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Some Fixed Point Results Involving a General LW-Type Cyclic Mapping in Complete B-Metric-Like Spaces

S. Q. Weng*

Ideological and Political Foundation, SiChuan Sanhe College of Professionals, Hejiang, Luzhou, 646200, Sichuan, China.

A B S T R A C T

In recent years, the research of the fixed point theorem is a hot topic all the time. In this paper, we propose the notion of new mapping, that is, a general LW-type cyclic mapping, in a complete b-metric-like spaces. Then, we obtain the existence and uniqueness theorem of its fixed point. Moreover, we give an example to illustrate the main results of this paper.

Keywords: B-Metric-like space, Fixed point theorem, LW-type Cyclic mapping.

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1. Introduction

Fixed point theory plays an important role in the field of functional analysis, even in the whole mathematical field. The first important fixed point theorem belongs to Banach (that is, Banach contraction mapping principle) [1]. He proved that in a complete metric space every Banach contraction mapping exists a fixed point, which is unique. Let (X,d) be a metric space, and T be a self-mapping in X, then is said to be a Banach contraction mapping if there exists a constant $k\in(0,1)$ such that $d(Tx,Ty)\leq kd(x,y),\forall x,y\in X.$ Banach contraction mapping is a typical and important nonlinear mapping with a wide range of practical background. Banach contraction mapping is an abstract expression of the classic Picard iteration method and a typical algebraic fixed point theorem.

According to this theorem, we can not only determine the existence and uniqueness of fixed points, but also construct an iterative scheme to approximate fixed points to any degree of

* Corresponding author
E-mail address: wengsq96@163.com
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accuracy. Therefore, the Banach fixed point theorem is used in many branches of modern mathematics, especially in the applied mathematics. Since Banach put forward this principle in the 1920s, nearly a century ago, Banach contraction mapping principle has been developed from various aspects and different perspectives. Many people have proposed a series of new concepts of compressed mappings and a series of new fixed point theorems of compressed mappings. Let (X,d) be a metric space, and T be a self-mapping in X, then T is said to be a Rakotch contraction mapping [2] if there exists a decreasing function $k(t):(0,\infty)\to(0,1)$ such that $d(Tx,Ty)\le k(d(x,y))d(x,y), \forall x,y\in X.$ With the development of Banach compression principle, the generalization of metric space has also been greatly developed.

With the development of generation of metric space, the fixed point theory is further extended. Next, we introduce the concept of metric spaces as follows.

Definition 1. Let X be a nonempty set, $d:X\times X\to[0,\infty)$ be a function such that for all $x,y\in X$ the following three conditions satisfy:

$$(d1)\quad d(x,y)\ge 0, d(x,y)=0 \Leftrightarrow x=y;$$
$$(d2)\quad d(x,y)=d(y,x);$$
$$(d3)\quad d(x,y)\le d(x,z)+d(z,y).$$

Then, the pair (X,d) is called a metric space.

Usually, we define the metric by the absolute value, that is, $d(x,y)=|x-y|$. It is easy to verify that a metric defined like this satisfies the conditions $(d1)-(d3)$.

Matthews [3] introduced the concept of a partial metric space as follows:

Definition 2. A mapping $p:X\times X\to R_+$, where X is a nonempty set, is said to be a partial metric on X if for any $x,y,z\in X$ the following four conditions hold true:

$$(p1)\quad x=y \text{ if and only if } p(x,x)=p(y,y)=p(x,y);$$
$$(p2)\quad p(x,x)\le p(x,y);$$
$$(p3)\quad p(x,y)=p(y,x);$$
$$(p4)\quad p(x,z)\le p(x,y)+p(y,z)-p(y,y).$$

The pair (X,p) is then called a partial metric space.

In a partial metric space, the partial metric is usually defined by reference [4].

Definition 3. [5]. A mapping $\sigma:X\times X\to R_+$, where X is a nonempty set, is said to be a metric-like on X if for any $x,y,z\in X$ the following four conditions hold true:

$$(\sigma1)\quad \text{if } D(x,y)=0 \Leftrightarrow x=y;$$
$$(\sigma2)\quad \sigma(x,y)=\sigma(y,x);$$
$$(\sigma3)\quad \sigma(x,z)\le\sigma(x,y)+\sigma(y,z).$$

The pair (X,σ) is then called a metric-like space.

Let $X=\{0,1\}$ and let $\sigma(x,y)=\begin{cases}2, \text{if} \cdot\ x=y=0,\\ 1, \text{otherwise},\end{cases}$ then (X,σ) that based on this metric is a metric-like space.

The concept of b-metric space was introduced by Czerwik in [6].

Definition 4. A b-metric on a nonempty set X is a function $D: X\times X \to [0,+\infty)$ such that for all $x,y,z\in X$ and a constant $K\geq 1$ the following three conditions hold true:

$$\text{(D1) if } D(x,y)=0 \Leftrightarrow x=y;$$
$$\text{(D2) } D(x,y)=D(y,x);$$
$$\text{(D3) } D(x,y)\leq K(D(x,z)+D(z,y)).$$

The pair (X,D) is then called a b-metric space.

Let (X,d) be a metric space and $D(x,y)=(d(x,y))^p$ where $p>1$ is a real number. Then, it is easy to verify that D is a b-metric with $s=2^{p-1}$.

The concept of b-metric-like space was introduced in [7] as follows:

Definition 5. A b-metric-like on a nonempty set X is a function $r: X\times X\to[0,+\infty)$ such that for all $x,y,z\in X$ and a constant $K\geq 1$ the following three conditions hold true:

$$\text{(r1) if } r(x,y)=0 \Rightarrow x=y;$$
$$\text{(r2) } r(x,y)=r(y,x);$$
$$\text{(r3) } r(x,y)\leq K(r(x,z)+r(z,y)).$$

The pair (X,r) is then called a b-metric-like space.

Let $X=[0,\infty)$ and let $r: X\times X\to[0,+\infty)$ be defined by $r(x,y)=\left(x+y\right)^2$. It is easy to know that $s=2$ and $(x,r,2)$ is a complete b-metric-like space.

In a metric space, Banach contractions principle is a classical result in fixed point theory. Here, we recall and state the concept of contractive mapping as follows:

Definition 6. Let X be a metric space, $T: X\to X$ be a mapping, if there exists a constant $\alpha\in(0,1)$ such that for all $x,y\in X$, the following condition is satisfied:

$$d(Tx,Ty)\leq \alpha d(x,y).$$

Then, T is called a contractive mapping.

It is natural that Banach contractions principle [8] is extended in kinds of metric space, for example, partial metric space, b-metric space, b-metric-like space, and so on.

In 2017, Lei and Wu [9] proposed the concept of a LW-type cyclic mapping in a complete b-metric-like space as follows:

Definition 7. Let G_1, G_2 be nonempty closed sets in (X,r). If (B,S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real constants β, δ, t such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta r(x,Bx) + \delta r(y,Sy) + tr(Bx,Sy) \le r(x,y).$$

Then, (B,S) is called as a LW-type cyclic mapping.

Based on the above contents, we propose the notion of a mapping and named it as a general LW-type cyclic mapping as follows:

Definition 8. Let G_1, G_2 be nonempty closed sets in (X,r). If (B,S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real functions β, δ, t such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\beta(r(x,y))r(x,Bx) + \delta(r(x,y))r(y,Sy) + t(r(x,y))r(Bx,Sy) \le r(x,y),$$

where $\gamma, \delta : [0,+\infty) \to [0,+\infty), t : [0,+\infty) \to [1,+\infty)$ are not decreasing. Then, (B,S) is called as a general LW-type cyclic mapping.

2. Preliminaries

We collect some necessary definitions, which will be used in next section.

Definition 9. [7]. Let (X,r) be a b-metric-like space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if

$\lim_{n\to\infty} r(x,x_n) = r(x,x)$, and we say that the sequence $\{x_n\}$ is convergent to *x* and denote it by $x_n \to x$ as $n \to \infty$.

Definition 10. [7]. Let (X,r) be a b-metric-like space.

- A sequence $\{x_n\}$ is called Cauchy if and only if $\lim_{n\to\infty} r(x_n, x_m)$ exists and is finite.
- A b-metric-like space (X,r) is said to be complete if and only if every Cauchy sequence $\{x_n\}$ in X converges to $x \in X$ so that $\lim_{m,n\to\infty} r(x_n, x_m) = r(x,x) = \lim_{n\to\infty} r(x_n, x)$.

Definition 11. [7]. Let G_1, G_2 be nonempty sets of metric space, if $B(G_1) \subset G_2$, and $S(G_2) \subset G_1$, then the mapping $(B,S) : G_1 \times G_2 \to G_2 \times G_1$ is called as a pair semi-cyclic mapping, where B is said to be a lower semi-cyclic mapping, *S* is said to be a upper semi-cyclic mapping. If $B = S$, then B is said to be a cyclic mapping.

Definition 12. [10]. If $\prec$ is a partially ordered in b-metric-like spaces (X, r), then $(X, r, \prec)$ is a partially ordered b-metric-like space.

3. Main Results

Next, we give the results of proposed algorithms in this section.

Theorem 1. Suppose that (X, r) is a complete b-metric-like space. Let (B, S) be a general LW-type cyclic mapping, and G_1, G_2 be nonempty closed sets in (X, r), $G_1 \cap G_2 \neq \varnothing$. If there exists some λ such that the following conditions are satisfied:

$$\max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z \in [0, +\infty)\} \leq \lambda < 1, \tag{1}$$

$$\lambda s < 1. \tag{2}$$

Then there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Proof. The sequence $\{x_n\}$ is defined as follows:

$$x_0 \in G_1,\ \ x_1 = Bx_0,\ \ x_2 = Sx_1,\ \ x_3 = Bx_2,\ \ x_4 = Sx_3,\ \ldots, n \geq 0.$$

Step 1. Prove that the sequence $\{x_n\}$ is a cauchy sequence. Because (B, S) be a general LW-Type cyclic mapping, thus we have

$$r(x_0, x_1) \geq \beta(r(x_0, x_1))r(x_0, Bx_0) + \delta(r(x_0, x_1))r(x_1, Sx_1) + t(r(x_0, x_1))r(Bx_0, Sx_1)$$

$$= \beta(r(x_0, x_1))r(x_0, x_1) + \delta(r(x_0, x_1))r(x_1, x_2) + t(r(x_0, x_1))r(x_1, x_2),$$

so we obtain

$$r(x_1, x_2) \leq \frac{1-\beta(r(x_0, x_1))}{\delta(r(x_0, x_1)) + t(r(x_0, x_1))} r(x_0, x_1). \tag{3}$$

Similarly we have

$$r(x_2, x_1) \geq \beta(r(x_2, x_1))r(x_2, Bx_2) + \delta(r(x_2, x_1))r(x_1, Sx_1) + t(r(x_2, x_1))r(Bx_2, Sx_1)$$

$$= \beta(r(x_2, x_1))r(x_2, x_3) + \delta(r(x_2, x_1))r(x_1, x_2) + t(r(x_2, x_1))r(x_3, x_2),$$

and we have

$$r(x_2, x_3) \leq \frac{1-\delta(r(x_2, x_1))}{\beta(r(x_2, x_1)) + t(r(x_2, x_1))} r(x_1, x_2) \tag{4}$$

$$\leq \frac{1-\delta(r(x_2, x_1))}{\beta(r(x_2, x_1)) + t(r(x_2, x_1))} \cdot \frac{1-\beta(r(x_0, x_1))}{\delta(r(x_0, x_1)) + t(r(x_0, x_1))} r(x_0, x_1).$$

Because of $\max\{\frac{1-\beta(z)}{\delta(z)+t(z)},\frac{1-\delta(z)}{\beta(z)+t(z)},z\in[0,+\infty)\}\le\lambda<1,$ then

$$\max\{\frac{1\text{-}\delta(r(x_{i+1},x_i))}{\beta(r(x_{i+1},x_i))+t(r(x_{i+1},x_i))},\frac{1\text{-}\beta(r(x_{i-1},x_i))}{\delta(r(x_{i-1},x_i))+t(r(x_{i-1},x_i))},x_i\in X,i\in N_+\}\le\lambda<1, \tag{5}$$

By 4 and 5, let $r(x_0,x_1)=C,$ we have

$$r(x_2,x_3)\le\lambda^2 C. \tag{6}$$

Let $m>n,\forall m,n\in N,$ by $\lambda s<1$, this shows

$$\begin{aligned} r(x_n,x_m)&\le sr(x_n,x_{n+1})+s^2r(x_{n+1},x_{n+2})+...+s^{m-n}r(x_{m-1},x_m),\\ &\le s\lambda^n r(x_0,x_1)+s^2\lambda^{n+1}r(x_0,x_1)+...+s^{m-n}\lambda^{m-1}r(x_0,x_1),\\ &=[s\lambda^n+s^2\lambda^{n+1}+...+s^{m-n}\lambda^{m-1}]C,\\ &=[(s\lambda)+(s\lambda)^2+...+(s\lambda)^{m-n}]\lambda^{n-1}C,\\ &=\frac{1-(s\lambda)^{m-n}}{1-(s\lambda)}s\lambda^n C,\\ &\le\frac{1}{1-(s\lambda)}\lambda^{n-1}C. \end{aligned} \tag{7}$$

From ***Eq. (6)*** and ***Eq. (7)*** let $n\to\infty$, we get that

$$\lim_{n\to\infty}r(x_n,x_m)=0. \tag{8}$$

This implies from ***Eq. (8)*** that the sequence $\{x_n\}$ is a Cauchy sequence.

Due to, (X,r) is a complete b-metric-like space, then there exists a sequence $x^*\in X$ such that

$$x_n\to x^*(n\to\infty).$$

Therefore, $x_{2n}\to x^*;x_{2n+1}\to x^*(n\to\infty).$

Because $\{x_{2n}\}\subset G_1,\{x_{2n+1}\}\subset G_2,$ and G_1,G_2 is closed, then $x^*\in G_1\cap G_2.$

Step 2. Prove that x^* is a fixed point of the mapping B and S, that is, $Bx^*=x^*=Sx^*$. Because (B,S) be a general LW-type cyclic mapping, then we get that

$$\beta(r(x,x))r(x,Bx)+\delta(r(x,x))r(x,Sx)+t(r(x,x))r(Bx,Sx)\le r(x,x).$$

Due to $r(x,x)=\lim_{m,n\to\infty}r(x_m,x_n)=0.$ Then, by ***Eq. (8)*** we have

$$r(x,Bx)=0,r(x,Sx)=0. \tag{9}$$

It implies that

$$Bx^*=x^*=Sx^*. \tag{10}$$

Step 3. Prove that the mappings B and S have a unique common fixed point. Now, let $x, x^* \in X$ are the common fixed points of mappings B and S in X. Then, by ***Definition 8*** and ***Eq. (10)***, we have

$$r(x,x^*) \ge \beta(r(x,x^*))r(x,Bx) + \delta(r(x,x^*))r(x^*,Sx^*) + t(r(x,x^*))r(Bx,Sx^*)$$

$$\ge \beta(r(x,x^*))r(x,x) + \delta(r(x,x^*))r(x^*,x^*) + t(r(x,x^*))r(x,x^*)$$

$$=t(r(x,x^*))r(x,x^*).$$

That is,

$$r(x,x^*) \ge t(r(x,x^*))r(x,x^*).$$

Due to, $t(x) \ge 1$, this is a contradiction. Thus, we obtain that $r(x,x^*)=0$, that is, $x=x^*$.

This completes the proof.

Corollary 1. Suppose that (X,r) is a complete partially ordered b-metric-like space. Let (B,S) be a general LW-type cyclic mapping, and G_1, G_2 be nonempty closed sets in (X,r), $G_1 \cap G_2 \ne \emptyset$. If there exists some λ such that the following conditions are satisfied:

$$\max\{\frac{1-\beta(z)}{\delta(z)+t(z)}, \frac{1-\delta(z)}{\beta(z)+t(z)}, z \in [0,+\infty)\} \le \lambda < 1, \qquad \textbf{\textit{(11)}}$$

$$\lambda s < 1. \qquad \textbf{\textit{(12)}}$$

Then there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Proof. First, define the partially ordered as $x \prec Bx \Leftrightarrow x \in G_1$ or $y \prec Sv \Leftrightarrow y \in G_2$. Then, the next proof follows ***Theorem 1.*** This completes the proof.

4. Applications

In this section, we will give a concrete example to illustrate the effectiveness of a general LW-type cyclic mapping and show the rationality of the obtained theorems.

Example 1. Consider $X = \{0,1,2\}$ and let $r: X \times X \to [0,+\infty)$ be defined by

$$r(0,0) = 0, r(1,1) = \frac{1}{2}, r(2,2) = \frac{15}{4}, r(0,1) = r(1,0) = \frac{3}{4},$$
$$r(0,2) = r(2,0) = \frac{3}{2}, r(1,2) = r(2,1) = 3.$$

It is clear that (X,r) is a complete b-metric-like space with constant $s=\frac{4}{3}$. This can see the reference [11].

According to the definition of a general LW-type cyclic mapping, we will choose proper functions β,δ,t to satisfy the conditions in ***Theorem 1***.

We recall the definition of a general LW-type cyclic mapping, that is,

$$\beta(r(x,y))r(x,Bx)+\delta(r(x,y))r(y,Sy)+t(r(x,y))r(Bx,Sy)\le r(x,y).$$

and let

$$\beta(x)=\frac{x}{1+x},\delta(x)=\frac{x^2}{1+x^2},t(x)=1+x.$$

It is easy to verify that these functions $\beta(x),\delta(x),t(x)$ are non-decreasing. In fact, we need to find a proper number λ to satisfy the conditions of ***Theorem 1*** as follows:

$$\max\{\frac{1-\beta(z)}{\delta(z)+t(z)},\frac{1-\delta(z)}{\beta(z)+t(z)},z\in[0,+\infty)\}\le\lambda<1 \text{ and } \lambda s<1.$$

Now, we will find λ by some concrete calculation.

If we choose $z=r(0,0)=0$, it is obvious to make ***Theorem 1*** correct.

If we choose $z=r(1,1)=\frac{1}{2}$, then

$$\beta(\frac{1}{2})=\frac{1}{3},\delta(\frac{1}{2})=\frac{1}{5},t\left(\frac{1}{2}\right)=\frac{3}{2},$$

thus,

$$\max\{\frac{20}{51},\frac{24}{55}\}=\frac{24}{55}\le\lambda<1.$$

If we choose $z=r(2,2)=\frac{15}{4}$, then

$$\beta(\frac{15}{4})=\frac{15}{19},\delta(\frac{15}{4})=\frac{225}{241},t(\frac{15}{4})=\frac{19}{4},$$

thus,

$$\max\{\frac{3856}{106001},\frac{1216}{101461}\}=\frac{3856}{106001}\le\lambda<1.$$

If we choose $z=r(0,1)=r(1,0)=\frac{3}{4}$, then

$$\beta(\frac{3}{4})=\frac{3}{7},\delta(\frac{3}{4})=\frac{9}{25},t(\frac{3}{4})=\frac{7}{4},$$

thus,

$$\max\{\frac{40}{1477},\frac{448}{1525}\}=\frac{448}{1525}\le\lambda<1.$$

If we choose $z=r(0,2)=r(2,0)=\frac{3}{2}$, then

$$\beta(\frac{3}{2})=\frac{3}{5},\delta(\frac{3}{2})=\frac{9}{13},t(\frac{3}{2})=\frac{5}{2},$$

thus,

$$\max\{\frac{52}{415},\frac{40}{403}\}=\frac{52}{415}\le\lambda<1.$$

If we choose $z=r(1,2)=r(2,1)=3$, then

$$\beta(3)=\frac{3}{4},\delta(3)=\frac{9}{10},t(3)=4,$$

thus,

$$\max\{\frac{5}{98},\frac{2}{95}\}=\frac{5}{98}\le\lambda<1.$$

From above, it shows that we can choose $\lambda\in[\frac{24}{55},\frac{3}{4})$, by virtue of $s=\frac{4}{3}$, and we obtain that $\lambda s<1$. This implies that the conditions given in ***Theorem 1*** are feasible and meaningful. At the same time, we can know that the mappings B and S have a unique common fixed point under these conditions.

Example 2. In this example, we use the same complete b-metric-like space with constant $s=\frac{4}{3}$. But we give different functions as follows:

Let

$$\beta(x)=\frac{1}{3}x,\delta(x)=\frac{1}{4}x,t(x)=1+\frac{1}{5}x.$$

In fact, it is also easy to verify that these functions $\beta(x),\delta(x),t(x)$ are non-decreasing. In fact, we need to find a proper number λ to satisfy the conditions of ***Theorem 1*** as follows:

$$\max\{\frac{1-\beta(z)}{\delta(z)+t(z)},\frac{1-\delta(z)}{\beta(z)+t(z)},z\in[0,+\infty)\}\le\lambda<1 \text{ and } \lambda s<1.$$

Now, we will find λ by some concrete calculation.

If we choose $z=r(0,0)=0$, it is obvious to make ***Theorem 1*** correct.

If we choose $z=r(1,1)=\frac{1}{2}$, then

$$\beta(\frac{1}{2})=6,\delta(\frac{1}{2})=\frac{1}{8},t\left(\frac{1}{2}\right)=\frac{11}{10},$$

thus,

$$\max\{\frac{100}{147},\frac{105}{152}\}=\frac{105}{152}\le\lambda<1.$$

If we choose $z=r(2,2)=\frac{15}{4}$, then

$$\beta(\frac{15}{4})=\frac{5}{4},\delta(\frac{15}{4})=\frac{15}{16},t(\frac{15}{4})=\frac{7}{4},$$

thus,

$$\max\{-\frac{4}{43},\frac{1}{48}\}=\frac{1}{48}\le\lambda<1.$$

If we choose $z=r(0,1)=r(1,0)=\frac{3}{4}$, then

$$\beta(\frac{3}{4})=\frac{1}{4},\delta(\frac{3}{4})=\frac{3}{16},t(\frac{3}{4})=\frac{23}{20},$$

thus,

$$\max\{\frac{60}{107},\frac{75}{112}\}=\frac{75}{112}\le\lambda<1.$$

If we choose $z=r(0,2)=r(2,0)=\frac{3}{2}$, then

$$\beta(\frac{3}{2})=\frac{1}{2},\delta(\frac{3}{2})=\frac{3}{8},t(\frac{3}{2})=\frac{13}{10},$$

thus,

$$\max\{\frac{20}{134},\frac{25}{72}\}=\frac{25}{72}\le\lambda<1.$$

If we choose $z=r(1,2)=r(2,1)=3$, then

$$\beta(3)=1, \delta(3)=\frac{3}{4}, t(3)=\frac{8}{5},$$

thus,

$$\max\{0, \frac{5}{52}\}=\frac{5}{52}\leq\lambda<1.$$

From above, it shows that we can choose $\lambda\in[\frac{105}{152},\frac{3}{4})$, by virtue of $s=\frac{4}{3}$, and we obtain that $\lambda s<1$. Of course, this also implies that the conditions given in ***Theorem 1*** are feasible and meaningful. At the same time, we can know that the mappings B and S have a unique common fixed point under these conditions.

5. Conclusion

In this article, we first proposed a new mapping in a b-metric-like space, called it as a general LW-type cyclic mapping. We proved the existence and uniqueness theorem of fixed points of the general LW-type cyclic mapping. This suggests that it is theoretically possible. In the end, we gave some concrete real examples to show the effectiveness of a general LW-type cyclic mapping that was defined in ***Definition 8*** by us in b-metric-like spaces. In fact, by these examples, we can ascertain that the functions β, δ, t exist and satisfy the conditions. This shows that our work is meaningful. All the above work supported the effectiveness of a general LW-type cyclic mapping and showed the general LW-type cyclic mapping was more general than the LW-type cyclic mapping defined in reference [8]. Through this study, we advanced the work of researching fixed point theory in b-metric-like spaces.

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