Int. J. Res. Ind. Eng. Vol. 8, No. 3 (2019) 203–224

International Journal of Research in Industrial Engineering

www.riejournal.com

An Optimization Model for Aggregate Production Planning and Control: A Genetic Algorithm Approach

S. A. Ahmed*, T. K. Biswas, C. K. Nundy

Department of Industrial and Production Engineering, Jashore University of Science and Technology, Jahsore, Bangladesh.

A B S T R A C T

In this paper, an optimization model for aggregate planning of multi-product and multi-period production system has been formulated. Due to the involvement of too many stakeholders as well as uncertainties, the aggregate production planning sometimes becomes extremely complex in dealing with all relevant cost criteria. Most of the existing approaches have focused on minimizing only production related costs, consequently ignored other cost factors, for instance, supply chain related costs. However, these types of other cost factors are greatly affected by aggregate production planning and its mismanagement often results in increased overall costs of the business enterprises. Therefore, the proposed model has attempted to incorporate all the relevant cost factors into the optimization model which are directly or indirectly affected by the aggregate production planning. In addition, the considered supply chain related costs have been segregated into two major categories. While the raw material purchasing, ordering, and inventory costs have been grouped into an upstream category, finished goods inventory, and delivery costs in the downstream category. The most notable differences with the other existing models of aggregate production planning are in the consideration of the cost factors and formulation process in the mathematical model. A real-life industrial case problem is formulated and solved by using a genetic algorithm to demonstrate the applicability and feasibility of the proposed model. The results indicate that the proposed model is capable of solving any type of aggregate production planning efficiently and effectively.

Keywords: Aggregate production planning, Cost optimization, Genetic algorithm, Production system.

Article history: *Received: 06 May 2019* *Revised: 23 July 2019* *Accepted: 07 September 2019*

1. Introduction

Aggregate Production Planning (APP) refers to 3 to 18 months of medium-term capacity planning. The main objective of this planning is to meet fluctuating demand over the planning horizon and to achieve customer satisfaction [1]. It mainly includes taking decision on production quantity, inventory, and workforce for the time horizon to ensure low cost product and timely

* Corresponding author

E-mail address: tazim_ipe@just.edu.bd

DOI: 10.22105/riej.2019.192936.1090

delivery [2]. Al-e et al. [3] described APP as a method that aggregates all data related to manufacturing and determines the best way to fulfill expected requirements through the use of available physical resources. Dakka et al. [4] described the APP as a production approach predicting the existence of an aggregate production unit, such as volume, production time, or dollar value. In this dynamic business environment, the effective aggregate planning is seen as key to success of a manufacturing company. During the last three decades, both academic institutes and industries have put great effort into designing and developing effective approaches and methods for APP. One of the major objectives of these efforts were to develop cost optimization model for APP. Different types of algorithms have been used to develop these models. However, the manufacturing environment and cost parameters have been changed over time because of changes in customer requirements and technology improvements.

In this modern manufacturing scenario, matching of supply and demand has become a major challenge to thrive for the manufacturing companies [5]. However, the proper APP helps matching supply and demand while reducing total costs. The aggregate plan output consists of the total quantities of each product or product group to be manufactured during the scheduling period of the various manufacturing activities required to achieve the planned levels of production. It intends to set general manufacturing objectives and to help plan the accessibility of additional inputs and support operations to fulfill manufacturing objectives. There are various types of mathematical techniques and models to perform the task of APP.

Many researchers have developed integrated approach to address the aggregate production problems and presented many models integrating different algorithms and techniques to solve the problems [6-9]. Although, main objective of all these models was to minimize overall production costs, they also focused on other important decision variables. However, production cost structure and manufacturing environment have become very dynamic. As a result, models for solving aggregate production problem also require the consideration of these factors. Again, there exists variety in type of production and also in production time. Traditional models lack the considerations of these important factors in solving APP problems.

Therefore, the aim of this study is to develop a cost optimization model for multi-product and multi-period with certain demand and considering escalating factors. This model attempts to minimize production costs, inventory costs, ordering costs, training costs, and worker hiring and firing costs. Then, the cost optimization model is solved using Genetic Algorithm (GA). First, the single objective function is developed assuming 11 decision variables. After that, authors have developed some constraints for the model with these decision variables. Finally, this model has been solved using GA. A case study has been presented for an electronic industry. Uniqueness of this model is that unlike traditional aggregate production planning cost optimization model, inventory costs have been divided into raw material inventory costs and finished good inventory costs. This model also considers training costs of workers as it has become an indispensable part for manufacturing industry.

The rest of this paper is organized as follows. Section 2 describes the previous literatures on aggregate production planning. Section 2 describes the problem, details assumptions, and develops the aggregate production planning cost optimization model. Section 3 outlines the step by step procedure of GA. Section 4 presents a numerical example to demonstrate the application of proposed cost optimization model for an electronic company. Section 5 discusses the findings from the application of the model. The final section draws conclusions and make relevant recommendations.

2. Literature Review

Although the issue of APP was introduced in the 1950s, it is still extensively researched by many researchers. Over the past few decades, they have constructed various models, each with their own pros and cons, to effectively solve the aggregate production planning problem. They also classified each method as being capable of either generating an optimal or near-optimal solution.

Some researchers used linear programming approaches with different application cases to solve APP problem. Hsieh and Wu [10] created a deterministic linear programming model for APP with an imprecise nature. This research examines how the imprecise nature of the Computer-Integrated Production Management System (CIPMS) affects the outcomes of the planning. Wang and Fang [11] suggested Fuzzy Linear Programming (FLP) technique for solving the issue of APP with different objectives where the item price, the unit cost to subcontract, the workforce level, the manufacturing capability, and the market requirements are inherently fuzzy. However, the limitation of this model is that it applied the conventional mathematical programming technique to medium-term production planning**.** Wang and Liang [12] proposed an interactive multiple fuzzy objective linear programming model for solving the aggregate production decision problem in fuzzy environment. They considered the time value of money to construct constraints of this model. Gulsun et al. [13] outlined the LP model for aggregate production planning to determine the most appropriate approach while minimizing general production costs and minimizing the impact of hiring or layoff decisions on the level of motivation of the workers. An integrated model combining with linear programming, simulation, and interactive approach was proposed by Nowak [14] in which the linear programming models were used to generate initial solutions, simulation experiments were performed to check the fluctuation in demands and interactive procedure was used for identifying the final solution of the problem. Chakrabortty et al. [15] developed multi-period and multi-product APP which was formulated as an integer linear programming model using a triangular possibility distribution [15].

Various meta-heuristic algorithms like ant colony algorithm, particle swarm algorithm, GA have been used by many researchers in solving APP. Kumar and Haq [2] applied hybrid ant colony algorithm combined with GA to solve aggregate production problem. Many scheduling problems can also be optimized by ant colony algorithm [16, 17]. Pal et al. [18] developed a model to solve aggregate procurement, production, and shipment decision problems using particle swarm algorithm combined with artificial bee colony algorithm.

In this study, authors have used a popular meta-heuristic, GA to solve the proposed aggregate product model. GA is a nature inspired algorithm which has become very popular in solving aggregate production problem. The early evolution of GA can be observed from late 1950's and early-1960's [19]. Ramezanian et al. [20] used Mixed Integer Linear Programming (MILP) to formulate two-phase aggregate production planning problem and applied GA combined with tabu search to solve the problem. However, cost parameters were not explicitly considered in this case. Chakrabortty and Hasin [21] carried out a case study on a readymade garments in Bangladesh. They suggested an APP model with an adaptive Fuzzy-Based Genetic Algorithm (FBGA) technique to solve a two-product and two-period APP with some susceptible management constraints such as imprecise requirements, varying production expenses, etc. Hossain et al. [22] developed a mathematical model for solving APP using GA and big M method. Savsani et al. [23] applied GA to develop aggregate production planning model. However, this model lacks consideration of explicit cost structure in the optimization model. Mahmud et al. [24] developed an APP model in possibilistic environment applying multi-objective GA. Apart from these, many other mathematical models of aggregate production planning problem have been developed applying different meta-heuristic algorithms [25-27]. However, most of these works lack the consideration of explicit cost structures in their models. Therefore, current research aims to develop an optimization model for aggregate production planning considering explicit cost structures. To achieve this objective, GA has been applied.

3. Problem Formulation

3.1. Problem Assumptions

The aggregate production problem for multi-product and multi-period can be described as follows. Assume that a company manufactures n types of products over a planning horizon t to satisfy market demand. This model aims to build a single objective GA to determine the optimum aggregate strategy for meeting specified demand by changing periodic and overtime manufacturing rates, inventory levels, labor levels, subcontracting and back-ordering rates, order amount, waste level, and other controllable variables. The mathematical model herein is created on the basis of the above characteristics on the following assumptions.

- All parameter values are fixed over the next t planning horizon.
- The intensifying variables are resolved over the next t planning horizon in each of the cost categories.
- Current levels of labor, machine capacity, inventory, backorder, order quantity per period and warehouse space cannot exceed their respective peak levels in each period.
- The fixed demand over a given period can either be satisfied or backordered, but in the next period, the backorder must be met.
- For the development of the model, some confidential information that is not provided to others from the sector has been assumed.
- For any sort of product, outsourcing is not acceptable.

- Workers are trained in every period of time to obtain the necessary level of expertise.
- Work In Process (WIP) inventory cost is not considered.
- The specified time horizon contains two monthly periods.
- Each type of products is assigned to just one production line.

In this research, authors have considered maximum types of costs and formulated the APP model and try to solve the APP model with GA.

3.2. Problem Notation

n –- Specific product types.
t – No. of periods.
D_t – Demand uncertain for nth product in period (t) (units) (Forecasted).
R_{xt} – Regular time production of n^{th} Product in period t (units).
R_{ct} – Regular time production cost per unit for n^{th} product in period t (TK/unit).
i_r – Escalating factors regular time production cost (%).
O_{xt} – Overtime production of n^{th} product in period t (units).
O_{ct} – Overtime production cost per unit of n^{th} product in period t (TK/unit).
i_o – Escalating factors overtime production cost (%).
S_{xt} – Subcontracting production of n^{th} product in period t (units).
S_{ct} – Subcontracting production cost per unit for n^{th} product in period t (TK/unit).
i_s – Escalating factors subcontracting production cost (%).
If_{nt} – Inventory level of finished goods in n^{th} product in period t (unit)
If_{ct} – Inventory carrying cost of finished goods per unit for n^{th} product in period t (TK/unit).
Ir_{nt} – Inventory level of raw material per unit n^{th} product in period t (unit).
Ir_{ct} – Inventory carrying cost of raw material per unit for n^{th} product in period t (TK/unit).
i_{fr} – Escalating factors inventory carrying cost (%).
B_{nt} – Back order of n^{th} product in period t (unit).
B_{ct} – Back order cost per unit for n^{th} product in period t.
i_b – Escalating factors for back order cost.
H_{nt} – Hired worker in period t (man hour).
H_{ct} – Cost of hired in period t (TK/man hour).
F_{nt} – Worker fired in period t (man hour).
F_{ct} – Cost of fired worker in period t (TK/man hour).
i_f – Escalating factor to hire and fire cost (%).
TR_{nt} – No. of training workers.
TR_{ct} – Average cost for training per unit labor (TK/unit).
i_t – Escalating factors of training cost (%).
Wx_{nt} – Wastage level of n^{th} product in period t (unit).
W_{ct} – Wastage cost per unit in n^{th} product in period t.
AW_{nt} – Allowable wastage produce in factory (unit).
WP_{nt} – Average percentage of wastage of n^{th} product in period t (unit).
Od_{ct} – Average ordering cost per order for n^{th} product in period t.
Od_{nt}- No. of order for nth product in period t.
i_{or} – Escalating factors of ordering cost (%).
L_{nt} – Hours of labor usage per unit of n^{th} product in period t (Machine-hour/unit).
Ho_{nt} –Hours of machine usage per unit of n^{th} product in period t (machine-hour/unit).
WH_{nt} – Warehouse spaces per unit of n^{th} product in period t (feet).

L_{max} − Maximum labor level available in period t (man hour).
MC_{tmax} − Maximum machine capacity available in period t (machine-hour).
WH_{tmax} − Maximum warehouse spaces available in period t (feet).
min_{if} − Minimum quantity of finished goods inventory per product.
min_{ir} − Minimum quantity of raw material inventory per product.
max_{if} − Maximum quantity of finished goods inventory per product.
max_{ir} − Maximum quantity of raw material inventory per product.
max_{w}- Maximum number of workers for product in period t.
B_{ntmax}- Maximum number of unit backordered nth product in period t.
Tw_{nt} − Total number of workers in the industry (average).
$B_{n(t-1)}$ − Number of unit backordered n^{th} product in period $t-1$ (Units).
$Ir_{n(t-1)}$ − Number of units held in raw material inventory n^{th} product in period $t-1$ (Units).
$If_{n(t-1)}$ − Number of units held in finished goods inventory n^{th} product in period $t-1$ (Units).

3.3. Decisions Variables

Rx_{nt} − Regular time production in n^{th} product in period t (Units).
Ox_{nt} − Over time production in n^{th} product in period t (Units).
Sx_{nt} − Subcontracting volume of n^{th} product in period t (Units).
B_{nt} − Number of backordered for n^{th} product in period t (Units).
Ir_{nt} − Number of units held in raw material inventory n^{th} product in period t (Units).
If_{nt} − Number of units held in finished goods inventory n^{th} Product in period t (Units)
Wx_{nt} − Waste level of nth product in period t (Units).
H_{nt} − Worker hired n^{th} product in period t (Man hour).
F_{nt} − Worker fired n^{th} product in period t (Man hour).
Od_{nt} − Order quantity per order for n^{th} product in period t (units).
Tr_{nt} − Workers training for n^{th} product in period t (Man hour).

3.4. Single Objective APP Model

Mainly, the aim of owner of the industry is profit maximization or cost minimization to survive in the competitive market. For this reason, the authors have developed single objective cost minimization model for an electronic industry through summation of five types of different costs. Authors have also considered escalating factor for all type of cost as money value may change with time.

- Minimization of production costs:

$$Z_1=\sum_{n=1}^{n} \sum_{t=1}^{t} R_{xt}R_{ct}(1+i_r)^t + B_{xt}B_{ct}(1+i_b)^t + S_{xt}S_{ct}(1+i_s)^t + O_{xt}O_{ct}(1+i_o)^t. \quad (1)$$

- Minimization of inventory costs:

$$Z_2 = \sum_{n=1}^{n}\sum_{t=1}^{t}(Ir_{nt}Ir_{ct} + If_{nt}If_{ct})(1+i_{fr})^t. \quad (2)$$

- Minimization of ordering costs:

$$Z_3=\sum_{n=1}^{n}\sum_{t=1}^{t} (Od_{ct} * ((D_t + (D_t * WP_{nt}))/Q_{nt})(1+i_{or})^t. \quad (3)$$

- Minimization of training costs:

$$Z_4=\sum_{n=1}^{n}\sum_{t=1}^{t} \mathrm{Tr}_{nt}\mathrm{Tr}_{ct}\,(1+\mathrm{i}_t)^t. \tag{4}$$

- Minimization of worker hiring and firing costs:

$$Z_4=\sum_{n=1}^{n}\sum_{t=1}^{t} (\mathrm{H}_t\mathrm{H}_{ct}+\mathrm{F}_t\mathrm{F}_{ct)}(1+\mathrm{i}_t)^t. \tag{5}$$

Now, combined with upper all objectives:

$$\begin{aligned} Z_{min} = {} & \sum_{n=1}^{n}\sum_{t=1}^{t} (\mathrm{Rx}_{nt}\mathrm{R}_{ct}(1+\mathrm{i}_r)^t+\mathrm{Bx}_{nt}\mathrm{B}_{ct}(1+\mathrm{i}_b)^t+\mathrm{Sx}_{nt}\mathrm{S}_{ct}(1+\mathrm{i}_s)^t+\mathrm{Ox}_{nt}\mathrm{O}_{ct}(1+\mathrm{i}_o)^t)+ \\ & \sum_{n=1}^{n}\sum_{t=1}^{t}(\mathrm{Ir}_{nt}\mathrm{Ir}_{ct}+\mathrm{If}_{nt}\mathrm{If}_{ct})(1+\mathrm{i}_{fr})^t+\sum_{n=1}^{n}\sum_{t=1}^{t} ((\mathrm{H}_{nt}\mathrm{H}_{ct}+\mathrm{F}_{nt}\mathrm{F}_{ct)}(1+\mathrm{i}_{hf})^t+ \\ & \sum_{n=1}^{n}\sum_{t=1}^{t} (\mathrm{W}_{xt}\mathrm{W}_{ct})\,(1+\mathrm{i}_{wi})^t+\sum_{n=1}^{n}\sum_{t=1}^{t} (\mathrm{Od}_{ct}*\left(\frac{\mathrm{D}_t+(\mathrm{D}_t*\mathrm{WP}_{nt})}{\mathrm{Q}_{nt}}\right)(1+\mathrm{i}_o)^t+ \\ & \sum_{n=1}^{n}\sum_{t=1}^{t} \mathrm{Tr}_{nt}\mathrm{Tr}_{ct}\,(1+\mathrm{i}_t)^t. \end{aligned} \tag{6}$$

3.5. Constraints

3.5.1. Constraints on finished goods inventory

$$\mathrm{D}_t= \mathrm{R}_{nt}+\mathrm{O}_{nt}+\mathrm{S}_{nt}+\mathrm{B}_{nt}+\mathrm{If}_{n(t-1)}-\mathrm{If}_{nt}-\mathrm{B}_{n(t-1)} \quad \text{for } \forall n,\ \forall t. \tag{7}$$

Certain demand for nth product in period (t) is equal to summation of regular time production in nth product in period and over time production in nth product in period t and subcontracting volume of nth product in period t and number of units backordered nth product in period t and number of units held in inventory nth product in period (t-1) and minus the summation of number of units held in inventory nth product in period t and number of units backordered nth product in period t-1.

$$If_{nt} \geq min_i \quad \text{for } \forall n,\ \forall t. \tag{8}$$

Number of units held in inventory nth product in period t is greater than minimum quantity of inventory per product of product t.

$$\mathrm{If}_{nt} \leq \max_{if} \quad \text{for } \forall n,\ \forall t. \tag{9}$$

Number of units held in finished goods inventory nth product in period t is less than maximum quantity of finished goods inventory per product of product t.

3.5.2. Constraints on raw material inventory

$$\mathrm{Ir}_{n(t-1)}\text{-}\,\mathrm{Ir}_{nt}+\mathrm{R}_{nt}+\mathrm{O}_{nt}+\mathrm{S}_{nt}=\mathrm{D}_t+(\mathrm{D}_t*\mathrm{WP}_{nt}) \quad \text{for } \forall n,\ \forall t. \tag{10}$$

Total demand with considering average percentage of wastage is equal to the summation of raw material inventory in period t-1 with other production unit and subtract the raw material inventory in period t.

$$\mathrm{If}_{nt} \leq \max_{ir} \quad \text{for } \forall n,\ \forall t. \tag{11}$$

Number of units held in raw material inventory nth product in period t is less than maximum quantity of raw material inventory per product of product t.

$$\mathrm{If}_{nt} \geq \mathrm{min}_i \quad \text{for } \forall n,\ \forall t. \tag{12}$$

Number of units held in raw material inventory nth product in period t is greater than minimum quantity of raw material inventory per product of product t.

3.5.3. Constraints on quantity per order, backordered, and subcontracting volume

$$\mathrm{O}_{nt} + \mathrm{S}_{nt} \leq \mathrm{R}_{nt} \quad \text{for } \forall n,\ \forall t. \tag{13}$$

Regular time production for nth product in period t is greater than the summation of overtime production in nth product in period t and subcontracting volume of nth product in period t.

$$\mathrm{B}_{nt} \leq \mathrm{B}_{ntmax} \quad \text{for } \forall n,\ \forall t. \tag{14}$$

Number of unit backordered nth product in period t is less than maximum number of unit backordered nth product in period t.

$$\mathrm{Q}_{nt} \geq 500 \quad \text{for } \forall n,\ \forall t. \tag{15}$$

Quantity per order for nth product in period t is always greater than 500.

3.5.4. Constraints on machine capacity and warehouse space

$$\mathrm{WHf}_{nt} * \mathrm{If}_{nt} \leq \mathrm{WHf}_{tmax} \quad \text{for } \forall n,\ \forall t. \tag{16}$$

Maximum warehouse spaces available in period t is greater than the multiplication of warehouse spaces per unit of nth product in period t (feet) and number of units held in finish good inventory nth product in period t.

$$\mathrm{WHr}_{nt} * \mathrm{Ir}_{nt} \leq \mathrm{WHr}_{tmax} \quad \text{for } \forall n,\ \forall t. \tag{17}$$

Maximum warehouse spaces available in period t is greater than the multiplication of warehouse spaces per unit of nth product in period t (feet) and number of units held in raw material inventory nth product in period t

$$(\mathrm{Rx}_{nt} + \mathrm{Ox}_{nt}) * \mathrm{Ho}_{nt} \leq \mathrm{MC}_{tmax} \quad \text{for } \forall n,\ \forall t. \tag{18}$$

Total value of the summation of regular time production in nth product in period and over time production in nth product in period t with the multiplication of hours of machine usage per unit of nth product in period t is less than the maximum machine capacity available in period t (machine-hour).

3.5.5. Constraints on labor levels

$$\sum_{n=1}^{n}\sum_{t=1}^{t} L_{n(t-1)} * (R_{n(t-1)} + O_{n(t-1)}) + H_t - F_t = \sum_{n=1}^{n}\sum_{t=1}^{t} L_{n(t-1)} * (R_{nt} + O_{nt}) \quad \text{for } \forall n, \forall t. \tag{19}$$

Here the equation represents a set of constraints in which the labor levels are identified by man hour in period t equal the labor levels in period t-1 plus new hires and subtraction of fires in period t.

$$\sum_{n=1}^{n}\sum_{t=1}^{t} L_{n(t-1)} * (R_{nt} + O_{nt}) \leq L_{max} \quad \text{for } \forall n, \forall t. \tag{20}$$

Actual labor levels cannot exceed the maximum available labor levels in each period.

3.5.6. Constraints on wastage unit

$$(R_{nt} + O_{nt}) * WP_{nt} \leq AW_{nt} \quad \text{for } \forall n, \forall t. \tag{21}$$

Total value of the summation of regular time production in nth product in period and overtime production in nth product in period t with multiplication of the percentage of wastage of nth product in period t (unit) is less than waste level of nth product in period t.

3.5.7. Constraint on training labor

$$TR_{nt} = \sum_{n=1}^{n} H_{nt} \quad \text{for } \forall n. \tag{22}$$

Number of workers training for nth product in period t is equal to summation of the workers hired for nth product for the same period t.

3.5.8. Constraint on non-negativity variables

$$Od_{nt} - H_{nt} - F_{nt} - If_{nt} - Ir_{nt} - B_{nt} - Rx_{nt} - Ox_{nt} - Sx_{nt} - Wx_{nt} - Tr_{nt} \geq 0 \quad \text{for } \forall n, \forall t. \tag{23}$$

The value of all decision variable must be greater than zero.

4. A Brief Outline of GA

GAs workability is based on Darwinian's most fitting survival theory. GAs may contain a population, fitness, breeding, mutation, and selection of a chromosome, gene, and set of population. GAs start with a set of solutions, called population, represented by chromosomes. Solutions from a single population are taken and used to form a new population motivated by the potential for the new population to be better than the old population. Further, solutions are selected according to their fitness to form new solutions, that is, offspring. Repetition of the above process will be continued until some condition is fulfilled. Algorithmically, the basic GA is outlined as below [28]:

Step 1. Generate a random population of chromosome, which is the right solution to the problem.

Step 2. Assess the fitness of the population of each chromosome (Fitness).

Step 3. Create a new population by repeating following steps until the new population is completed.

- Selection: Select two parent chromosomes from a population according to their fitness. Better fitness and bigger chance to be selected to be the parent.
- Crossover: With a crossover probability, cross over the parents to form new offspring, that is, children. If no crossover was performed, offspring is the exact copy of parents.
- Mutation: With a mutation probability, mutate new offspring at each locus.
- Accepting: Place new offspring in the new population.

Step 4. Use new generated population for a further run of the algorithm.

Step 5. If the end condition is satisfied, stop, and return the best solution in current population.

Step 6. Go to Step 2.

Some associated terms of GA have been discussed below [29].

4.1. Crossover Options

Crossover options specify how the GA combines two individuals, or parents, to form a crossover child for the next generation. Here we have chosen five different crossover (scattered crossover, single point crossover, two point crossover, arithmetic crossover, and constraint-dependent crossover) options for five scenarios.

Scattered crossover**:** It creates a random binary vector and selects the genes where the vector is a 1 from the first parent, and the genes where the vector is a 0 from the second parent, and combines the genes to form the child. For example, if p1 and p2 are the parents such as p1 = [a b c d e f g h] and p2 = [1 2 3 4 5 6 7 8] and the binary vector is [1 1 0 0 1 0 0 0], then the function returns the following child 1 = [a b 3 4 e 6 7 8].

Single point crossover: It chooses a random integer *n* between 1 and number of variables and then selects vector entries numbered less than or equal to *n* from the first parent and selects vector entries numbered greater than *n* from the second parent. For example, if p1 and p2 are the parents such as p1 = [a b c d e f g h] and p2 = [1 2 3 4 5 6 7 8] and the crossover point is 3, the function returns the following child = [a b c 4 5 6 7 8].

Two point crossover: It selects two random integer m and n between 1 and number of variables. The function selects vector entries numbered less than or equal to m from the first parent, vector entries numbered from m+1 to*n*, inclusive, from the second parent, vector entries numbered greater than *n* from the first parent. The algorithm then concatenates these genes to form a single gene. For example, if p1 and p2 are the parents such as p1 = [a b c d e f g h] and p2 = [1 2 3 4 5 6 7 8] and the crossover points are 3 and 6, the function returns the following child = [a b c 4 5 6 g h].

Arithmetic crossover: It is a crossover operator that linearly combines two parent chromosome vectors to produce two new offspring according to the following equations:

$$\text{Offspring1} = a * \text{Parent1} + (1-a) * \text{Parent2}$$
$$\text{Offspring2} = (1-a) * \text{Parent1} + a * \text{Parent2}$$

Where ‘a’ is a random weighting factor (chosen before each crossover operation).

4.2. Mutation Options

Mutation options specify how the GA makes small random changes in the individuals in the population to create mutation children. Mutation provides genetic diversity and enables the GA to search a broader space. Here the authors use constraint dependent mutation and adapt feasible mutation options. Adaptive feasible randomly generates directions that are adaptive with respect to the last successful or unsuccessful generation. The feasible region is bounded by the constraints and inequality constraints. A step length is chosen along each direction so that linear constraints and bounds are satisfied.

4.3. Creation Function

Creation function creates the initial population for GA. Here the authors choose feasible population & Constraint dependent options. Feasible population creates a random initial population that satisfies all bounds and linear constraints. It is biased to create individuals that are on the boundaries of the constraints and create well-dispersed populations. This is the default if there are linear constraints.

4.4. Selection Options

Selection options specify how the GA chooses parents for the next generation. Here the authors used only Tournament selection option for tournament size 2 and 4. Tournament selection chooses each parent by choosing Tournament size players at random and then choosing the best individual out of that set to be a parent.

4.5. Migration Options

Migration options specify how individuals move between subpopulations. Migration occurs if we set population size to be a vector of length greater than 1. When migration occurs, the best individuals from one subpopulation replace the worst individuals in another subpopulation. Individuals that migrate from one subpopulation to another are copied. They are not removed from the source subpopulation.

The GAs performance is largely influenced by crossover and mutation operators. The block diagram representation of GA is shown in ***Figure 1.***

4.6. Genetic algorithm parameters

The authors have used MATLAB (2015a) computer software to solve the proposed Single Objective Genetic Algorithm (SOGA) approach for this case study. Total five runs were implemented considering five scenarios with different SOGA parameters shown in ***Table 1***. Also ***Table 5*** lists the single objective values for five SOGA runs through MATLAB.

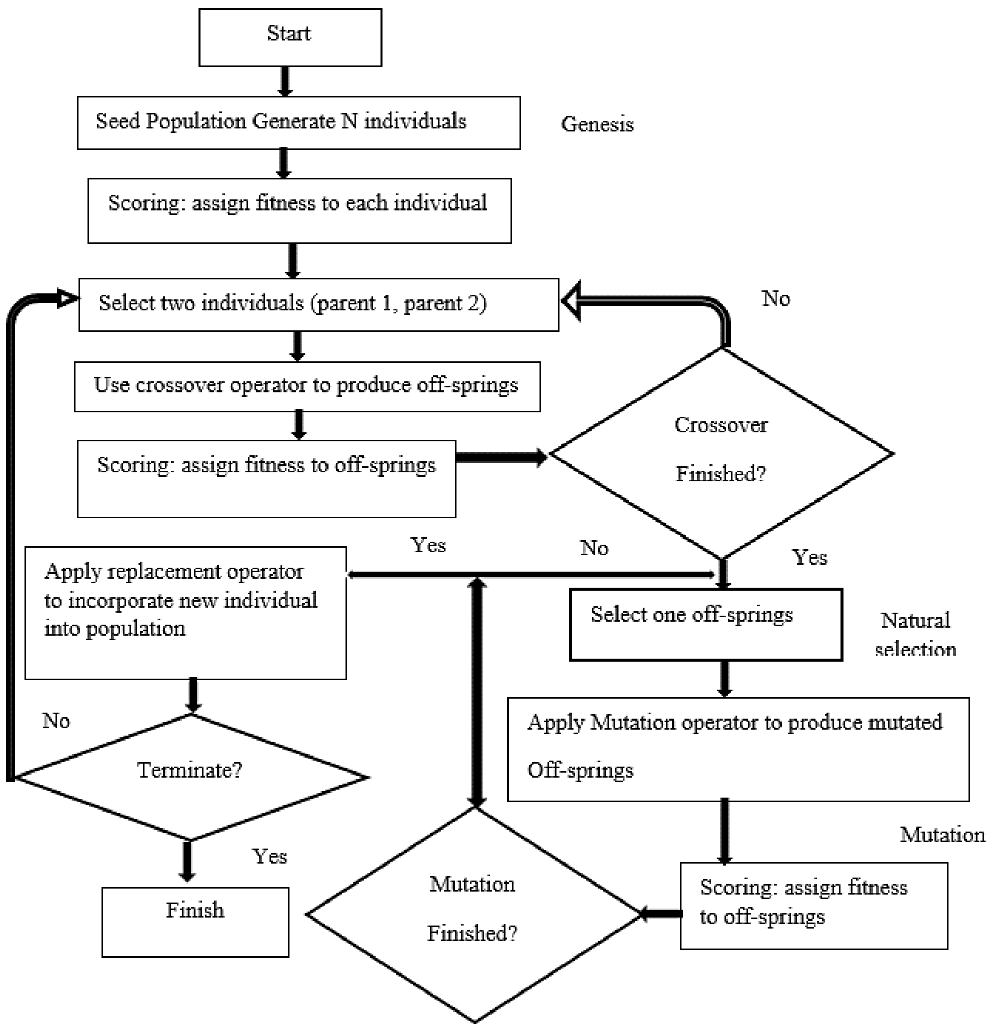

Figure 1. The block diagram representation of genetic algorithms [28].

Table 1. Different genetic algorithm options used for five scenarios.

Population Type	Double vector	Double vector	Double vector	Double vector	Double vector
Population Size	200	100	50	120	220
Creation Function	Constraint dependent	Constraint Dependent	Feasible population	Feasible population	Constraint dependent
Scaling function	Rank	Rank	Rank	Rank	Rank
Selection Fraction	Tournament (4)	Tournament (4)	Tournament (2)	Tournament (2)	Tournament (2)
Reproduction (Fraction)	Crossover (0.8)	Crossover (0.8)	Crossover (0.5)	Crossover (0.5)	Crossover (0.8)
Mutation	Constraint dependent	Constraint dependent	Adaptive feasible	Adaptive feasible	Adaptive feasible
Crossover	Single point	Two point	Constraint dependent	Arithmetic	Scattered
Migration (Fraction)	Forward (0.2)	Both (0.2)	Both (0.2)	Forward (0.2)	Both (0.2)

5. Model Implementation

A well-known electronic industry was used as a case study to demonstrate the practicality of the proposed methodology. This company readily produces various electronic items and among them some are novel and some are expensive. Therefore, it requires a lot of appropriate observations and accurate manufacturing practices to gain the market and satisfy the buyers within specified lead time. Among various types of products, authors have collected data only for fan production sector and formulated aggregate production plan for table fan (Product 1) and another type of ceiling fan (Product 2).

The APP decision problem for manufacturing plant (electronic industry) have been presented in this research and solved with GA approach for minimizing the total costs. The planning horizon is 2 months long including April and May. Relevant data for this problem are presented in ***Table 2***, ***Table 3***, and ***Table 4***.

Table 2. List of relevant cost for all decision variables.

	Period (1)	Period (2)	Period (1)	Period (2)
R_{ct}	40	40	52	52
O_{ct}	70	70	80	80
B_{ct}	20	20	25	25
S_{ct}	60	60	65	65
Ir_{ct}	5	5	6	6
If_{ct}	4.5	4.5	5.5	5.5
W_{ct}	800	800	1000	1000
Od_{ct}	2000	2000	2200	2200
TR_{ct}	5	5	5	5
H_{ct}	50	50	50	50
F_{ct}	20	20	20	20

It is needed to know cost of the different product per unit for different period for solving APP problem as this is the cost minimization problem.

Table 3. Initial data for two products.

$If_{n(t-1)}$ (units)	2500	2800
$Ir_{n(t-1)}$ (units)	3000	3200
$R_{n(t-1)}$	4800	5500
$O_{n(t-1)}$	950	1375
Total workers (people)	20	25
Regular + overtime working hour	10	10
No. of machine (units)	18	22
Total working day per period	25	25
Escalating factor for all types of cost for all period =1%		

Table 4. Other relevant data for calculation.

	Period (1)	Period (2)	Period (1)	Period (2)
D_t	6500 units	6200 units	7000 units	7200 units
max_{if}	3500 units	3500	4000 units	4000
max_{ir}	3200 units	3200	3500 units	3500
min_{if}	500 units	500	500	500
min_{ir}	500 units	500	500 units	500
B_{ntmax}	350 units	350	300	300
WHf_{tmax}	3150 feet	3150	3200 feet	3200
WHf_{nt}	0.9 feet	-	0.8 feet	-
WHr_{tmax}	3840	3840	3500	3500
WHr_{nt}	1.2	1.2	1.0	1.0
MC_{tmax}	3600 m/c hour	-	6875 m/c hour	-
Ho_{nt}	0.63	0.63	0.8	0.8
WP_{nt}	.025	.025	.030	.030
AW_{xt}	200 units	200 units	250 units	250 units
$L_{n(t-1)}$	1.15	1.15	1.10	1.10
L_{max}	7000 Man-hour	7000	7500 Man-hour	7500

By putting above combination (from ***Table 1***) in MATLAB software for optimization using with GA, the authors have got five different objective values for five different scenarios which are shown in ***Table 5***.

Table 5. Calculated single-objective values for different scenario.

Scenario	Objective value (Z)
1	8.462226845090458E9
2	8.462225795686462E9
3	8.462226017043371E9
4	8.462224824834982E9
5	8.462226609028172E9

The authors have got graph comparing with fitness value to generation from GA tool from MATLAB for five different GA scenario. These 5 scenarios have been shown in ***Figure 2***.

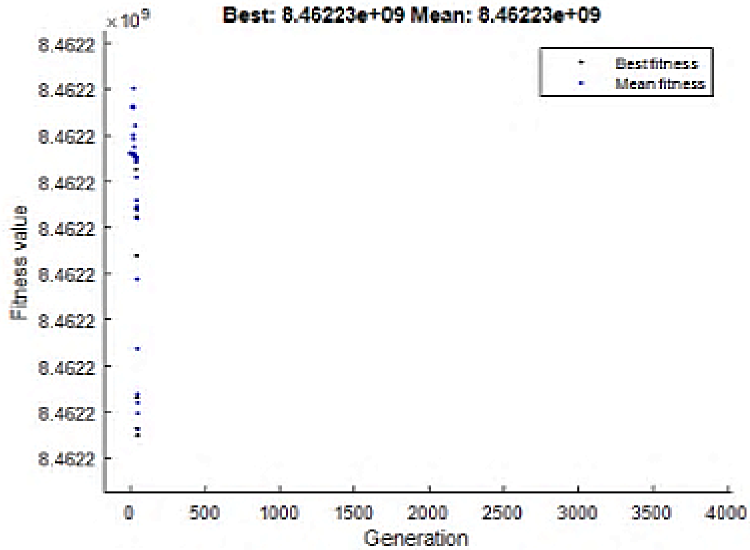

Figure 2a. Scenario 1**.**

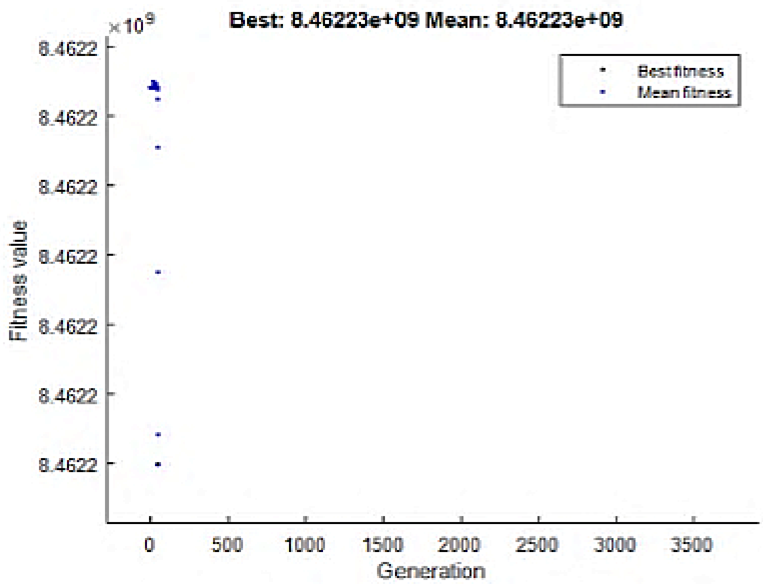

Figure 2b. Scenario 2.

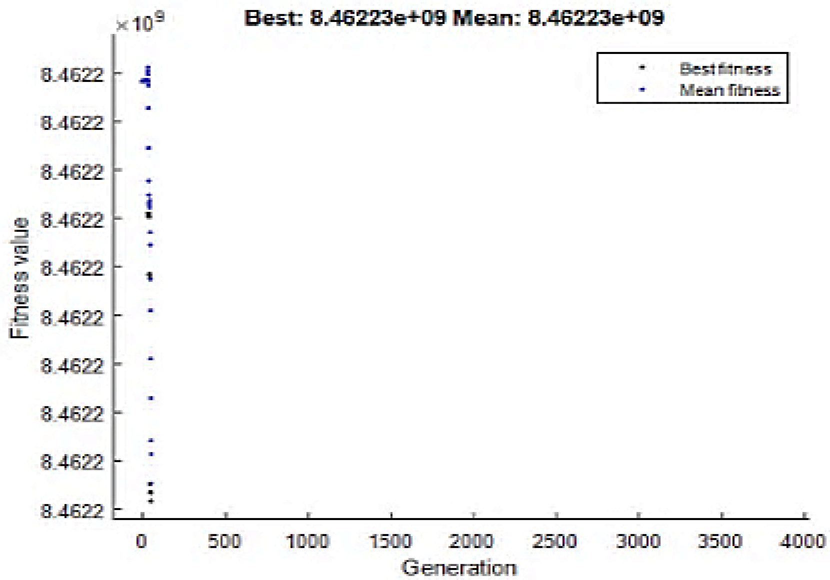

Figure 2c. Scenario 3.

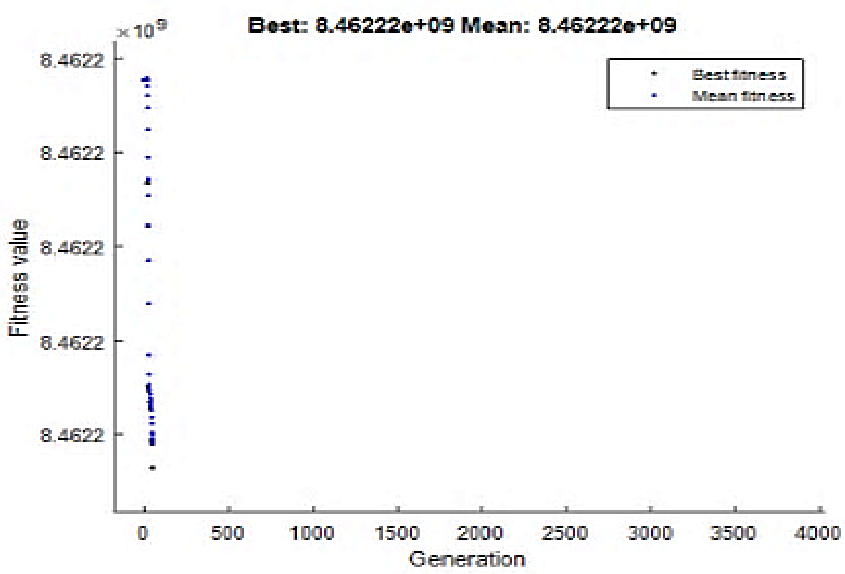

Figure 2d. Scenario 2.

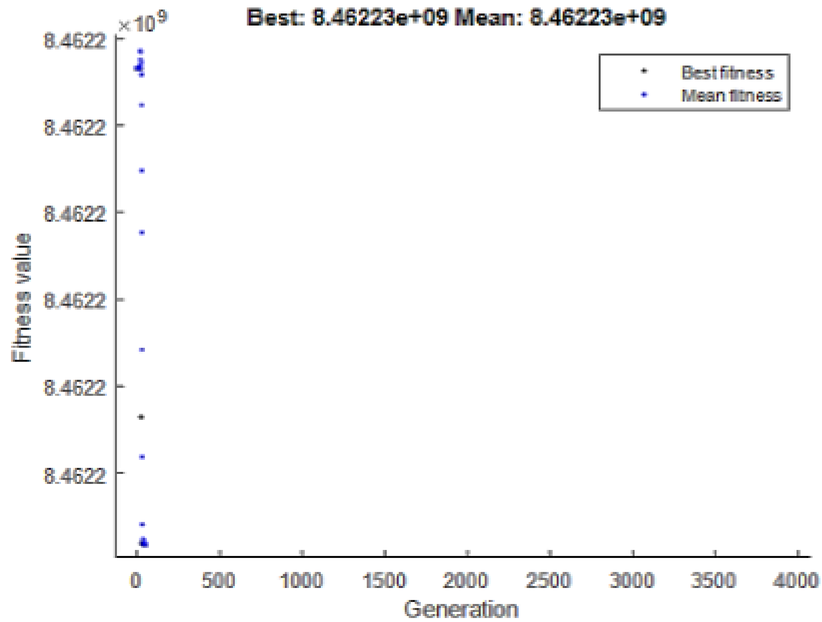

Figure 2e. Scenario 2.

Figure 2. Graph for five different scenario (a), (b), (c), (d) and (e) (source: MATLAB-2015a).

6. Results and Discussion

In order to solve the proposed mathematical model of APP, a well-established meta-heuristic technique named genetic algorithm is employed. As meta-heuristic methods are problem independent and provides reasonably good solution within a defined computational timeframe, they are preferred to apply in various optimization problems. Since genetic algorithm does not ensure the global optimal solution and often trap into local optimal, to improve the results, we set our experiments for five different scenarios with different parameter settings as shown in ***Table 5***. It shows that the minimum cost is obtained for the fourth scenario where 120 population size and arithmetic crossover option were considered. Since results of genetic algorithm in each run varies slightly, therefore we have run the program three times for each scenario and taken the average. Improved result might be possible for different other combinations of the GA parameters. ***Table 6*** shows the output values of decision variables for multi-products multi-periods APP of the studied electronic factory. For the sake of illustration, the proposed APP model is solved for two product and two periods time horizon only. However, the developed model is quite general and it expected to be applicable for many products and many periods as well as other types of production process.

Table 6. Outputs of multi-product & multi-period APP plan for the case study (scenario 4).

Regular time production in nth product in period t		Number of units held in raw material inventory nth product in period t	
Rx_{11}	4987.277	Ir_{11}	1324.277
Rx_{12}	5099.086	Ir_{12}	1319.278
Rx_{21}	5558.594	Ir_{21}	1548.595
Rx_{22}	5847.942	Ir_{22}	1532.595
Over time production in nth product in period t		Waste level of nth product in period t	
Ox_{11}	0	Wx_{11}	23.728
Ox_{12}	90.087	Wx_{12}	33.568
Ox_{21}	.001	Wx_{21}	49.97
Ox_{22}	134.942	Wx_{22}	4.638
Subcontracting volume of nth product in period t		Worker hired nth product in period t (Man hour)	
Sx_{11}	0	H_{11}	769.713
Sx_{12}	1160.826	H_{12}	232.181
Sx_{21}	.001	H_{21}	998.73
Sx_{22}	1417.117	H_{22}	466.717
Number of backordered for nth product in period t		Worker fired nth product in period t (Man hour)	
B_{11}	.001	F_{11}	36.844
B_{12}	350	F_{12}	0
B_{21}	0	F_{21}	109.276
B_{22}	300	F_{22}	0
Number of units held in finished goods inventory nth product in period t		Order quantity per order for nth product in period t (units)	
If_{11}	837.278	Od_{11}	515.319
If_{12}	500	Od_{12}	510.305
If_{21}	1178.595	Od_{21}	544.09
If_{22}	500	Od_{22}	501.561
		Workers training for nth product in period t (Man hour)	
		$Tr_{(1,2)1}$	1768.444
		$Tr_{(1,2)2}$	698.898

Note that, all the input variables of the proposed APP model are involved with substantial uncertainties, consideration of deterministic values may lead to poor performance of the model. Also, anticipations of the values of these variables with large quantity of historical datasets can improve the solution quality.

7. Conclusions

Over the last few decades, researchers have formulated many aggregate planning models considering various decision variables and using different solution technique. Most of them have primarily focused on minimizing production related costs, and ignored others type of costs like supply chain related costs. However, many supply chain related costs, both upstream and downstream are directly or indirectly affected by aggregate production planning. In this research, authors addressed this gap along with other production related costs which have often been overlooked in past studies such as training costs, hiring costs, and wastage costs. The novelty of this work lies on the formulation of the mathematical model and consideration of the cost factors. The results of the case study indicated that the proposed model can be effectively applied in real-life multi-product multi-period aggregate production planning. Although the proposed model is applied in electronic factory, it is quite general and expected to be applied in any other types of factory with minor modifications. This provides decision support to managers in setting up APP in order to achieve maximum profit by minimizing the total costs. The developed APP model has been solved by using GA, however other meta-heuristic optimization techniques including Particle Swarm Optimization (PSO), Simulated Annealing, Ant Colony Optimization and Artificial Bee Colony Optimization can also be employed [30-32]. Designing an APP model considering uncertain cost factors for large size problem can be a potential future research direction.

References

[1] Gansterer, M. (2015). Aggregate planning and forecasting in make-to-order production systems. *International journal of production economics*, *170*, 521-528.

[2] Kumar, G. M., & Haq, A. N. (2005). Hybrid genetic—ant colony algorithms for solving aggregate production plan. *Journal of advanced manufacturing systems*, *4*(01), 103-111.

[3] Al-e, S. M. J. M., Aryanezhad, M. B., & Sadjadi, S. J. (2012). An efficient algorithm to solve a multi-objective robust aggregate production planning in an uncertain environment. *The international journal of advanced manufacturing technology*, *58*(5-8), 765-782.

[4] Dakka, F, Aswin, M., & Siswojo, B. (2017). Multi-Plant multi-product aggregate production planning using genetic algorithm. *International journal of engineering research and management*, *4*, 2349-2058.

[5] Swinney, R. (2011). Selling to strategic consumers when product value is uncertain: The value of matching supply and demand. *Management science*, *57*(10), 1737-1751.

[6] Fahimnia, B., Farahani, R. Z., Marian, R., & Luong, L. (2013). A review and critique on integrated production–distribution planning models and techniques. *Journal of manufacturing systems*, *32*(1), 1-19.

[7] Entezaminia, A., Heydari, M., & Rahmani, D. (2016). A multi-objective model for multi-product multi-site aggregate production planning in a green supply chain: Considering collection and recycling centers. *Journal of manufacturing systems*, *40*, 63-75.

[8] Modarres, M., & Izadpanahi, E. (2016). Aggregate production planning by focusing on energy saving: A robust optimization approach. *Journal of cleaner production*, *133*, 1074-1085.

[9] Makui, A., Heydari, M., Aazami, A., & Dehghani, E. (2016). Accelerating benders decomposition approach for robust aggregate production planning of products with a very limited expiration date. *Computers & industrial engineering*, *100*, 34-51.

[10] Hsieh, S., & Wu, M. S. (2000). Demand and cost forecast error sensitivity analyses in aggregate production planning by possibilistic linear programming models. *Journal of intelligent manufacturing*, *11*(4), 355-364.

[11] Wang, R. C., & Fang, H. H. (2001). Aggregate production planning with multiple objectives in a fuzzy environment. *European journal of operational research*, *133*(3), 521-536.

[12] Wang, R. C., & Liang, T. F. (2005). Aggregate production planning with multiple fuzzy goals. *The international journal of advanced manufacturing technology*, *25*(5-6), 589-597.

[13] Gulsun, B., Tuzkaya, G., Tuzkaya, U. R., & Onut, S. (2009). An aggregate production planning strategy selection methodology based on linear physical programming. *International journal of industrial engineering*, *16*(2), 135-146.

[14] Nowak, M. (2013). An interactive procedure for aggregate production planning. *Croatian operational research review*, *4*(1), 247-257.

[15] Chakrabortty, R. K., Hasin, M. A. A., Sarker, R. A., & Essam, D. L. (2015). A possibilistic environment based particle swarm optimization for aggregate production planning. *Computers & industrial engineering*, *88*, 366-377.

[16] Shyu, S. J., Lin, B. M., & Yin, P. Y. (2004). Application of ant colony optimization for no-wait flowshop scheduling problem to minimize the total completion time. *Computers & industrial engineering*, *47*(2-3), 181-193.

[17] Montgomery, J., Fayad, C., & Petrovic, S. (2006). Solution representation for job shop scheduling problems in ant colony optimisation. *International workshop on ant colony optimization and swarm intelligence* (pp. 484-491). Berlin, Heidelberg: Springer.

[18] Pal, A., Chan, F. T. S., Mahanty, B., & Tiwari, M. K. (2011). Aggregate procurement, production, and shipment planning decision problem for a three-echelon supply chain using swarm-based heuristics. *International journal of production research*, *49*(10), 2873-2905.

[19] Bremermann, H. J., Oehme, R., & Taylor, J. G. (1958). Proof of dispersion relations in quantized field theories. *Physical review*, *109*(6), 2178.

[20] Ramezanian, R., Rahmani, D., & Barzinpour, F. (2012). An aggregate production planning model for two phase production systems: Solving with genetic algorithm and tabu search. *Expert systems with applications*, *39*(1), 1256-1263.

[21] Chakrabortty, R. K., & Hasin, M. A. A. (2013). Solving an aggregate production planning problem by fuzzy based genetic algorithm approach. *International journal of fuzzy logic systems*, *3*(1), 1-15.

[22] Hossain, M. M., Nahar, K., Reza, S., & Shaifullah, K. M. (2016). Multi-period, multi-product, aggregate production planning under demand uncertainty by considering wastage cost and incentives. *World review of business research*, *6*(2), 170-185.

[23] Savsani, P., Banthia, G., Gupta, J., & Ronak, V. (2016). Optimal aggregate production planning by using genetic algorithm. *Proceedings of the international conference on industrial engineering and operations management*, *(IEOM*) (pp. 863-874).

[24] Mahmud, S., Hossain, M. S., & Hossain, M. M. (2018). Application of multi-objective genetic algorithm to aggregate production planning in a possibilistic environment. *International journal of industrial and systems engineering*, *30*(1), 40-59.

[25] Jamalnia, A., Yang, J. B., Feili, A., Xu, D. L., & Jamali, G. (2019). Aggregate production planning under uncertainty: a comprehensive literature survey and future research directions. *The international journal of advanced manufacturing technology*, *102*(1-4), 159-181.

[26] Al Aziz, R., Paul, H. K., Karim, T. M., Ahmed, I., & Azeem, A. (2018). Modeling and optimization of multi-layer aggregate production planning. *Journal of operations and supply chain management*, *11*(2), 1-15.

[27] Mehdizadeh, E., Niaki, S. T. A., & Hemati, M. (2018). A bi-objective aggregate production planning problem with learning effect and machine deterioration: Modeling and solution. *Computers & operations research*, *91*, 21-36.

[28] Malhotra, R., Singh, N., & Singh, Y. (2011). Genetic algorithms: Concepts, design for optimization of process controllers. *Computer and information science*, *4*(2), 39.

[29] Chakrabortty, R. K., & Hasin, M. A. A. (2013). Solving an aggregate production planning problem by using multi-objective genetic algorithm (MOGA) approach. *International journal of industrial engineering computations*, *4*, 1-12.

[30] Mohammadi-Andargoli, H., Tavakkoli-Moghaddam, R., Shahsavari Pour, N., & Abolhasani-Ashkezari, M. H. (2012). Duplicate genetic algorithm for scheduling a bi-objective flexible job shop problem. *International journal of research in industrial engineering*, *1*(2), 10-26.

[31] Moradi, N., & Shadrokh, S. (2019). A simulated annealing optimization algorithm for equal and un-equal area construction site layout problem. *International journal of research in industrial engineering*, *8*(2), 89-104.

[32] Ali, S. M., & Nakade, K. (2015). A mathematical optimization approach to supply chain disruptions management considering disruptions to suppliers and distribution centers. *Operations and supply chain management*, *8*(2), 57-66.