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Calculation of Fuzzy Matrices Determinant

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A B S T R A C T

Matrix and its determinant are two basic tools, which are important in financial, accounting, and economic affairs. Therefore, in this paper, a simple and effective method is proposed to obtain the determinant of fuzzy matrices. First, using arithmetic operations based on Transmission Average (TA), the second order fuzzy determinant is calculated. Then, Sarrus rule is defined to calculate third order fuzzy determinant. Finally, by defining minor of fuzzy matrix and ij_{th} adjugate of the fuzzy matrix, n_{th} order fuzzy determinant is calculated. The effectiveness and applicability of the proposed method are verified by solving some numerical examples.

Keywords: Arithmetic operations, Fuzzy approximation, Fuzzy determinant.

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1. Introduction

Modeling of many problems in engineering, economic science, etc. leads to a matrix and determinant [1-6]. Since uncertainty is involved in the data achieved from the real environment and determining the exact value of the parameters is difficult, the output of the mathematical models generally does not has the necessary efficiency. For this reason, it is essential to use an appropriate tool to deal with uncertain data in optimization issues. One of the most important tools to deal with uncertainty is fuzzy theory, which has been widely used in various issues [7-13]. For example, Kim et al. [14] proposed determinant theory for square fuzzy matrices involving Bully matrices using Cayley Hamilton theorem. Also, in 1994, determinant and adjoint of a square fuzzy matrices were defined in [15]. Recently, TA-based arithmetic operation has been proposed in [16] and Allahviranloo et al. [17] solved the fuzzy linear equations using this method. Moreover, Dhar [18] calculated the fuzzy determinant using reference functions. In this

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paper, using TA-based arithmetic operations, a simple method is proposed to calculate the fuzzy determinant.

The structure of the paper is as follows. In Section 2, some necessary basic definitions based on TA are presented. In Section 3, the proposed method for calculation of the fuzzy determinant is introduced. Numerical examples are presented in Section 4 and conclusion and future works are presented in Section 5.

2. Basic Definitions

In this section, some basic definitions which are required in the following are defined. These notations can be found in [19-23].

Definition 1. Let A be a fuzzy set in R $\left(A = \left\{\left(x, \mu_A(x)\right) \middle| x \in R\right\}\right)$. Then

- A is called normal if there exists an $x \in R$such that $\mu_{\tilde{A}}(x) = 1$. Otherwise, A is subnormal,
- The support of A, denoted $\mathrm{supp}(A)$, is the subset of R whose elements all have nonzero membership grades in A. In the other words, $\mathrm{supp}(A) = \{x \in R | \mu_A(x) > 0\}$.
- An α-level set (or α –cut) of a fuzzy set A in R is a non-fuzzy set denoted by A_α and defined by

$$A_\alpha = \begin{cases} \{ x \in R | \mu_{\tilde{A}}(x) > 0 \} & \alpha > 0, \\ \mathrm{cl}\left(\mathrm{supp}\ (A)\right) & \alpha = 0. \end{cases}$$

Where $\mathrm{cl}\left(\mathrm{supp}(A)\right)$ denotes the closure of the support of A.

Definition 2. Let $\tilde{A}$ be a Normal, Convex and Continuous (NCC) fuzzy set on the universal set U. Then,

$$\mathrm{ac}\ (\tilde{A}) = \frac{1}{2}\left\{\min\left(\mathrm{core}(\tilde{A})\right) + \max\left(\mathrm{core}(\tilde{A})\right)\right\}.$$

Definition 3. A fuzzy number $\tilde{A}$ is called a pseudo-triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} l_{\tilde{A}}(x) & \underline{a} \le x \le a\,, \\ r_{\tilde{A}}(x) & a \le x \le \overline{a}\,, \\ 0 & \text{otherwise}\,. \end{cases}$$

where $l_{\tilde{A}}(x)$ and $r_{\tilde{A}}(x)$ are non-decreasing and non-increasing functions, respectively. The pseudo-triangular fuzzy number $\tilde{A}$ is denoted by the quintuplet $\tilde{A} = \left(\underline{a}, a, \overline{a}, l_{\tilde{A}}(x), r_{\tilde{A}}(x)\right)$ and the triangular fuzzy number by $\tilde{A} = (\underline{a}, a, \overline{a}, -, -)$.

Definition 4. A fuzzy number $\tilde{A}$ is called a pseudo-trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x)$ is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} l_{\tilde{A}}(x) & \underline{a} \le x \le a_1, \\ 1 & a_1 \le x \le a_2\,, \\ r_{\tilde{A}}(x) & a_2 \le x \le \overline{a}\,, \\ 0 & \text{otherwise}. \end{cases}$$

Where $l_{\widetilde{A}}(x)$ and $r_{\widetilde{A}}(x)$ are non-decreasing and non-increasing functions, respectively. The pseudo-trapezoidal fuzzy number $\widetilde{A}$ is denoted by the Senary $\widetilde{A} = \left(\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)\right)$ and the trapezoidal fuzzy number by the Senary $\widetilde{A} = (\underline{a}, a_1, a_2, \overline{a}, -, -)$.

Definition 5. Consider two pseudo-triangular fuzzy numbers

$$\widetilde{A} = \left(\underline{a}, a, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)\right), \qquad \widetilde{B} = \left(\underline{b}, b, \overline{b}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x)\right)$$

with the following α-cut forms

$$\widetilde{A} = \cup_\alpha A_\alpha,\ A_\alpha = [\underline{A}_\alpha, \overline{A}_\alpha],\ \widetilde{B} = \cup_\alpha B_\alpha,\ B_\alpha = [\underline{B}_\alpha, \overline{B}_\alpha].$$

In the following, we define fuzzy arithmetic operations based on TA.

$$\begin{aligned} &\widetilde{A} + \widetilde{B} = \cup_\alpha \left(\widetilde{A} + \widetilde{B}\right)_\alpha, \\ &\left(\widetilde{A} + \widetilde{B}\right)_\alpha = \left[\frac{a+b}{2} + \left(\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2}\right), \frac{a+b}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2}\right)\right], \end{aligned} \tag{1}$$

$$\begin{aligned} &\widetilde{A} - \widetilde{B} = \cup_\alpha \left(\widetilde{A} - \widetilde{B}\right)_\alpha, \\ &\left(\widetilde{A} - \widetilde{B}\right)_\alpha = \left[\frac{a-3b}{2} + \left(\frac{\underline{A}_\alpha + \underline{B}_\alpha}{2}\right), \frac{a-3b}{2} + \left(\frac{\overline{A}_\alpha + \overline{B}_\alpha}{2}\right)\right], \end{aligned} \tag{2}$$

$$\begin{aligned} &\widetilde{A}.\widetilde{B} = \cup_\alpha \left(\widetilde{A}.\widetilde{B}\right)_\alpha, \\ &\left(\widetilde{A}.\widetilde{B}\right)_\alpha = \begin{cases} \left[\left(\frac{b}{2}\right)\underline{A}_\alpha + \left(\frac{a}{2}\right)\underline{B}_\alpha, \left(\frac{b}{2}\right)\overline{A}_\alpha + \left(\frac{a}{2}\right)\overline{B}_\alpha\right], & a \ge 0, b \ge 0, \\ \left[\left(\frac{b}{2}\right)\overline{A}_\alpha + \left(\frac{a}{2}\right)\underline{B}_\alpha, \left(\frac{b}{2}\right)\underline{A}_\alpha + \left(\frac{a}{2}\right)\overline{B}_\alpha\right], & a \ge 0, b \le 0, \\ \left[\left(\frac{b}{2}\right)\overline{A}_\alpha + \left(\frac{a}{2}\right)\overline{B}_\alpha, \left(\frac{b}{2}\right)\underline{A}_\alpha + \left(\frac{a}{2}\right)\underline{B}_\alpha\right], & a \le 0, b \le 0, \\ \left[\left(\frac{b}{2}\right)\underline{A}_\alpha + \left(\frac{a}{2}\right)\overline{B}_\alpha, \left(\frac{b}{2}\right)\overline{A}_\alpha + \left(\frac{a}{2}\right)\underline{B}_\alpha\right], & a \le 0, b \ge 0. \end{cases} \end{aligned} \tag{3}$$

$$\widetilde{A}^{-1} = \cup_\alpha \left(\widetilde{A}^{-1}\right)_\alpha, \quad \left(\widetilde{A}^{-1}\right)_\alpha = \left[\frac{1}{a^2}\underline{A}_\alpha, \frac{1}{a^2}\overline{A}_\alpha\right]. \tag{4}$$

$$\begin{aligned} &\widetilde{A}.\widetilde{B}^{-1} = \cup_\alpha \left(\widetilde{A}.\widetilde{B}^{-1}\right)_\alpha, \\ &\left(\widetilde{A}.\widetilde{B}^{-1}\right)_\alpha = \begin{cases} \left[\left(\frac{1}{2b}\right)\underline{A}_\alpha + \left(\frac{a}{2b^2}\right)\underline{B}_\alpha, \left(\frac{1}{2b}\right)\overline{A}_\alpha + \left(\frac{a}{2b^2}\right)\overline{B}_\alpha\right], & a \ge 0, b > 0, \\ \left[\left(\frac{1}{2b}\right)\overline{A}_\alpha + \left(\frac{a}{2b^2}\right)\underline{B}_\alpha, \left(\frac{1}{2b}\right)\underline{A}_\alpha + \left(\frac{a}{2b^2}\right)\overline{B}_\alpha\right], & a \ge 0, b < 0, \\ \left[\left(\frac{1}{2b}\right)\overline{A}_\alpha + \left(\frac{a}{2b^2}\right)\overline{B}_\alpha, \left(\frac{1}{2b}\right)\underline{A}_\alpha + \left(\frac{a}{2b^2}\right)\underline{B}_\alpha\right], & a \le 0, b < 0, \\ \left[\left(\frac{1}{2b}\right)\underline{A}_\alpha + \left(\frac{a}{2b^2}\right)\overline{B}_\alpha, \left(\frac{1}{2b}\right)\overline{A}_\alpha + \left(\frac{a}{2}\right)\underline{B}_\alpha\right], & a \le 0, b > 0. \end{cases} \end{aligned} \tag{5}$$

Definition 6. Consider two pseudo-trapezoidal fuzzy numbers:

$$\widetilde{A} = \left(\underline{a}, a_1, a_2, \overline{a}, l_{\widetilde{A}}(x), r_{\widetilde{A}}(x)\right), \qquad \widetilde{B} = \left(\underline{b}, b_1, b_2, \overline{b}, l_{\widetilde{B}}(x), r_{\widetilde{B}}(x)\right),$$

with the following α-cut forms:

$$\tilde{A} = \cup_{\alpha} A_{\alpha}, \qquad A_{\alpha} = [\underline{A}_{\alpha}, \overline{A}_{\alpha}],$$
$$0 \le \alpha \le 1, \qquad A_1 = [a_1, a_2],$$
$$\tilde{B} = \cup_{\alpha} B_{\alpha}, \qquad B_{\alpha} = [\underline{B}_{\alpha}, \overline{B}_{\alpha}],$$
$$0 \le \alpha \le 1, \qquad B_1 = [b_1, b_2].$$

Let

$$\phi = \frac{a_1 + a_2}{2}, \qquad \varphi = \frac{b_1 + b_2}{2}.$$

In the following, the fuzzy arithmetic operations are defined based on TA.

$$\tilde{A} + \tilde{B} = \cup_{\alpha} (\tilde{A} + \tilde{B})_{\alpha},$$
$$(\tilde{A} + \tilde{B})_{\alpha} = \left[\frac{\phi + \varphi}{2} + \left(\frac{\underline{A}_{\alpha} + \underline{B}_{\alpha}}{2}\right), \frac{\phi + \varphi}{2} + \left(\frac{\overline{A}_{\alpha} + \overline{B}_{\alpha}}{2}\right)\right], \tag{6}$$

$$\tilde{A} - \tilde{B} = \cup_{\alpha} \left(\tilde{A} - \tilde{B}\right)_{\alpha},$$
$$\left(\tilde{A} - \tilde{B}\right)_{\alpha} = \left[\frac{\phi - 3\varphi}{2} + \left(\frac{\underline{A}_{\alpha} + \underline{B}_{\alpha}}{2}\right), \frac{\phi - 3\varphi}{2} + \left(\frac{\overline{A}_{\alpha} + \overline{B}_{\alpha}}{2}\right)\right], \tag{7}$$

$$\tilde{A}.\tilde{B} = \cup_{\alpha} \left(\tilde{A}.\tilde{B}\right)_{\alpha},$$
$$\left(\tilde{A}.\tilde{B}\right)_{\alpha} = \begin{cases} \left[\left(\frac{\varphi}{2}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\underline{B}_{\alpha}, \left(\frac{\varphi}{2}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\overline{B}_{\alpha}\right], & \phi \ge 0, \varphi \ge 0, \\ \left[\left(\frac{\varphi}{2}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\underline{B}_{\alpha}, \left(\frac{\varphi}{2}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\overline{B}_{\alpha}\right], & \phi \ge 0, \varphi \le 0, \\ \left[\left(\frac{\varphi}{2}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\overline{B}_{\alpha}, \left(\frac{\varphi}{2}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\underline{B}_{\alpha}\right], & \phi \le 0, \varphi \le 0, \\ \left[\left(\frac{\varphi}{2}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\overline{B}_{\alpha}, \left(\frac{\varphi}{2}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2}\right)\underline{B}_{\alpha}\right], & \phi \le 0, \varphi \ge 0. \end{cases} \tag{8}$$

$$\tilde{A}^{-1} = \cup_{\alpha} \left(\tilde{A}^{-1}\right)_{\alpha}, \qquad \left(\tilde{A}^{-1}\right)_{\alpha} = \left[\left(\frac{1}{\phi^2}\right)\underline{A}_{\alpha}, \left(\frac{1}{\phi^2}\right)\overline{A}_{\alpha}\right]. \tag{9}$$

$$\tilde{A}.\tilde{B}^{-1} = \cup_{\alpha} \left(\tilde{A}.\tilde{B}^{-1}\right)_{\alpha},$$
$$\left(\tilde{A}.\tilde{B}^{-1}\right)_{\alpha} = \begin{cases} \left[\left(\frac{1}{2\varphi}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\underline{B}_{\alpha}, \left(\frac{1}{2\varphi}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\overline{B}_{\alpha}\right], & \phi \ge 0, \varphi > 0, \\ \left[\left(\frac{1}{2\varphi}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\underline{B}_{\alpha}, \left(\frac{1}{2\varphi}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\overline{B}_{\alpha}\right], & \phi \ge 0, \varphi < 0, \\ \left[\left(\frac{1}{2\varphi}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\overline{B}_{\alpha}, \left(\frac{1}{2\varphi}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\underline{B}_{\alpha}\right], & \phi \le 0, \varphi < 0, \\ \left[\left(\frac{1}{2\varphi}\right)\underline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\overline{B}_{\alpha}, \left(\frac{1}{2\varphi}\right)\overline{A}_{\alpha} + \left(\frac{\phi}{2\varphi^2}\right)\underline{B}_{\alpha}\right], & \phi \le 0, \varphi > 0. \end{cases} \tag{10}$$

3. Proposed Method

Definition 7. If all elements of a matrix are fuzzy numbers, the matrix is called fuzzy matrix.

Definition 8. Determinant of a 2×2 fuzzy matrix as $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} \\ \tilde{a}_{21} \tilde{a}_{22} \end{bmatrix}$ is shown by $|\tilde{A}|$ and is defined as follow:

$$(|\tilde{A}|)_\alpha = ((\tilde{a}_{11})_\alpha . (\tilde{a}_{22})_\alpha) - ((\tilde{a}_{12})_\alpha . (\tilde{a}_{21})_\alpha), \qquad |\tilde{A}| = \cup_\alpha (|\tilde{A}|)_\alpha. \tag{11}$$

Definition 9. (Sarrus rules for calculating determinant of a 3×3 fuzzy matrix) Consider matrix $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} \tilde{a}_{13} \\ \tilde{a}_{21} \tilde{a}_{22} \tilde{a}_{23} \\ \tilde{a}_{31} \tilde{a}_{32} \tilde{a}_{33} \end{bmatrix}$. To obtain the determinant, first $(\tilde{A})_\alpha$ is achieved:

$$\begin{aligned} (|\tilde{A}|)_\alpha = & (((\tilde{a}_{11})_\alpha . (\tilde{a}_{22})_\alpha . (\tilde{a}_{33})_\alpha) + ((\tilde{a}_{12})_\alpha . (\tilde{a}_{23})_\alpha . (\tilde{a}_{31})_\alpha) + \\ & +((\tilde{a}_{13})_\alpha . (\tilde{a}_{21})_\alpha . (\tilde{a}_{32})_\alpha) - (((\tilde{a}_{13})_\alpha . (\tilde{a}_{22})_\alpha . (\tilde{a}_{31})_\alpha) \\ & +((\tilde{a}_{11})_\alpha . (\tilde{a}_{23})_\alpha . (\tilde{a}_{32})_\alpha) + ((\tilde{a}_{12})_\alpha . (\tilde{a}_{21})_\alpha . (\tilde{a}_{33})_\alpha)). \end{aligned} \tag{12}$$

Definition 10. Let $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots \quad \vdots & \vdots & \vdots \\ \tilde{a}_{n1} \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix}$, ijth minor of fuzzy matrix is denoted by $\tilde{M}_{ij}$ and is a matrix cut down from $\tilde{A}$ by removing its ith row and jth column.

Definition 11. Consider $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots \quad \vdots & \vdots & \vdots \\ \tilde{a}_{n1} \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix}$, ijth adjutant of the fuzzy matrix is shown by $\tilde{A}_{ij}$ and is defined as follow:

$$(\tilde{A}_{ij})_\alpha = (-1)^{i+j} (|\tilde{M}_{ij}|)_\alpha \tilde{A}_{ij} = \cup_\alpha (\tilde{A}_{ij})_\alpha .$$

In the following, the expansion method is explained that can be used for calculating determinant of $n \times n$ fuzzy matrices.

Definition 12. Consider fuzzy matrix $\tilde{A} = \begin{bmatrix} \tilde{a}_{11} \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots \quad \vdots & \vdots & \vdots \\ \tilde{a}_{n1} \tilde{a}_{n2} & \cdots & \tilde{a}_{nn} \end{bmatrix}$. The determinant of thematrix can be evaluated by expanding each row or column of the matrix. For example, by expanding with respect to the first row:

$$\begin{aligned} & (|\tilde{A}|)_\alpha = (\tilde{a}_{11})_\alpha (\tilde{A}_{11})_\alpha + (\tilde{a}_{12})_\alpha (\tilde{A}_{12})_\alpha + . \cdots + (\tilde{a}_{1n})_\alpha (\tilde{A}_{1n})_\alpha , \\ & |\tilde{A}| = \cup_\alpha (|\tilde{A}|)_\alpha . \end{aligned} \tag{13}$$

4. Numerical Examples

In this section, the performance of the proposed method is shown through some numerical examples.

Example 1. Consider the following matrix:

$$\widetilde{A}=\begin{bmatrix}(0,4,6,\frac{1}{4}x,(1-\frac{1}{4}(x-4)^2)^{\frac{1}{2}}) & (-3,-2,-1,-,-) & (1,2,7,-,-)\\ (3,4,5,-,-) & (1,2,4,-,-) & (6,8,11,-,-)\\ (1,2,3,-,-) & (1,2,3,-,-) & (3,4,5,-,-)\end{bmatrix},$$

there is:

$$(\tilde{A})_\alpha=\begin{bmatrix}\left[4\alpha,4+\sqrt{4-4\alpha^2}\right] & [-3+\alpha,-1-\alpha] & [1+\alpha,7-5\alpha]\\ [3+\alpha,5-\alpha] & [1+\alpha,4-2\alpha] & [6+2\alpha,11-3\alpha]\\ [1+\alpha,3-\alpha] & [1+\alpha,3-\alpha] & [3+\alpha,5-\alpha]\end{bmatrix},$$

From ***Eq. (12)***:

$$\left(|\widetilde{A}|\right)_\alpha=\left[-\frac{297}{8}+\frac{105}{8}\alpha,-\frac{85}{8}+\frac{\sqrt{1-\alpha^2}}{2}-\frac{107}{8}\alpha\right].$$

Therefore

$$|\widetilde{A}|=\cup_\alpha\left[-\frac{297}{8}+\frac{105}{8}\alpha,-\frac{85}{8}+\frac{\sqrt{1-\alpha^2}}{2}-\frac{107}{8}\alpha\right].$$

Example 2. Consider the following matrix:

$$\widetilde{A}=\begin{bmatrix}(-4,1,2,-,-) & (5,6,7,-,-) & (1,2,4,(1-(x-2)^2)^{\frac{1}{2}},(1-\frac{1}{4}(x-2)^2)^{\frac{1}{2}})\\ (1,2,3,-,-) & (6,6,7,-,-) & (1,2,3,-,-)\\ (-4,1,2,-,-) & (4,5,7,-,-) & (-7,2,3,-,-)\end{bmatrix},$$

from ***Eq. (13)*** the following is achieved:

$$\begin{aligned}\left(|\widetilde{A}|\right)_\alpha&=(\tilde{a}_{11})_\alpha(\widetilde{A}_{11})_\alpha+(\tilde{a}_{12})_\alpha(\widetilde{A}_{12})_\alpha+(\tilde{a}_{13})_\alpha(\widetilde{A}_{13})_\alpha=\\ &=\left[-\frac{13}{2}+\frac{5}{2}\alpha-\frac{5}{4}\sqrt{1-\alpha^2},\frac{17}{4}-\frac{33}{4}\alpha+\frac{5}{8}\sqrt{1-\alpha^2}\right]+\\ &\left[-\frac{3}{4}\sqrt{1-\alpha^2},\frac{3}{2}\sqrt{1-\alpha^2}\right]+\left[-\frac{2}{8}+\frac{18}{8}\alpha,\frac{77}{8}-\frac{61}{8}\alpha\right]=\\ &\left[-\frac{60}{16}-\frac{1}{2}\sqrt{1-\alpha^2}+\frac{28}{16}\alpha,\frac{29}{16}-\frac{61}{16}\alpha+\frac{17}{32}\sqrt{1-\alpha^2}\right].\end{aligned}$$

Therefore

$$|\widetilde{A}|=\cup_\alpha\left[-\frac{60}{16}-\frac{1}{2}\sqrt{1-\alpha^2}+\frac{28}{16}\alpha,\frac{29}{16}-\frac{61}{16}\alpha+\frac{17}{32}\sqrt{1-\alpha^2}\right].$$

5. Conclusion

Since determinant is a key tool in calculating the matrix properties, in this paper, Sarrus rules was investigated to calculate the determinant of 3×3 fuzzy matrices. Moreover, the expansion method was explained to generally calculate the determinant using TA-based arithmetic operations. The advantage of the proposed method is that the determinant of a fuzzy matrix is always a fuzzy number. The superiority of the proposed method with respect to the method in [7] is that in the aforementioned paper, the determinant is calculated for matrices whose elements belong to [0,1]. However, in the present paper, the determinant is achieved for matrices with semi-trapezoidal and semi-triangular elements. In the next paper, the fuzzy equation set is going to be solved using fuzzy determinant.

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