-B)} + Q(-c+w_b) \quad (34)$$

To determine the quantity of economic order and the optimal value of product price, derivative from the retailer's profit function must be taken with respect to the value of the decision variables Q and p and set equal to zero.

$$\frac{\partial E[\pi_r]}{\partial Q} = \frac{(A-Q+\alpha-p\beta)(-2b+2h+2p+Ap-3pQ+p\alpha-p^2\beta-(A-3Q+\alpha-p\beta)w_b)}{2(A-B)} \tag{35}$$

$$\frac{\partial E[\pi_r]}{\partial p} = \frac{1}{2(A-B)} ((-1+Q)(A-Q+\alpha-p\beta)^2 + b\beta\gamma(A-Q+\alpha-2p\beta-p\beta\gamma) - b\beta(2+\gamma)(A-Q+\alpha+p\beta\gamma) - 2\beta(A-Q+\alpha-p\beta)(-h+p(-1+Q)-Qw_b)) \tag{36}$$

$$Q_{co}^*(w_b, b) = \frac{-2b+2h+2p+Ap+p\alpha-p^2\beta-(A+\alpha-p\beta)w_b}{3(p-w_b)} \tag{37}$$

$$P_{co}^*(w_b, b, \gamma) = \frac{1}{3(-1+Q)\beta^2} (-2A\beta+2Q\beta+2AQ\beta-2Q^2\beta-2\alpha\beta+2Q\alpha\beta+h\beta^2 + 2b\beta^2\gamma+b\beta^2\gamma^2+Q\beta^2w_b + \sqrt{(\beta^2((2A(-1+Q)+2Q-2Q^2-2\alpha+2Q\alpha+h\beta+2b\beta\gamma+b\beta\gamma^2+Q\beta w_b))^2 - 3(-1+Q)(A-Q+\alpha)(A(-1+Q)+Q-Q^2-\alpha+Q\alpha-2b\beta+2h\beta+2Q\beta w_b))}) \tag{38}$$

Then, the optimal quantity of the order and the value of the product price in the contract must be equal to the optimal quantity of the order and the value of the product price in the centralized state before the contract, i.e.,

$$Q_{co}^*(w_b, b) = Q_c^* \tag{39}$$

$$P_{co}^*(w_b, b, \gamma) = P_c^* \tag{40}$$

Therefore, by equating the two expressions above, the value of the parameter w_b and γ based on a function of $f(b)$ is obtained as described in the following relation:

$$w_b = \frac{-2h^2+2b(h+p)+2hp(-2+A+\alpha-p\beta)-p(3B(c-p)+A(-3c+p)+2p(1-\alpha+p\beta))}{-3B(c-p)+A(3c+2h-p)+2(h+p)(\alpha-p\beta)} \tag{41}$$

$$b = \frac{1}{2(h+p)} (2h^2+2hp(2-A-\alpha+p\beta)+p(3B(c-p)+A(-3c+p)+2p(1-\alpha+p\beta)) + (-3B(c-p)+A(3c+2h-p)+2(h+p)(\alpha-p\beta))w_b) \tag{42}$$

$$\gamma = -((A^2b-2bBQ+bQ^2+2Ab\alpha-2bQ\alpha+b\alpha^2-2Abh\beta + 2bhQ\beta-2bh\alpha\beta + \sqrt{(-\frac{1}{\beta^2}b(A^2-2BQ+2A(\alpha-h\beta)+(Q-\alpha)(Q-\alpha+2h\beta)) - 2A^2Q\beta+2ABQ\beta+2AQ^2\beta+2A^2Q^2\beta-2BQ^2\beta-2ABQ^2\beta-2AQ^3\beta+2BQ^3\beta - 2AQ\alpha\beta+2BQ\alpha\beta+2AQ^2\alpha\beta-2BQ^2\alpha\beta+A^2b\beta^2-4AbQ\beta^2+2bBQ\beta^2+AhQ\beta^2 - A^2hQ\beta^2-BhQ\beta^2+bQ^2\beta^2+3AhQ^2\beta^2-BhQ^2\beta^2-hQ^3\beta^2+2Ab\alpha\beta^2-2bQ\alpha\beta^2 -$$

$$\begin{aligned}
 & 2AhQ\alpha\beta^2 + 2hQ^2\alpha\beta^2 + b\alpha^2\beta^2 - hQ\alpha^2\beta^2 + Abh\beta^3 - bhQ\beta^3 - Ah^2Q\beta^3 + h^2Q^2\beta^3 + \\
 & bh\alpha\beta^3 - h^2Q\alpha\beta^3 + AQ\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - \\
 & BQ\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - \\
 & AQ^2\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} + \\
 & BQ^2\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - \\
 & Ab\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} + \\
 & bQ\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} + \\
 & AhQ\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - \\
 & hQ^2\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - \\
 & b\alpha\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} + \\
 & hQ\alpha\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)} - A^2Q\beta^2w_b + \\
 & 4AQ^2\beta^2w_b - 2BQ^2\beta^2w_b - Q^3\beta^2w_b - 2AQ\alpha\beta^2w_b + 2Q^2\alpha\beta^2w_b - Q\alpha^2\beta^2w_b \\
 & - AhQ\beta^3w_b + hQ^2\beta^3w_b - \\
 & hQ\alpha\beta^3w_b + AQ\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)}w_b - \\
 & Q^2\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)}w_b + \\
 & Q\alpha\beta\sqrt{\beta^2(A^2 + 6BQ + 2A(-4Q + \alpha + h\beta) + (-Q + \alpha + h\beta)^2)}w_b(((/ \\
 & (b(A^2 - 2BQ + 2A(\alpha - h\beta) + (Q - \alpha)(Q - \alpha + 2h\beta))).
 \end{aligned}$$

(43)

Finally, the following relation must be solved to determine the effective parameter range in the contract, i.e. *b*. That is, the profit of the retailer and the producer after the contract is higher than the profit of the retailer and wholesaler in the decentralized state before the contract, i.e.,

$$\pi_r^{co}(w_b, b, \gamma) \geq \pi_r^{dc} \rightarrow \pi_r^{co}(b) \geq \pi_r^{dc}, \tag{44}$$

$$\pi_m^{co}(b, \gamma) \geq \pi_m^{dc} \rightarrow \pi_m^{co}(b) \geq \pi_m^{dc}. \tag{45}$$

After presenting a mathematical model for a single-cycle supply chain problem in centralized and decentralized mode and modeling the contract between the retailer and the manufacturer, this chapter deals with problem solving and analysis. Thus, after presenting different numerical examples, the sensitivity of each sample problem in the pre-contract and post-contract states is analyzed and compared.

4 | Solving Sample Problems in the Pre-Contract State

To assess the modeling results, five sample problems with different parameters presented in *Table 1* are discussed. Therefore, at the beginning of this section, the values of decision variables, the optimal quantity of economic order, and product price are shown in the centralized and decentralized scenarios. In the second section, the sensitivity of the sample problems under variation of its various parameters is analyzed.

4.1 | Numerical Results of Modeling in the Pre-Contract State

Several numerical examples are provided to examine the profit value of the retailer, producer, and the optimal quantity of economic order in the centralized and decentralized scenarios. *Table 1* shows five numerical examples for examining centralized and decentralized scenarios.

Table 1. Numerical parameters of sample problems in centralized and decentralized scenarios.

Instance	h	c	B	A	β	α	w
1	5	10	20	10	1.2	30	20
2	10	12	20	5	1.2	30	20
3	6	12	30	10	0.8	20	20
4	6	12	30	15	0.6	25	24
5	10	15	40	15	0.6	25	25

Table 2 shows the optimal quantity of economic order, product prices in the centralized and decentralized scenarios as well as firm profits in these two scenarios.

Table 2. The quantity of economic order, product prices, and profits of firms in pre-contract centralized and decentralized scenarios.

Instance	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
1	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
2	25.47	6.74	53.97	30.59	84.56	22.25	13.06	104.50	5	109.50
3	32.08	10.67	85.41	90.70	176.11	29.35	16.31	130.55	67.55	198.27
4	49.77	17.06	204.80	350.56	555.36	44.79	22.80	273.66	315.50	589.17
5	53.03	19.29	192.96	385.15	578.12	49.40	24.83	248.35	357.04	605.40

The total profit of the chain and the order quantity in all examples increased in the centralized mode compared to the decentralized mode, and the price of the product decreased. Due to the fact that in the decentralized state, the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized state and the producer's profit increases.

4.2 | Sensitivity Analysis of the Problem in Pre-Contract State

First, the sensitivity of the problem parameters on the sample problem 1 is analyzed. Hence, the value of problem decision variables against parameters variations is examined. First, the variations of decision-making variables of the problem are examined in relation to the variations of maintenance costs. Therefore, the value of decision variables is as follows.

Table 3. Sensitivity analysis of the values of the objective functions for the maintenance cost parameter.

Parameter variations	Centralized					decentralized				
	π_{sc}^c	π_r^c	π_m^c	Q_c	P_c	π_{sc}^{dc}	π_r^{dc}	π_m^{dc}	Q_{dc}	P_{dc}
2	195.09	21.92	173.16	17.31	23.28	156.52	62.04	94.48	9.44	27.59
4	192.54	22.39	170.15	17.01	23.19	154.81	61.43	93.38	9.33	27.54
5	191.40	22.58	168.82	16.88	23.15	154.05	61.15	92.89	9.29	27.51
6	190.35	22.75	167.60	16.76	23.12	153.33	60.89	92.44	9.24	27.49
8	188.46	23.03	165.43	16.54	23.05	152.03	60.42	91.61	9.16	27.45
10	186.80	23.25	163.55	16.35	22.98	150.89	60.00	90.88	9.08	27.41

According to *Table 3*, with the increase of maintenance costs, the quantity of economic order decreased and in turn the price of the product decreased in the centralized and decentralized chain. Since the maintenance cost has a positive effect on the profit of the centralized and decentralized supply chain network regarding the value of the two objective functions, the profit of the whole network increases with its increase and the profit of the producer decreases due to decrease in product price. *Fig. 1* shows the trend of variations in decision variables.

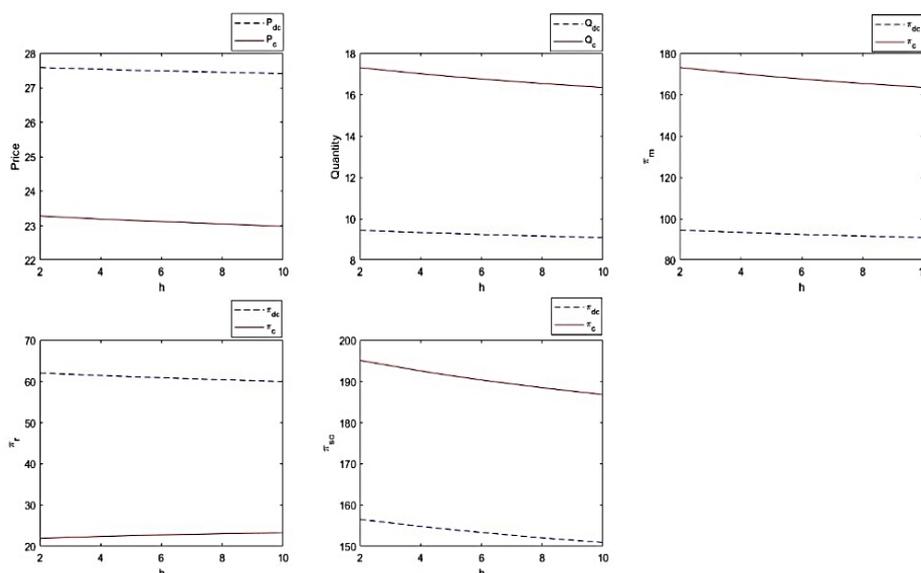


Fig. 1. The trend of variations in decision variables due to variations of maintenance costs.

In the following, production cost sensitivity analysis is examined. Therefore, the value of decision variables is below.

Table 4. Sensitivity analysis of the values of the objective functions for the production cost parameter.

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
8	27.51	9.29	111.47	61.15	172.63	22.27	18.50	222.02	4.76	226.78
10	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
12	27.51	9.29	74.31	61.15	135.47	24.03	15.30	122.41	36.81	159.22
15	27.51	9.29	46.44	61.15	107.60	25.34	12.99	64.97	51.82	116.80
20	27.51	9.29	0	61.15	61.15	27.51	9.29	0	61.15	61.15

According to the results, with the increase of production cost, there has been no change in the optimal quantity of order and retailer price in the decentralized supply chain. However, with the increase of production costs, the price of the product increased in the centralized scenario and the optimal quantity

of order decreased. Therefore, the total profit of the producer decreased and the retailer profit increased due to increase in production costs. Fig. 2 shows the trend of variations in this parameter.

Fig. 2. The trend of variations in decision variables due to variations of production costs.

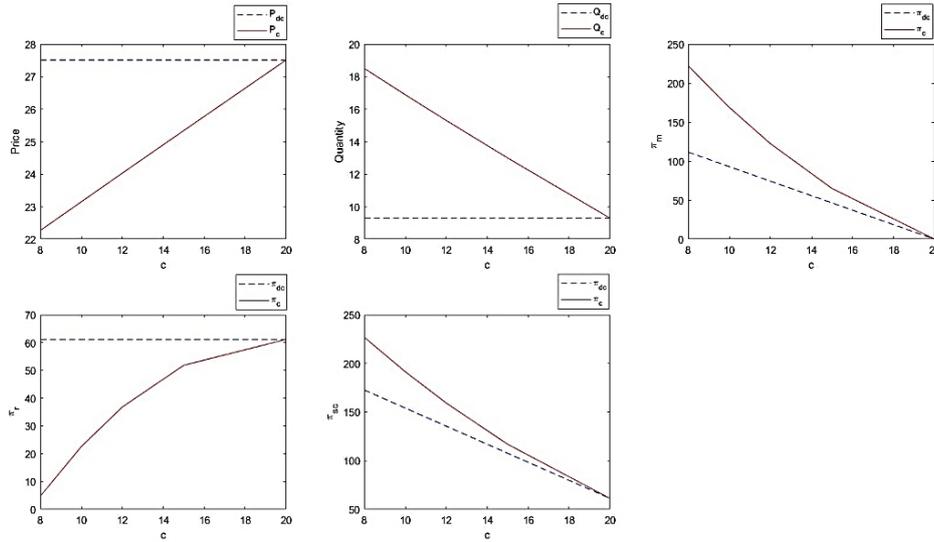


Table 5 and Fig. 3 show the variations of decision variables by wholesale price variations.

Table 5. Sensitivity analysis of the objective functions' values for the wholesale price parameter.

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
10	23.15	16.88	0	191.40	191.40	23.15	16.88	0	191.40	191.40
12	24.03	15.30	30.60	159.22	189.83	23.15	16.88	33.76	157.64	191.40
15	25.34	12.99	64.97	116.80	181.77	23.15	16.88	84.41	106.99	191.40
18	26.65	10.75	86.02	81.19	167.21	23.15	16.88	135.06	56.34	191.40
20	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40

According to the results, with increase in the wholesale price, the profit of the whole supply chain network did not change. In the decentralized scenario, however, the retailer profit increased due to the increase in the wholesale price and decreased in the centralized scenario. Fig. 3 shows the trend of variations in this parameter.

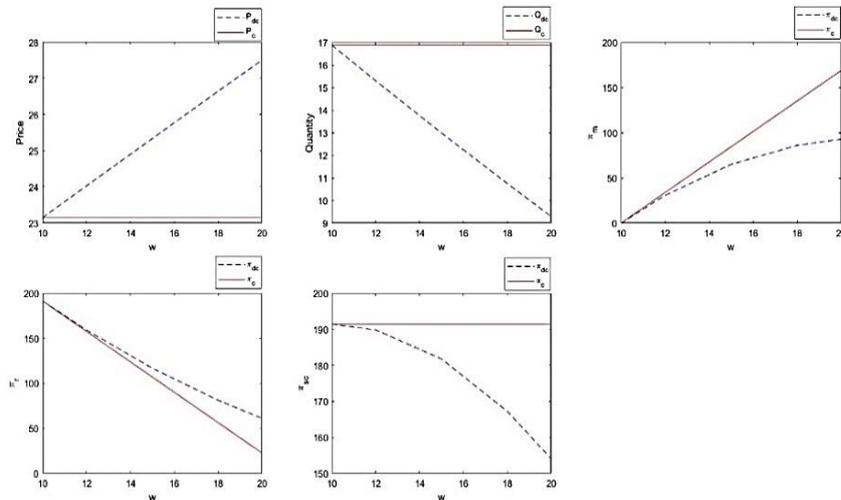


Fig. 3. The trend of variations in decision variables due to wholesale prices variations.

Table 6 and Fig. 4 show the variations in decision variables by parameter α variations.

Table 6. Sensitivity analysis of the objective functions' values for parameter α .

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
30	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40
32	28.41	10.41	104.18	77.09	181.27	24.02	18	180	38.59	218.59
35	29.75	12.1	121.00	104.35	225.35	25.32	19.66	196.65	65.97	262.62
38	31.08	13.77	137.71	135.61	273.33	26.61	21.31	213.18	97.34	310.53
40	31.96	14.88	148.80	158.66	307.46	27.47	22.41	224.14	120.47	344.61

According to the results, with increase in parameter α , the profit of retailer, producer, and the whole supply chain network increased. This occurred due to the increase in product price and the quantity of economic order because of increase in the value of parameter α . Fig. 4 shows the trend of variations in this parameter.

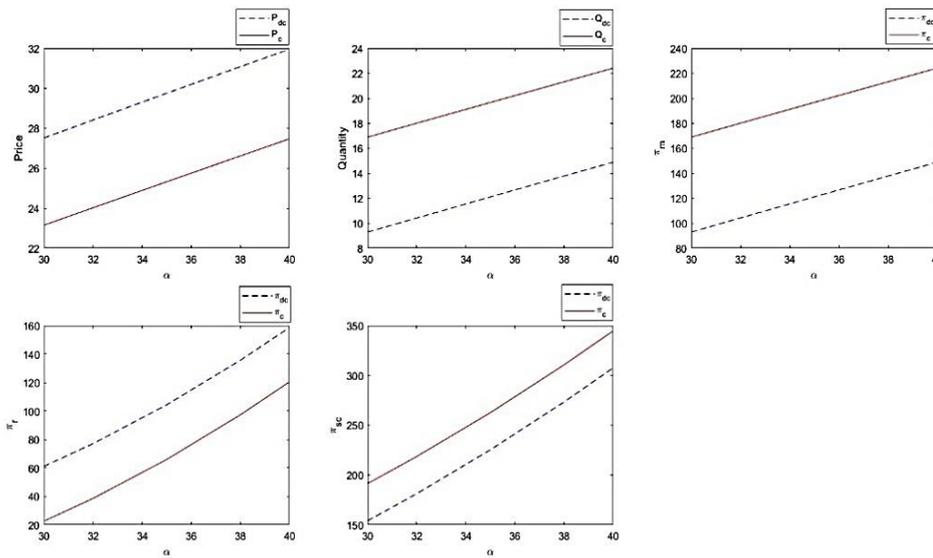


Figure 4. The trend of variations in decision variables due to variations in parameter α .

Table 7 and Fig. 5 illustrate the variations in decision variables by parameter β variations.

Table 7. Sensitivity analysis of the values of objective functions for parameter β .

Parameter variations	Decentralized					Centralized				
	P_{dc}	Q_{dc}	π_m^{dc}	π_r^{dc}	π_{sc}^{dc}	P_c	Q_c	π_m^c	π_r^c	π_{sc}^c
0.8	37.01	14.43	144.35	211.20	355.55	32.62	19.91	199.10	183.40	382.51
0.9	33.85	13.1	131.00	156.74	287.74	29.47	19.12	191.22	126.16	317.38
1	31.31	11.80	118.00	115.89	233.90	26.94	18.35	183.56	82.59	266.16
1.1	29.24	10.53	105.31	84.87	190.19	24.88	17.61	176.10	48.91	225.01
1.2	27.51	9.29	92.89	61.15	154.05	23.15	16.88	168.82	22.58	191.40

It is observed that with increasing parameter β , the profit of the retailer, producer, and the whole supply chain network decreased, which is due to the increase in product price and the quantity of economic order due to the decrease in the value of parameter β . Fig. 5 shows the trend of variations in this parameter.

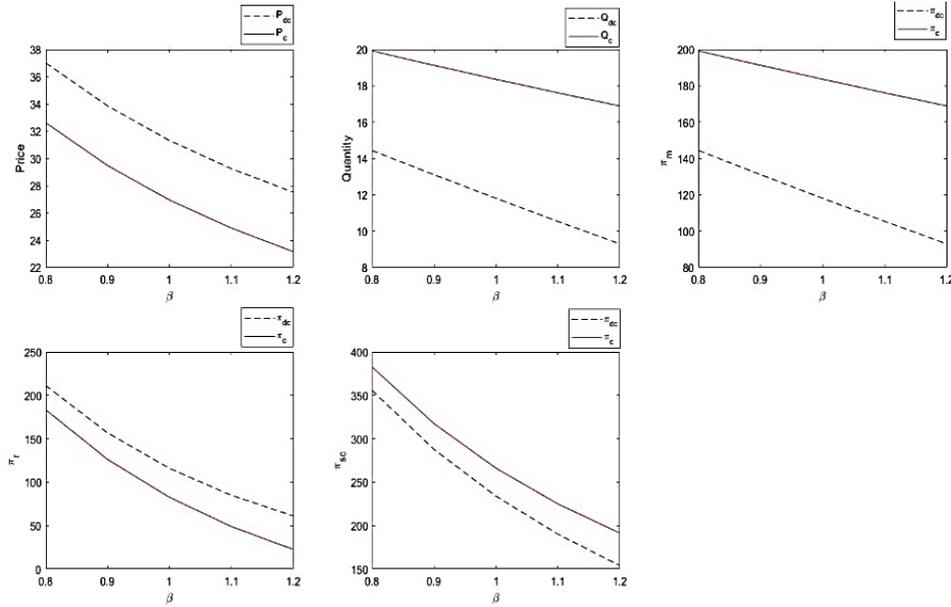


Fig. 5. The trend of variations in decision variables due to variations in parameter β .

5 | Solving Sample Problems in the Post-Contract Mode

After evaluation of the sample problem for centralized and decentralized scenarios and examining output variables and carrying out sensitivity analysis on all input parameters, the model is examined and solved in the post-contract state. In this section, the sensitivity of the parameters is analyzed and the exact range of the contract b parameter is determined after solving the sample problem.

5.1 | Results of the Numerical Example After Signing the Contract

In this section, the results of output variables are examined, and also the exact range of contract b parameter for sample problem 1 is determined. *Table 8* shows the output variables including product price, optimal quantity of economic order, and profit of all firms before and after the contract.

Table 8. The quantity of economic order, product price, and profit of firms' post-contract.

Scenario	Contract b parameter	P	Q	π_m	π_r	π_{sc}
Centralized	-	23.15	16.88	168.82	22.58	191.40
Decentralized	-	27.51	9.29	92.89	61.15	154.05
Signing a contract	370.34	23.15	16.88	160.26	61.49	221.42
	485.71			92.89	137.21	230.11

According to *Table 8*, the product price and the quantity of the economic order in the centralized scenario are equal to the post-contract state. However, the profit of the retailer, producer, and whole supply chain at the time of signing contract is more than the decentralized scenario. *Fig. 6* shows the trend of profit variations of firms in sample problem 1 in the range of contract b parameter.

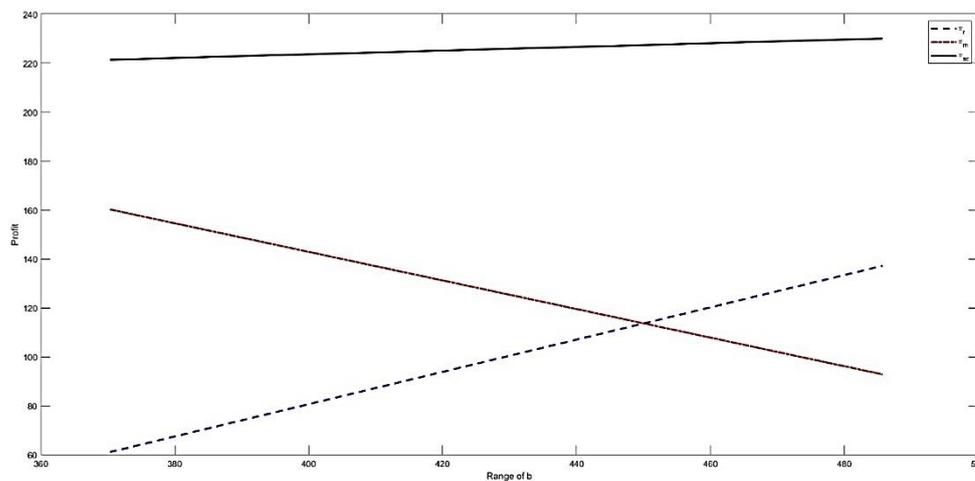


Fig. 6. The trend of variations of firms' profit after signing the contract in the range of contract *b* parameter.

Fig 6 displays when two members of the supply chain are in the range of contract *b* parameter, the profit of both firms increases compare with the decentralized state. Also, the profit of the whole chain is higher than the centralized and decentralized scenarios.

5.2 | Sensitivity Analysis of the Problem After Signing the Contract

In this section, the problem sensitivity is analyzed separately for post-contract state for the input parameters of the problem. The purpose of this analysis is to assess the effect of input parameters on the range of contract *b* parameter. First, the sensitivity of the sample problem 1 is analyzed under the maintenance cost parameter. *Table 9* shows the lower and upper bounds of the contract *b* parameter and the profits of firms. For further analysis, *Fig. 7* shows the trend of variations of firms' profit and the range of contract *b* parameter by variations of maintenance costs.

Table 9. Variations in firms' profits due to changes in maintenance costs post-contract.

Parameter variations	Contract <i>b</i> parameter	π_{sc}	π_r	π_m	
8	Lower bound	296.79	230.44	62.04	168.40
	Upper bound	359.61	241.34	146.86	94.48
10	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89
12	Lower bound	386.88	206.64	60.00	146.64
	Upper bound	762.77	212.36	121.48	90.88

Accordingly, with increasing maintenance costs, the bound of contract *b* parameter ranges increased and also the total profit of firms including the profit of retailer, producer, and supply chain network decreased. However, the profit of the retailer, producer, and the total profit is higher compared to the total profit of the centralized and decentralized scenarios. As a result, the profit of firms decreases by increasing the maintenance cost. The following figure shows the trend of variations of firms' profit in different values of maintenance costs for post-contract.

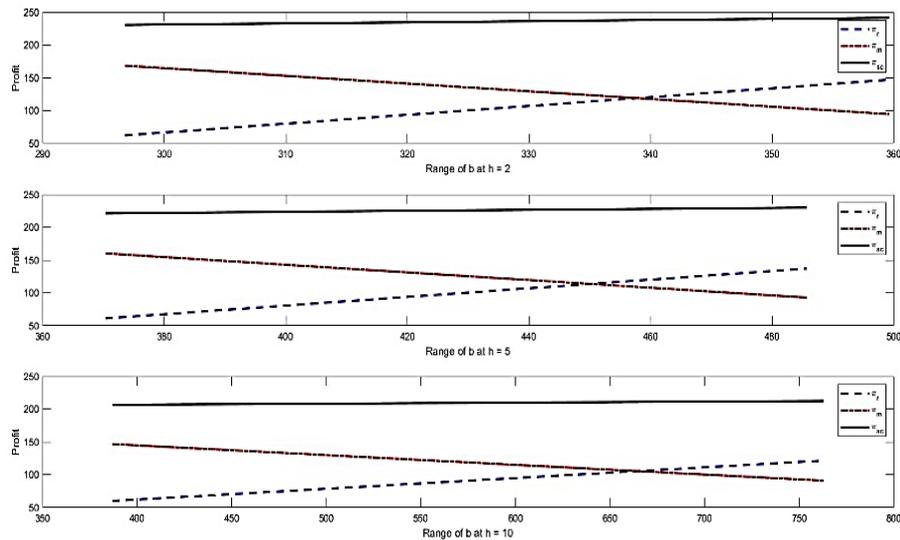


Fig. 7. The trend of variations in firms' profits due to variations in maintenance costs after signing the contract.

The following show the variations in the profits of supply chain members and the entire network post contract in exchange for changes in production costs. Table 10 and Fig. 8 show the variations trend for sample problem 1 for different production costs.

Table 10. The trend of variations in firms' profits due to variations in production costs post-contract.

Parameter variations	Contract b parameter	π_{sc}	π_r	π_m
8	Lower bound	254.15	234.14	61.15
	Upper bound	364.15	248.71	131.24
10	Lower bound	370.34	221.42	61.15
	Upper bound	485.71	230.11	137.21
12	Lower bound	479.38	204.69	61.15
	Upper bound	552.03	218.96	144.65

According to the table above, with increasing production costs, the retailer's profit increases, while the producer's profit decreases. However, the profit of both firms is higher in the decentralized scenario, while the price of the product and the quantity of production are the same as in the centralized scenario. The profit of the whole network in post contract also decreased with the increase of production costs.

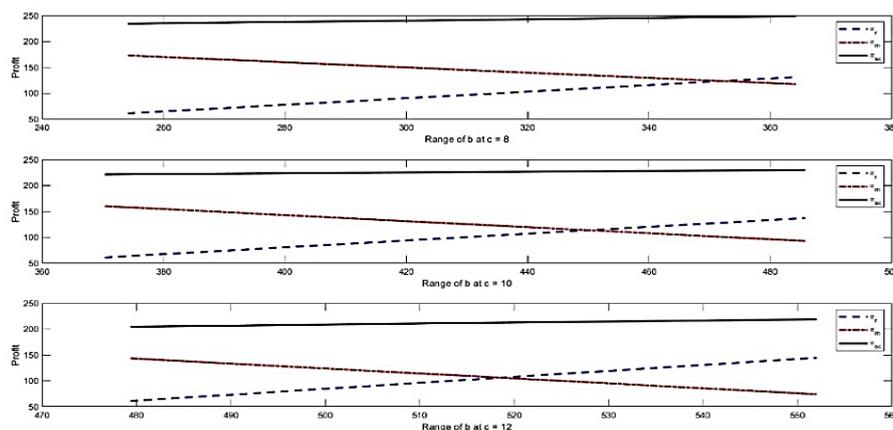


Fig. 8. The trend of variations in firms' profits due to variations in production costs post-contract.

Table 11 examines the profits of chain members and the total profits of firms in exchange for variations in the wholesale price. This table also specifies the contract parameter interval as b. Fig. 9 also shows the

variations trend of the profits of chain members and the firm in exchange for changes in the wholesale price in the post-contract period.

Table 11. The trend of variations in firms' profits due to variations in wholesale prices post-contract.

Parameter variations	Contract b parameter	π_{sc}	π_r	π_m	
12	Lower bound	249.76	187.63	159.22	28.41
	Upper bound	368.46	218.63	188.03	30.60
15	Lower bound	310.12	203.47	116.80	86.67
	Upper bound	423.10	224.36	159.39	64.97
20	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89

Based on the results of the analysis obtained from the wholesale price variations on the firms' profits post contract, it is observed that the profit of the whole network and the profit of each member of the chain in the contract state is higher than the members' profit in the decentralized state. Also, with the increase in maintenance costs, the variations trend of the profit of the entire supply chain network decreases.

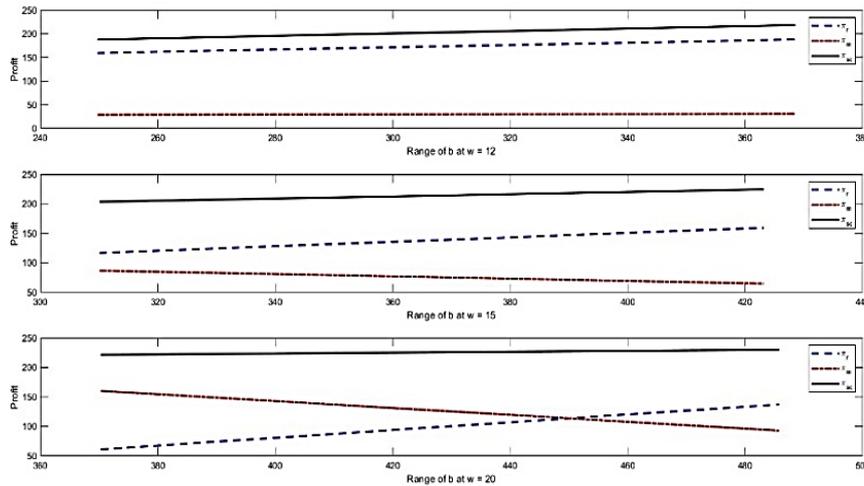


Fig. 9.

The trend of variations in firms' profits due to variations in wholesale prices post-contract.

In the following, the problem sensitivity is analyzed under the input parameter α . The profits of the chain members by variations in the value of a are shown in *Table 12*, including the profits of the retailer, producer, and the whole chain.

Table 12. The trend of variations in firms' profits due to variations in parameter α post-contract.

Parameter variations	Contract b parameter	π_{sc}	π_r	π_m	
30	Lower bound	370.34	221.42	61.15	160.26
	Upper bound	485.71	230.11	137.21	92.89
35	Lower bound	420.38	243.57	104.35	139.22
	Upper bound	586.14	276.45	155.45	121.00
40	Lower bound	624.81	268.94	158.66	110.28
	Upper bound	971.36	312.48	163.68	148.80

Fig. 10 also shows the variations trend in firms' profits due to wholesale price changes after signing the contract for the sample problem 1.

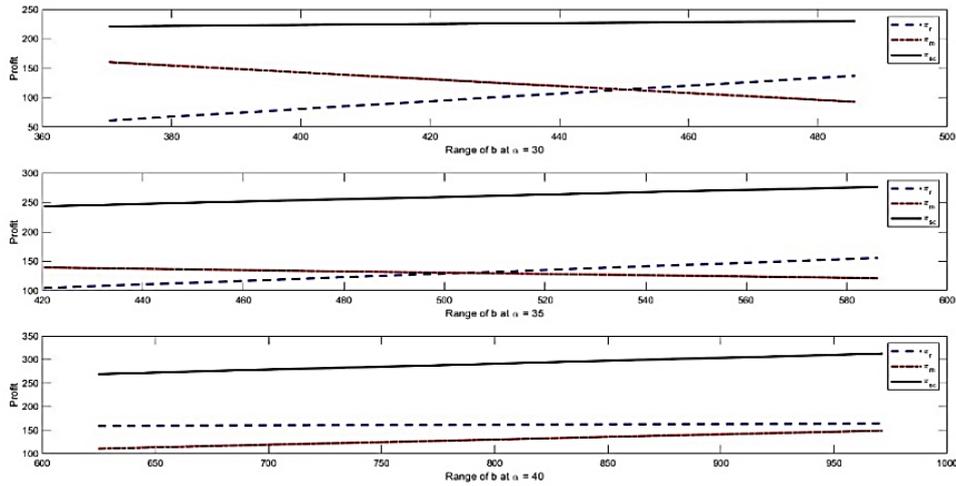


Fig. 10. The trend of variations in firms' profits due to variations in parameter α post-contract.

Analyses revealed that with increasing the coefficient of parameter a , the profit of all members in the contract increased, and also with increasing this coefficient, the profit of the whole network increased in the contract. *Table 13* shows the trend of variations in firms' profits due to variations in parameter β post-contract for sample problem 1.

Table 13. The trend of variations in firms' profits due to variations in parameter β post-contract.

Parameter variations	Contract b parameter	π_{sc}	π_r	π_m
12	Lower bound	635.82	312.14	211.20
	Upper bound	763.58	324.69	180.34
15	Lower bound	420.36	255.12	115.89
	Upper bound	512.36	286.23	168.23
20	Lower bound	370.34	221.42	61.15
	Upper bound	485.71	230.11	137.21

According to the analysis of *Table 13*, with increase in the input parameter β , the profit of the whole chain decreased. Nevertheless, the profit of members increased compared to the decentralized scenario. *Fig. 11* also shows the trend of variations in firms' profits due to variations in parameter β post-contract for sample problem 1.

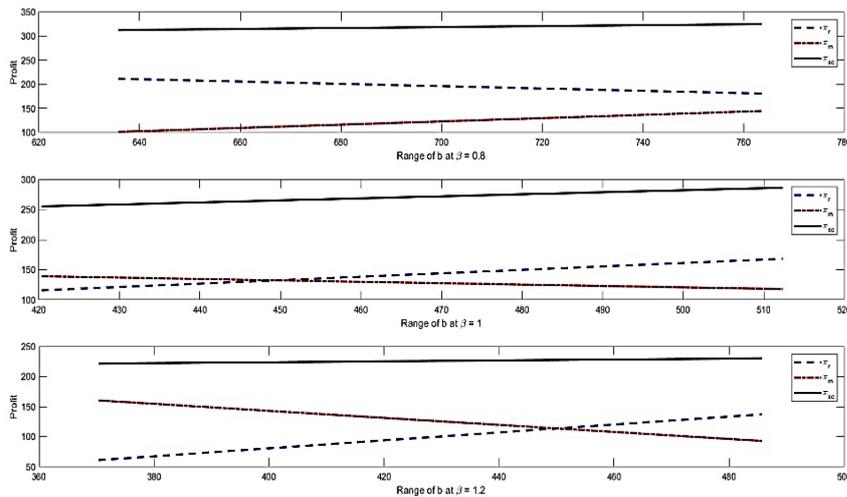


Fig. 11. The trend of variations in firms' profits due to variations in parameter β after signing the contract.

6 | Conclusion

In this paper, the design of a repurchase agreement related to the amount of goods remaining in the two-echelon supply chain between the retailer and the manufacturer is examined, in which both members of the supply chain seek to maximize the profit from the sale of their products. Two scenarios are considered quite separately; In the first scenario (decentralized) in which the retailer determines the price of the product and the optimal amount of the economic order and the producer is persuaded to follow this method. In the second (centralized) scenario, the goal is to maximize the profit of the whole chain, in which case the price of the product and the amount of the economic order are determined based on the profit of the whole chain. Then, a model of repurchase agreement related to the remaining goods was considered based on cooperative play and contract between two members of the supply chain, in which the goal is to maximize the profit of chain members.

Due to the uncertainty of the competitive environment, in this paper, the demand parameter is uncertain and the uniform distribution function is used in modeling to determine the optimal level of cooperation in a competitive and cooperative market. After presenting a mathematical model for the pre-contract state in both centralized and decentralized scenarios and providing numerical examples, the profit of whole chain and the order quantity increased in all examples in the centralized scenario and the product price decreased, compared to the decentralized scenario. As in the decentralized scenario, the retailer determines the values of the optimal variables, the profit of this member decreases in the centralized scenario and the producer's profit increases. After performing the sensitivity analysis on the input parameters and on all numerical examples, the quantity of economic order decreased with increase on the maintenance cost, and consequently, the product price in the centralized and decentralized chain decreased. Since the maintenance cost has a positive effect on the profit of the centralized and decentralized supply chain network regarding the value of the two objective functions, the profit of the whole network increased with its increase, and the producer profit decreased due to decline in the product price. Also, with the increase in production costs, no change was observed in the optimal order quantity and retailer price in the decentralized supply chain.

Nevertheless, with the increase of production costs, the product price in the centralized scenario increased and the optimal quantity of the order decreased. Therefore, due to the increase in production costs, the total profit of the producer decreased and the profit of the retailer increased. It was further found that with the increase in the wholesale price, the profit of the whole supply chain network did not change. In the decentralized scenario, however, the retailer profit increased due to the increase in the wholesale price and decreased in the centralized scenario. Finally, by performing sensitivity analysis on parameters a and β , it was found that with increasing parameter a , the profit of the retailer, producer, and the whole supply chain network increased. This occurred due to the increase in product price and the quantity of economic order because of increase in the value of parameter a . Also, with increase in parameter β , the profit of the retailer, producer, and the whole supply chain network decreased. This is due to the increase in product price and the quantity of economic order due to the decrease in the value of parameter β . Then, a cooperative model was designed between the two members of supply chain, and numerical examples were designed under the input parameters of the contract and executed on the model. The result was to determine the contract parameter range on the problem. Their profit increased compared to the decentralized scenario by creating a cooperative relationship between two parties of the supply chain. Sensitivity analysis was also performed on all parameters in a cooperative manner.

To improve the work, it is suggested that the proposed model be designed in a three-echelon supply chain network. Also, due to the uncertainty in real environments, it is suggested that other parameters in the model be considered uncertain. Finally, to be as close as possible to the real world, considering the discount in the model can be considered as one of the future studies. The results of the models presented in this paper can be used to improve the business of small and medium units as well as their cooperation to gain more profit from the market in conditions of demand uncertainty.

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