



Applying Mathematical Modeling to Create Job Rotation for Improve Workforce Performance in Semi-Automatic Systems


M. Bakhshi, S. E. Hashemi*, H. Dezhdar

Department of Industrial Engineering, Arak Branch, Islamic Azad University, Arak, Iran.
(*Corresponding Author's Email Address: emad.hashemi88@gmail.com)

ABSTRACT

In this research, we present a mathematical model for allocating people to different jobs and shifting employees between related jobs. This action will reduce the repetitive activities workload and ergonomic risks at the planned time horizon, and finally increases the organization's efficiency. In this proposed model, the devices are semi-automatic and it is possible to allocated more than one task to one person. Regarding the modeling and the case study of the constraints, it is shown that the complexity of this problem type is NP-Hard, and the result of accurate methods for solving the problem is not possible in a reasonable time. Due to this Simulated Annealing (SA) algorithm is used to study the proposed model and comparison of the results of SA algorithm with the results of precise optimization methods shows the better performance of the Simulated Annealing algorithm in terms of the time and answer quality.

Keywords: Job rotation, Mathematical modeling, Physical injuries, Simulated annealing algorithm.

 **Article history:** Received:10 August 2020 Reviewed:03 September 2020 Revised:25 October 2020 Accepted:29 November 2020

1. Introduction

Workforce is considered as the best, most important and effective factor for the survival and development of research organization, another items such as devices and equipment, asset and technology, etc. are placed in the next category. It is clear, when it comes to workforce the related factors such as thought, creativity, knowledge, being up to date, etc. are also considered [1]. An efficient and well workforce which has job specification is most important asset of these organizations that can not be replaced quickly and easily. The point is that employees can show their talents in work tasks that they have most adaptability with them. Unfortunately, this adaptability does not occur always. Many talents have been lost due to lack of adaptability in the jobs that is not related to their records, interest and education and a lot of job dissatisfaction has happened because of this issue and it has affected the goals. However, it is beneficial to organizations, especially research organizations, if this issue will be modified at any time and from anywhere [2]. The success of organizations depends on employee satisfaction, motivating and creativity, who are striving to achieve this by knowing the goals of the organization to fulfill

their demands. In this case, managers will be able to create an enabling environment, utilize the capabilities of individuals, and ultimately increase the productivity of the organization [3]. One of the ways to increase productivity in an organization is job rotation, job rotation means moving people between two or more job in a planned manner so that each job requires different skills and responsibilities [4, 5], also in the jobs that are quite similar, job rotation is for changing the position of employees in different environments, for example, in a production workshop or assembly line, the ambient light or sound caused by assembly and production, does not spread equally in the hall, for this reason, the relocation of persons helps them to be equally exposed to such conditions [6]. Job rotation refers to the phenomenon that an employer rotates employees assigned positions from period to period. It happens in both private sectors and public sectors [7]. Job rotation is used as an organizational approach by manufacturing industries aimed at mitigating exposure to high physical workload and monotony [8]. Job rotation is an aspect of training undergone in an organization involving both managers and non-managers. It occurs when an employee is moved from one job, department, or task to another after spending sometime in a particular job, task, assignment, or department with a view of enabling the employees to acquire more skills, talents and knowledge. In a way, job rotation is influenced by coordination and is meant to broaden ones outlook, knowledge, skill, and contact [9].

In research [10], it has been shown that during a case study of organizations, employees suffer from long-term damage to the waist or other parts of their body involved with repetitive activities. The implementation of job rotation helps to perform repetitive activities to their lowest state and reduces the damages by moving people between jobs. In general, the job rotation reduces the work load applied to each employee, it also provides the general attitude of work process for each employee, which will increase job satisfaction and finally reduce the cost of workforce [11]. In order to implement a job rotation, the factors such as the number of employees, devices and shifts work, the employees skill level and other effective factors need to considered [12]. In this study, in addition to applying the factors and restrictions mentioned, the semi-automatic limitations of the devices are also considered, which illustrates the completeness of the main model.

2. Problem Definition and Modeling

2.1. Model Assumptions

Each worker has a certain level of skill that their skill level still remains constant during this study. The number of machines studied is clear. The number of employees that required for each group is constant. In this workshop, assigned a device for two workers for each one individually, and assigned two devices to the third person. Each worker is allowed to take r number of leave of absence. Each person is permitted to have a certain number of days off from work.

2.2. Parameters and Problem Variables

Z: Workload cost.

I: Number of employees.

N: Number of available machines.

D: Weekdays.

T_d: Number of shifts on day D.

K: The number of different skill levels of employees (low, medium and high).

P_i: The minimum number of machines that i person can work on them simultaneously.

r_{ik}: The minimum number of shift works for the i person with the skill level k.

C_{ijtd}: Cost variable that represents the cost in terms of workload for the worker i with each skill level working with the set of machinery.

2.3. Main Model

$$\text{s.t.} \quad \text{Min } Z, \tag{1}$$

$$\sum_{d=1}^D \sum_{j=1}^N \sum_t^{T_d} C_{ijtd} X_{ijtdk} \leq Z, \quad \forall i, k \tag{2}$$

$$\sum_{j=1}^N \sum_{t=1}^T \sum_d^D X_{ijtdk} \geq r_{ik}, \quad \forall i, k \tag{3}$$

$$X_{ijtdk} \leq s_{ikt}, \quad \forall i, t, k, j, d \tag{4}$$

$$X_{ijtdk} + X_{idjk(t+1)} \leq 1, \quad \forall i, t, k, j, d \tag{5}$$

$$j * \sum_i^I X_{ijtdk} \leq \sum_i^I \sum_j^J X_{ijtdk} \geq r_{ik}, \quad \forall i, t, k, j, d \tag{6}$$

$$X_{ijtdk} \leq h_{ikt}, \quad \forall i, t, k, j, d \tag{7}$$

$$\sum_j^{j+p_i} X_{i,j+p,t} \geq p_i, \quad \forall i, k, t, d \tag{8}$$

$$X_{ijtdk} \in 0 \text{ or } 1 \quad C_{ijtd} \in R. \quad \forall i, j, t, d, k \tag{9}$$

2.4. Interpretation of Objective Function and Problem Constraints

2.4.1. Objective function problem

In *Eq. (1)* and *(2)*, the purpose is to determine the maximum workload z found in employees scheduling to be minimized. For this purpose the problem requires the integer to find a set of values of X_{ijtdk} which is related to the job rotation schedule to minimize the maximum amount of E among employees.

2.4.2. Problem constraints

Eq. (3) expresses that each person is allowed to a maximum number of days off. *Eq. (4)* each person only works with the device which has the skill to work with it. According to *Eq. (5)* each person should not have the same activity in the two continuous shifts. *Eq. (6)* in order to prevent persons and machines to being unemployed, the total number of activities in the time horizons must be a multiple of j . *Eq. (7)* is a constraint to ensure that no person is absent on day d in the shift t . Each person works at the same time with the j number of the semi-automatic device. With the help of *Eq. (8)* the semi-automatic production and allocation more than one machine to an operator are considered. So that each person can be allocated to machines which are next to each other. And finally *Eq. (9)* considers that our main variable X_{ijtdk} is of type zero-one and the costs of each Employees activity C_{ijtd} can take any real number.

Das et al. in one paper, a model for a linear programming problem with absolute value functions is presented that can be used to solve linear problems [13].

3. Results

3.1. Results of the GAMS

The proposed mathematical model has been studied by using numerical data In the GAMS software. Since the proposed mathematical model is Integer Programming, the answer that comes from solving the model in the GAMS software by Solver CPLEX which is based on the Branch and Cut method, will be the best public answer. For this purpose, a T600 processor with a capacity of 2.00 GHz has been used.

3.2. Results of Simulated Annealing Algorithm

Due to the problem is NP-Hard, and it is mentioned in related articles [14] to solve this model, the Simulated Annealing (SA) has been proposed.

SA is similar to heating solids in this way, if we heat a solid and bring it to the melting point, then make it cool, its construction details depends on how it cools down. If we cool that solid slowly, we will have big crystals which can be formed as we want but if we cool it fast we do not

get what we want and the process of cooling the system (material) by reducing the temperature until it changed into a frozen state, was simulated. This method moves step-by-step to the optimal answer by creating and evaluating consecutive answers. To move, a new neighborhood is created and evaluated randomly. In this method, the points near the given point in the search space are checked. If the new point is a better point, it reduces the cost function [14].

The SA optimization method is that starting with a random initial response for decision variables, the new answer is generated randomly in the vicinity of the previous answer using an appropriate neighbor construct. So one of the important issues in SA is the neighborhood production method.

3.2.1. Starting point

The point in the search space is where we start the search from there. This point is usually chosen randomly.

In this case, a procedure for assigning tasks to individuals in a discipline is generated. For example, in the problem sample considered in this paper, an assignment is a zero-order code vector and a result of $i \times j \times t \times d \times k$, indicating the allocation of the device to the personnel with the desired skill on the working day and the corresponding shift, forming the answer string. This field seems to be the answer to a natural choice.

3.2.2. Motion generator

This generator is responsible for generating the next scenarios and, based on the calculation of the current point cost and the cost of the next point, determines the position of the algorithm.

3.2.3. Cooling schedule

Parameters that specify how to cool down the algorithm. How often and how much temperature will be reduce and how much start and finish temperatures are. SA optimization method is a numerical method with intelligent random structure. The flexibility of minimizing the length of random steps in the SA algorithm prevents any inconsistency and disparity in combination with the model. In addition, SA's ability to quit local optimizations and convergence towards a global optimization of the theoretical aspect and in practical applications has been proven. Optimization of nonlinear functions and non-complete problems with classical methods is difficult and sometimes impossible, and numerical optimization methods should be used. To solve the problem using SA method, mathematical modeling is initially done.

3.2.4. Acceptance criteria (one move)

In local optimization algorithms, the new answer is only accepted if the target function is improved. Whereas, in SA, not only the answers that improve the target function are accepted, also inappropriate responses are accepted. A thermodynamic law explains the relationship

between temperature “t” and the probability of increasing the size of the energy “ΔE”, this law is as follows:

$$p(\Delta E) = \text{exxp}\left(\frac{-\Delta E}{kt}\right).$$

Where “K” is a constant which called the Boltzmann constant. Using this thermodynamic law, the probability of accepting bad moves is calculated by the following equation:

$$p(\text{pm}_i \rightarrow \text{pm}_{i+1}) = \exp\left(\frac{-\Delta c}{t}\right) > r.$$

Which is here:

Δc: Change in evaluation function.

t: Temperature.

r: A random number between zero and one.

p: The probability of moving to the new answer.

Moving to the new answer will be done if the new answer is better than the current answer or the value of the probability function of movement is greater than the random number of the domain [0,1). Otherwise, the searcher will generate and evaluate another new answer. This move continues step by step until the condition for stopping the algorithm is reached. An important problem in the proposed algorithm of “SA” is the examination of the condition of the equilibrium and the condition of the proposed stopping criteria.

3.2.5. Equilibrium condition

In general, in the SA method, the number of accepted responses or the total number of generated responses at each temperature is considered as the basis for examining the equilibrium condition at that temperature. The number of replacements at each temperature to check the condition of equilibrium is called "period". This number is a parameter of the “SA” pattern which must be determined.

3.2.6. Stop condition

Two criteria are used to check the stop condition. A criterion is for reaching the final temperature. Another criterion is based on the ratio of dispersion rate of the accepted responses at the current temperature to the average difference between the values of the target function for the responses accepted at the initial temperature and the current temperature. If this ratio is low, that's mean the system got to freeze point and stops. Otherwise, the proposed algorithm will continue with decreasing temperature.

In order to implement the “SA” algorithm in the allocation problem, the necessary codes are written with “MATLAB” and compare the results with the implemented model in “GAMS”. The results of the algorithm show that for the above problem, the value of the objective function is 351 and the corresponding allocation vector is obtained in a shorter time period than the “GAMS” model. However, it is also necessary to compare the results for larger problems in this regard.

To do this, we select different parameters by designing the experiments. We also examine a large range of the problem, from 11 machines, 8 personnel to 40 personnel and 100 machines. All problems solved by MATLAB and the specified computer specifications.

3.3. SA Results Simulation

The efficiency of “SA” largely depends on controlling the parameters, the initial temperature T , the probability of moving to the new answer P_c , and the probability of accepting the next point P_m , and in order to eliminate the probable deviations from the production of the initial “SA” responses and to ensure the robustness of the “SA”, for each combination of parameters (Totally 8 combinations) The number of needed and necessary tests are produced by the 5 initial sequence generated randomly and tested in each test. In order to reduce the variance for each 8 factorial experiments, these 5 initial sequences will remain unchanged. A more robustness approach is to generate 5 random problems, and for each of these problems a random sequence is selected. During the implementation of the SA algorithm for each combination of parameters (Totally 8 combinations) these problems and initial sequences will remain unchanged. The combination of parameters for a problem with a specific size is determined for each combination of parameters and for each experimental run, consider the lowest observed value for the production time after 300 executing, then get the average of these values on 5 runs, select the combination of parameters that give the lowest average production time.

Table 1 shows two levels of factors investigated in DOE. In each experiment, the algorithm is repeated 300 times, and it is assumed that after this number of replicates, the answer is sufficiently close to the optimal answer.

Table 1. Levels of DOE factors.

No	Factors	Level 1	Level 2
1	T	1E3	2E3
2	P_c	0.9	0.7
3	P_m	0.05	0.005

In this research, problems of small, medium and large scale are examined. The dimension of the examined problems are 4 machines and 3 personnel, 8 machines and 6 personnel, 10 machines and 8 personnel, for small-scale problems, 20 machines and 10 personnel, 30 machines and 12 personnel, 15 personnel and 49 machines, for problems with medium scale and finally 25 personnel and 60 machines, 30 personnel and 80 machines, 35 personnel and 90 machines, 40

personnel and 100 machines for large-scale problems. The planning period is considered a two-shift week and skill level is also low, medium, and high.

By solving 10 types of problems, the levels of DOE factors and dimensions of the problem are presented in *Table 2*.

Table 2. Levels of DOE factors and issues dimensions.

Dimensions of the problem	Coding	Parameters		
		Work time	Machine	Workforce
small	1	10	4	3
small	2	10	8	6
small	3	10	10	8
medium	4	10	20	10
medium	5	10	30	12
medium	6	10	49	15
large	7	10	60	25
large	8	10	80	30
large	9	10	90	35
Large	10	10	100	40

4. Comparing Results

As can be seen from *Table 3* and *Figure 1*, the solving results using SA are close to the optimal final results obtained by solving the GAMS software and it is been achieved in a short time, indicating that the SA algorithm, in terms of both efficiency and solving problem time, has advantage over the results of GAMS.

Table 3. Compare the results of problem solving with GAMS and SA software.

GAMS Result	Time Solution	SA Result	Time Solution	Gap ANALYZE
720	10 (s)	720	3 (s)	0 %
1430	50 (s)	1430	10 (s)	0 %
1715	2.5 (min)	1715	1 (min)	0 %
3070	8 (min)	3.147	2 (min)	2.5 %
4104	15 (min)	4.277	3.5 (min)	4.3 %
6327	25 (min)	6.681	6 (min)	5.6 %
7505	40 (min)	8.090	8 (min)	7.8 %
8720	60 (min)	9.435	10 (min)	8.2 %
9580	90 (min)	10.471	17 (min)	9.3 %
10504	150 (min)	11.607	25 (min)	10.5 %

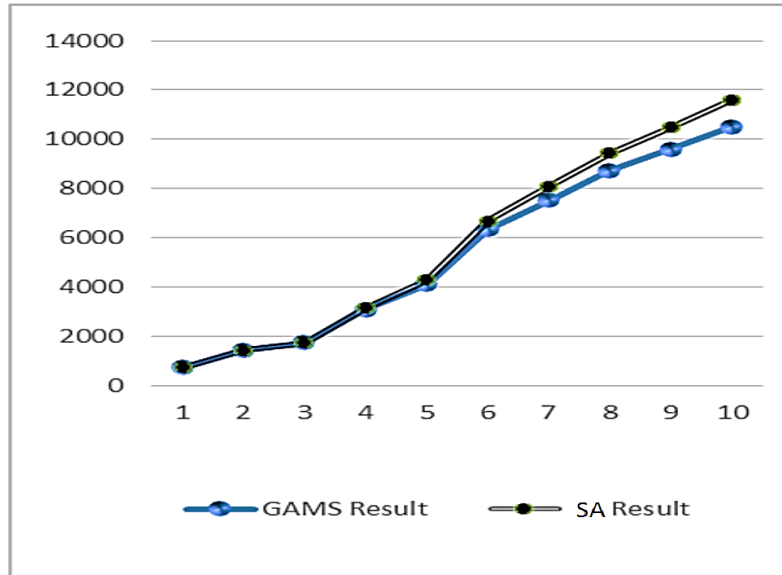


Figure 1. Compare the results of problem solving with GAMS and SA software.

5. Conclusion

In this article, after addressing the importance of the workforce, we introduced job rotation as one of the effective ways to increase productivity in organizations. According to research, job rotation reduces the workload imposed on each employee, which leads to increased job satisfaction and ultimately reduced costs of manpower. Then, a mathematical model was proposed to allocate people to different occupations and to move employees between occupations of the same class. The proposed model is designed so that the devices are semi-automatic and it is possible to assign several activities to one person. The proposed model is intended to minimize the damage caused by employees exposed to the ergonomic hazards of the work environment, in this regard, the skill of the staff and the difference in SA to compare the obtained results. The results of this solving of the model indicate that in problems by large sizes the GAMS software is not able to solve them or it will take a long time, on the other side the SA algorithm solve them in a short time with very close results to the optimized answer.

Today, with the advancement of science, work systems are moving towards automation. This causes working conditions to change automatically or semi-automatically. Hence, people can sometimes interact with several systems. This has not been discussed in the models presented in previous articles, but this article provides a more comprehensive review of this model. On the other hand, the level of skill, experience, expertise and even talent of no two human beings is the same. This factor is one of the influential factors in the model, which was sometimes ignored, but has been implemented in detail in this model. Applying more parameters and variables and considering all the constraints will lead to a more comprehensive model. This is not possible except by careful examination and even implementing the model in a real environment and case

study. On the other hand, to minimize the error rate and reduce the access time to the final answer, the model can be used with different metaheuristic algorithms, and the results can be examined in detail.

References

- [1] Olorunsola, R. (2000). Job rotation in academic libraries: the situation in a Nigerian university library. *Library management*, 21(2), 94-98. <https://doi.org/10.1108/01435120010309434>
- [2] Wickens, C. D., Gordon, S. E., & Liu, Y. (1998). *An introduction to human factors engineering*. Pearson.
- [3] Yaoyuenyong, S., & Nanthavanij, S. (2006). Hybrid procedure to determine optimal workforce without noise hazard exposure. *Computers and industrial engineering*, 51(4), 743-764. DOI: 10.1016/j.cie.2006.08.018
- [4] Azizi, N., Zolfaghari, S., & Liang, M. (2010). Modeling job rotation in manufacturing systems: The study of employee's boredom and skill variations. *International journal of production economics*, 123(1), 69-85.
- [5] Costa, A. M., & Miralles, C. (2009). Job rotation in assembly lines employing disabled workers. *International journal of production economics*, 120(2), 625-632.
- [6] Yoon, S. Y., Ko, J., & Jung, M. C. (2016). A model for developing job rotation schedules that eliminate sequential high workloads and minimize between-worker variability in cumulative daily workloads: Application to automotive assembly lines. *Applied ergonomics*, 55, 8-15. Doi: 10.1016/j.apergo.2016.01.011
- [7] Yu, J., & Zhang, J. (2020). A market design approach to job rotation. *Games and economic behavior*, 120, 180-192. DOI: 10.1016/j.geb.2020.01.002
- [8] Moussavi, S. E., Zare, M., Mahdjoub, M., & Grunder, O. (2019). Balancing high operator's workload through a new job rotation approach: Application to an automotive assembly line. *International journal of industrial ergonomics*, 71, 136-144.
- [9] Oparanma, A. O., & Nwaeke, L. I. (2015). Impact of job rotation on organizational performance. *Journal of economics, management and trade*, 183-187. DOI:10.9734/BJEMT/2015/12051
- [10] Frazer, M., Norman, R., Wells, R., & Neumann, P. (2003). The effects of job rotation on the risk of reporting low back pain. *Ergonomics*, 46(9), 904-919. DOI: 10.1080/001401303000090161
- [11] Kuijjer, P. P. F., Visser, B., & Kemper, H. C. (1999). Job rotation as a factor in reducing physical workload at a refuse collecting department. *Ergonomics*, 42(9), 1167-1178. DOI: 10.1080/001401399185054
- [12] Kheir Khah, A., Malek Ahmadi, A., & Masibidgoli, M. (2017). Presenting a model of intercellular ergonomic workflow problem in cell production systems taking into account leave and staff skills. *9th international conference of the Iranian association for operations research*. https://research.basu.ac.ir/_Pages/Research.aspx?ID=1022046
- [13] Das, S. K., Edalatpanah, S. A., & Mandal, T. (2019). A new method for solving linear fractional programming problem with absolute value functions. *International journal of operational research*, 36(4), 455-466.
- [14] Seçkiner, S. U., & Kurt, M. (2007). A simulated annealing approach to the solution of job rotation scheduling problems. *Applied mathematics and computation*, 188(1), 31-45. <https://doi.org/10.1016/j.amc.2006.09.082>

