Int. J. Res. Ind. Eng. Vol. 9, No. 2 (2020) 183–201

International Journal of Research in Industrial Engineering

www.riejournal.com

Fuzzified Synthetic Extent Weighted Average for Appraisal of Design Concepts

O. M. Olabanji *

Department of Mechanical Engineering, Federal University of Technology, Akure, Nigeria.

A B S T R A C T

Decision-making models such as Analytical Hierarchy Process (AHP), Weighted Decision Matrix (WDM), Pugh Matrix and the likes have been able to assist in the decision process considering the objectives of each evaluation criteria in the alternatives. However, these models need to consider the qualitative and subjective nature of the design features. In order to reduce the unbalanced scale of judgment and the uncertainty associated with the crisp information in the decision process, fuzzified and hybridized models are necessary. Existing hybridized decision models applied for machine concept selection deploy several design features and sub-features at the conceptual product design, which thus make the decision making process to be tedious. In light of this, this article presents a hybridized decision-making model, which harness the comparative strength and computational integrity of fuzzy pairwise comparison matrix and fuzzy weighted average, to numerically analyse a reasonable amount of machine design features, thereby making decision making process less tedious. Design for reconfigurability and functionality which are peculiar to reconfigurable machines was introduced using a Reconfigurable Assembly Fixture (RAF) as a case study while other design features related to design concept evaluation were grouped under design for X. The result of the hybridized model shows that, concept three is the optimal design from four sets of designs. This is compared to previous publication using the RAF design concepts with different design features and sub-features. The comparison indicates that there is a close range in the final values of the designs due to the inclusion of several sub-features in the decision process which were not used in the previous study.

Keywords: Fuzzified decision-making computations, Synthetic Extent Weighted Average (SEWA), Design for reconfigurability, Design for functionality, Design for X, Multi-criteria decision making.

Article history: *Received: 01 January 2020* *Revised: 09 March 2020* *Accepted: 26 May 2020*

1. Introduction

The emergence of reconfigurable manufacturing paradigm due to the dynamic nature of the market has called for the extensive design of equipment and machines. Manufacturers are faced with the challenge of demands for customized designs that are having shorter life cycle and may

* Corresponding author

E-mail address: omolabanji@futa.edu.ng

DOI: 10.22105/riej.2020.214239.1111

not have a larger share of the competitive market [1, 2]. In order to proffer solution to the demand for customization in design, machines serving a particular function are designed around the same part family in order to be suitable for varieties of parts. Given this, the design of reconfigurable machines has emerged in recent times to serve as enabling equipment in a Reconfigurable Manufacturing System (RMS) [3]. An example of this reconfigurable machine is a Reconfigurable Assembly Fixture (RAF) that is primarily designed as aiding equipment in a Reconfigurable Assembly System (RAS) [4, 5]. An RAF is designed to uniquely locate and position a workpiece while other assembly processes are carried out on the workpiece [3, 6].

1.1. Decision Making in Engineering Design

Decision-making in the engineering design process (***Figure 1***) for selection of optimal design concept usually involves the application of design for X tools to different design alternatives in order to select the optimal design [7, 8]. These features or tools include design for manufacturing, design for assembly and disassembly, design for reliability, design for maintainability, design for serviceability, design for environment, and design for life cycle cost [9]. These tools have been implemented in decision-making using different multicriteria decision-making tools. They have provided improved decision-making processes to the selection of optimal design alternative at the conceptual phase of designs [10, 11]. However, the design of the reconfigurable machines requires the introduction of more features to cater to their agile and changeability characteristics. Given this, the design for reconfigurability and functionality is considered as peculiar features to reconfigurable machines in order to provide a robust and systematic approach to the decision on the optimal design of reconfigurable machines.

The design for reconfigurability and functionality can be disintegrated into sub-features in order to further improve the decision process. An excellent method to analyze the performance of a reconfigurable machine in terms of reconfigurability is to consider the characteristics of reconfigurable manufacturing principle as sub-features. These characteristics are modularity, integrability, customization, convertibility, scalability, and diagnosability [12]. The design for functionality can be considered from the performance of the design in terms of the operational requirements. It highlights some performance index as sub-features that are related to the purpose for which the reconfigurable machine is designed. It is worthwhile to state that these indices differ for different machines and this makes it a task for the design engineer to identify and analyze performance indices that can be accrued to the optimal design. The contributions of the sub-features to the performance of the main design features and the relative importance of the main design features in the optimal design are needed to be determined in the form of weights because they are needed in the decision making on optimal design [13, 14].

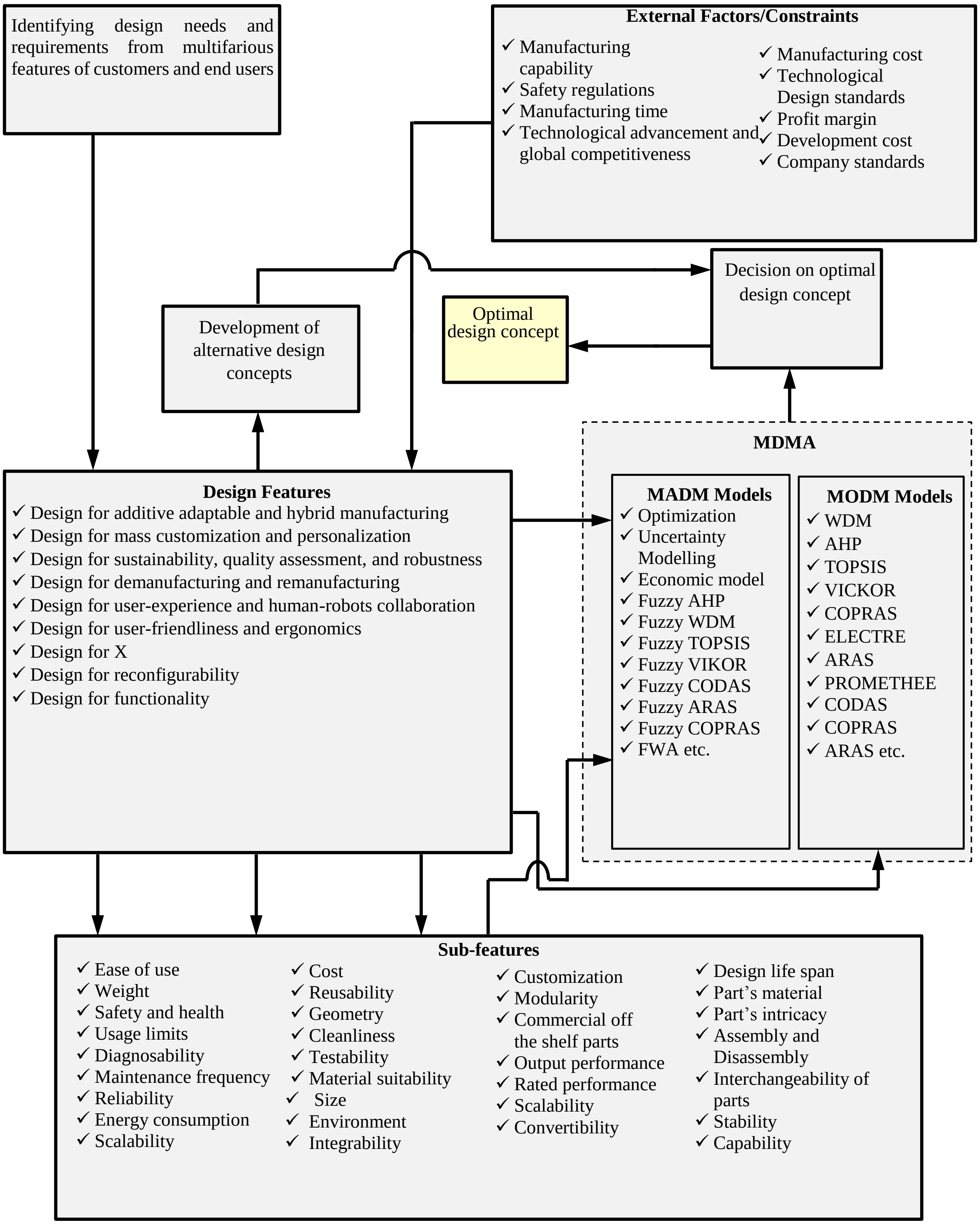

Figure 1. Decision-making in the engineering design process.

1.2. Approaches to Decision Making

Multicriteria Decision Making Analysis (MDMA) is a prominent tool that is applied in different fields, such as engineering, science, and management, to facilitate decision making. The MDMA models are classified into Multi-Objective Decision Making (MODM) analysis and Multi-Attribute Decision Making (MADM) analysis [15]. The MODM models are usually employed when the details of sub-criteria are given less consideration in the decision process. Examples of the MODM models are the weighted decision matrix (WDM), Technique for Order Performance by Similarity to Ideal Solutions (TOPSIS), and Analytic Hierarchy Process (AHP). A general form of the AHP is the Analytic Network Process (ANP) which considers the interdependence among the sub-criteria and alternatives using a system of pairwise comparison matrices to obtain weights for the decision criteria and sub-criteria. However, when it is required to consider various dimensions of the design features and their sub-features, the MADM models are applied. MADM models are developed by fuzzifying and hybridizing the MODM models in order to produce an optimised decision-making process [16-18].

Selection of optimal conceptual design from a set of alternative designs usually requires that all necessary design features and their sub-features are considered. The primary goal of the design engineer at the concept selection stage is to select a design that embodies most of the design features in order to satisfy the users' requirement. Considering the fact that these sub-features have different units and dimensions with interrelated functions, there is a tendency of encountering bias judgement in quantifying these features during the comparison. The design engineer tends to avoid bias and ambiguity in the decision process by applying the MADM models using fuzzy sets and linguistic terms. It provides better results than the MODM models, which involves apportioning of crisp values in the decision process. Also, hybridizing two or more MADM models will enhance the computational integrity of the decision process because each of the MADM models has their strengths and weaknesses. Hence the hybridization will harness the strengths of the models and develop a new framework for integration of the models. Several efforts has been made in the application of hybridized models to decision making process [29-31]. There is a need to continually apply these hybridized models for decision making in the design process because of the importance of design concept selection. In essence, a good way to identify the optimal design is applying these hybridized models by considering several design features and sub-features that are applicable to the optimal design [7, 32]. This will ensure that all design features and sub-features have been considered before choosing an optimal design for fabrication. However, the application of numerous design features with various sub-features in [37] makes the decision process tedious, hence design engineers need to continually seek for a way to analyse the design features and sub-features such that they are not numerous but encompassing. Also, from another perspective, the comparison of hybridized models in order to know the best hybridized model will be another topic of interest that will not be addressed in this study because different hybridized models harnesses the computational strengths of two or more MADM models [18, 37].

In order to achieve effective operation of RMS, various efforts have been made by researchers to evaluate and select optimal system configuration using various decision-making models [19-21]. Gupta et al. [22] proposed a decision model for the selection of optimal system configuration of machine types in order to improve the efficiency of a reconfigurable manufacturing system. The criteria of the decision hierarchy considered were productivity, convertibility, scalability, and cost. The weights of these criteria were determined by Shannon's entropy and six alternative system configurations were compared in pairwise comparison matrices whose principal eigenvectors represent the relative importance of the configurations. Results obtained for the system's configuration are in terms of expected production, minimum increment for convertibility, cost per increment capacity, and initial investment. Similar to this approach is the application of grey relational grade method for evaluation of different system configuration in order to have a system that is sensitive to small deviations in multiple control measures [23]. In this case, four alternative configurations are compared using system operational parameters such as lot size and customer's and orders priority rule. The decision model provided four scenarios from the grey relational analysis. Each scenario had a different ranking of the configurations but indicated the same configuration as optimal.

Also, from another point of view, a reconfiguration decision making based on game theory algorithm using the Gale-Shapely model was proposed by Renna [24]. The reconfigurable machines, in this case, are divided into men and women set base on their workload and capability of the machines to perform base technical operation or perform with auxiliary modules. The model allocates technical operations to machines in the system in order to allow reconfiguration of the reconfigurable machines. Results obtained from the simulation indicated reduction in average workload and job shop time, however, the bottleneck shiftiness value increases. More also, in line with the work of Rehman [23] and Gupta et al. [22], a multi-attribute decision-making method based on VIKOR was proposed by Yi et al. [25] to evaluate design schemes of reconfigurable machine tools. Three evaluation indicators were established based on module similarity between the reconfigurable machine tool and the prototype machine tool to evaluate five design schemes. These indicators are in the form of modules which are chain module, interface complexity module, and reconfiguration cost module. The proposed decision model was compared with Simple Average Weighting (SAW) and TOPSIS, and the three decision models indicate the same optimal design scheme.

To this end, the application of decision models in reconfigurable manufacturing systems has gained attention selecting optimal system configuration and evaluation of design schemes of reconfigurable machine tools, but the consideration of these decision models in the selection of the optimal design of other reconfigurable machines aside from the reconfigurable machines tools requires attention. Hence, this article is proposing the application of a MADM model to identify the optimal design of a RAF. The design of MADM models for the decision on optimal design is an area of research that needs attention because the implication of not having an optimal design before engaging in fabrication is threatening to the manufacturer. Hence, this article suggests that, aside from the design for X tools that are generally known as design features, the design for reconfigurability and functionality can be integrated to decision making in the engineering design process particularly for the development of reconfigurable machines. In order to implement this, a Synthetic Extent Weighted Average (SEWA) is developed based on left and right score of a Triangular Fuzzy Number (TFN) and using the reconfigurable assembly fixture as a case study.

2. Mathematical Formulation for the SEWA

Consider k number of design alternatives (D_{ak}) that are selected for decision making with a mixed scenario of m number of design features (d_{fm}) and n number of sub-features (S_{fn}) allotted to each design features. In order to compensate for the multifarious dimensions of the sub-features and design features, it is necessary to assign linguistic terms (***Table 1***) using an interval-valued fuzzy set with fuzzy number P using a TFN which membership function $\mu_p(y)$ is contained in [0 1] and defined as ***Eq. (1)*** (***Figure 2***) [26, 27].

$$\mu_\rho(Y) = \begin{cases} \frac{1}{v-u} y - \frac{u}{v-u} & y \in [u \quad v], \\ \frac{1}{v-w} y - \frac{w}{v-w} & y \in [v \quad w], \\ 0 & \text{Otherwise.} \end{cases} \tag{1}$$

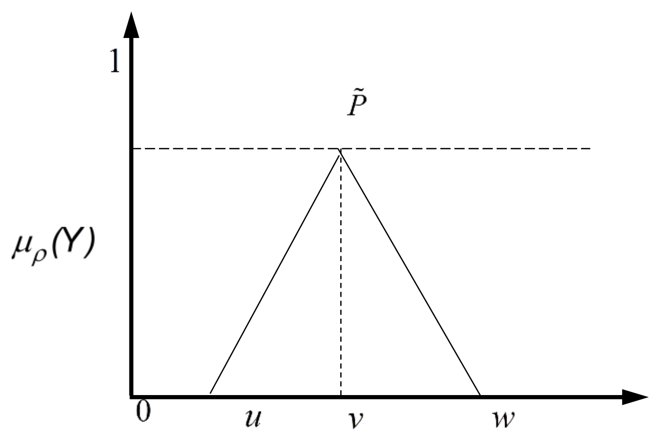

Figure 2. Membership function for a TFN.

Where $u \leq v \leq w$ and u, v and w represent the lower, modal, and upper values of the fuzzy number P, respectively. The design features can be rated according to their level of importance as needed in the optimal design [14]. The inclusion of sub-features membership functions for all the design alternatives will provide a robust decision-making process because an analysis of the design characteristics of all the concepts would have been considered during the pairwise comparison process [1, 28].

Table 1. TFNs and linguistics terms for pairwise matrices and weighted average.

Pairwise Matrix			Weighted Average		
Linguistic terms for rating of relative significance of design features in the optimal design	Triangular fuzzy scale membership function	Crisp value of ranking	Linguistic terms for ranking of availability of sub-features in the design concepts	Triangular fuzzy scale membership function	Crisp value of rating
Equally Important (EQI)	1 1 1	1	Very High (VH)	2.5 3 3.5	5
Weakly Important (WEI)	1 1.5 2	2	High (H)	2 2.5 3	4
Essentially Important (ESI)	1.5 2 2.5	3	Medium (M)	1.5 2 2.5	3
Very Strong Important (VSI)	2 2.5 3	4	Low (L)	1 1.5 2	2
Absolutely Important (ABI)	2.5 3 3.5	5	Very Low (VL)	0.5 1 1.5	1

The motive for applying the fuzzified pairwise comparison matrix is to determine the weights of the design features and sub-features from the Fuzzy Synthetic Evaluation (FSE) with respect to their relative importance in the optimal design. The pairwise comparisons for the design features (***Eq. (2)***) and sub-features (***Eq. (3)***) are obtained from judgement matrix of the form $\tilde{C}=\left\{\tilde{c}_{gi}^{j}\right\}$ which can be presented as [29]

$$\tilde{D}_f=\begin{pmatrix} \tilde{d}_{fg1}^{1} & \tilde{d}_{fg1}^{2} & \dots\dots\dots & \tilde{d}_{fg1}^{s} \\ \tilde{d}_{fg2}^{1} & \tilde{d}_{fg2}^{2} & \dots\dots\dots & \tilde{d}_{g2}^{s} \\ \vdots & \vdots & & \vdots \\ \tilde{d}_{fgk}^{1} & \tilde{d}_{fgk}^{2} & \dots\dots\dots & \tilde{d}_{fgk}^{s} \end{pmatrix}, \tag{2}$$

$$\tilde{S}_f=\begin{pmatrix} \tilde{s}_{fg1}^{1} & \tilde{s}_{fg1}^{2} & \dots\dots\dots & \tilde{s}_{fg1}^{s} \\ \tilde{s}_{fg2}^{1} & \tilde{s}_{fg2}^{2} & \dots\dots\dots & \tilde{s}_{fg2}^{s} \\ \vdots & \vdots & & \vdots \\ \tilde{s}_{fgk}^{1} & \tilde{s}_{fgk}^{2} & \dots\dots\dots & \tilde{s}_{fgk}^{s} \end{pmatrix}. \tag{3}$$

Where $\tilde{c}_{ij}$ is a TFN that can be represented by $\left(u_{ij} \ \ v_{ij} \ \ w_{ij}\right)$ as presented in ***Eq. (1)***. For i=1, 2, 3…. k, j=1, 2, 3……s, such that, when i = j, then $\tilde{c}_{gi}^{j}=\{1 \ 1 \ 1\}$. The value of the FSE is required from the fuzzy pairwise comparison matrices for all the design features and the sub-features. Since the FSEs represents the weights, hence the weights of the relative importance of the design features (Wd_f) and sub-features (WS_f) is defined as [30, 31]

$$W\tilde{d}_f\Big|_m=\sum_{j=1}^{s}\tilde{d}_{fgk}^{s}\otimes\left\{\sum_{i=1}^{k}\sum_{j=1}^{s}\tilde{d}_{fgk}^{s}\right\}^{-1}, \tag{4}$$

$$W\tilde{S}_{fn}\Big|_{m} = \sum_{j=1}^{s}\tilde{s}_{fgk}^{s} \otimes \left\{\sum_{i=1}^{k}\sum_{j=1}^{s}\tilde{s}_{fgk}^{s}\right\}^{-1}. \tag{5}$$

Assigning TFNs to the availability of the sub-features in the alternative design concepts based on the parts analysis will produce a comparison matrix which aggregate will form a basis for the comparison matrix of the design alternatives. A matrix of the aggregate TFNs for the design alternatives from all the sub-features for m number of design features can be represented by [32]

$$\begin{array}{cc} & \text{Design Alternatives} \\ & \begin{array}{cccc} D_{a1} & D_{a2} & .... & D_{ak} \end{array} \\ \begin{bmatrix} W\tilde{S}_{f_1} \\ W\tilde{S}_{f_2} \\ \vdots \\ W\tilde{S}_{f_n} \end{bmatrix} * & \begin{pmatrix} \tilde{b}_{sf1}^{1} & \tilde{b}_{sf1}^{2} & .... & \tilde{b}_{sf1}^{k} \\ \tilde{b}_{sf2}^{1} & \tilde{b}_{sf2}^{2} & .... & \tilde{b}_{sf2}^{k} \\ \vdots & \vdots & & \\ \tilde{b}_{sfn}^{1} & \tilde{b}_{sfn}^{2} & .... & \tilde{b}_{sfn}^{k} \end{pmatrix}. \\ \tilde{D}_{ak}\Big|_{Agg} & \begin{array}{cccc} \cdots & \cdots & & \cdots \end{array} \end{array} \tag{6}$$

Where $\tilde{b}_{sfn}^{k}$ is a TFN that is equal to the product of the weight of a sub-feature and the availability of the sub-feature in a design alternative. The aggregate of the design concepts considering all the sub-features in a design feature ($D_{ak}\big|_{Agg}$) can be obtained from ***Eq. (7)***.

$$\tilde{D}_{ak}\Big|_{Agg} = \sum_{n=1}^{n=n}\left[\tilde{W}_{Sf_n} * \tilde{b}_{sfn}^{k}\right]. \tag{7}$$

All the aggregates are harnessed to form the decision matrix of the form

$$\begin{array}{cc} & \begin{array}{cccc} W\tilde{d}_{f1} & W\tilde{d}_{f1} & .... & W\tilde{d}_{fm} \end{array} \\ \begin{array}{c} D_{a1} = \\ D_{a2} = \\ \vdots \\ D_{ak} = \end{array} & \begin{pmatrix} \tilde{D}_{ak}\Big|_{Agg\,1}^{1} & \tilde{D}_{ak}\Big|_{Agg\,1}^{2} & .... & \tilde{D}_{ak}\Big|_{Agg\,1}^{m} \\ \tilde{D}_{ak}\Big|_{Agg\,2}^{1} & \tilde{D}_{ak}\Big|_{Agg\,2}^{2} & .... & \tilde{D}_{ak}\Big|_{Agg\,2}^{m} \\ \vdots & \vdots & & \vdots \\ \tilde{D}_{ak}\Big|_{Agg\,k}^{1} & \tilde{D}_{ak}\Big|_{Agg\,k}^{2} & .... & \tilde{D}_{ak}\Big|_{Agg\,k}^{m} \end{pmatrix}. \end{array} \tag{8}$$

In order to normalise the decision matrix, consider a fuzzy number $y_{ij} = (u_{ij}\ \ v_{ij}\ \ w_{ij})$ for $(i = 1.....n\ \ j = 1.....m)$ the normalisation process can be presented as shown in ***Eqs. (9)-(11)*** [26, 33].

$$\left(y_{ij}\right)_N = \left[\left(u_{ij}\right)_N \quad \left(v_{ij}\right)_N \quad \left(w_{ij}\right)_N\right], \tag{9}$$

$$\left(y_{ij}\right)_N = \left[\frac{u_{ij} - u_j^{Min}}{\Delta_{Min}^{Max}},\ \frac{v_{ij} - u_j^{Min}}{\Delta_{Min}^{Max}},\ \frac{w_{ij} - u_j^{Min}}{\Delta_{Min}^{Max}}\right], \quad i = 1,......n;\ \ j \in \Omega_b, \tag{10}$$

$$\left(y_{ij}\right)_N = \left[\frac{w_{ij} - w_j^{Max}}{\Delta_{Min}^{Max}}, \quad \frac{v_{ij} - w_j^{Max}}{\Delta_{Min}^{Max}}, \quad \frac{u_{ij} - w_j^{Max}}{\Delta_{Min}^{Max}}\right], \quad i = 1,......n; \;\; j \in \Omega_c . \qquad \textbf{\textit{(11)}}$$

Where $u_j^{Min} = \text{Min } u_{ij}$ and $w_j^{Max} = \text{Max } w_{ij}$ for $i = 1,......n$; $\Delta_{Min}^{Max} = w_j^{Max} - u_j^{Min}$. Also, Ω_b and Ω_c are sets of benefit and cost attributes, respectively. The left and right scores of the normalised decision matrix and the weighted priority are important for computations of the fuzzy weighted average. It is necessary to present an analysis on the determination of left and right score from the TFNs which can be obtained from ***Eqs. (12)-(13)*** [34, 35].

$$\left(L_S\right)_{ij} = \frac{\left(v_{ij}\right)_N}{1 + \left(v_{ij}\right)_N - \left(u_{ij}\right)_N} . \qquad \textbf{\textit{(12)}}$$

$$\left(R_S\right)_{ij} = \frac{\left(w_{ij}\right)_N}{1 + \left(w_{ij}\right)_N - \left(v_{ij}\right)_N} . \qquad \textbf{\textit{(13)}}$$

Applying ***Eqs. (12)-(13)*** to the normalised form of ***Eq. (8)***, two matrices that include intervals of the left and right score can be constructed for the normalised decision matrix and the weights of the design features. The value for the weighted average of each design alternative is also obtainable in the form of the intervals of the left and right scores. For ease of analysis let

$d_{ij} = \left[\left(L_S\right), \left(R_S\right)\right]_{ij} = \left[\left(L_S\right)_{ij}, \left(R_S\right)_{ij}\right]$ and $w_j = \left[\left(L_S\right), \left(R_S\right)\right]_j = \left[\left(L_S\right)_j, \left(R_S\right)_j\right]$, then the FWA $\left(\theta_i\right)$ for design alternative D_{ak} can be obtained from ***Eq. (16)***.

$$\left[\left(L_S\right), \left(R_S\right)\right]_{\left(D_{ak}\right)N} = \begin{bmatrix} \left[\left(L_S\right), \left(R_S\right)\right]_{11} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{12} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{1n} \\ \vdots & & \vdots & & \vdots \\ \left[\left(L_S\right), \left(R_S\right)\right]_{21} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{22} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{2n} \\ \vdots & & \vdots & & \vdots \\ \left[\left(L_S\right), \left(R_S\right)\right]_{j1} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{j2} & \cdots & \left[\left(L_S\right), \left(R_S\right)\right]_{jn} \end{bmatrix} . \qquad \textbf{\textit{(14)}}$$

$$\left[\left(L_S\right), \left(R_S\right)\right]_{\left(\tilde{W}_{dfm}\right)N} = \left[\left[\left(L_S\right), \left(R_S\right)\right]_1 \;\; \cdots \;\; \left[\left(L_S\right), \left(R_S\right)\right]_2 \;\; \cdots \;\; \left[\left(L_S\right), \left(R_S\right)\right]_n\right] . \qquad \textbf{\textit{(15)}}$$

$$\theta_i = \frac{\sum_{j=1}^{n}\left(w_j * d_{ij}\right)}{\sum_{j=1}^{n} w_j} = \frac{w_1 d_{i1} + w_2 d_{i2} + ... + w_n d_{in}}{w_1 + w_2 + ... + w_n} \;\; ; \; i = 1,...m . \qquad \textbf{\textit{(16)}}$$

Eq. (16) is subject to

$$\begin{aligned} \left(L_s\right)_j &\le w_j \le \left(R_s\right)_j , \;\; j = 1.....n \\ \left(L_s\right)_j &\le d_{ij} \le \left(R_s\right)_j , \;\; j = 1.....n \end{aligned} . \qquad \textbf{\textit{(17)}}$$

Since the components for each of the design alternative obtained in ***Eq. (16)*** is a function of the intervals of the left and right scores, then the FWA can be considered as a lower and upper bound of a fractional programming model. Besides, since the FWA is a monotonically increasing function of d_{ij} which reaches its minimum and maximum at $d_{ij} = (L_S)_{ij}$ and $d_{ij} = (R_S)_{ij}$, respectively then the pair of fractional programming model can be presented as

$$\theta_i^{\,L} = \mathrm{Min}\left[\frac{\sum_{j=1}^{n}\left(w_j * (L_S)_{ij}\right)}{\sum_{j=1}^{n} w_j}\right] \text{ Subject to } (L_s)_j \le w_j \le (R_s)_j,\ \ j = 1.....n, \tag{18}$$

$$\theta_i^{\,U} = \mathrm{Max}\left[\frac{\sum_{j=1}^{n}\left(w_j * (R_S)_{ij}\right)}{\sum_{j=1}^{n} w_j}\right] \text{ Subject to } (L_s)_j \le w_j \le (R_s)_j,\ \ j = 1.....n. \tag{19}$$

Eqs. (18)-(19) can be transformed into a linear programming model using transportation equations as presented in ***Eqs. (20)-(21)***.

$$z = \frac{1}{\sum_{j=1}^{n} w_j}. \tag{20}$$

$$t_j = z * w_j \quad\ j = 1.......n. \tag{21}$$

$$\begin{gathered}(\theta_i)^L = \mathrm{Min}\sum_{j=1}^{n}\left(t_j * (L_S)_{ij}\right) \text{ subject to } \sum_{j=1}^{n} t_j = 1\\ \left(z * (L_S)_{ij}\right) \le t_j \le \left(z * (L_S)_{ij}\right),\ \ j = 1.....n.\end{gathered} \tag{22}$$

$$\begin{gathered}(\theta_i)^U = \mathrm{Max}\sum_{j=1}^{n}\left(t_j * (R_S)_{ij}\right) \text{ subject to } \sum_{j=1}^{n} t_j = 1\\ \left(z * (L_S)_{ij}\right) \le t_j \le \left(z * (L_S)_{ij}\right),\ \ j = 1.....n.\end{gathered} \tag{23}$$

An interval $\left[(\theta_i)^L,\ (\theta_i)^U\right]$ is created from ***Eqs. (22)- (23)*** for each design alternative whose average value ($\theta_{i\ \ avg}$) will provide the weight for each design alternative as presented in ***Eq. (24)*** [26].

$$(\theta_i)_{avg} = \frac{(\theta_i)^L + (\theta_i)^U}{2}. \tag{24}$$

In order to simplify the analysis, a framework for the fuzzified SEWA and its application to the identification of optimal design concept is presented in ***Figure 3***. It is evident from ***Figure 3*** that the hybridized model utilizes the comparative strength of the fuzzified analytic network process by obtaining weights of the design features and sub-features from the fuzzified pairwise comparison matrix. These weights are obtained from the fuzzified pairwise comparison matrix of the design features and sub-features using the fuzzy synthetic extent value expression, which may be analogous to the local weights obtained from priority vectors when dealing with crisp values in the ANP. Also, considering the multi-dimensional nature and units of the design features and their sub-features, apportioning of crisp values in the pairwise comparison matrix may not be suitable as using fuzzy sets and linguistic terms. The hybridized model surpasses from the conventional AHP model by further exploiting the computational strength of the fuzzy weighted average based on left and right scores in order to aggregate the availability of the sub-features in the design alternatives and consider the weights of the design features in determining the optimal design.

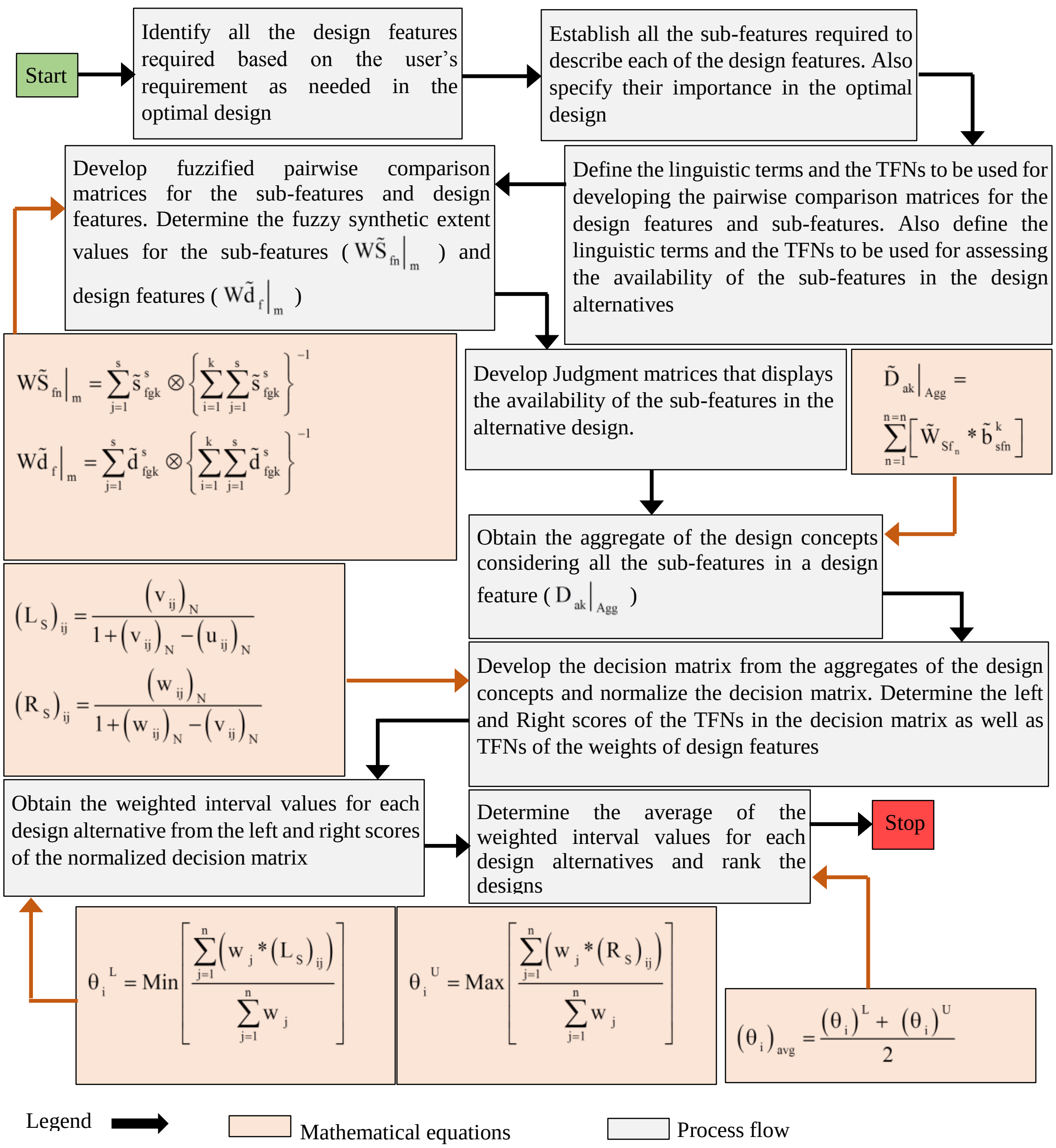

Figure 3. Framework for identifying optimal design via fuzzified synthetic extent weighted average.

3. Application to the Design of RAF

The SEWA can be implemented using the RAF. Four design alternatives of the RAF are presented in ***Figure 4***. Details of the RAF designs are obtainable from [36]. The sub-features of the three design features are also presented in ***Figure 3***. Pairwise comparison matrices for the sub-features and design features are shown in ***Tables (2)-(5)***. Due to a large number of sub-features, the TFNs values cannot fit in the columns, hence the symbols from ***Table 1*** are used to represent the TFNs. As stated earlier, the sub-features of design for functionality are peculiar to the required performance of the reconfigurable machine. They are associated with the RAF in terms of the morphology of locators, position synchronization and others as specified in ***Figure 4***. The FSE is obtained for the design features and sub-features using ***Eqs. (4)-(5)***, respectively. ***Table 6*** presents the TFNs estimated from all the comparison matrices for the sub-features and design features.

Table 2. Comparison matrix for sub-features of DfF.

Functionality F									
	DW	ML	SL	PA	CW	SW	AF	CG	WS
DW	EQI	ESI	WEI	VSI	WEI	WEI	ESI	VSI	WEI
ML	ESI^{-1}	EQI	WEI	WEI	VSI	VSI	ABI	ESI	WEI^{-1}
SL	WEI^{-1}	WEI^{-1}	EQI	ESI	ESI	ESI^{-1}	WEI^{-1}	ESI	WEI
PA	VSI^{-1}	WEI^{-1}	ESI^{-1}	EQI	ABI	ESI^{-1}	WEI^{-1}	ESI^{-1}	ABI
CW	WEI^{-1}	VSI^{-1}	ESI^{-1}	ABI^{-1}	EQI	ESI^{-1}	WEI	WEI	ABI
SW	WEI^{-1}	VSI^{-1}	ESI	ESI	ESI	EQI	VSI	WEI	ABI
AF	ESI^{-1}	ABI^{-1}	WEI	WEI	WEI^{-1}	VSI^{-1}	EQI	ESI	ESI^{-1}
CG	VSI^{-1}	ESI^{-1}	ESI^{-1}	ESI	WEI^{-1}	WEI^{-1}	ESI^{-1}	EQI	VSI
WS	WEI^{-1}	WEI	WEI^{-1}	ABI^{-1}	ABI^{-1}	ABI^{-1}	ESI	VSI^{-1}	EQI

Table 3. Comparison matrix for sub-features of DfR.

Reconfigurability R						
	MO	IN	CU	CO	SC	DI
MO	EQI	ESI	VSI	WEI	ABI	WEI
IN	ESI^{-1}	EQI	WEI	ESI	WEI	VSI
CU	VSI^{-1}	WEI^{-1}	EQI	WEI	ESI	VSI
CO	WEI^{-1}	ESI^{-1}	WEI^{-1}	EQI	VSI	ESI
SC	ABI^{-1}	WEI^{-1}	ESI^{-1}	VSI^{-1}	EQI	ABI
DI	WEI^{-1}	VSI^{-1}	VSI^{-1}	ESI^{-1}	ABI^{-1}	EQI

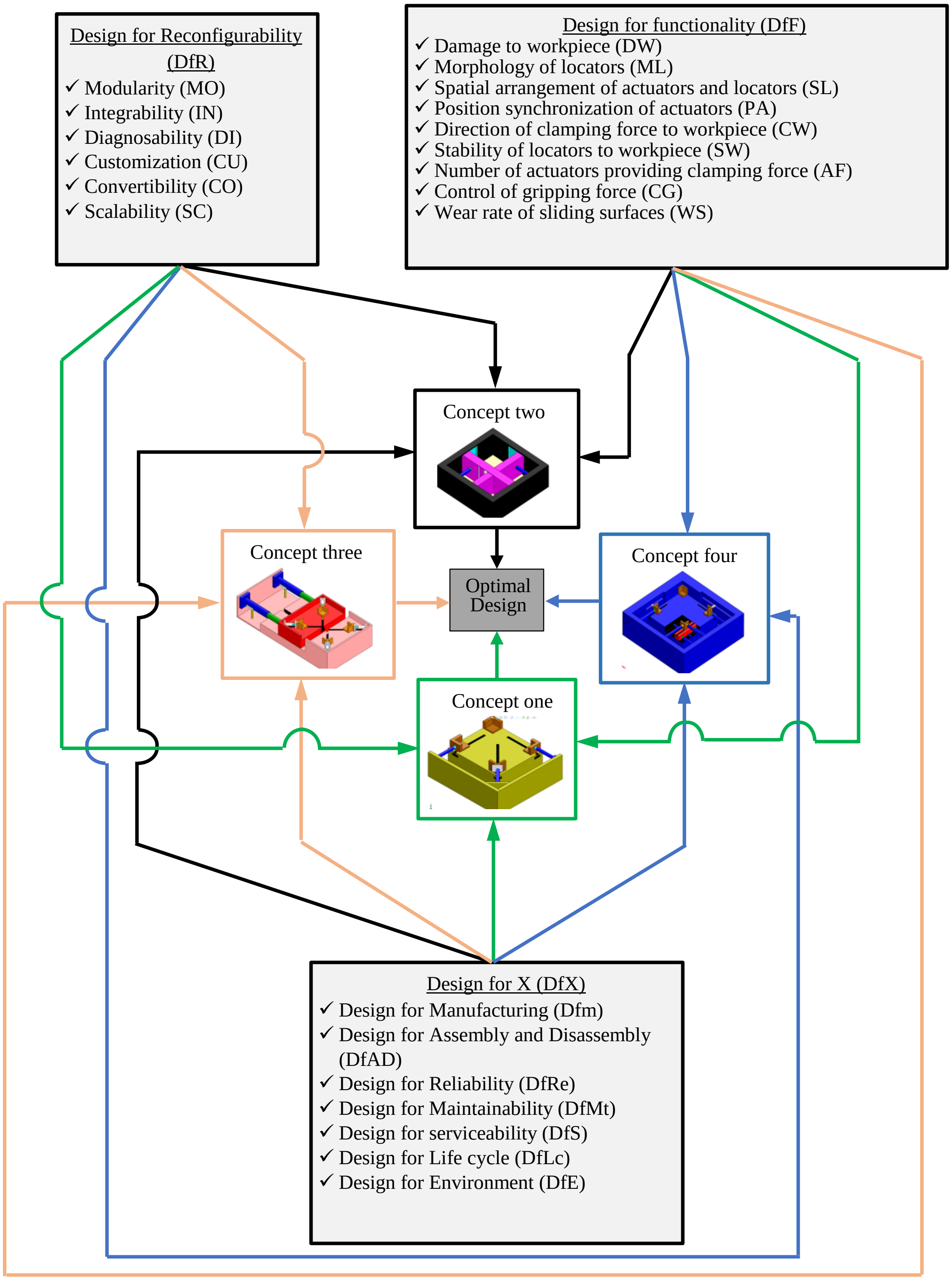

Figure 4. Decision tree for evaluation of the RAF.

Table 4. Comparison matrix for sub-features of DfX.

Design for X							
	DfM	DfAD	DfRe	DfMt	DfS	DfLc	DfE
DfM	EQI	WEI	ESI	ESI	VSI	ABI	ESI
DfAD	WEI^{-1}	EQI	ESI	WEI	WEI	VSI	VSI
DfRe	ESI^{-1}	ESI^{-1}	EQI	WEI^{-1}	WEI^{-1}	WEI	ESI
DfMt	ESI^{-1}	WEI^{-1}	WEI	EQI	WEI	ESI	WEI
DfS	VSI^{-1}	WEI^{-1}	WEI	WEI^{-1}	EQI	WEI	VSI
DfLc	ABI^{-1}	VSI^{-1}	WEI^{-1}	ESI^{-1}	WEI^{-1}	EQI	ESI
DfE	ESI^{-1}	VSI^{-1}	ESI^{-1}	WEI^{-1}	VSI^{-1}	ESI^{-1}	EQI

Table 5. Comparison matrix for design features.

Design Features			
	DfX	DfR	DfF
DfX	EQI	ESI^{-1}	WEI^{-1}
DfR	ESI	EQI	WEI
DfF	WEI	WEI^{-1}	EQI
FSE	0.16 0.22 0.34	0.29 0.46 0.70	0.21 0.32 0.51

Table 6. Values of FSE for the sub-features.

Functionality		Reconfigurability		Design for X	
Sub-Features	FSE	Sub Features	FSE	Sub-Features	FSE
DW	0.095 0.158 0.252	MO	0.16 0.25 0.38	DfM	0.15 0.24 0.37
ML	0.094 0.149 0.236	IN	0.12 0.20 0.31	DfAD	0.12 0.20 0.32
SL	0.066 0.109 0.179	CU	0.11 0.18 0.25	DfRe	0.07 0.12 0.19
PA	0.068 0.101 0.158	CO	0.12 0.18 0.28	DfMt	0.09 0.15 0.24
CW	0.059 0.093 0.148	SC	0.09 0.13 0.19	DfS	0.09 0.14 0.23
SW	0.094 0.149 0.234	DI	0.05 0.07 0.11	DfLc	0.06 0.09 0.15
AF	0.051 0.083 0.135			DfE	0.05 0.07 0.11
CG	0.056 0.086 0.139				
WS	0.045 0.071 0.116				

The next stage is to assess the design concepts based on these sub-features considering parts analysis and morphology of the component in each of the design alternatives. An assessment of the design alternatives considering the sub-features is presented in ***Tables (7)-(9)***.

Table 7. Assessing design concepts based on sub-features of reconfigurability.

Reconfigurability (R)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
MO (0.16 0.25 0.38)	1.5 2 2.5	1 1.5 2	2 2.5 3	1 1.5 2
IN (0.12 0.20 0.31)	2 2.5 3	2 2.5 3	2.5 3 3.5	2 2.5 3
CU (0.11 0.18 0.25)	1.5 2 2.5	2 2.5 3	1.5 2 2.5	1.5 2 2.5
CO (0.12 0.18 0.28)	1.5 2 2.5	1.5 2 2.5	2 2.5 3	2 2.5 3
SC (0.09 0.13 0.19)	1.5 2 2.5	1.5 2 2.5	1.5 2 2.5	1.5 2 2.5
DI (0.05 0.07 0.11)	2 2.5 3	2 2.5 3	2 2.5 3	2 2.5 3
Cumulative TFN	1.07 2.13 4.03	1.04 2.10 3.96	1.27 2.45 4.51	1.05 2.10 3.98

Table 8. Assessing design concepts based on sub-features of functionality.

Functionality (F)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
DW (0.095 0.158 0.252)	1.5 2 2.5	2 2.5 3	2 2.5 3	2 2.5 3
ML (0.094 0.149 0.236)	1.5 2 2.5	1.5 2 2.5	2 2.5 3	1.5 2 2.5
SL (0.066 0.109 0.179)	1.5 2 2.5	1 1.5 2	2 2.5 3	1.5 2 2.5
PA (0.068 0.101 0.158)	1.5 2 2.5	1 1.5 2	2.5 3 3.5	2 2.5 3
CW (0.059 0.093 0.148)	2 2.5 3	1.5 2 2.5	2 2.5 3	2 2.5 3
SW (0.094 0.149 0.234)	2 2.5 3	1 1.5 2	1.5 2 2.5	2 2.5 3
AF (0.051 0.083 0.135)	1 1.5 2	1 1.5 2	2 2.5 3	2 2.5 3
CG (0.056 0.086 0.139)	2 2.5 3	2 2.5 3	1 1.5 2	1.5 2 2.5
WS (0.045 0.071 0.116)	1.5 2 2.5	1.5 2 2.5	2.5 3 3.5	2 2.5 3
Cumulative TFN	1.02 2.12 4.18	0.88 1.90 3.83	1.21 2.43 4.67	1.14 2.33 4.51

Table 9. Assessing design concepts based on sub-features of design for X.

Design for X (DfX)	Design Alternatives			
	Concept 1	Concept 2	Concept 3	Concept 4
DfM (0.15 0.24 0.37)	2 2.5 3	2.5 3 3.5	1.5 2 2.5	2 2.5 3
DfAD (0.12 0.20 0.32)	2.5 3 3.5	2.5 3 3.5	2 2.5 3	2.5 3 3.5
DfRe (0.07 0.12 0.19)	2 2.5 3	1.5 2 2.5	2.5 3 3.5	2 2.5 3
DfMt (0.09 0.15 0.24)	2 2.5 3	2 2.5 3	2 2.5 3	2 2.5 3
DfS (0.09 0.14 0.23)	1.5 2 2.5	2 2.5 3	1.5 2 2.5	2 2.5 3
DfLc (0.06 0.09 0.15)	2 2.5 3	1.5 2 2.5	2.5 3 3.5	1.5 2 2.5
DfE (0.05 0.07 0.11)	1 1.5 2	1 1.5 2	1 1.5 2	1 1.5 2
Cumulative TFN	1.21 2.46 4.77	1.26 2.55 4.90	1.13 2.32 4.55	11.23 2.48 4.81

The fuzzified decision matrix for the design alternatives can be obtained from the cumulative TFNs obtained from the assessments in ***Tables (7)-(9)***. The weights of the design features are also presented together with the decision matrix using the FSEs of the comparison matrix for the design alternatives. ***Table 10*** presents the decision matrix.

Table 10. Fuzzified decision matrix.

Design Alternatives	Design Features		
	DfX (0.16 0.22 0.34)	DfR (0.29 0.46 0.70)	DfF (0.21 0.32 0.51)
Concept 1	1.21 2.46 4.77	1.07 2.13 4.03	1.02 2.12 4.18
Concept 2	1.26 2.55 4.90	1.04 2.10 3.96	0.88 1.90 3.83
Concept 3	1.13 2.32 4.55	1.27 2.45 4.51	1.21 2.43 4.67
Concept 4	1.23 2.48 4.81	1.05 2.10 3.98	1.14 2.33 4.51

The TFNs of the fuzzified decision matrix is normalized in order to ensure that they are weighted in range [0 1] as presented in ***Table 11***. In order to estimate the intervals of the TFNs, they will be converted to left and right scores applying ***Eqs. (12)-(13)*** as shown in ***Table 12***. By applying the transportation model in ***Eqs. (20)-(21)***, the weights of the design features are normalized in order to determine their contributions to the determination of intervals for the design alternatives using ***Eqs. (22)-(23)***. The weighted average for each of the design alternatives is obtained from this interval. A ranking of the design concept is done based on the values of this weighted average. ***Table 12*** presents the normalized left and right scores of the weights for the design features, weighted interval, and average for each design alternative and ranking.

Table 11. Normalized fuzzy decision matrix.

Design Alternatives	Design Features		
	DfX (0.16 0.22 0.34)	DfR (0.29 0.46 0.70)	DfF (0.21 0.32 0.51)
Concept 1	0.02 0.35 0.97	0.01 0.31 0.86	0.04 0.33 0.87
Concept 2	0.03 0.38 1.00	0.00 0.30 0.84	0.00 0.27 0.78
Concept 3	0.00 0.32 0.91	0.06 0.40 1.00	0.09 0.41 1.00
Concept 4	0.03 0.36 0.98	0.00 0.30 0.84	0.07 0.38 0.96

Table 12. Weighted average computation and ranking of design alternatives.

Design Alternatives	Design Features			Weighted Interval	Weighted Average	Ranking
	DfX (0.21 0.30)	DfR (0.39 0.56)	DfF (0.29 0.43)			
	Normalized Weight of Design Features					
	[0.23 0.23]	[0.44 0.44]	[0.33 0.33]			
Concept 1	0.27 0.60	0.24 0.56	0.25 0.56	0.25 0.57	0.41	3rd
Concept 2	0.28 0.62	0.23 0.55	0.21 0.52	0.24 0.55	0.39	4th
Concept 3	0.24 0.57	0.30 0.63	0.31 0.63	0.29 0.61	0.45	1st
Concept 4	0.27 0.60	0.23 0.55	0.29 0.61	0.26 0.58	0.42	2nd

4. Conclusions

Considering the final values of the weighted average, concept three appears to be the optimal design of the RAF. The determination of weights of sub-features and design features from the synthetic extent of the pairwise comparison matrix created a means of showing the connection between the sub-features and design features and their contributions in the optimal design of RAF. This method supports the saaty's theory by ensuring that a design concept is not over scored compared to others as it can be observed in the final values of the concepts that the differences between the final values are not unnecessarily large. This result can be compared with the results obtained from Olabanji and Mpofu [14] using WDM and AHP on the same RAF designs where the final values between the design alternatives are high thus creating a large margin between the optimal design and other design alternatives. However, the lagre margin in [14] does not depict that the AHP overscores the optimal design. The margin is due to the choice of the design features which have no sub-features in the previous study. Hence, there is an implication that the strengths of all design alternatives have been considered in the decision process. The decision approach used in this article appears to be a robust computational process because normalizing the weights of the design features from the left and right score forced the weighted intervals of the design alternatives to be in the desired range. It can be observed from the final values of the design features that appear to be equal after normalization by applying the transportation equation. The introduction of design for reconfigurability and functionality assisted in harnessing the detail performance of the design alternatives compared with the conventional design for X tools/sub-features because the design for reconfigurability and functionality speaks to the details of the RAF operations.

In essence, the role of design concept evaluation in the design of a reconfigurable machine is crucial because it ensures that an appropriate and efficient design is selected for detailed design and fabrication. Aside the fact that design concept evaluation assists in the selection of the optimal design, it also assists in ensuring that a holistic approach is given to all design features and their relevance in all design alternatives before a decision is made. Hence, the introduction of design for reconfigurability and functionality as design features have been able to identify the optimal design of the RAF using the synthetic extent weighted average. In view of this, it can be hypothetically stated that introducing design for reconfigurability and functionality for deciding on the optimal design of reconfigurable machines will provide improved results particularly when a robust multicriteria decision-making model is used for the computational process. However, future work is still possible by further analyzing the characteristics of the reconfigurable manufacturing principle into several sub-features such that the characteristics become a design feature. Also, a method that needs to simplify the rigorous computation of the fuzzified synthetic extent weighted average proposed in this study, for just-in-time determination of the optimal design concept still need to be explored.

References

[1] Olabanji, O. M. (2018). Reconnoitering the suitability of fuzzified weighted decision matrix for design process of a reconfigurable assembly fixture. *International journal of design engineering*, *8*(1), 38-56.

[2] Renzi, C., Leali, F., Pellicciari, M., Andrisano, A. O., & Berselli, G. (2015). Selecting alternatives in the conceptual design phase: an application of Fuzzy-AHP and Pugh's controlled convergence. *International journal on interactive design and manufacturing (IJIDeM)*, *9*(1), 1-17.

[3] Olabanji, O., Mpofu, K., & Battaïa, O. (2016). Design, simulation and experimental investigation of a novel reconfigurable assembly fixture for press brakes. *The international journal of advanced manufacturing technology*, *82*(1-4), 663-679.

[4] Benkamoun, N., Huyet, A. L., & Kouiss, K. (2013, October). Reconfigurable assembly system configuration design approaches for product change. *Proceedings of 2013 international conference on industrial engineering and systems management (IESM)* (pp. 1-8). IEEE.

[5] Li, Z., Pasek, Z. J., & Adams, J. (2004). *Reconfigurable fixtures: concept and examples*. Symposium conducted at the meeting of USA symposium on flexible automation, Denver, Colorado.

[6] Yu, K., Wang, S., Wang, Y., & Yang, Z. (2018). A flexible fixture design method research for similar automotive body parts of different automobiles. *Advances in mechanical engineering*, *10*(2), 1687814018761272.

[7] Renzi, C., & Leali, F. (2016). A multicriteria decision‐ making application to the conceptual design of mechanical components. *Journal of multi‐ criteria decision analysis*, *23*(3-4), 87-111.
[8] Girod, M., Elliott, A. C., Burns, N. D., & Wright, I. C. (2003). Decision making in conceptual engineering design: an empirical investigation. *Proceedings of the institution of mechanical engineers, Part B: journal of engineering manufacture*, *217*(9), 1215-1228.
[9] Kuo, T. C., Huang, S. H., & Zhang, H. C. (2001). Design for manufacture and design for 'X': concepts, applications, and perspectives. *Computers & industrial engineering*, *41*(3), 241-260.
[10] Dombrowski, U., Schmidt, S., & Schmidtchen, K. (2014). Analysis and integration of design for X approaches in lean design as basis for a lifecycle optimized product design. *Procedia CIRP*, *15*, 385-390.
[11] Skander, A., Roucoules, L., & Meyer, J. S. K. (2008). Design and manufacturing interface modelling for manufacturing processes selection and knowledge synthesis in design. *The international journal of advanced manufacturing technology*, *37*(5-6), 443-454.
[12] Harari, N. S., Fundin, A., & Carlsson, A. L. (2018). Components of the design process of flexible and reconfigurable assembly systems. *Procedia manufacturing*, *25*, 549-556.
[13] Renzi, C., Leali, F., & Di Angelo, L. (2017). A review on decision-making methods in engineering design for the automotive industry. *Journal of engineering design*, *28*(2), 118-143.
[14] Olabanji, O. M., & Mpofu, K. (2014). Comparison of weighted decision matrix, and analytical hierarchy process for CAD design of reconfigurable assembly fixture. *Procedia CIRP*, *23*, 264-269.
[15] Okudan, G. E., & Shirwaiker, R. A. (2006, July). A multi-stage problem formulation for concept selection for improved product design. *2006 technology management for the global future-PICMET 2006 conference* (Vol. 6, pp. 2528-2538). IEEE.
[16] Okudan, G. E., & Tauhid, S. (2008). Concept selection methods–a literature review from 1980 to 2008. *International journal of design engineering*, *1*(3), 243-277.
[17] Wang, J. (2001). Ranking engineering design concepts using a fuzzy outranking preference model. *Fuzzy sets and systems*, *119*(1), 161-170.
[18] Olabanji, O., & Mpofu, K. (2020). Pugh matrix and aggregated by extent analysis using trapezoidal fuzzy number for assessing conceptual designs. *Decision science letters*, *9*(1), 21-36.
[19] Goyal, K. K., Jain, P. K., & Jain, M. (2012). Optimal configuration selection for reconfigurable manufacturing system using NSGA II and TOPSIS. *International journal of production research*, *50*(15), 4175-4191.
[20] Youssef, A. M., & ElMaraghy, H. A. (2007). Optimal configuration selection for reconfigurable manufacturing systems. *International journal of flexible manufacturing systems*, *19*(2), 67-106.
[21] Ashraf, M., & Hasan, F. (2018). Configuration selection for a reconfigurable manufacturing flow line involving part production with operation constraints. *The international journal of advanced manufacturing technology*, *98*(5-8), 2137-2156.
[22] Gupta, A., Jain, P. K., & Kumar, D. (2015). Configuration selection of reconfigurable manufacturing system based on performance. *International journal of industrial and systems engineering*, *20*(2), 209-230.
[23] Rehman, A. U. (2017). Multi-criteria grey relational approach to evaluating reconfigurable manufacturing configurations. *South African Journal of industrial engineering*, *28*(1), 36-46.
[24] Renna, P. (2017). Decision-making method of reconfigurable manufacturing systems' reconfiguration by a Gale-Shapley model. *Journal of manufacturing systems*, *45*, 149-158.
[25] Yi, G., Wang, Y., & Zhao, X. (2018). Evaluation and optimization of the design schemes of reconfigurable machine tools based on multiple-attribute decision-making. *Advances in mechanical engineering*, *10*(12), 1687814018813054.
[26] Mokhtarian, M. N. (2011). A new fuzzy weighted average (FWA) method based on left and right scores: An application for determining a suitable location for a gas oil station. *Computers & mathematics with applications*, *61*(10), 3136-3145.
[27] Wang, J. (2002). Improved engineering design concept selection using fuzzy sets. *International journal of computer integrated manufacturing*, *15*(1), 18-27.

[28] Yeo, S. H., Mak, M. W., & Balon, S. A. P. (2004). Analysis of decision-making methodologies for desirability score of conceptual design. *Journal of engineering design*, *15*(2), 195-208.

[29] Balin, A., Demirel, H., & Alarcin, F. (2016). A novel hybrid MCDM model based on fuzzy AHP and fuzzy TOPSIS for the most affected gas turbine component selection by the failures. *Journal of marine engineering & technology*, *15*(2), 69-78.

[30] Alarcin, F., Balin, A., & Demirel, H. (2014). Fuzzy AHP and fuzzy TOPSIS integrated hybrid method for auxiliary systems of ship main engines. *Journal of marine engineering & technology*, *13*(1), 3-11.

[31] Nazam, M., Xu, J., Tao, Z., Ahmad, J., & Hashim, M. (2015). A fuzzy AHP-TOPSIS framework for the risk assessment of green supply chain implementation in the textile industry. *International journal of supply and operations management*, *2*(1), 548-568.

[32] Chakraborty, K., Mondal, S., & Mukherjee, K. (2017). Analysis of product design characteristics for remanufacturing using fuzzy AHP and axiomatic design. *Journal of engineering design*, *28*(5), 338-368.

[33] Aryanezhad, M., Tarokh, M. J., Mokhtarian, M. N., & Zaheri, F. (2011). A fuzzy TOPSIS method based on left and right scores. *International journal of industrial engineering & production research*, *22*(1), 53-60.

[34] [Amiri-Aref, M., Javadian, N., & Kazemi, M. (2012). A new fuzzy positive and negative ideal solution for fuzzy TOPSIS. *WSEAS Transactions on Circuits and Systems*, *11*(3), 92-103.

[35] Mokhtarian, M. N., & Hadi-Vencheh, A. (2012). A new fuzzy TOPSIS method based on left and right scores: An application for determining an industrial zone for dairy products factory. *Applied soft computing*, *12*(8), 2496-2505.

[36] Olabanji, O. M. (2015). *Development of a reconfigurable assembly system for the assembly of press brakes* (Doctoral Thesis in the Department of industrial Engineering, Tshwane University of Technology, Pretoria West, South Africa). Retrieved from https://books.google.com.ng/books?id=5VNDAQAACAAJ

[37] Olabanji, O. M., & Mpofu, K. (2019). Adopting hybridized multicriteria decision model as a decision tool in engineering design. *Journal of engineering, design and technology*, *18*(2).