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Option Pricing on Sesame Price Using Jump Diffusion Models

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A B S T R A C T

In this paper, we aim at developing a model for option pricing to reduce the risks associated with Ethiopian sesame price fluctuations. The White Humera Gondar Sesame Grade 3 (WHGS3) price, which is recorded from 5 November 2010 to 30 March 2018 at Ethiopia Commodity Exchange (ECX) market, is used to analyze the price fluctuation. The nature of log-returns of the price is asymmetric (positively skewed) and exhibits high kurtosis. We used jump diffusion models for modeling and option pricing of sesame price. The method of maximum likelihood is applied to estimate the parameters of the models. We used the Root Mean Square Error (RMSE) to test the goodness of fitting for the two models to the data. This test indicates that the models fit the data well. The techniques of analytical and Monte Carlo simulation are used to find the call option pricing of WHGS3 sesame price. From the results, we concluded that Double Exponential Jump Diffusion (DEJD) model is more efficient than Merton's model for modeling and option pricing of this sesame price.

Keywords: Jump diffusion model, Option pricing, Kurtosis, Skewness, Risk-neutral measure, WHGS3 sesame price.

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1. Introduction

Oilseed, which is one of the export commodities in Ethiopia, has great contribution to the growth of Ethiopian economy. It is rapidly growing to meet both local and foreign demand and it plays a vital role in generating foreign exchange earnings and income for the country. Sesame is the main oilseed crop in terms of production value. The cultivation of sesame has been grown gradually and has owed to its high value on the export market and good adaptability in the country. Sesame occupied 0.62% of the total area covered by grain crops and 1.61% of the total grains produced during 2010/11 [8]. The total area cultivated, production and productivity in Ethiopia during 2012 was 337,505 hectares, 44783 tons and 7.253 quintal/hectare, respectively [11]. It is mainly harvested by small-scale farmers as a major cash crop in the northern and

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northwestern lowlands of Ethiopia. Some of the major producing areas are found in and around the locations of Humera, Metema, Benshungul Gumuz, and Wellega [18]. The Whitish Humera sesame type has great demands in world markets than any other sesame in Ethiopia [24].

The international market demand for Ethiopian sesame seed has increasingly grown due to its new seed buyers coming to the market and it also implies the prospect for sesame production growth and market expansion in the future [9]. Ethiopian export share, 1.5% in volume and 1.9% in value to world market in 1997, had grown to 8.9% and 8.3% in 2004 [1].

The major destinations for Ethiopian sesame seed are China, Israel, Turkey, Japan, and other European countries [18]. It contributes the highest market values from the oilseeds exports. It is remarkable that 80% of the export earning of oilseeds comes from sesame seeds and it has become a major foreign currency earning next to coffee [1]. Thus, Sesame plays a significant role for the development of the economy of the country as well as for the livelihood of sesame growing farmers. To this effect, the Ethiopian ministry of agriculture has developed a master plan to enhance market oriented production for priority crops. Sesame is one of the priority crops identified in the plan for accelerated and sustainable development to end poverty.

Ethiopia was among the top five sesame producing countries in the world ranked at fourth place in the fiscal year 2011/2012 [11]. Moreover, it is the third largest exporter of sesame seed after India and Sudan [2]. The sesame market price is inherently noisy in nature and is volatile too [19, 24]. The sesame export has been fluctuated tremendously by volume and value in the country and this leads to high risk for the income of the country as well as for those who depend on sesame for their livelihood [21]. Therefore, it is of great importance to develop a model for option pricing to reduce the risks associated with WHGS3 price fluctuations.

To capture the behavior of market prices various, authors used different models. The first to mention is the Black-Scholes model based on Brownian motion and normal distribution. However, the empirical phenomena have received much attention recently on the asymmetric leptokurtic features that is the return distribution has fat tails and high kurtosis than those of the normal distribution. It is widely recognized that the implied volatility is not constant as in Black-Scholes model, but it is a convex function of the strike price resembling a smile. Thus, many models have been proposed in order to reflect the above phenomena under a market measure. However, the leptokurtic feature under risk-neutral measure leads to the volatility smile in option pricing.

In Merton model [20], the asset return follows a Brownian motion with drift punctuated by jumps arriving according to a compound Poisson process with constant intensity and with normally distributed jump sizes. Due to normality of the jump size distribution, Merton was able to obtain explicit analytical solutions for European style call options in this model. Kou [15] recently proposed a double exponential jump-diffusion model where jump sizes are double exponentially distributed. This model has a memoryless property inherited from the exponential distribution [15]. This property explains the reason why analytical or approximated solutions for different option pricing problems are viable with this model. Both Merton's and double exponential jump-diffusion models are used to model hedge fund indices in continuous time [23]. Thus, these models are proposed to reflect the asymmetric leptokurtic features of asset prices which lead to determine the call option pricing of some asset prices to reduce risks associated with price fluctuations under a risk neutral measure.

However, no study has been done on option pricing of Ethiopian sesame price using jump diffusion models. Therefore, we used Merton's and double exponential jump diffusion models for modeling and call option pricing of WHGS3 price to reduce the risk caused by price fluctuation under risk neutral measure. The method of maximum likelihood is used to estimate the parameters for these models. Based on the results, we compared the models fitting them to the empirical data. Here, we also used the RMSE, the Q-Q plot and non-parametric fit with normal kernel to test the validation of these models. Finally, we investigated the option pricing formula to compute the European style call option pricing of Ethiopian sesame price.

2. Analysis of WHGS3 Sesame Price Data

The WHGS3 sesame price, which is recorded from 5 November 2010 to 30 March 2018 at ECX market, is considered to study its price movement. The price unit in Ethiopian agricultural commodity market price is Birr per kilogram.

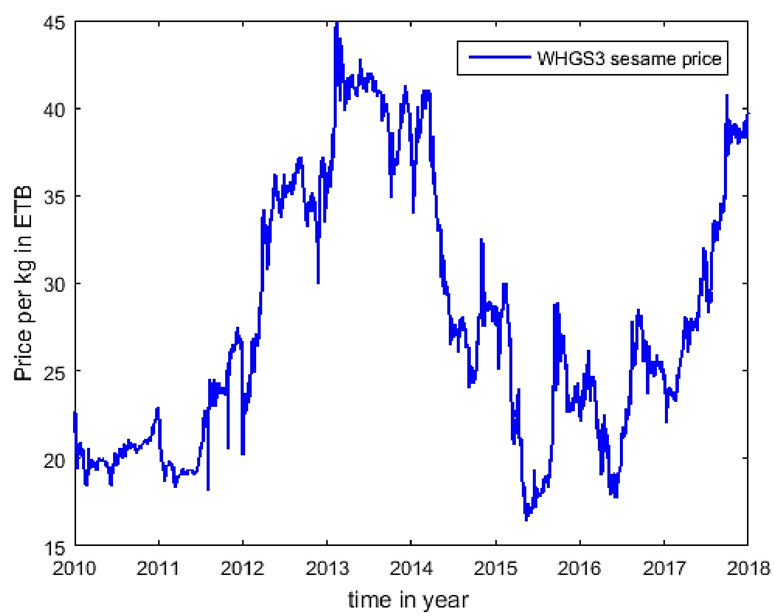

Figure 1. The WHGS3 sesame price from 2010 to 2018.

Assume that S_t is to represent WHGS3 sesame prices. We focus on the dynamic behavior of sesame price by analyzing log-return price, which is defined as: $x_t = \Delta \ln(S_t) = \ln(S_t) - \ln(S_{t-1})$. The descriptive statistics of x_t are shown in ***Table 1***. We plot the graph of the log-return of sesame price in ***Figure 2***. This graph indicates that spikes are observed significantly in the empirical data. We also plot the Quantile-Quantile (Q-Q) and histogram of the daily log-return price as shown in ***Figure 3*** and ***Figure 4,*** respectively. These indicate the presence fat tails and high kurtosis in the empirical distribution of WHGS3 sesame price. Thus, based on the analysis, we conclude that the sesame price is not normally distributed.

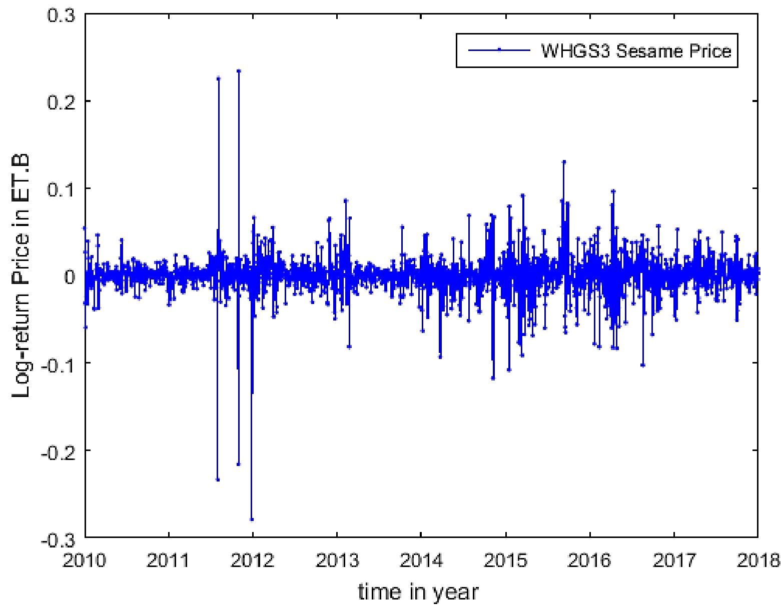

Figure 2. Log-return WHGS3 sesame price from 2010 to 2018.

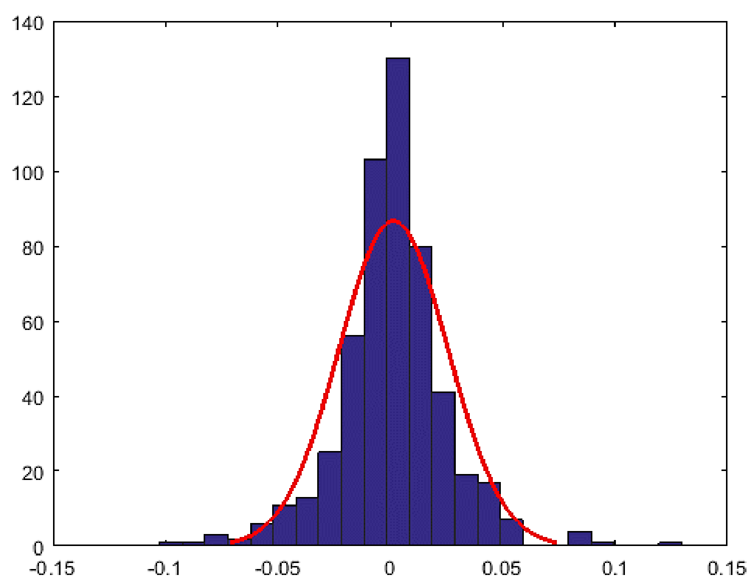

Figure.3. The histogram and normal density of log-return WHGS3 price from 2010 to 2018.

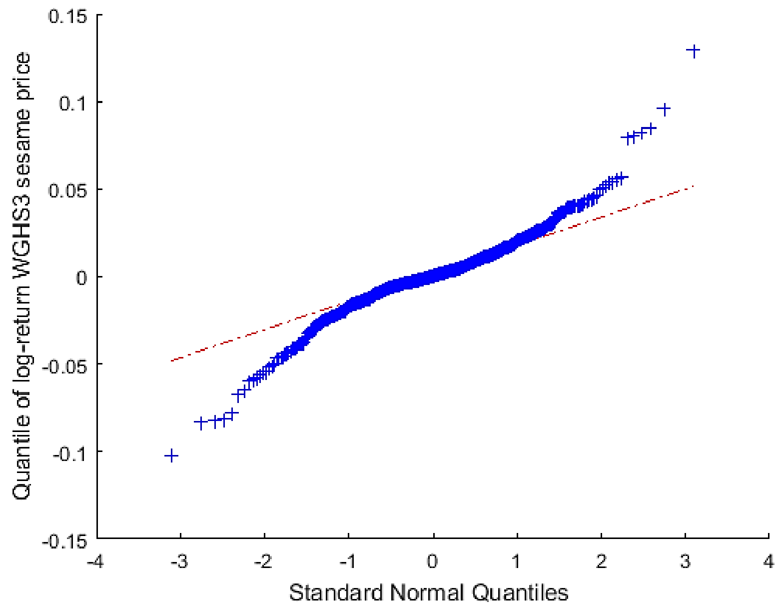

Figure 4. The Q-Q plot of log-return WHGS3 price from 2010-2018.

Table 1. Descriptive statistics of log-return WHGS3 sesame price.

Descriptive statistics	Value
Mean	0.00144
Standard Deviation	0.02421
Skewness	0.05762
Kurtosis	6.59725

3. Jump Diffusion Models

Based on the empirical findings discussed in the previous section, namely the presence of skewness and kurtosis in the empirical distribution of the sesame prices returns; an adequate model for the sesame prices would be a jump diffusion model. In fact, Merton's work [20], recognizing the presence of jumps in asset prices and more accurate option pricing, proposed modeling the prices as a jump diffusion process instead of a pure diffusion model. Pure diffusion based models could not adequately explain the smile effect in short-dated option prices and emphasized the importance of adding a jump component in modeling asset price dynamics [4]. Models with jumps generically lead to significant skews for short-term maturities. More generally, adding jumps to returns in diffusion based stochastic volatility model, the resulting model can generate sufficient variability and asymmetry in the short-term returns to match implied volatility skews for short term maturities [3].

3.1. Merton Jump Diffusion Model

Under Merton jump diffusion model, the sesame price process S_t with the physical probability measure P is assumed to follow the stochastic differential equation:

$$\frac{dS_t}{S_{t-}} = \mu dt + \sigma dB_t + (y_t - 1)dN_t, \tag{1}$$

where μ is instantaneous expected return, σ is the instantaneous volatility of the price return and S_{t-} is the value of price process before the jump whenever jump occurs. The continuous component is given by a standard Brownian motion, B_t, distributed as $dB_t \sim (0, dt)$. The discontinuities of the price process are described by a Poisson counter N_t, characterized by its intensity, λ and jump size y_t. The assumption is that the Brownian motion B_t, the Poisson process N_t and the jump size y_t are independent. The intensity of the Poisson process describes the mean number of arrivals of abnormal information per unit of time dt and is expressed as:

$$\text{Prob}[dN_t = 1] = \lambda dt \textit{ and } \text{Prob}[dN_t = 0] = 1 - \lambda dt. \tag{2}$$

The sesame price S_{t-} is the value of price process before the jump whenever the price jumps from S_{t-} to $S_t = y_t S_{t-}$ in a small time interval dt. The percentage change is measured by $(y_t - 1)$. The price S_t presents log-normal jumps y_t on each random time t which represents the moments of jumping of a Poisson process [13, 14]. Introduction of the jump diffusion model adds three extra parameters $(\beta, \delta^2, \lambda)$ to the Black-Sholes process model which contains two parameters (μ, σ^2). Merton assumes that the log-price jump size $Y_t = \ln(y_t)$ is normal random variables. Letting $X_t = \ln(S_t)$ and using Ito's lemma, the log-price return process becomes:

$$dX_t = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dB_t + Y_t dN_t. \tag{3}$$

Discretized over $[t, t + \Delta t]$ this model takes the form:

$$\Delta X_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j, \tag{4}$$

where $\Delta B_t = B_{t+\Delta t} - B_t \sim N(0, \Delta t)$ and $\Delta N_t = N_{t+\Delta t} - N_t$ is the number of jumps occurring during the time interval $[t, t + \Delta t]$ and Y_t are independently and identically distributed as $Y_t \sim N(\beta, \delta^2)$ with probability density:

$$f(y) = \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[-\frac{(y-\beta)^2}{2\delta^2}\right], \quad y \in \mathbb{R}. \tag{5}$$

The log-return, $x_t = \Delta X_t$, therefore includes the sum of two independent components: a diffusion component with drift and a jump component. The probability density of ΔX_t can be expressed [3] as:

$$f_{\Delta t}(x) = \sum_{n=0}^{\infty} \frac{(\lambda\Delta t)^n e^{-\lambda\Delta t}}{n!}\left[\frac{1}{\sqrt{2\pi(\sigma^2\Delta t + n\delta^2)}}\exp\left(-\frac{(x-\left(\mu-\frac{1}{2}\sigma^2\right)\Delta t - n\beta)^2}{2(\sigma^2\Delta t + n\delta^2)}\right)\right], \tag{6}$$

with $n = 0,1,2, \ldots$.

Putting $\Delta t = 1$ that is the time interval is $[t,\ t+1]$, the density function becomes:

$$f(x) = \sum_{n=0}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!}\left[\frac{1}{\sqrt{2\pi(\sigma^2 + n\delta^2)}}\exp\left(-\frac{\left(x-\left(\mu-\frac{1}{2}\sigma^2\right) - n\beta\right)^2}{2(\sigma^2 + n\delta^2)}\right)\right]. \tag{7}$$

3.2. Double Exponential Jump Diffusion Model

The model that we also used for the price of WHGS3 sesame consists of two parts, a continuous part driven by a geometric Brownian motion and a jump part, with the logarithm of jump sizes having a double exponential distribution and the jump times corresponding to the event times of a Poisson process. Thus, under the physical probability measure P, the dynamics of the sesame price is assumed to follow the stochastic differential equation:

$$\frac{dS_t}{S_{t-}} = \mu_1 dt + \sigma_1 dB_t + d\left(\sum_{i=1}^{N_t}(V_i - 1)\right), \tag{8}$$

where B_t is a standard Brownian motion, N_t is a Poisson process with rate λ_1 and $\{V_t\}$ is sequence of independent identically distributed nonnegative random variables such that $\Upsilon_t = \ln(V_t)$ has an asymmetric double exponential distribution with density:

$$f(y) = p\eta_1 e^{-\eta_1 y} 1_{\{y\geq 0\}} + q\eta_2 e^{\eta_2 y} 1_{\{y<0\}}, \quad \eta_1 > 1,\ \eta_2 > 0,$$

where $p,\ q \geq 0,\ p + q = 1$ are constants and represent the probabilities of upward and downward jumps, respectively. This can be put in other way:

$$\Upsilon_t = \begin{cases} \xi^+, & \text{with probability } p \\ -\xi^-, & \text{with probability } q \end{cases}, \tag{9}$$

where ξ^+ and ξ^- exponentially random variables with mean $\frac{1}{\eta_1}$ and $\frac{1}{\eta_2}$, respectively.

The random variables N_t, B_t and Υ_t are assumed to be independent and identically distributed in the model. It is proposed that the drift μ_1 and the volatility σ_1 are constants, while the Brownian motion and jumps are one-dimensional [15]. The solution of the stochastic differential ***Eq. (8)*** using Ito's formula which gives the dynamics of the sesame price can be expressed as:

$$S_t = S_0 \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1{}^2\right)t + \sigma_1 B_t + \sum_{j=0}^{N_t} \Upsilon_j\right), \qquad \textbf{(10)}$$

where $E[\Upsilon_t] = \frac{p}{\eta_1} - \frac{q}{\eta_2}$, $Var[\Upsilon_t] = pq\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 + \left(\frac{p}{\eta_1{}^2} + \frac{q}{\eta_2{}^2}\right)$ and $E[\Upsilon_t] = E\left[e^{\Upsilon_t}\right] = \left(p\frac{\eta_1}{\eta_1 - 1} + q\frac{\eta_2}{\eta_2 + 1}\right)$, $\eta_1 > 1$, $\eta_2 > 0$. Here, $\eta_1 > 1$ guarantees for $E[\Upsilon_t] < \infty$ and $E[S_t] < \infty$. This means that the average upward jump cannot greater than 100%, which is quite reasonable [16].

Based on ***Eq. (10)***, the rate of sesame price return over the time interval Δt is given by:

$$\frac{\Delta S_t}{S_t} = \frac{S_{t+\Delta t}}{S_t} - 1 = \exp\left(\left(\mu_1 - \frac{1}{2}\sigma_1{}^2\right)\Delta t + \sigma_1 \Delta B_t + \sum_{j=0}^{\Delta N_t} \Upsilon_j\right) - 1.$$

If Δt becomes small enough, by neglecting the terms with order higher than Δt, the daily sesame price return can be approximated in distribution using expansion $e^x \approx 1 + x + \frac{1}{1}x^2$ by:

$$\frac{\Delta S_t}{S_t} \approx \mu_1 \Delta t + \sigma_1 Z\sqrt{\Delta t} + R\Upsilon_t, \qquad \textbf{(11)}$$

where Z is standard normal and R is Bernoulli random variable with $P(R = 1) = \lambda\Delta t$, $P(R = 0) = 1 - \lambda\Delta t$ and Υ_t is given by ***Eq. (9)***. The density function g of the right side of ***Eq. (11)*** which is an approximation of the sesame price return $\frac{\Delta S_t}{S_t}$ is given by:

$$\begin{aligned} g(x) = \frac{1 - \lambda_1 \Delta t}{\sigma_1\sqrt{\Delta t}} \phi\left(\frac{1 - \lambda_1 \Delta t}{\sigma_1\sqrt{\Delta t}}\right) &+ \lambda_1 \Delta t\left[p\eta_1 e^{\frac{\sigma_1{}^2\eta_1{}^2\Delta t}{2}} e^{-(x-\mu_1\Delta t)\eta_1} \times \Phi\left(\frac{x - \mu_1\Delta t - \sigma_1{}^2\eta_1\Delta t}{\sigma_1\sqrt{\Delta t}}\right)\right] \\ &+ \lambda_1 \Delta t\left[q\eta_2 e^{\frac{\sigma_1{}^2\eta_2{}^2\Delta t}{2}} e^{-(x-\mu_1\Delta t)\eta_2} \times \Phi\left(\frac{x - \mu_1\Delta t + \sigma_1{}^2\eta_2\Delta t}{\sigma_1\sqrt{\Delta t}}\right)\right]. \end{aligned} \qquad \textbf{(12)}$$

Setting $\Delta t = 1$, this density function can also be written as:

$$\begin{aligned} g(x) = \frac{1 - \lambda_1}{\sigma_1} \phi\left(\frac{1 - \lambda_1}{\sigma_1}\right) &+ \lambda_1\left[p\eta_1 e^{\frac{\sigma_1{}^2\eta_1{}^2}{2}} e^{-(x-\mu_1)\eta_1} \times \Phi\left(\frac{x - \mu_1 - \sigma_1{}^2\eta_1}{\sigma_1}\right)\right] \\ &+ \lambda_1\left[q\eta_2 e^{\frac{\sigma_1{}^2\eta_2{}^2}{2}} e^{-(x-\mu_1)\eta_2} \times \Phi\left(\frac{x - \mu_1 + \sigma_1{}^2\eta_2}{\sigma_1}\right)\right], \end{aligned} \qquad \textbf{(13)}$$

where $\phi(.)$ is density function of standard normal and $\Phi(.)$ is its distribution function.

Using Le'vy-Khintchine theorem, the characteristic function of the double exponential jump diffusion process of the log-return sesame price $\Delta \ln(S_t) = X_{\Delta t}$ over the time interval $[t, \ t+1]$ is represented by:

$$\phi_{X_{\Delta t}}(u) = E\left[e^{iuX_{\Delta t}}\right] = \exp\left[iu\mu_1 - \frac{{\sigma_1}^2 u^2}{2} + \lambda_1\left(\frac{p\eta_1}{\eta_1 - iu} + \frac{q\eta_2}{\eta_2 + iu} - 1\right)\right]. \qquad \textbf{\textit{(14)}}$$

4. Parameter Estimation

The parameter vectors, which are associated with the sesame price process, are denoted by $\theta = (\mu, \sigma, \ \beta, \delta, \ \lambda)$ and $\Theta = (\mu_1, \sigma_1, \eta_1, \eta_2, p, \lambda_1)$. The method of Maximum Likelihood Estimation (MLE) is used to estimate the parameters for Merton's and DEJD models by maximizing the likelihood function specified in ***Eq. (7)*** and the likelihood function is obtained by applying inverse Fourier transform on characteristic function specified in ***Eq. (14)***, respectively. We considered 70% of the log-return WHGS3 sesame price for parameters estimation and 30% for testing purpose that is for validation of these models. We truncate the number of jumps at n = 10 to estimate the parameter values as it is pointed out by [5]. In this paper, the parameters are estimated with corresponding 95% confidence intervals using the method. The error and variance for the parameters come from a maximum likelihood estimate. The variance is approximately equal to the inverse of Fisher's information matrix, evaluated at the estimates. The standard error, which is the square root of this variance, is estimated corresponding to each parameter using the method to check the reliability of the estimation. Finally, the estimated values are shown in ***Table 2*** and ***Table 3***. Lb and Ub are used to denote the lower and upper bound of confidence intervals, respectively.

Table 2. Estimated parameters associated with WHGS3 sesame price under Merton's model.

Parameters	Values	95% Confidence Interval		Standard Error
		Lb	Ub	
μ	-0.00073335	-0.00146962	0.00000292	0.00037565
σ	0.00954214	0.00868980	0.01039448	0.00043487
β	0.00332072	-0.00118924	0.00783069	0.00023011
δ	0.03737942	0.03155192	0.04320692	0.00297326
λ	0.26558969	0.19341604	0.33776333	0.03682396

Table 3. Estimated parameters associated with WHGS3 sesame price under DEJD model.

Parameters	Values	95% Confidence Lb	Interval Ub	Standard Error
μ_1	-0.00098422	-0.00197702	0.00000857	0.00050653
σ_1	0.00886518	0.00791537	0.00981499	0.00048460
η_1	45.32141145	34.57828814	56.06453475	2.48128608
η_2	45.29452607	32.54639635	58.04265580	3.50426733
p	0.56407188	0.443179541	0.68496422	0.06168089
λ_1	0.39925769	0.27624844	0.52226695	0.06276097

5. Model Simulation

In this section, we used the Euler discretized version of DEJD and Merton's models to simulate the sesame price. The discretized form of the Merton's model specified in ***Eq. (2)*** over the time interval $(t, t + \Delta t)$ can be expressed as:

$$X_{t+\Delta t} = X_t + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j, \quad \textbf{\textit{(15)}}$$

where $\Delta B_t = \sqrt{\Delta t} Z$ and $Z \sim N(0, 1)$.

Putting $\Delta t = 1$ and replacing $\ln(S_t)$ for X_t and $\ln(S_{t+1})$ for X_{t+1}, we obtain:

$$\ln(S_{t+1}) = \ln(S_t) + \left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j. \quad \textbf{\textit{(16)}}$$

This implies that:

$$S_{t+1} = S_t \exp\left(\left(\mu - \frac{\sigma^2}{2}\right) + \sigma Z + \sum_{j=0}^{N_t} Y_j\right). \quad \textbf{\textit{(17)}}$$

Moreover, the discretized form of DEJD model specified in ***Eq. (8)*** over the time interval $[t, t + \Delta t]$ and using Ito's lemma is given by:

$$\ln(S_{t+1}) = \ln(S_t) + \left(\mu_1 - \frac{{\sigma_1}^2}{2}\right)\Delta t + \sigma_1 \Delta B_t + \sum_{j=0}^{\Delta N_t} Y_j, \quad \textbf{\textit{(18)}}$$

where $\Delta B_t = \sqrt{\Delta t} Z$ and $Z \sim N(0, 1)$.

Setting $\Delta t = 1$, we get:

$$\ln(S_{t+1}) = \ln(S_t) + \left(\mu_1 - \frac{1}{2}{\sigma_1}^2\right) + \sigma_1 Z + \sum_{j=0}^{N_t} Y_j. \tag{19}$$

This expression can be also written as:

$$S_{t+1} = S_t \exp\left(\left(\mu_1 - \frac{1}{2}{\sigma_1}^2\right) + \sigma_1 Z + \sum_{j=0}^{N_t} Y_j\right). \tag{20}$$

We used the models specified in ***Eqs. (16)-(20)*** for the simulation of the sesame price. The fitted values from the simulations are plotted against the observed sesame price in ***Figure 6*** and ***Figure 7***.

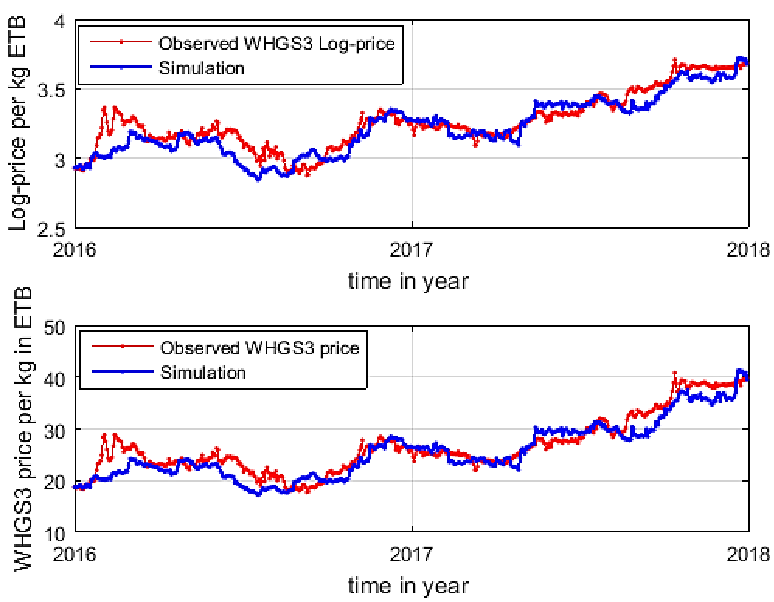

Figure 5. Simulated price fitted to WHGS3 log-price (Upper panel) and WHGS3 price (Lower panel) under Merton's model from 2016 to 2018.

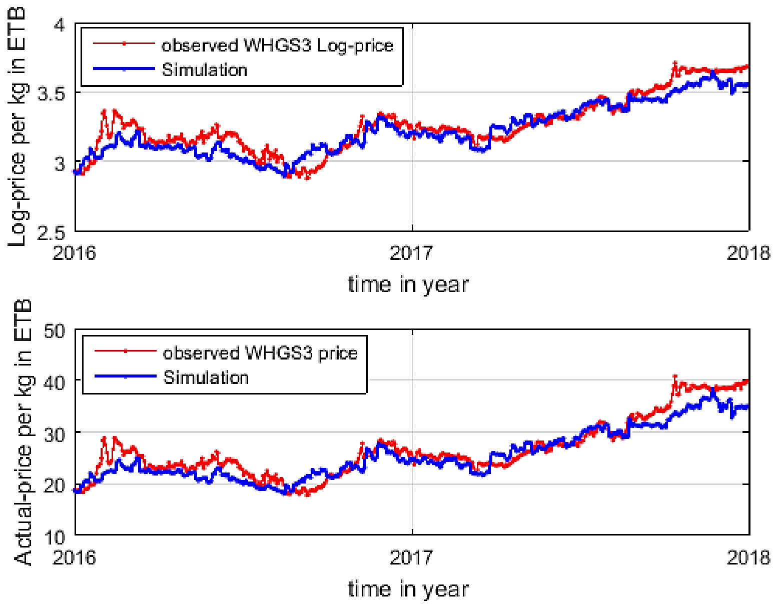

Figure 6. Simulated price fitted to WHGS3 log-price (Upper panel) and WHGS3 price (Lower panel) under DEJD model from 2016 to 2018.

In this paper, the RMSE, Q-Q plot and non-parametric fit with normal kernel are used to test the goodness of fit of the Merton's and DEJD distributions to the dynamic behavior of the sesame price. From the results, we conclude that the models perform well. However, DEJD model has better fitness to sesame data than Merton's model as shown in ***Figures (7)-(9)*** and ***Table 4*** below.

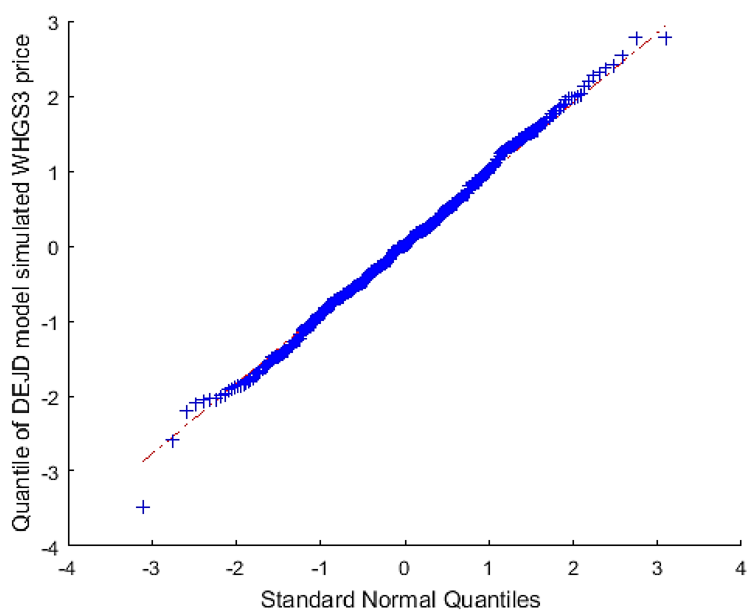

Figure 7. The Q-Q plot of DEJD model simulated WHGS3 sesame price.

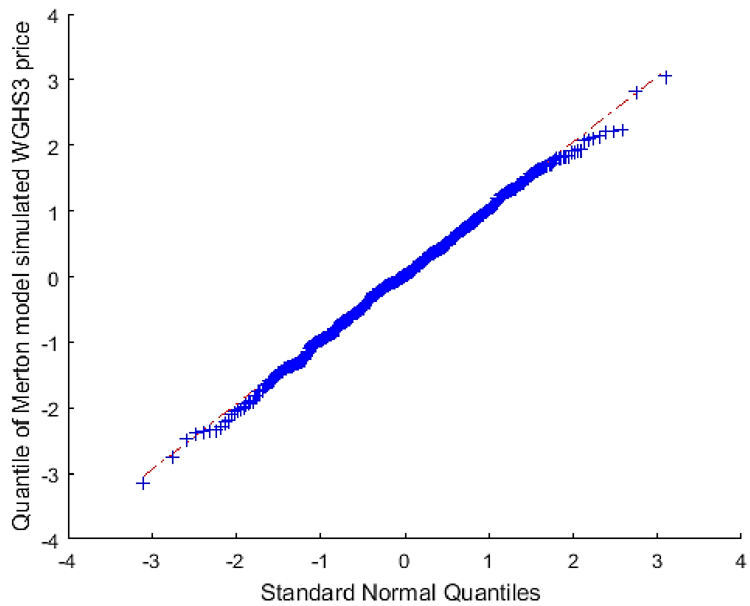

Figure 8. The Q-Q plot of Merton's model simulated WHGS3 sesame price.

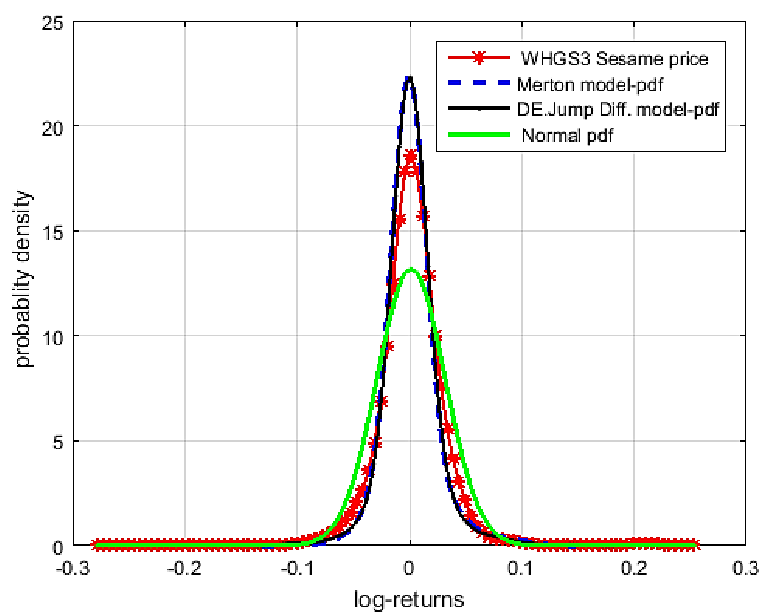

Figure 9. The probability densities of the models fitted with the log-return of WHGS3 price.

Table 4. RMSE values under DEJD and Merton's models.

Sesame price	RMSE: DEJD model	RMSE: Merton's model
WHGS3 price	0.08616489	0.09320617

6. Option Pricing

6.1. Option Pricing Using Merton Jump Diffusion Model

In this section, we used Merton jump diffusion model to determine the call option price for WHGS3 sesame price. However, this model which is to the contrary of Black-Scholes model, is incomplete. So, there are many possible choice to define a risk neutral measure Q equivalent to the physical probability measure P such that the discounted sesame price, $e^{-rt}S_t$ is a martingale where r is a risk free interest rate. Moreover, in order to make the discounted sesame price a martingale, the drift parameter μ must be set to $\mu = r - \lambda\kappa$, defining the risk neutral measure.

The stochastic differential equation which represents the dynamics of the sesame price can be expressed under the risk neutral measure Q as:

$$\frac{dS_t}{S_{t-}} = (r - \lambda\kappa)dt + \sigma d\widetilde{B}_t + (y_t - 1)d, \qquad (21)$$

where $\widetilde{B}_t$ is a standard Brownian motion under a risk neutral probability measure Q and $E[y_t - 1] = \exp\left(\beta + \frac{\delta^2}{2}\right) - 1 = \kappa$. $\lambda\kappa dt$ is the expected relative price change $E\left[\frac{dS_t}{S_t}\right]$ from the jump part dN_t in the time interval dt. This is the predictable part of the jump. This is why the instantaneous expected return under the risk neutral probability measure rdt is adjusted by $-\lambda\kappa dt$ in the drift term of the jump diffusion process to make the jump part unpredictable innovation. Solving the stochastic differential (***Eq. (21)***) gives the dynamics of sesame price under a risk neutral probability measure Q:

$$S_t = S_0 \exp\left(\left(r - \frac{\sigma^2}{2} - \lambda\kappa\right)t + \sigma\,\widetilde{B}_t + \sum_{i=0}^{N_t} Y_i\right), \qquad 0 \le t \le T, \qquad (22)$$

where $Y_t = \ln(y_t)$ is the log-return sesame price jump size. Assuming that the jumps are log-normally distributed that is $Y_t \sim N(\beta,\ \delta^2)$. The sesame price of the European style call option for the given strike price K, spot price S_0 at time t_0 and the terminal price S_T at maturity time T can be expressed [7, 20] as:

$$C(S_0, T) = \sum_{n=0}^{\infty} \frac{(\hat{\lambda}T)^n e^{-\hat{\lambda}T}}{n!} E^Q[e^{-rT}(S_T - K)^+ | S_t = S_0], \qquad (23)$$

$$C(S_0, T) = \sum_{n=0}^{\infty} \frac{(\hat{\lambda}T)^n e^{-\hat{\lambda} T}}{n!} \left[S_0 \Phi(d_{1,n}) - Ke^{-r_n T} \Phi(d_{2,n})\right], \qquad \textbf{\textit{(24)}}$$

where $\hat{\lambda} = \lambda(1+\kappa)$, $\sigma_n = \sqrt{\sigma^2 + \frac{n\delta^2}{T}}$, $r_n = r - \lambda\kappa + n\frac{\ln(1+\kappa)}{T}$, $d_{1,n} = \frac{\ln\left(\frac{S_0}{K}\right) + (r_n + \frac{1}{2}\sigma^2)}{\sigma_n \sqrt{T}}$, $d_{2,n} = d_{1,n} - \sigma_n\sqrt{T}$.

6.2. Option Pricing Using Double Exponential Jump Diffusion Model

Here, we used double exponential jump diffusion model to find the call option price of WHGS3 sesame price. However, this model is not complete because of its jump component. We considered the rational expectations arguments with a hyperbolic absolute risk aversion type utility function for the representative agent, as it is suggested by [17]. So, one can choose a risk neutral probability measure $\hat{Q}$ equivalent to the physical probability measure P so that the equilibrium price of an option is given by the rational expectation of the discounted option payoff with this risk neutral measure. The sesame price S_t still follows a double exponential jump diffusion process under the a risk neutral probability measure. The stochastic differential equation which describes the dynamic behavior of the sesame price under this measure is given by:

$$\frac{dS_t}{S_{t-}} = (r - \hat{\lambda}_1 \hat{\zeta})dt + \sigma_1 d\hat{B}_t + d\left(\sum_{i=1}^{\hat{N}_t} (\hat{V}_t - 1)\right). \qquad \textbf{\textit{(25)}}$$

Letting $X_t = \ln\left(\frac{S_t}{S_0}\right)$ and using Ito's lemma, the log-return sesame price over the time interval $[0,\ t]$, can be written as:

$$X_t = \left(r - \frac{{\sigma_1}^2}{2} - \hat{\lambda}_1 \hat{\zeta}\right)t + \sigma_1 \hat{B}_t + \sum_{i=0}^{\hat{N}_t} \hat{Y}_i, \quad X_0 = 0, \qquad \textbf{\textit{(26)}}$$

where $\hat{B}_t$ is a standard normal Brownian motion, $\hat{N}_t$ is a poison process with intensity $\hat{\lambda}_1$ and the log-jump size $\hat{Y}_t$ which forms a sequence of random variables with a new double exponential density function $\hat{f}(y)$ that are under the measure $\hat{Q}$. More precisely, this function can be written as:

$$\hat{f}(y) = \hat{p}\hat{\eta}_1 e^{-\hat{\eta}_1 y} 1_{\{y \geq 0\}} + \hat{q}\hat{\eta}_2 e^{\hat{\eta}_2 y} 1_{\{y<0\}}, \qquad \textbf{\textit{(26)}}$$

where $\hat{p}, \hat{q} \geq 0$, $\hat{\lambda}_1 > 0$, $\hat{p} + \hat{q} = 1$, $\hat{\eta}_1 > 1$, $\eta_2 > 0$ are constants and $\hat{\zeta} := E[\hat{V}_t] - 1 = \hat{p}\frac{\hat{\eta_1}}{\hat{\eta}_1 - 1} + \hat{q}\frac{\hat{\eta}_2}{\hat{\eta}_2 + 1} - 1$ the expected relative jump size in the DEJD model under $\hat{Q}$. For simplicity, we omit the superscript ˆ in the parameters and processes as we focus on option pricing. It is assumed that the sources of randomness, N_t , B_t , and Y_t are independent under $\hat{Q}$.

In this paper, we also used the method of Monte Carlo simulation to obtain an approximate solution to the call option sesame price. This method is one of the most popular numerical methods for pricing financial options because of the current advances in applying the tool [6, 22]. The method is mainly used to find an approximate solution to a complex financial problem, particularly European-style and exotic options for which no analytical pricing formula is available [10]. A Monte Carlo method is a technique that involves using random numbers and probability to solve problems and simulates paths for asset prices [12]. Since, the dynamic of the sesame price is modeled by double exponential jump diffusion model under a risk neutral measure, we used this method to find the expectation of the discount payoff of sesame price. We consider a call option giving an opportunity for the holder the right to buy the sesame at a fixed price K and fixed time T in the future. If at time T the sesame price S_T exceeds the strike price K , the holder exercises the option for a profit of $(S_T - K)^+$, otherwise the option expires worthless. Thus, the payoff to the option holder at time T is given by $(S_T - K)^+ = \max(S_T - K, 0)$. The solution which represents the daily return of sesame price as specified in ***Eq. (25)*** can be expressed by:

$$S_t = S_0 \exp\left(\left(r - \frac{{\sigma_1}^2}{2} - \lambda_1 \zeta\right) t + \sigma_1 B_t + \sum_{i=0}^{N_t} \Upsilon_i\right), \quad 0 \leq t \leq T. \tag{27}$$

It is assumed that S_0 is the current price of the sesame at $t = 0$, the random variable $B_t = \sqrt{T}\,Z$, $Z \sim N(0,1)$, and the log-jump size Υ_t having double exponential distributed. So, the terminal sesame price S_T over the time interval $[0,\ T]$ can be represented as:

$$S_T = S_0 \exp\left[\left(r - \frac{{\sigma_1}^2}{2} - \lambda_1 \zeta\right) T + \sigma_1 \sqrt{T}\, Z + \sum_{i=0}^{N_t} \Upsilon_i\right]. \tag{28}$$

We considered the first 10,000 sample paths simulation of the sesame price S_T and we used the following algorithm to estimate the expectation $E[e^{-rT}(S_T - K)^+]$:

Take $n = 10;000$;
for $j = 1, \ldots, n$
generate Z_j and Υ_j from the respective distribution,
set $S_T(j) = S_0 \exp\left[\left(r - \frac{{\sigma_1}^2}{2} - \lambda_1 \zeta\right) T + \sigma_1 \sqrt{T}\, Z(j) + \sum_{i=0}^{N_t(j)} \Upsilon_i\right]$,
set $C(j) = e^{-rT} \max(S_0 - K, 0)$
set $\hat{C}_n = \frac{C_1 + C_2 + C_3 + \cdots + C_n}{n}$.

For ≥ 1, the estimator $\hat{C}_n$ is unbiased which implies that its expectation $E[\hat{C}_n] = C \equiv E[e^{-rT}(S_T - K)^+]$ and it is consistent meaning that as $n \to \infty$, $\hat{C}_n \to C$ with probability 1.

Thus, we applied Monte Caro simulation technique to determine the call option prices of WHGS3 sesame price under DEJD and Merton's models. We used spot price $S0 = 23$, strike price K and interest rate $r = 0.07$ to calculate the call option prices at maturity time $T = 0.2535$ and these are indicated in ***Figures (10)-(12)*** and ***Table 5*** below.

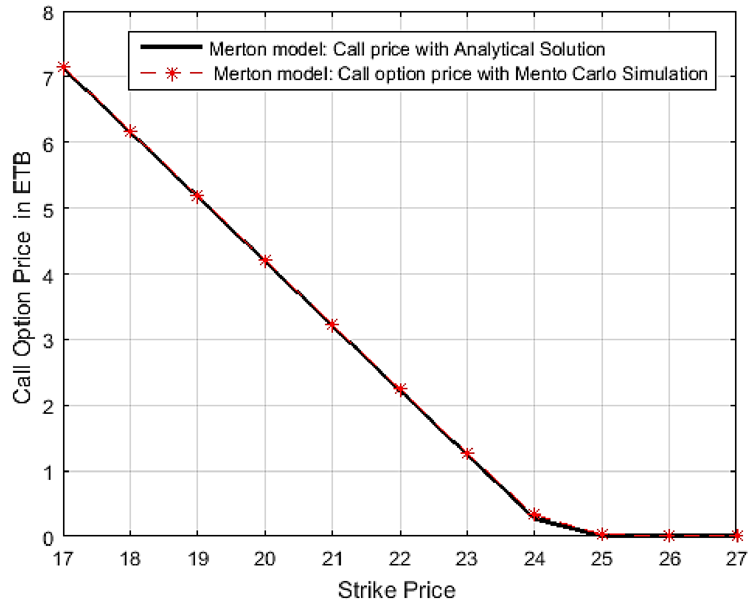

Figure 10. Call option WHGS3 price under Merton's model.

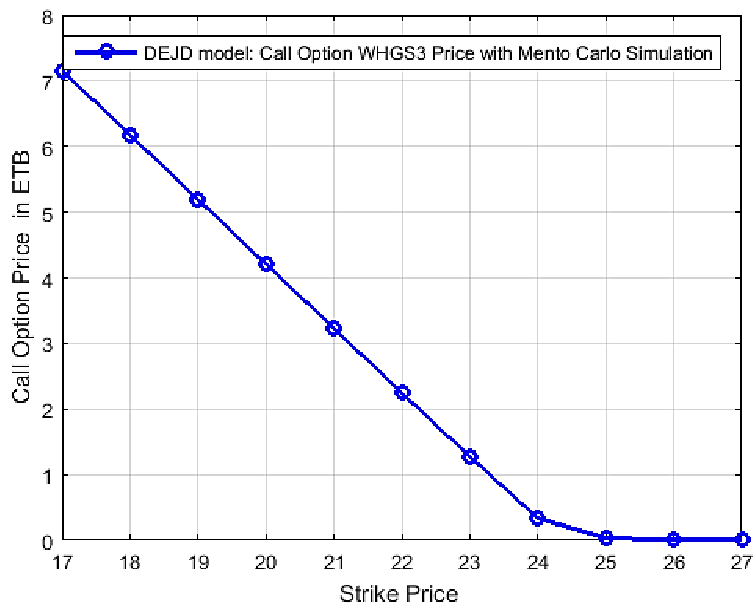

Figure 11. Call option WHGS3 price under DEJD model.

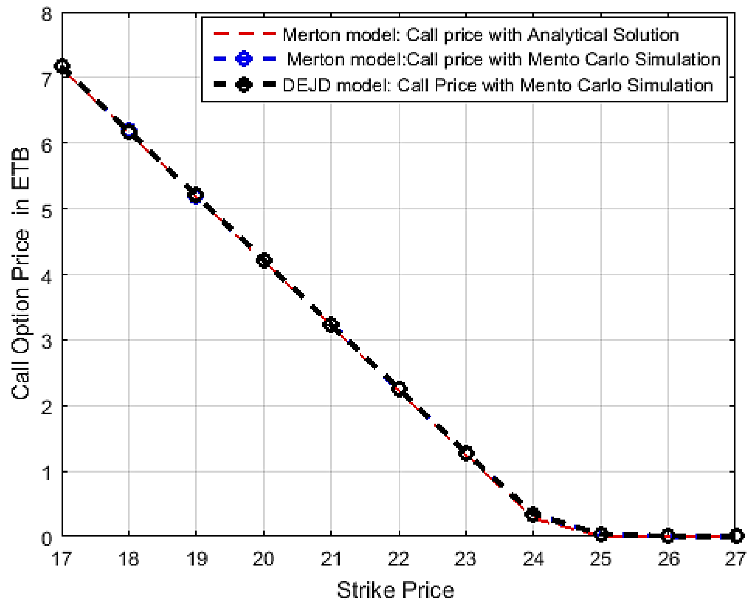

Figure 12. Call option WHGS3 price under DEJD and Merton's models.

Table 5. Call option WHGS3 price values estimated by MC simulation under DEJD and Merton's models.

Strike price K	17	18	19	20	21	22	23	24	25
MC-DEJD M.	7.161	6.177	5.191	4.216	3.226	2.247	1.271	0.337	0.0361
MC-Merton M.	7.158	6.171	5.190	4.210	3.222	2.246	1.269	0.329	0.0334

7. Comparison of Models

We used DEJD and Merton's models for modeling and call option pricing of sesame price. These models are applied to describe the dynamic behavior the price to capture heavy skewness and high kurtosis of the WHGS3 price. The RMSE, Q-Q plot and the method of non-parametric fit with normal kernel are used to test the goodness of fit of the models with to the observed WHGS3 sesame price. The tests indicate that these two models perform well to describe the dynamic behavior of sesame price. However, DEJD model shows good performance than Merton's model. The call option prices of sesame price under double exponential jump diffusion model is over estimated in the money option contracts when we compared to Merton's model as shown in ***Figure 12*** and ***Table 5***. Finally, from the results, we conclude that the models show good performance for modeling and option pricing of the WHGS3 price to reduce. But, the double exponential jump diffusion model is comparatively more efficient than Merton's model.

8. Result and Discussion

In this paper, we used the daily recorded WHGS3 sesame price from 31 November 5, 2010 to 30 March 2018 at ECX market. The method of maximum likelihood is being used to estimate the parameters for DEJD and Merton's model as shown in ***Table 2*** and ***Table 3***, respectively. This result indicated that the dynamics of the sesame price process were influenced by both diffusion and jump components, however, the price was dominated by a jump component with large discontinuities occurring at high intensity. The high volatility of the jump component reflects the presence of jumps of large magnitude and was in accordance with excess kurtosis in the empirical distribution of the data. The WHGS3 price simulated under the models are as shown in ***Figure 5*** and ***Figure 6***. The RMSE, Q-Q plot and the method of non-parametric fit with normal kernel used, test the goodness for fitting of the models to the historical price as shown in ***Table 4*** and ***Figures (7)-(9)***, respectively. The Q-Q plot is used to compare the quantile of simulated WHGS3 price under the models to the quantile of historical sesame price. Furthermore, the method of non-parametric fit with normal kernel is used to plot the graphs of the probability density functions of the models to assess the goodness of fit of distributions of the models to the dynamic behavior of the price. Thus, the tests indicate that the models perform well. Analytical and Monte Carlo simulation method under Merton's model and Monte Carlo simulation technique under double exponential jump diffusion model are used to find the call option pricing of WHGS3 sesame price. So, the call option prices of WHGS3 sesame price, which are determined at a maturity time under these models, are plotted as shown in ***Figures (10)-(12)*** and ***Table 5***. As a result, the comparison between call price values of the Merton's and double exponential jump diffusion models is indicated in ***Figure 12*** and ***Table 5***.

9. Conclusion

WHGS3 sesame price from 2010 to 2018 are characterized by large fluctuations in value. The nature of log-returns of the sesame price exhibits fat tails and high kurtosis. We used Merton's and double exponential jump diffusion models for modeling and option pricing of WHGS3 sesame price to reduce the price fluctuation associated with the price. The method of maximum likelihood is applied to estimate the parameters under the models. The Q-Q plot is used to compare the quantile of simulated WHGS3 price under the models to the quantile of historical sesame price and the graph of simulated prices approximately lie on the line $y = x$. The method of non-parametric fit with normal kernel is applied to assess the goodness of fit of the models by comparing the estimated probability density functions of the models to the historical density function of WHGS3 price. The RMSE is used to test the validation of models. The tests show that the models perform well. The method of analytical and Monte Carlo simulation under Merton's model and Monte Carlo simulation technique under the double exponential jump diffusion model are applied to find the call option prices of WHGS3 sesame price. Finally, from the results, we conclude that the models are suitable for modeling and option pricing of the

WHGS3 price to reduce the risk associated with the price fluctuation, though, the double exponential jump diffusion model is relatively more efficient than Merton's model.

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