E-ISSN: 2717-2937 | P-ISSN: 2783-1337

International Journal of Research in Industrial Engineering

IJRIE

www.riejournal.com

Int. J. Res. Ind. Eng. Vol. 10, No. 2 (2021) 165–173.

Paper Type: Research Paper

Using a New Algorithm to Improve the Search Answer in Quadratic Assignment Problem (QAP)

Hossein Jafari [1,*], **Abbas Sheykhan**[2]

[1] Young Researchers and Elite Club, Arak Branch, Islamic Azad University, Arak, Iran; hossein_jafari_123@yahoo.com

[2] Department of Industrial Engineering, Islamic Azad University of Arak, Arak, Iran; a-sheykhan@iau-arak.ac.ir

Citation:

Jafari, H., & Sheykhan, A. (2021). Using a new algorithm to improve the search answer in quadratic assignment problem (QAP). *International journal of research in industrial engineering, 10*(2), 165-173.

Received: 14/02/2021 Reviewed: 03/04/2021 Revised: 07/05/2021 Accept: 28/05/2021

Abstract

Layout design problem is one of the useful field of study used to increase the efficiency of sources in organizations. In order to achieve an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Therefore, using computer in solving the related problems is important in the view of the researchers of this area of study. However, the designs produced by a computer to solve big problems require more time, so, this paper suggests an algorithm that can be useful in better performance of the known algorithms such as Branch and Bound. The proposed study aims to improve the performance of the Branch and Bound (BB) algorithm in solving Quadratic Assignment Problem (QAP) problems. The findings show that the proposed method enables the BB algorithm to produce an optimal solution in the minimum amount of time.

Keywords: Computer algorithms, Exact methods, Branch and bound, Feasible answer.

1 | Introduction

Facility layout design is a position layout of the equipment of good production or service offering. Koopmans and Beckmann [1] were the pioneers defining facility layout design problem as a common industrial problem which aims at configuring facility so that the cost of the transportable materials will be minimized. The facility layout or the Quadratic Assignment Problem (QAP) is a spatial layout of goods production or service provision facilities.

The design of the layout is an optimization problem that tries to make deployment more efficient, taking into account the various interactions between the facilities and materials transportation system [2].

Azadivar and Wang [2] defined layout design problem as a problem determining relative displacement and allocating space to the existed facilities. It is often hypothesized that material

Corresponding Author: hossein_jafari_123@yahoo.com

10.22105/RIEJ.2021.271276.1185

flow among departments is fixed and the designed layout will be applicable for a long time. But due to competitive atmosphere of the market and change in customers' taste, dynamism is considered as an inevitable element in industry today, that as a result of the production companies, they have to be able to answer it [3]. Due to this point, it can be inferred that facility layout for short time needs can inconsiderably increase the costs resulted from primary facility for a long time. Thus, it seems that considering a dynamic factor is necessary and important [4].

2 | Quadratic Assignment Problem (QAP)

Koopmans and Beckmann [1] defined and formulated QAP to be used in economic activities. Because of its quadratic nature, this problem is known as Quadratic Assignment Problem, which has attracted researchers' attention working in several areas of study. Lots of researchers and scientists used it in areas of mathematics, computer, operations research and economics to model optimization problems. Assignment means that each facility should be conformed into one position and vice versa. In QAP, the number of facilities has to be equal to the number of positions. Mathematic form of this problem is as follows [5]:

$$\text{Min } C=\sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{j=1}^{n}\sum_{s=1}^{n} d_{i,k}w_{j,s}(x_{i,j}\ x_{k,s}),$$

Subject to:

$$\sum_{j=1}^{n} x_{i,j}=1\ ;\ i=1,2,\ldots,n, \tag{1}$$

$$\sum_{i=1}^{n} x_{i,j}=1\ ;\ j=1,2,\ldots,n,$$

$$x_{i,j}=0 \text{ or } 1\ ;\ i=1,2,\ldots,n \text{ and } j=1,2,\ldots,n.$$

$d_{i,k}$ refers to the distance between ith and kth cells.

$w_{j,s}$ refers to the transportation flow between the $j$the and sth machines.

$x_{i,j}$ determines whether jth machine is in the ith cell or not.

$x_{k,s}$ determines whether sth machine is in the kth cell or not.

In the event that facility of j is located in cell of i and facility of s is located in cell of k ($x_{i,j}=1$ *and* $x_{k,s}=1$), by calculating the previous condition, the cost of displacement in this route is $d_{i,k}*x_{i,j}*x_{k,s}=d_{i,k}$. In the event that importance of this route is considered, average displacement cost in this route is $d_{i,k}*w_{j,s}*x_{i,j}*x_{k,s}=d_{i,k}w_{j,s}$.

3 | Statement of Problem

QAP is one of the most complex optimization problems of nonlinear integer [6]. In general, (in wide dimension problems) QAP does not include an exact solution because it is located in the group of NP-Hard problems and to solve it, Meta-heuristic algorithm and Invasive weed optimization are often used.

To solve QAP, some exact algorithms such as Dynamic programming, cut page method, Branch and Bound method can be used [7] and [8]. Branch and Bound method proves better function than the previous two methods does [9]. One of the problems of the mentioned three methods is their incapability in solving wide dimension problems. In other words, using the mentioned algorithms are not possible for the problems with size more than 15 [10].

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In real world, all of facilities cannot be settled in all Locations. Thus, by making search space small, we can reach an optimum answer faster in problems having wide dimensions and such limitations.

4 | Literature Review

Stützle [11] offered a new method called Iterated Local Search (ILS) to solve QAP. ILS is a simple random search method. First, some random points are created in search space, then based on the competence of the mentioned points, searching around them is started. One of the biggest challenge in Stützle's method is the radius in local search.

Hicks [12] in a paper developed Genetic algorithm to be used in facility layout in a set of productive cells. The results showed that the approach of redesigning facilities determines intracellular layout, then it localizes the cells among empty departments.

Mak et al. [13] in a paper used Genetic algorithm as a general method to solve layout design problems. They developed a mathematical model to study layout of the devices and material flow pattern for workshop and product manufacturing environment. The suggested Genetic algorithm with the aim of minimizing material displacement cost, extracts an optimum machinery layout.

Pichka et al. [14] solved the Vehicle Routing Problem (VRP) and suggested the use of the Simulated Annealing Algorithm (SAA) to find the optimal routes between the customers and warehouses.

Moradi and Shadrokh [15] investigated the Site Layout Planning (SLP) with equal and unequal surface areas. The SA algorithm was used to find the optimal layout. Comparison of the SA results with those of other algorithms showed the superiority of the SA in finding optimal solutions with high speed in a shorter time.

Jafari et al. [16] investigate the facility layout problem in an industrial workshop. Their problem-solving recommendation was to use a Developed Simulated Annealing Algorithm (DSAA). This new algorithm is an iterative form of the Basic Simulated Annealing Algorithm (BSAA). The results indicate the ability of the proposed algorithm to find better solutions.

Shadkam and Ghavidel [17] investigate the balancing assembly lines problem. The purpose of this paper is to present a multi-objective integer linear mathematical programming model for balancing assembly lines, which is solved using the general criteria method. The three objective functions considered in this model are: (1) Minimizing cycle time (2) Minimize the idle time of each station and (3) increase the efficiency of the assembly line. In order to investigate the model, Iran-Shargh Neishabour Company has been considered as a case study. After implementing the proposed model of the paper, the results show the optimal performance of the proposed model and the studied parameters in line balancing have been significantly improved.

Kane et al. [18] investigate the transportation problem. The aim of this paper is to introduce a formulation of TP involving Triangular fuzzy numbers for the transportation costs and values of supplies and demands. They propose a two-step method for solving fuzzy transportation problem where all of the parameters are represented by non-negative triangular fuzzy numbers i.e., an Interval Transportation Problems and a Classical Transport Problem. To illustrate the proposed approach two application examples are solved. The results show that the proposed method is simpler and computationally more efficient than existing methods in the literature.

Zanjani et al. [19] investigate the Hybrid Flow Shop (HFS) scheduling problem. This study develops a multi-objective Robust Mixed-Integer Linear Programming (RMILP) model to accommodate the problem with the real-world conditions in which due date and processing time are assumed uncertain. The developed model is able to assign a set of jobs to available machines in order to obtain the best trade-off between two objectives including total tardiness and makespan under uncertain parameters. Fuzzy Goal Programming (FGP) is applied to solve this multi objective problem. Finally, to study and validate the efficiency of the developed RMILP model, some instances of different size are generated and solved using CPLEX solver of GAMS software under different uncertainty levels. Experimental results show that the developed model can find a solution to show the least modifications against uncertainty in processing time and due date in an HFS problem.

5 | Branch and Bound Algorithm

Branch and Bound is a public algorithm used to find the optimum solutions of different problems, especially in discrete optimization and combinational optimization. This algorithm counts all the solutions of a problem, meanwhile, there are lots of useless solutions that, by deleting them through estimating upper and lower boundaries, can be optimized. This method was first introduced for discrete programming by Land and Doig [20]. In this algorithm all the states preparing the probability of reaching better answers, will be studied and finally, the best answer will be chosen out of all the studied answers.

6 | Introducing Feasible Search Algorithm

The new algorithm is explained in the following order:

1. Start.
2. Put $K=1$.
3. Put $n=N$.
4. Put $MaxCost= +\infty$.
5. Put $NE=0$.
6. Put $i=1$.
7. Put $j=1$.
8. Choose a possible state (feasible) for $X_{i,j}$ from the set $S_{i,j}$ as if solution X is not repetitive.
9. Put $NE=NE+1$.
10. Calculate objective function for the present layout and copy it in variable Cost_{NE}.
11. If $Cost_{NE} < MaxCost$, put Best Cost=Cost_{NE} and Bestsolution=x.
12. If $j \leq n-1$, add one unit to j and go to step 8, otherwise go to step 10.
13. If $i \leq n-1$, add one unit to i, and go to step 7, otherwise go to step 14.
14. Print NE.
15. Put Best solution.
16. Print Best cost.
17. The end.

N means the number of facilities, $S_{i,j}$ means all the possible (feasible) states for $x_{i,j}$, also NE depicts the number of evaluations or the measured solutions.

7 | Case Study

The case study in this paper includes an industrial workshop producing different kinds of wooden and metal products. This workshop includes 17 facilities and 17 Locations. The aim of this paper is to reach an optimized settlement of the facilities in the locations based on the distance among the locations and the transportation flow among machines.

8 | Distance of the Locations

The distance among the Locations is shown in *Table 1*.

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Table 1. Distance of locations (meter).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	25	38	50	60	77	22	35	49	63	77	28	43	55	67	79	101
2	25	0	16	28	40	55	5.5	17.5	31.5	44.5	58.5	23	38	50	62	74	98
3	38	16	0	17	29	41	17.5	6.5	19.5	33.5	47.5	23	24	36	48	60	68
4	50	28	17	0	16	31	29.5	15.5	7.5	21.5	35.5	35	20	28	32	44	56
5	60	40	29	16	0	19	41.5	27.5	13.5	9.5	23.5	47	32	20	24	36	44
6	77	55	41	31	19	0	56.5	42.5	28.5	14.5	8.5	62	47	35	23	23	25
7	22	5.5	17.5	29.5	41.5	56.5	0	7	31	45	59	8	23	35	47	59	81
8	35	17.5	6.5	15.5	27.5	42.5	7	0	7	31	45	14	9	21	32	44	68
9	49	31.5	19.5	7.5	13.5	28.5	31	7	0	7	31	28	13	7	19	31	55
10	63	44.5	33.5	21.5	9.5	14.5	45	31	7	0	7	44	27	15	5	17	41
11	77	58.5	47.5	35.5	23.5	8.5	59	45	31	7	0	56	41	29	17	5	26
12	28	23	23	35	47	62	8	14	28	44	56	0	20	32	44	56	80
13	43	38	24	20	32	47	23	9	13	27	41	20	0	16	27	39	63
14	55	50	36	28	20	35	35	21	7	15	29	32	16	0	16	28	52
15	67	62	48	32	24	23	47	32	19	5	17	44	27	16	0	16	40
16	79	74	60	44	36	23	59	44	31	17	5	56	39	28	16	0	28
17	101	98	68	56	44	25	81	68	55	41	26	80	63	52	40	28	0

9 | Percentage of Transportation Flows

Percentage of displacements among machines is shown in *Table2*.

Table 2. Matrix of the transportation percentage among facilities.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	0	0	0	0	0	0	0	0	16.9	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	12.15	0	0	0	0	0	0	0	4.97
3	0	0	0	0	0	0	0	0	1.83	0	10.35	0	0	0	0	0	3.65
4	0	0	0	0	0	12.27	2.79	2.79	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	12.61	0	0	2.23	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0.01	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0.01	0	0	0	0.01
9	0	0	0	0	0	0	0	0	0	0	0.03	0.03	0	0	0	0	0.09
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0.37	0	0	0	0	0	0.41	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	3.67	1.2	5.27	2.39	3.99	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In the following discussion, the required model is defined generally and parametrically.

$$\text{Cost}=\sum_{i=1}^{17}\sum_{k=1}^{17}\sum_{j=1}^{17}\sum_{s=1}^{17} d_{i,k} w_{j,s} x_{i,j}\, x_{k,s}.$$

Subject to:

$$\sum_{j=1}^{17} x_{i,j}=1\ ;\ i=1,2,\ldots,17. \qquad (2)$$

$\sum_{i=1}^{17} x_{i,j}=1\ ;\ j=1,2,\ldots,17\ .$

$x_{12,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\},$

$x_{6,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16\},$

$x_{1,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\},$

$x_{17,j}=0\ ;j\in\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,17\},$

$x_{7,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{8,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{9,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{10,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{11,j}=0\ ;j\in\{1,3,4,5,8,10,11,13,14,15,16,17\},$

$x_{2,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{3,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{4,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{5,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{13,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{14,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{15,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{16,j}=0\ ;j\in\{2,6,7,9,12,14,15,16,17\},$

$x_{12,14}=x_{1,16},$

$x_{12,17}=x_{1,15},$

$x_{6,14}=x_{17,16},$

$x_{6,17}=x_{17,15},$

$x_{i,j}=0$ or 1 ; $i=1,2,\ldots,17$ and $j=1,2,\ldots,17$.

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Due to this point that in the current problem, based on real condition of the studied workshop, some new stipulations are added to that do not exist in base QAP, thus these stipulations are explained briefly in *Table 3*.

Table 3. Explanation of new stipulations.

Mathematical Stipulation	Limitation in Reality
$x_{12,j}=0$;j∈{1,2,3,4,5,6,7,8,9,10,11,12,13,15,16}.	Location 12 can only accept facility 14 and 17.
$x_{7,j}=0$;j∈{1,3,4,5,8,10,11,13,14,15,16,17}.	Location 7 can only accept facilities {6, 7, 9, 12, 2}.
$x_{2,j}=0$;j∈{2,6,7,9,12,14,15,16,17}.	Location 2 can only accept facilities {1, 11, 10, 8, 4, 3, 2, 5, 13}.
$x_{12,14}=x_{1,16}$.	If facility 14 is settled in Location 12, facility 16 shouldbe settled in Location 1.

11 | Comparison of the Results of the Proposed Algorithm and Branch and Bound Algorithm

Table 4 shows the results of performing the two algorithms by a common computer (CPU: 3.2 GHz & RAM: 4096MB).

Table 4. Results of the two algorithms' performance.

Branch and Bound Algorithm			Proposed Algorithm		
Time of performance (seconds)	Number of Evaluations made	Optimum amount	Time of performance (seconds)	Number of evaluations made	Optimum amount
2256.3	19353600	1400.845	1003.1	9676800	1400.845

As it is shown in *Table 4*, the new algorithm could reach the optimum answer by spending less time and making less evaluations. In fact, the proposed algorithm (1003.1 sec) finds the optimal solution in a shorter amount of time than the Branch and Bound (BB) algorithm (2256.2 sec). Moreover, the iteration number of the proposed algorithm (9676800) is lower than the iteration number of the BB algorithm (19353600). The optimal layouts are similar in both layouts. The objective functions of both algorithms are equal to 1400.845. The optimized facility layout is inserted in *Table 5*. For example, facility 16 should be located in the first Location.

Table 5. Optimum facility layout.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0

12 | Conclusion

To reach an appropriate layout design, it is necessary to define and solve the related nonlinear programming problems. Thus, using computer to solve the related problems seems to be important to the researchers of this area of study. But usually, the designs produced by computers for solving big problems need more time. In fact, this is a QAP; therefore, if we use a BB algorithm, we should analyze all states. According to the proposed approach, it is impossible to install some pieces of machinery in certain locations (stations) in some QAPs. Therefore, it is better to use an algorithm that ignores unfeasible installations. This minor change in BB algorithm can accelerate it and reduce its runtime.

In this paper, an algorithm is proposed that can be useful for better performance of the known algorithms such as BB and so it can identify the best answers by spending less time.

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