Int. J. Res. Ind. Eng. Vol. 8, No. 1 (2019) 28–39

International Journal of Research in Industrial Engineering

www.riejournal.com

Developing 360 Degree Performance Evaluation Method for School Teachers

M. A. Movafaghpour *

Department of Industrial Engineering, Facuty of Mechanical Engineering, Jundi-Shapur University of Technology, Dezful, Iran.

A B S T R A C T

The purpose of this study is to provide a proper method for evaluating performance of teachers, which leads to authorities awareness about the quality and quantity of activities that are acceptable to the organization; also creating the grounds for empowering human resources, reduction of dissatisfaction, and complaints. This can result in eliminating discrimination or unfair judgments and therefore reduction of tensions, and conflicts between managers and employees. The Analytic Hierarchy Process (AHP) which was developed by Saaty is a decision analysis tool. It has been applied to many different decision fields. In this paper, we use AHP as a performance assessment tool to inquire diverse assessments in the organization about the performance of each employee. Since the number of the items to be compared in AHP is increased, the number of pairwise comparisons that each assessor answer, are increased drastically. So, we develop a method called EVMP to elicit the preference vector of pairwise comparison matrices with missing entries. The newly developed method is tested in a school; the generated results have been proved by experts.

Keywords: Performance assessment, Analytic hierarchy process, Non-Linear programming, Missing judgments.

Article history: *Received: 12 October 2018* *Revised: 13 February 2019* *Accepted: 20 February 2019*

1. Introduction

The purpose of this study is to provide the proper method for evaluating performance of teachers, which leads to reducing illegal discrimination and the reduction of tensions and conflicts between managers and employees. In order to achieve the above objectives, in this research, the performance measurement indicators has been identified in government agencies, the importance of indicators has been determined, and then performance evaluation in the organization has been performed by self-assessment method.

* Corresponding author

E-mail address: movafaghpour@jsu.ac.ir
DOI: 10.22105/riej.2019.169895.1074

The statistical population used in this research consists of the managers and teachers of the schools in Dezful city. The data collected in this study has been analyzed by means of the mathematical model for eliciting performance vector from pairwise comparison matrix.

The subject of self-assessment in recent years is considered as one of the continuous improvement techniques for all organizational processes. The proper design and accurate implementation of the system of self-assessment in an organization allows management to decide on the facts rather than relying on subjectivities and guesses. In order to be able to effectively and open-mindedly plan for our organization and to accurately portray the future position of the organization, we need to know where and in what position. To achieve this, one method that can be very useful, efficient, and effective is the implementation of the self-assessment process.

In order to control the process of assessment in the path to organizational success, all activities must be controlled at all times, which can only be achieved through the development of self-control culture by the employees themselves, and is possible through the self-assessment mechanism. In recent years, the self-assessment is considered as one of the most important management mechanisms for continuously improving all business and organizational capabilities. It can be said that many organizations perform their performance appraisal to assess the maturity level of the quality management system and identify areas for improvement or even to receive quality awards. This assessment can be carried out by external experts or external consultants or by experts and the internal assessment team of the organization itself. The accurate implementation of self-assessment allows management to decide on the basis of objective facts instead of making decisions based on subjective perceptions. In this research, we design a framework in which all employees can assess each other. The result of this cross-assessment is earning a full map of human interactions between all personnel as good as the real performance indicator of all employees.

2. Calculating Preference Weights from PCMs

The pairwise comparison method has been widely applied for representing judgments about criteria/alternatives in Multi-Criteria Decision Making (MCDM), especially in the Analytical Hierarchy Process (AHP). Such quantitative judgments are usually declared as Pairwise Comparison Matrices (PCM). The preference relations in the PCMs are filled in by the decision maker judgments, and presented using different measurement scales such as nine-point scale developed by Saaty [17]. The judgments may be inconsistent and/or incomplete because of the limits of decision makers' expertise and capabilities [14].

Besides the conventional applications of AHP and Analytical Network Process (ANP) in the field of decision making, recently some researchers have applied it in the field of performance analysis such as Farias et al. [21], Kyriakidis et al. [22], Liu et al. [23], and Pipatprapa et al. [24]. Human judgments tend to be inconsistent for decisions involving numerous criteria or alternatives. The Pairwise Comparison (PC) method is often used to elicit preferences for better decision making.

In PC, a Decision Maker (DM) is asked to compare only two objects (criteria or alternatives) at a time, and a prioritization method is used to estimate the preference vector from a given set of judgments [19]. In this research we elicit the performance score of each personnel for each question in the appraisal form as a relative number as comparison with a set of 2 to 4 other colleagues.

Although the PC method tries to reduce the mental effort for DM when expressing his/her subjective preferences beside increasing the accuracy of estimating the intent of DM in form of the preference vector, but when the number of objects to be compared increase, the number of comparisons that DM has to make increase beyond the capabilities of human performance.

According to the principle of pairwise comparison of AHP proposed by Saaty and Vargas [17], if there are n objects, all pair-compared results are arranged in a matrix A $=[a_{ij}]_{n*n}$ where $a_{ij} > 0$, $a_{ii} = 1$, $a_{ij} = 1/a_{ji}$, and decision makers need to complete $n(n - 1)/2$ pairwise comparisons [10].

In some real world situations, the decision makers could not fill in all $n(n - 1)/2$ pairwise comparisons because of time pressure. Time pressure means that we prefer to have an instant solution based on a subset of present data which is dynamically fulfilled, instead of waiting for the completion of data gathering among a widespread group of experts. Other situations that force us to deal with incomplete PCM are as unwillingness to make a direct comparisons between alternatives, hesitancy, or being unsure of some of the comparisons, incomplete information and limited expertise; hence there are one or more pairs of missing entries in PCM. In such cases PCM is called an incomplete or a PCM with missing values [10].

Dealing with incomplete information is an important problem in decision making [8]. In this paper, we present a new method for estimating preference vector without eliciting missing values of an incomplete PCM which is based on the maximization of consistency via a Linear programming model.

The problem of quantifying the comparative judgments could be tackled by two main approaches. The first approach is known as multiplicative. In this framework, each entry a_{ij} in PCM estimates the relative preference of the alternative i over j as w_i/w_j. The obtained multiplicative PCM A $=[a_{ij}]$ is positive, reciprocal ($a_{ji} = 1/a_{ij}$), with $a_{ii} = 1$, $i = 1, 2, \ldots, n$ [11]. In other words, a multiplicative PCM is called consistent if and only if:

$$a_{ih}a_{hj} = a_{ij}, \qquad 1 \le i, j, h \le n \tag{1}$$

In the second approach, also known as additive approach, the expert's preferences are represented by a fuzzy preference relation $r(i, j) \in [0, 1]$. [1 and 11]. In this paper, we focus on multiplicative PCM.

In this research, literature surveys have been used to formulate the basics, definitions, theoretical concepts. The field method has been used to collect the required data in relation to the identification of performance evaluation criteria, the importance of research criteria, and the determination of the status of research options. In the performance evaluation questionnaire that was analyzed using its mathematical model, respondents were asked to evaluate their colleagues and each respondent would divide 100 of the available scores between 4 of their colleagues depending on the degree of competence of individuals in that criterion.

Suppose that there exists a preference vector $W = (w_1, w_2, \ldots, w_n)$ such that w_i represents the preference intensity of *ith* object where $i = 1, 2, \ldots, n$. The preference vector W is unknown to the DM and must be estimated trough answering to a set of pairwise comparisons. When the DM is perfectly consistent in his/her judgments, then the judgments a_{ij} have perfect values $a_{ij} = w_i / w_j$. In such a case, the PCM is said to be (perfectly) consistent and can be represented as $A = [a_{ij}]_{n\times n} = [w_i / w_j]_{n\times n}$. In the case of inconsistent PC judgments (*i.e.* $a_{ij} \neq a_{ik} \times a_{kj}$ for some i, j, k), the calculated preference vector $\hat{W}$ approximates the unknown preference vector intended by DM [19].

The preference vector W could be calculated from the PC judgments using several mathematical techniques. In the case of error-free (consistent) judgments, all prioritization methods give the same results; and the results are different when judgments are inconsistent. A summary of prioritization methods is given by Choo and Wedley [9], where 18 different methods are analyzed and numerically compared. Some of the well-knowns are reviewed here.

The preference vector W, can be approximated by normalizing the columns of PC and averaging the values in each row. This method of Additive Normalization (AN) is mathematically formulated as [19]:

$$\bar{a}_{ij} = \frac{a_{ij}}{\sum_i a_{ij}}, \qquad \hat{w}_i = \frac{1}{n}\sum_j \bar{a}_{ij} \tag{2}$$

The AN method is considered to be appropriate because it has closed-form formulas for easy calculation and its very good performances on PCMs with small and large inconsistencies [9].

A well-known method proposed by Saaty and Vargas [17] uses the principal EigenVector (EV) of a given PCM to be used as the estimate of the preference vector W [19].

$$A\hat{W} = \lambda_{\max}\hat{W}. \tag{3}$$

Where $\lambda_{\max}$ denotes the maximal eigenvalue, also known as Perron eigenvalue of A, and $\hat{W}$ denotes the right-hand side eigenvector of a corresponding to $\lambda_{\max}$. Saaty and Vargas [17] defined the inconsistency ratio as:

$$CR = \frac{\lambda_{\max} - n}{RI_n\left(n-1\right)}, \tag{4}$$

where RI_n is average random consistency index for $A_{n\times n}$ [5].

The calculated results remain acceptable if the inconsistency ratio assigned to the computed results is less than 0.1. The EV solutions are unsatisfactory when inconsistency of the given PCM is larger (CR > 0.1) [19].

The Direct Least Squares (DLS) method minimizes the total squared error (residuals) defined as the total distance between the judgments and estimations. The objective function, $d(w)$, is the aggregation of individual deviations between the given judgment and the estimated preference vector, formulated as [19]:

$$d_{DLS}(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij} - \frac{\hat{w}_i}{\hat{w}_j}\right)^2 \tag{5}$$

$$\text{subjct to. } \sum_{i=1}^{n}\hat{w}_i = 1 \tag{6}$$

Because of some applied limitation of DLS as a non-linear optimization problem, the Weighted Least Squares (WLS) method as a modification to DLS that has been proposed. The objective function for WLS is mathematically formulated as [19]:

$$d_{WLS}(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\hat{w}_j a_{ij} - \hat{w}_i\right)^2 \tag{7}$$

$$\text{subjct to. } \sum_{i=1}^{n}\hat{w}_i = 1 \tag{8}$$

Unlike DLS, the WLS reduces the problem to a system of linear equations that can easily be solved, providing the optimum solution [19].

The Logarithmic Least Squares (LLS) method makes use of the multiplicative properties of PC. The LLS minimization assumes that the best solution is the preference vector that has the minimal sum of the logarithmic squared deviations from a given set of data [19] *i.e.*

$$d_{LLS}(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\log a_{ij} - \log \hat{w}_i + \log \hat{w}_j\right)^2 \tag{9}$$

$$\text{subjct to. } \sum_{i=1}^{n}\hat{w}_i = 1 \tag{10}$$

The solution for LLS is always unique and can be found simply as the Geometric Mean (GM) of the rows of PCM, provided that the set of given judgments is complete *i.e.* all $n(n-1)/2$ comparisons are provided. The GM can be formulated as [19]:

$$\hat{w}_i = \left(\prod_{j=1}^{n} a_{ij} \right)^{1/n} \tag{11}$$

Another proposed method is Logarithmic Least Absolute Values (LLAV). It was shown that the LLAV method also provides a unique solution. The objective function for LLAV is defined as [19]:

$$d_{LLAV}(w) = \sum_{i=1}^{n}\sum_{j=1}^{n} \left| \log a_{ij} - \log \hat{w}_i + \log \hat{w}_j \right| \tag{12}$$

$$\text{subjct to. } \sum_{i=1}^{n} \hat{w}_i = 1 \tag{13}$$

Harker [12] investigated an incomplete set of judgments, where DMs are allowed to respond with ‘‘do not know’’ or ‘‘not sure’’ to some judgments. It has been highlighted that the probability of acquiring incomplete PCs increases as *n* increases. In such cases, the AN, EV, and GM methods cannot estimate preferences without applying an intermediate method to elicit the missing judgments. The GM and LLS approaches are both equivalent for complete PCs, however, only the LLS approach is applicable to incomplete PCs [19].

Harker [13] proposed a method for determining the weights for such incomplete PCMs. He replaced the given incomplete PCM with modified matrix (called A), in which the missing values were filled with a zero value, and the diagonal entries were changed; he assumed the largest eigenvalue of A as the largest eigenvalue of the original incomplete PCM. Although, this heuristic method had no proof for reaching the optimum minimum error, but its main advantage was its capability to handle PCMs with several missing values. Shiraishi et al. [18] assumed a PCM with only one missing entry and developed a geometric mean to estimate the missing entry. This method was generalized by Kwiesielewicz [15]. Kwiesielewicz [15] considered the Logarithmic Least Squares Method for incomplete matrices and proposed a weighting method based on the generalized pseudoinverse matrices. Wedley [20] proposed a consistency prediction method for incomplete PCMs.

Kou et al. [14] considered the PCM as an adjacency matrix of a graph and suggested to generate all possible spanning trees out of this graph. Since every spanning tree results in a unique preference vector $\hat{W}$, in their proposed model, the mean of all preferences, and the variance were used to approximate the preference vector and the inconsistency measurement, respectively. The main advantage of their proposed method is its capability for handling incomplete PCM’s [14].

Benitez et al. [2] developed an approach to complete the incomplete judgments by minimizing the Frobenius norm based matrix distance. Chen et al. [6] proved that the Connecting Path Method (CPM) can guarantee minimal geometric consistency index, and proposed a PCM based method to estimate the missing judgments whilst improve the consistency for an incomplete

PCM. Ergu et al. [10] proposed a revised Geometric Mean Induced Bias Matrix (GMIBM) to estimate the missing values for the incomplete decision matrix in the case of emergency management. The consistency ratio can be efficiently improved by the proposed model [14].

The Geometric Mean Induced Bias Matrix (GMIBM) proposed by Ergu et al. [10] is a generalization of GM, in which missing values are filled with variables x_i. Then Geometric Mean (GM) of the rows of PCM is assumed as w_i. By estimating $a_{ij} = w_i/w_j$ they establish an over determined system of n^2 equations. In case of perfect consistency, a single solution is obtained. Otherwise, the resulted system of non-linear equations has no solution. In such cases, the Least Absolute Error (LAE) or Least Square Method (LSM) solution is calculated as the estimate of preference vector. Since the non-linear programming models developed to find LAE and LSM solutions, could not be solved exactly to the optimal solution, and they reported feasible solutions for two numerical examples after some iterations.

Bozóki et al. [4] developed a Linear Programming Model to elicit missing entries for an incomplete PCM. They minimized the inconsistency index as a nonlinear term in the objective function, and the summation of w_i's was constrained to be equal to 1.

Although their proposed model could elicit the missing entries for a given incomplete PCM, but their model do not provide any estimate for preference weights W_i's or inconsistency ratio CR. The effort for completing the PCM's with missing values is still being pursued by researchers such as [26].

Bozóki et al. [5] developed a Non-Linear Programming (NLP) model to minimize $\lambda_{\max}$. Since they could not provide any efficient algorithm to solve this NLP model, they proposed the iterative method of cyclic coordinates, in each step only one variable is allowed to be optimized and the other variables are fixed to the initial values; the optimal solution is then computed by a univariate minimization algorithm. With refer to the local optimality of their proposed algorithm there was no guaranty for reaching the global optimality of minimizing inconsistency ratio.

There are many other methods to estimate preference vector from PC judgments. Different methods perform differently and no method outperforms other methods in all situations [19]. Determining the best prioritization method entailing all aspects of efficiency remains an open problem [19]. However, Bozoki and Fulop [3] define the efficiency or Pareto optimality of a preference vector as no other preference vector is at least as good in approximating the elements of the pairwise comparison matrix. Based on this definition of an efficient preference vector, finding an efficient or Pareto optimal preference vector is equivalent to solve a multi-criteria decision problem. They defined dominant and non-dominant preference vectors. They showed that the preference weights generated by EigenValue (EV) method are inefficient and therefore dominated solutions.

Recently Oliva et al. [16] considered the problem of incomplete PCMs as calculating the eigenvector of a sparse matrix corresponding to an undirected connected graph. They proposed a general definition of consistency that takes into account the sparseness of data and provided a necessary and sufficient consistency condition. In this paper, we consider an incomplete PCM and develop a NLP model which can be efficiently solved to the optimal solution. Our proposed mathematical model minimizes *CR* and finds the optimum estimates for the preference vector *W* without eliciting missing entries of the PCM. Since the objective function of our developed method minimizes *CR* which is linearly dependent with $\lambda_{\max}$, we call our developed method as EigenValue calculation by Mathematical Programming (EVMP).

3. Problem Formulation

When filling a PCM, in the inconsistent case, the entry a_{ij} of the matrix *A* is an estimation of the ratio W_i / W_j. Since it is an estimate, when an error $\varepsilon_{i,j}$ is messing with the judgments, the relative importance *ith* object to *jth* one is expressed as:

$$a_{i,j} = \frac{w_i + \varepsilon_{i,j}}{w_j} \tag{14}$$

In the above relation, $\varepsilon_{i,j}$ denotes the deviation of $a_{i,j}$ from being an accurate judgment. Obviously, if $\varepsilon_{i,j} = 0$, then the $a_{i,j}$ is perfectly estimated. If all $a_{i,j}$'s are perfectly estimated, then A is consistent and its maximal eigenvalue ($\lambda_{\max}$) equals *n* (the number of rows of square matrix A). Since there are some imperfections in judgments about $a_{i,j}$'s, therefor $\lambda_{\max}$ is greater than or equal *n*:

$$\lambda_{\max} \geq n \xrightarrow{\text{if PC matrix is completely consistent}} \lambda_{\max} = n \Rightarrow \lambda_{\max} = n + \varepsilon. \tag{15}$$

In order to minimize the error of estimating correct preference vector, we have to minimize the inconsistency ratio or equivalently ε:

$$Min \;\; CR = \frac{\lambda_{\max} - n}{RI\,(n-1)} \cong Min \frac{\varepsilon}{n-1} \cong Min\,\varepsilon. \tag{16}$$

Since

$$AW = \lambda_{\max} W, \tag{17}$$

we have

$$A^{(i)}W = \lambda_{\max} w_i, \;\; \forall i \tag{18}$$

$$\Rightarrow A^{(i)}W = (n+\varepsilon) w_i, \quad \forall i \tag{19}$$

Therefore, we can minimize the error of estimating preference vector W with regard to following set of constraints:

$$subject\ to\ \begin{cases} \sum_i w_i = 1 \\ A^{(i)}\ W = \lambda_{\max} w_i & \forall i \quad \Rightarrow \sum_j a_{i,j} w_i = \lambda_{\max} w_i \quad \forall i \end{cases} \tag{20}$$

By replacing $\lambda_{\max}$ with $(n+\varepsilon)$ we have

$$\begin{aligned} &Min\ \varepsilon \\ &subject\ \ to\ \begin{cases} \sum_i w_i = 1 \\ \sum_j a_{i,j} w_j \le (n+\varepsilon) w_i & \forall i \\ w_i, \varepsilon \ge 0 & \forall i \end{cases} \end{aligned} \tag{21}$$

We can reformulate the right hand side of (34) as:

$$\sum_j a_{i,j} w_j \le n w_i (1+\frac{\varepsilon}{n}), \forall i \tag{22}$$

By replacing $\frac{\varepsilon}{n}$ with ε_i we have

$$\begin{aligned} &\sum_{j|a_{i,j}>0} a_{i,j} w_j \le \left(\sum_{j|a_{i,j}>0} 1 \right) w_i (1+\varepsilon_i), \forall i \\ &-\frac{\varepsilon}{n} \le \varepsilon_i \le \frac{\varepsilon}{n} \end{aligned} \tag{23}$$

If the PCM is incomplete, then we can update the linear programming model (22-25) to find preference weights w_i's from an incomplete PCM A as:

$$\begin{aligned} &Min \varepsilon \\ &subject\ \ to\ \begin{cases} \sum_i w_i = 1 \\ \sum_{j|a_{i,j}>0} a_{i,j} w_j \le \left(\sum_{j|a_{i,j}>0} 1 \right) w_i (1+\varepsilon_i) & \forall i \\ -\frac{\varepsilon}{n} \le \varepsilon_i \le \frac{\varepsilon}{n} & \forall i \\ w_i, \varepsilon \ge 0,\ \varepsilon_i \text{ free in sign} & \forall i \end{cases} \end{aligned} \tag{24}$$

4. Experimental Results

At this stage, as previously stated, each employee was asked to evaluate four of his colleagues. In this way, the 100 points available for each criterion are divided by the suitability of each person. With the help of the mathematical model described, the score tables for each individual will be analyzed in each criterion, the output of which will be the weight vector that will represent each individual's score in that criterion. The scores of individuals in each school are gathered in a table in each criterion, and a weight matrix will be provided depending on the importance of the criteria. The average weight matrix for each individual will be the final score. Finally, the ranking of individuals in each school is done with the help of the final grades and it can be said that the best employees will be according to their colleagues.

5. Conclusions

One of the shortcomings in the field of performance management is the lack of integrity and coherence among the components of this system at various organizational levels, and this is especially evident in the establishment of coherence between performance assessment systems at the organizational level and employee performance appraisal. Most of the managers have accepted the philosophy of evaluation, but inadequacy of the existing performance assessment questionnaires, results in lack of trust to the result of traditional performance assessment methods. In this research we developed a novel method for filling the same questionnaire used in traditional performance measuring systems. In our proposed method each questionnaire form was filled for several employees based on their relative performance; therefore the results scores were relative numbers. Since the performance of each people may be assessed by several colleagues, we used the geometric mean to aggregate several judgments for individual entry a_{ij} of PCM. The purpose of this research is to identify the preference weight vector of all employees for each item (question) in the assessment form.

Considering that all of the current performance indicators of the organization's performance appraisal as well as some of the desirable indicators of performance evaluation are accepted by managers, it is recommended that the performance evaluation forms be redesigned in order to let the assessor to fill the form based on comparative judgment instead of individual judgments about every single personnel. Due to the fact that prioritization of performance of the personnel is relative to the organizations strategic plan, it is recommended that special attention should be given to defining new relevant performance indices. Performance appraisal of employees should provide a healthy competitive environment among employees and reinforce the motivation and effort of the individual. Performance appraisal should modify the behavior. After evaluating the performance, it is necessary to give feedback to the individual. This feedback can be arranged in a friendly and coordinated session so that during a friendly meeting, the positive and negative points of the staffing are discussed, then the evaluation will result.

6. Acknowledgment

The writer would like to acknowledge the “Khuzestan Water and Power Authority” for financial support within the frame of the project number 95/3929. This research has been supported by

"Khuzestan Water and Power Authority" and "Exploiting Northern Khuzestan Irrigation Networks Company" grant in Iran.

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