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Implementation of Fuzzy Rule-based Algorithms in P Control Chart to Improve the Performance of Statistical Process Control

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A B S T R A C T

In the statistical process control when the process is very sensitive and the control limit shifts are the prime concerns, there the fuzzy control chart can be a better solution. In decision making, the extra 'rather in control' and 'rather out of control' decisions facilitate to find out the slight changes in the control chart. The automation of the fuzzy control chart in the Excel VBA makes the data input and decisions making process faster. The vagueness of the data is removed as the charts deal with the triangular or trapezoidal area rather than some points in the control limits. Alongside the fuzzy control charts, the Marcucci approach has been followed to find out the goodness-of-fit of the samples and to find out the effectiveness of the fuzzy control charts.

Keywords: Fuzzy control charts, Control limits, Excel VBA, Marcucci approach, Goodness-of-fit.

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1. Introduction

Control charts are the preferred tools for SPC. Since 1924, when Shewhart presented, the Shewhart control charts and the various control chart techniques have been developed and widely applied in industries. The major contribution of control charts is to detect causes of defective production so that the necessary corrective actions can be taken before any large quantity of nonconforming product is manufactured. The variability of sequential data collected from a process to measure the quality characteristics reveals the existence of the common causes. So, the control charts can be used to separate out the cause of the variation, whether due to common

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causes or an assignable causes and thus determine whether the process is ‘in control’ or ‘out of control’.

Basic two types of control charts are variable control chart and attribute control chart. When the quality characteristics is measured in the numeric scale, then it is call variable control chart such as $\bar{X}$-S control charts, $\bar{X}$-R control charts, cumulative sum chart, etc. When the quality characteristics are measured in the qualitative form then it is called attribute control chart. The control charts of *p, np, c,* and *u* are charts for the fraction of nonconforming, nonconforming units, number of nonconformities, and nonconformities per unit, respectively; so they are attribute control chart. The quality characteristic can often be evaluated according to a binary classification of ‘conforming’ or ‘nonconforming’ depending on whether they meet the required specifications such that the number of nonconforming units can be determined. However, some amount of uncertainty or vagueness may exist due to the operator judgment variation or measurement device because the related quality characteristics are evaluated subjectively. El-Shal and Morris [3] proposed a methodology for control charting by integrating the fuzzy logic and SPC zone rules. This methodology provides a means to avoiding generation of false alarms due to random variations in the environment and/or measuring instruments and sensors. Hsu and Chen [5] applied the concept of the fuzzy sets and membership functions for softening Nelson’s rules to detect unnatural patterns of symptoms. Moreover, this group presented a new methodology used to acquire knowledge of the relationships between cause and symptoms from the data. The maximum similarity method has been used and also used the genetic algorithms for the MSM in the application of $\bar{X}$ chart. Statistical Process Control (SPC) is a method used to monitor, evaluate and, improve processes in both the production and service industries to ensure process stability. A two-phase approach is followed while SPC is practiced. Phase I focuses on obtaining knowledge about the ‘in-control’ (IC) characteristics from a fixed sized data and removes the cause of the variation. On the other hand, in Phase II the process is monitored based on the estimates of the IC process parameters established in Phase I [6].

The implementation and methodologies of the quality tools are essential to alleviate items with nonconformity which results in the reduction of aggregate quality costs that enhances company’s competitive performance. Reduction of process variability can help to obtain this [7]. Control charts are very popular tools for ensuring process stability and capability [8]. This paper reports a research that has been focused on the application of fuzzy rule-based algorithms in p control chart to production lines of a garment industry.

2. Literature Review

Control chart is a core quality control tool which are extensively used to assure the process consistency, and the fuzzy systems are adjuvant while dealing with uncertainty. Tremendous works have been done where the control charts and the fuzzy systems are researched and analyzed individually and/or are in integrations. Some of these previous works are discussed in this section. Fadaei and Pooya [9] took an approach to investigate the performance of the fuzzy u-chart by applying the Fuzzy Operating Characteristic (FOC) curve and reached the decision that the fuzzy charts are competent for identifying process shifts and they are more efficient than the crisp chart. Zhang et al. [10] proposed a strategy to extract the controllable-domain-based fuzzy rule as a new scheme for better process control and ran a real life experiment for justification of their proposal. Naik et al. [11] proposed a Dynamic Fuzzy Rule Interpolation (D-FRI) approach which is more reliable, robust, and efficient than the conventional approach. Keivanpour et al. [12] integrated the game theory with the fuzzy rule-based approach for analyzing the strategic choice while performing the automobile manufacturers' green end-of-life vehicle practices. The fuzzy logic is utilized as a tool for investigating a problem and analyzing the interactions and inter-relationships between its variables [4].

Cheng [13] presented a novel methodology for the fuzzy process control for the purpose of monitoring a process with the fuzzy outcomes represented by the fuzzy numbers. The out-of-control conditions of these control charts were formulated in accordance with the possibility theory. Faraz and Moghadam [14] introduced a real illustrative example, and a power test shows that designing a fuzzy control chart for the process average of a continuous (variable) quality characteristic with a warning line is a better alternative to Shewhart $\bar{X}$ chart in many respects. Erginel [15] presented the theoretical structure of the fuzzy individual and moving range control charts with α cuts by considering the fuzziness that may be caused by the operator, the gauge or the environmental conditions. Colubi [16] represented a fuzzy random variable, an important role as the central summary measure, and for this reason, in the last years valuable, the statistical inferences about the means of the fuzzy random variables have been developed. Kanagawa et al. [17] used the linguistic data from a different viewpoint to construct the control chart. The presented control charts aim at controlling the process average and the variability based on an existing underlying probability distribution of the linguistic data.

Kaya and Kahraman [18] proposed the fuzzy formulation of the indices C_p and C_{pk} which are the most used two traditional Process Capability Indices (PCIs). Faraz and Shapiro [19] proposed a control chart that is an extension of Shewhart $\bar{X}$-S^2 control charts in the fuzzy space which avoids defuzzification methods. Laviolette et al. [20] reviewed some basic concepts of the fuzzy methods, point out some philosophical and practical problems, and offered simpler alternatives based on the traditional probability and statistical theory. Marcucci [21] presented two classes of the procedures to monitor a mutually exclusive process. One is the modified Pearson $\chi 2$ statistics and the second depends on the multinomial distribution. Saaty [22] questioned the concept of the equality of belonging of elements to sets and cleared the idea of sets criterion. Wang and Raz [23] studied two approaches for constructing the control charts for linguistic data. Woodall [24] represented an attempt to offer a comprehensive bibliography of references on control charting using the attribute data. Woodall et al. [25] presented a bibliography of the fuzzy quality charts. The present paper suggests a different approach for p control chart which provides the automation and precision in the industrial process control.

3. Problem Statement

Manufacturing or production industries like garments, steels, cables, beverages, automobiles, etc. use p control chart for monitoring their products quality. But p control chart provides only yes or no decisions about their products. But only yes or is not a satisfactory decision for sampling a huge number of products. The p control chart decisions do not show the variations of samples characteristics much. Moreover, the p control chart is not automatic too. Those problems can be solved by implementing the fuzzy rules in p control charts which is proposed in this paper.

4. Theoretical Framework

Theoretical framework according to the proposed method has been described steps by steps in the following.

4.1 Fuzzy P-Control Chart Based on a Constant Sample Size for a Triangular Fuzzy Number (TFN) Case

The nonconforming fraction is defined as the ratio of the number of nonconforming units to the total number of units in that population. The units may have several quality characteristics that are examined simultaneously by the operator. If the unit does not conform to the standards for one or more of these characteristics, the unit is classified as nonconforming [26].

In the fuzzy case, the number of nonconforming units is stated using the triangular fuzzy numbers such as (d_{aj}, d_{bj}, and d_{cj}) based on the triangular membership function parameters (a, b, and c). In this case, the nonconforming fraction can be expressed using the triangular fuzzy numbers such as (paj, pbj, and pcj). In this work, (p_a, p_b, and p_c) are the fuzzy averages of the nonconforming fractions, where n is the sample size, m is the number of sample, and j = 1, 2. . . m.

$$p_{aj} = \frac{d_{aj}}{n_j}, \quad p_{bj} = \frac{d_{bj}}{n_j}, \quad p_{aj} = \frac{d_{cj}}{n_j} \tag{1}$$

$$\bar{p_a} = \frac{\sum d_{aj}}{\sum n_j}, \quad \bar{p_b} = \frac{\sum d_{bj}}{\sum n_j}, \quad \bar{p_c} = \frac{\sum d_{cj}}{\sum n_j} \tag{2}$$

By considering the formulations of p control limits and the fuzzy numbers based on triangular membership functions, the fuzzy center line and the fuzzy upper and fuzzy lower limits of the p control chart are given as follows:

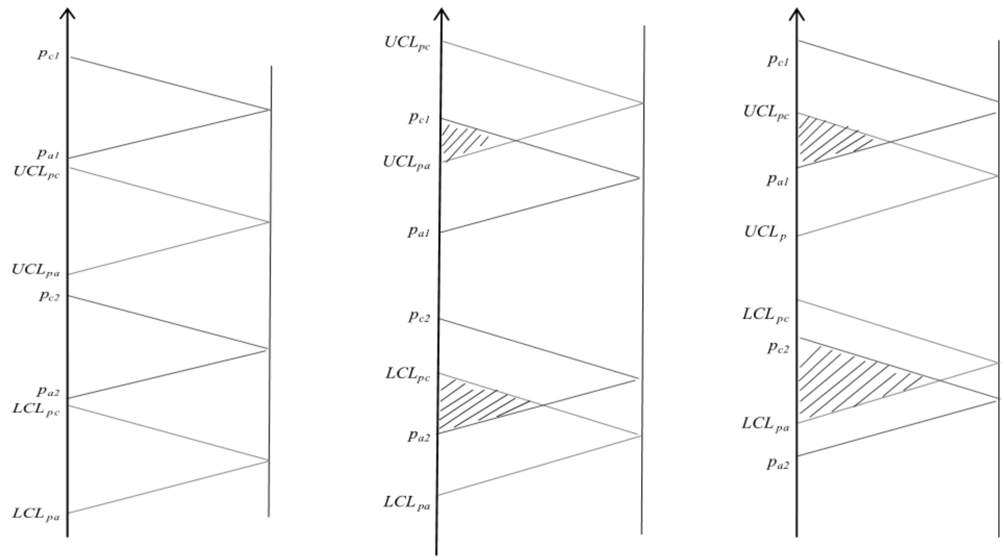
$$(U\tilde{CL}_{p_a}, U\tilde{CL}_{p_b}, U\tilde{CL}_{p_c}) = \left(\bar{p_a} + 3\sqrt{\frac{\bar{p_a}(1-\bar{p_a})}{n}}, \bar{p_b} + 3\sqrt{\frac{\bar{p_b}(1-\bar{p_b})}{n}}, \bar{p_c} + 3\sqrt{\frac{\bar{p_c}(1-\bar{p_c})}{n}}\right)$$

$$(\tilde{CL}_{p_a}, \tilde{CL}_{p_b}, \tilde{CL}_{p_c}) = (\bar{p_a}, \bar{p_b}, \bar{p_c}) \tag{3}$$

$$\left(LCL_{p_a}, LCL_{p_b}, LCL_{p_c}\right) = \left(\bar{p_a} - 3\sqrt{\frac{\bar{p_c}(1-\bar{p_c})}{n}}\ , \bar{p_b} - 3\sqrt{\frac{\bar{p_b}(1-\bar{p_b})}{n}}\ , \bar{p_c} - 3\sqrt{\frac{\bar{p_a}(1-\bar{p_a})}{n}}\ \right).$$

4.1.1 Rule for a fuzzy p-control chart based on a constant sample size for a TFN case

Rule 1. Rule 1 analyses the cases in which the fuzzy nonconforming fraction lies wholly between the fuzzy control limits or wholly outside the fuzzy control limits, as shown in Figure 1 [26].



(a) **(b)** **(c)**

Figure 1. Rules for TFN Case.

$$\text{Process control state} = \begin{cases} \text{in control if Pc} < \text{UCLpa And Pa} > \text{LCLpc} \\ \text{out of control if Pa} > \text{UCLpc Or Pc} < \text{LCLpa} \end{cases}. \quad (4)$$

In Figure 1, the triangular membership function (Pc2, Pb2, Pa2) representing the process control state is ‘process in control’, and the triangular membership function (Pc1, Pb1, Pa1) representing the process control state defines the ‘process out of control’ situations.

Rule 2. Rule 2 takes into account the case in which the fuzzy nonconforming fraction that closes the inner side to either of the control limits is a partially included corresponding control limit as shown in Figure 2. If the percentage area of a nonconforming fraction that remains inside the fuzzy control limits is equal to or greater than a predefined acceptable percentage (β*), then the process can be accepted as ‘process rather in control’; otherwise it can be stated as ‘process rather out of control’.

In Figure 1(b), the triangular membership function (Pc1, Pb1, Pa1) representing the process control state is 'rather in control', and the triangular membership function (Pc2, Pb2, Pa2) representing the process control state is 'rather out of control' [2].

$$\text{Process control state} = \begin{Bmatrix} \text{rather in control if } \beta \geq \beta^* \\ \text{rather out of control if } \beta < \beta^* \end{Bmatrix}.$$

$$\beta{:}\begin{Bmatrix} 1 - \dfrac{\text{Pc} - \text{UCLpa}}{\text{Pc} - \text{pa}} \text{ if Pc} > \text{UCLpa And Pa} < \text{UCLpa} \\ 1 - \dfrac{\text{LCLpc} - \text{pa}}{\text{Pc} - \text{pa}} \text{ if Pa} < \text{LCLpc And Pc} > \text{LCLpc} \end{Bmatrix}. \tag{5}$$

Rule 3. Rule 3 describes the case in which the fuzzy nonconforming fraction that closes the outside to either of the control limits is a partially included corresponding control limit as shown in Figure 1(c) [1].

$$\text{Process control state} = \begin{Bmatrix} \text{rather in control if } \beta \geq \beta^* \\ \text{rather out of control if } \beta < \beta^* \end{Bmatrix}.$$

$$\beta{:}\begin{Bmatrix} 1 - \dfrac{\text{Pc} - \text{UCLpc}}{\text{Pc} - \text{pa}} \text{ if Pc} > \text{UCLpc And Pa} < \text{UCLpc} \\ 1 - \dfrac{\text{Pc} - \text{LCLpc}}{\text{Pc} - \text{pa}} \text{ if Pa} < \text{LCLpa And Pc} > \text{LCLpa} \end{Bmatrix}. \tag{6}$$

In the constant-sample-size case, the control limits and the central limits are calculated by considering the average of the sample nonconforming fractions and are calculated only once. However, if the sample size has a large variation between samples, both the upper and lower control limits should be calculated for each variable sample size.

4.1.2 Marcucci approach

Marcucci approach finds out the goodness-of-fit of the samples according to the base sample sample quality. The target value is estimated for the base period where the process is assumed to be in control. Firstly, the base sample sample standard non-conforming is expressed as

P_{01}, P_{02}, P_{03}…. P_{0j} , where j is the grade number.

The statistic to be plotted in the control chart for each sample is [2]:

$$Z_i^{\ 2} = n_i n_0 \sum_{j=1}^{j_{max}} \frac{(p_{ij} - p_{0j})}{X_{ij} - X_{0j}} \ , \tag{7}$$

where $p_{ij}=X_{ij}/n_i$ and X_{ij} is the number of observation of sample i classified as j products; n_i= each sample size. The resulting generalized p-chart is illustrated in the Figure 11 . The upper control limit is taken to be the 70th percentile of the chi square (X^2) distribution which is 3.434444. Note that, contrarily to the p-chart, the control limit in a generalized p-chart does not depend on the number of samples but only on the numbers of categories. Then, the upper control limit in a generalized p-chart is the same for all multinomial processes that have the same number of categories. The control limit remains the same when the number of samples is reduced.

4.2 Fuzzy P-Control Chart Based on a Variable Sample Size for a TFN Case

If the sample size is not constant, a variable sample size should be used for calculating the control limits in a p-control chart. Two approaches exist for a variable sample size: Calculating the control limits using an approximate sample size and calculating the control limits for each sample size. n approximate sample size is used rather than a constant sample size. The following equations for the control limits given is suitable for calculating the approximate control limits [1]. However, if there is an unusually large variation in the size of a particular sample or if a point is plotted near the approximate control limits, then the exact control limits for that point should be determined and the point examined relative to that value [1]. Here, the approximate sample size is being used. Here, the fuzzy nonconforming fraction for each sample (p_{aj}, p_{bj}, p_{cj}) and the fuzzy average ($\bar{p_a}, \bar{p_b}, \bar{p_c}$) are calculated as follows:

$$p_{aj} = \frac{d_{aj}}{n_j}, \quad p_{bj} = \frac{d_{bj}}{n_j}, \; p_{aj} = \frac{d_{cj}}{n_j} \tag{8}$$

$$\bar{p_a} = \frac{\sum d_{aj}}{\sum n_j}, \quad \bar{p_b} = \frac{\sum d_{bj}}{\sum n_j}, \quad \bar{p_c} = \frac{\sum d_{cj}}{\sum n_j}$$

The control limits can be calculated in the fuzzy p control chart for each nj, expresses the jth sample size, uses a triangular membership function and the fuzzy average of the sample nonconforming fraction similar the way in Eq. (3).

4.2.1 Rules for fuzzy p control chart based on a variable sample size for a TFN case

Similar rules for a fuzzy p-control chart based on a constant sample size for a TFN case are defined on a variable sample size. However, the fuzzy control limit symbols include the variable sample size symbol (j). These rules are detailed as follows [1]:

Rule 1. Rule 1 analyses the cases in which the fuzzy nonconforming fraction lies completely between the fuzzy control limits or completely outside the fuzzy control limits.

$$\text{Process control state} = \begin{cases} \text{in control if Pc} < \text{UCLpa And Pa} > \text{LCLpc} \\ \text{out of control if Pa} > \text{UCLpc Or Pc} < \text{LCLpa} \end{cases}. \tag{9}$$

Rule 2. Rule 2 has the same formulation as for the fuzzy p control chart based on a constant sample size for a TFN case.

$$\text{Process control state} = \begin{Bmatrix} \text{rather in control if } \beta \geq \beta^* \\ \text{rather out of control if } \beta < \beta^* \end{Bmatrix}. \quad (10)$$

$$\beta: \begin{Bmatrix} 1 - \frac{\text{Pc} - \text{UCLpa}}{\text{Pc} - \text{pa}} \text{ if Pc} > \text{UCLpa And Pa} < \text{UCLpa} \\ 1 - \frac{\text{LCLpc} - \text{pa}}{\text{Pc} - \text{pa}} \text{ if Pa} < \text{LCLpc And Pc} > \text{LCLpc} \end{Bmatrix}.$$

Rule 3. Rule 3 describes the case in which the fuzzy nonconforming fraction that closes the outside to either of the control limits is a partially included corresponding to control limit.

$$\text{Process control state} = \begin{Bmatrix} \text{rather in control if } \beta \geq \beta^* \\ \text{rather out of control if } \beta < \beta^* \end{Bmatrix}.$$

$$\beta: \begin{Bmatrix} 1 - \frac{\text{Pc} - \text{UCLpc}}{\text{Pc} - \text{pa}} \text{ if Pc} > \text{UCLpc And Pa} < \text{UCLpc} \\ 1 - \frac{\text{Pc} - \text{LCLpc}}{\text{Pc} - \text{pa}} \text{ if Pa} < \text{LCLpa And Pc} > \text{LCLpa} \end{Bmatrix}. \quad (11)$$

5. Formation of Fuzzy P-Control Chart Based on a Constant and Variable Sample Size for a TFN Case

Formation of the fuzzy p-control chart based on a constant sample size and variable samples has been described sequentially in the following.

5.1 Formation of Fuzzy P-Control Chart Based on a Constant Sample Size for a TFN Case

The data of the samples are taken from the different production line in Epyllion Group garments. The sample size is 50 and is constant for the all the 20 sample. The data has been processed in the excel sheet. Visual Basic coding has been moduled in the excel sheet to take the input of the data in the excel sheet.

The Upper Control Limit (UCL), Centre Line (CL), Lower Control Limit (LCL) have been determined in the lower part of the excel sheet by calculating the respecting Eqs. (1), (2), and (3) with the help of the excel functions. Then the final decision of the process condition has been given by the transforming the Eqs. (4), (5), and (6) algorithms in the Visual Basic coding in the sheet module.

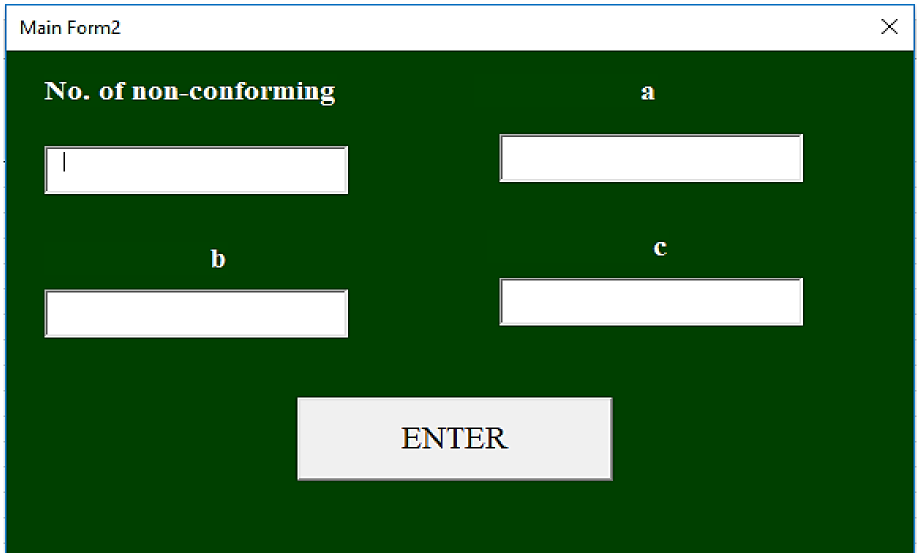


Figure 2. Input Box.

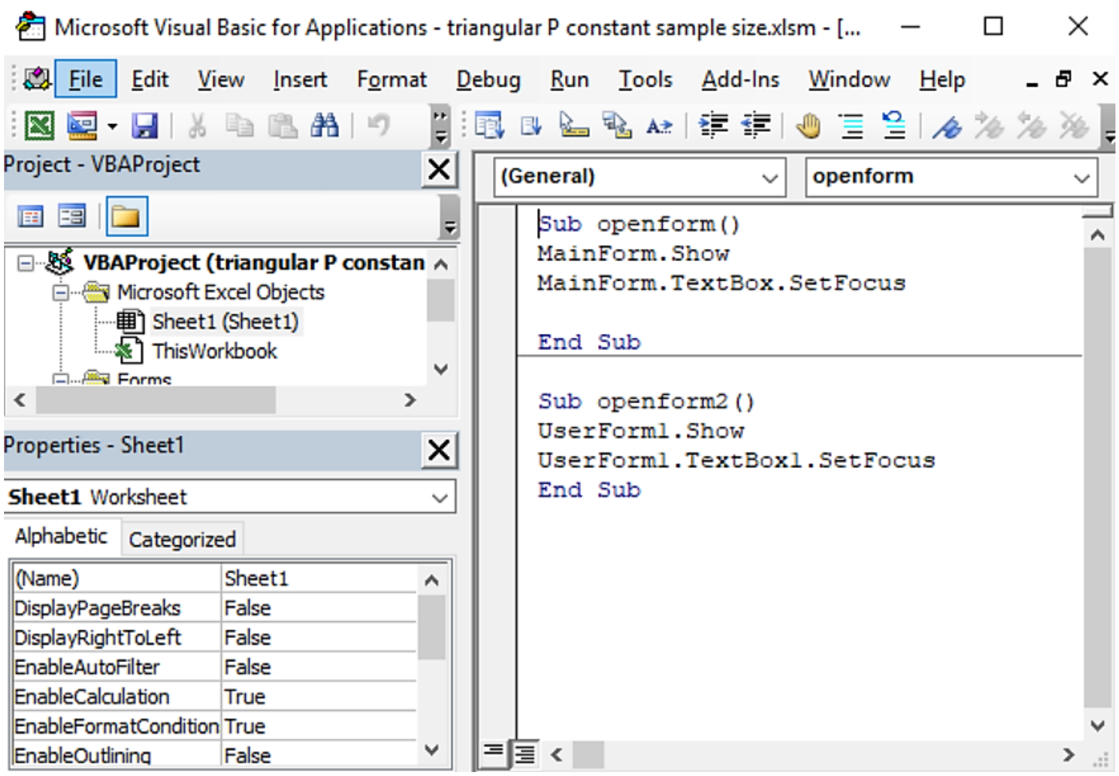


Figure 3. Visual Basic Coding for Input Box in the Excel Sheet.

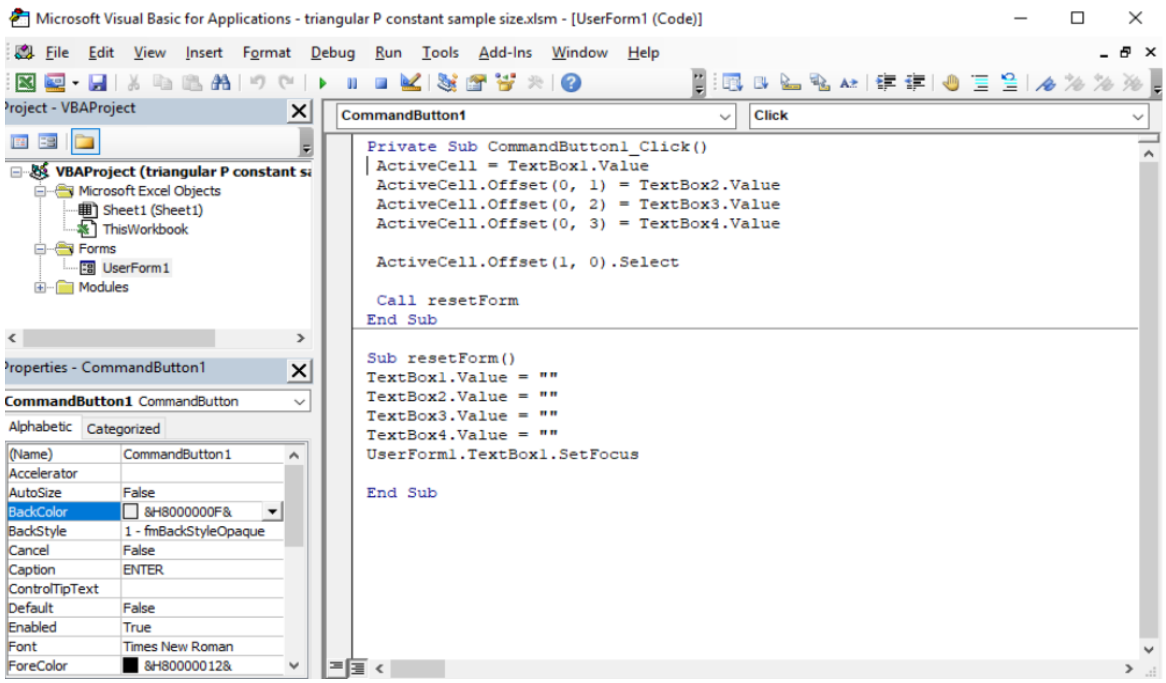


Figure 4. Visual Basic Coding Against the ENTER Command Button.

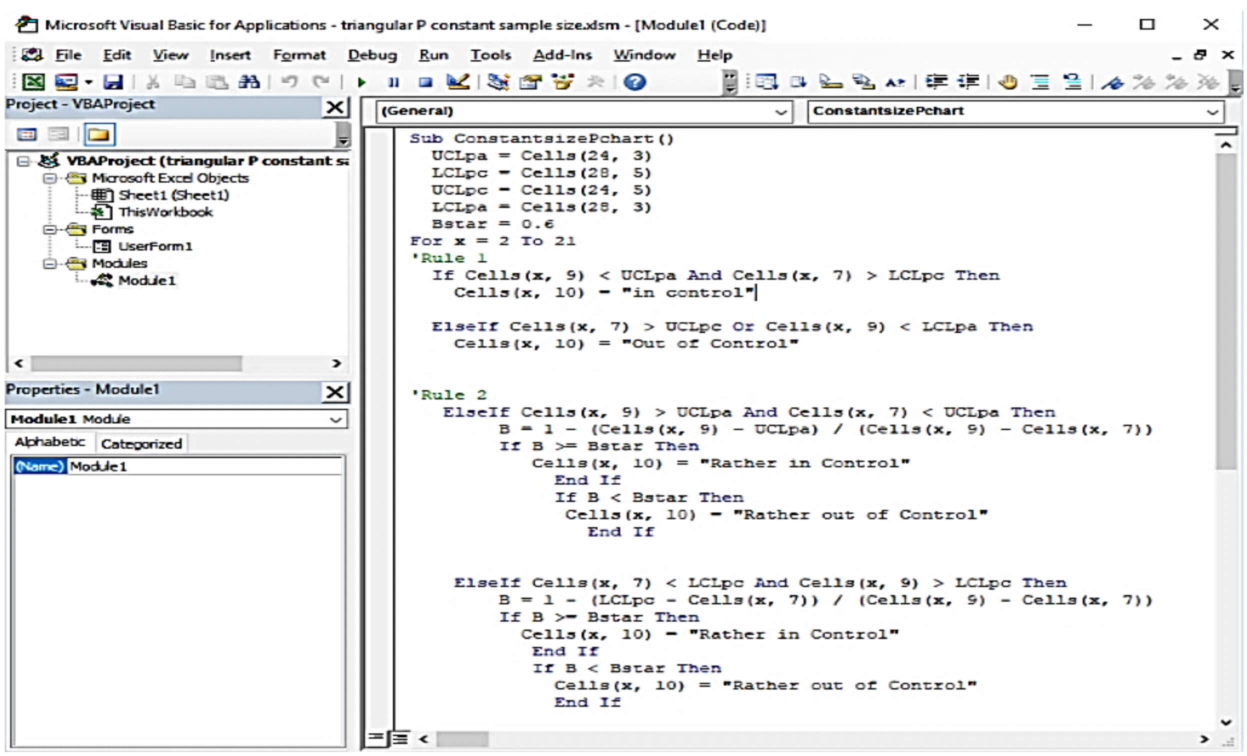


Figure 5. Visual Basic Coding for the Process Condition Algorithms (part 1).

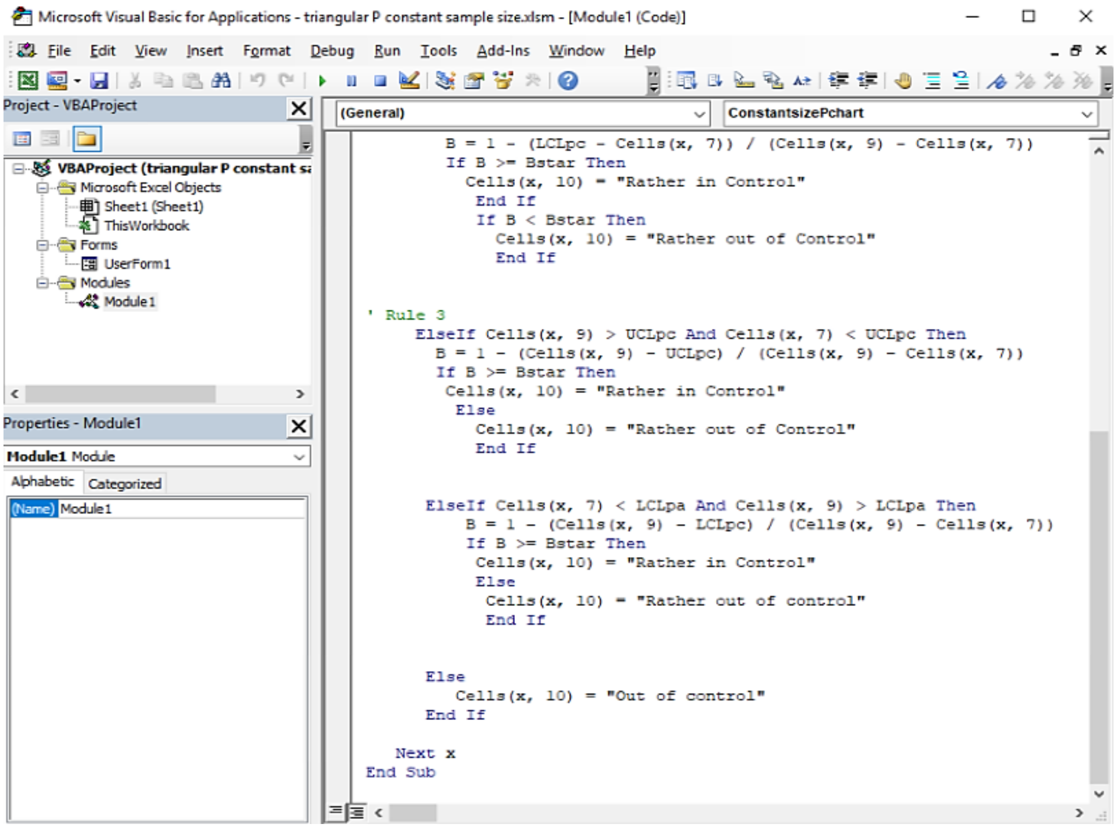


Figure 6. Visual Basic Coding for the Process Condition Algorithms (part 2).

5.2 Marcucci Approach Application for Fuzzy P-Control Chart Based on a Constant Sample Size for a TFN Case

The base sample is taken as the 4th sample as i=4 and is colored in the excel sheet where p_{01}= 0.9, p_{02} = 0.06, and p_{03} = 0.04. All the calculations of the data are being done according to the Eq. (7). The data has been taken through the data Input Box.

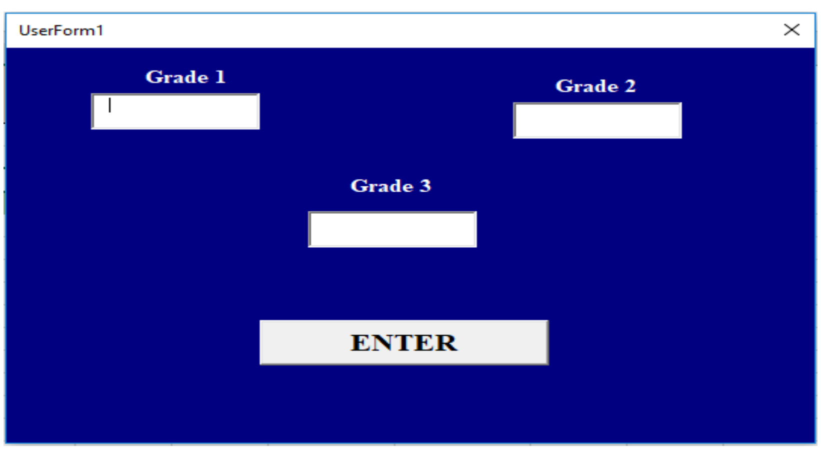


Figure 7. Input Box.

Table 1. P-Control Chart Based on Constant Sample Size for TFN Case.

Sample No	Sample Size	Number of non-conforming units	a	b	c	Pa	Pb	Pc	Process Condition	Button
1	50	10	8	9	10	0.16	0.18	0.2	in control	
2	50	5	3	4	5	0.06	0.08	0.1	in control	
3	50	13	11	12	13	0.22	0.24	0.26	rather in control	Input Data
4	50	5	3	4	5	0.06	0.08	0.1	in control	
5	50	8	6	7	8	0.12	0.14	0.16	in control	
6	50	15	13	14	15	0.26	0.28	0.3	out of control	
7	50	11	9	10	11	0.18	0.2	0.22	in control	
8	50	10	8	9	10	0.16	0.18	0.2	in control	Program
9	50	6	4	5	6	0.08	0.1	0.12	in control	
10	50	7	5	6	7	0.1	0.12	0.14	in control	
11	50	6	4	5	6	0.08	0.1	0.12	in control	
12	50	4	2	3	4	0.04	0.06	0.08	in control	
13	50	4	2	3	4	0.04	0.06	0.08	in control	
14	50	8	6	7	8	0.12	0.14	0.16	in control	
15	50	8	6	7	8	0.12	0.14	0.16	in control	
16	50	9	7	8	9	0.14	0.16	0.18	in control	
17	50	5	3	4	5	0.06	0.08	0.1	in control	
18	50	5	3	4	5	0.06	0.08	0.1	in control	
19	50	10	8	9	10	0.16	0.18	0.2	in control	
20	50	10	8	9	10	0.16	0.18	0.2	in control	
	UCLpa	UCLpb	UCLpc			∑Pa	∑Pb	∑Pc		
	0.25637183	0.285772681	0.31			2.38	2.78	3.18		
	Clpa	CLpb	CLpc			$\bar{P}a$	$\bar{P}b$	$\bar{P}c$		
	0.119	0.139	0.16			0.119	0.139	0.159		
	LCLpa	LCLpb	LCLpc							
	-0.0361432	-0.00777268	0.02							

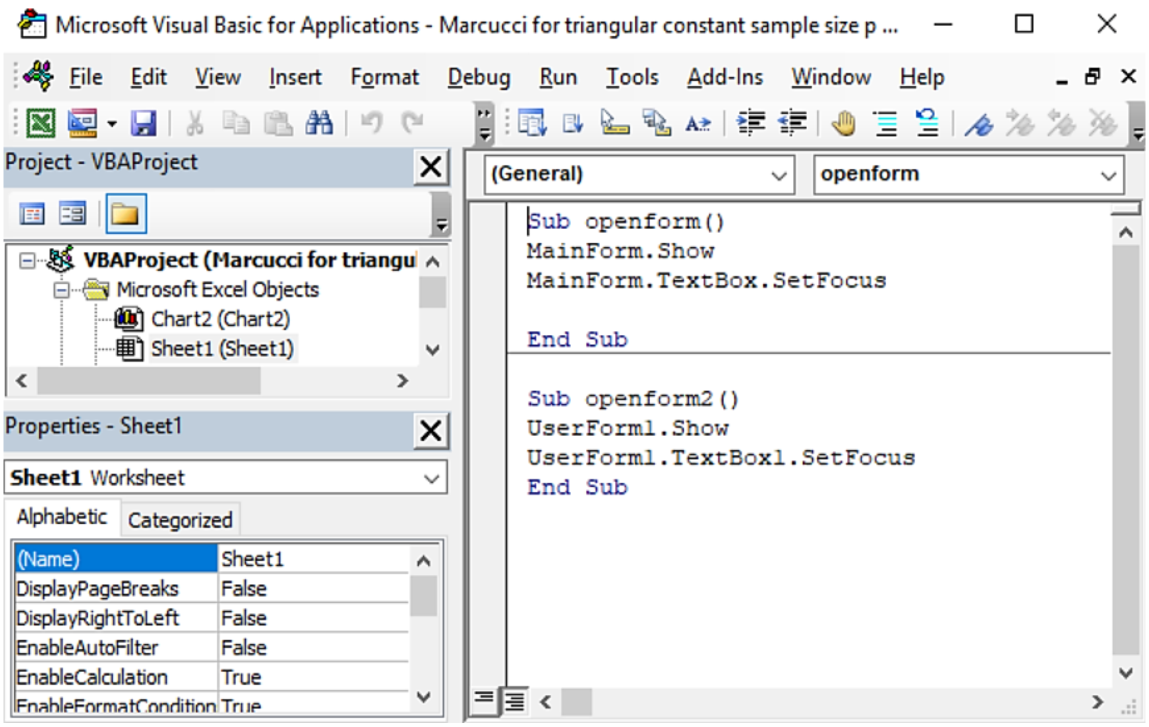


Figure 8. Visual Basic Coding for the Input Box in the Excel Sheet.

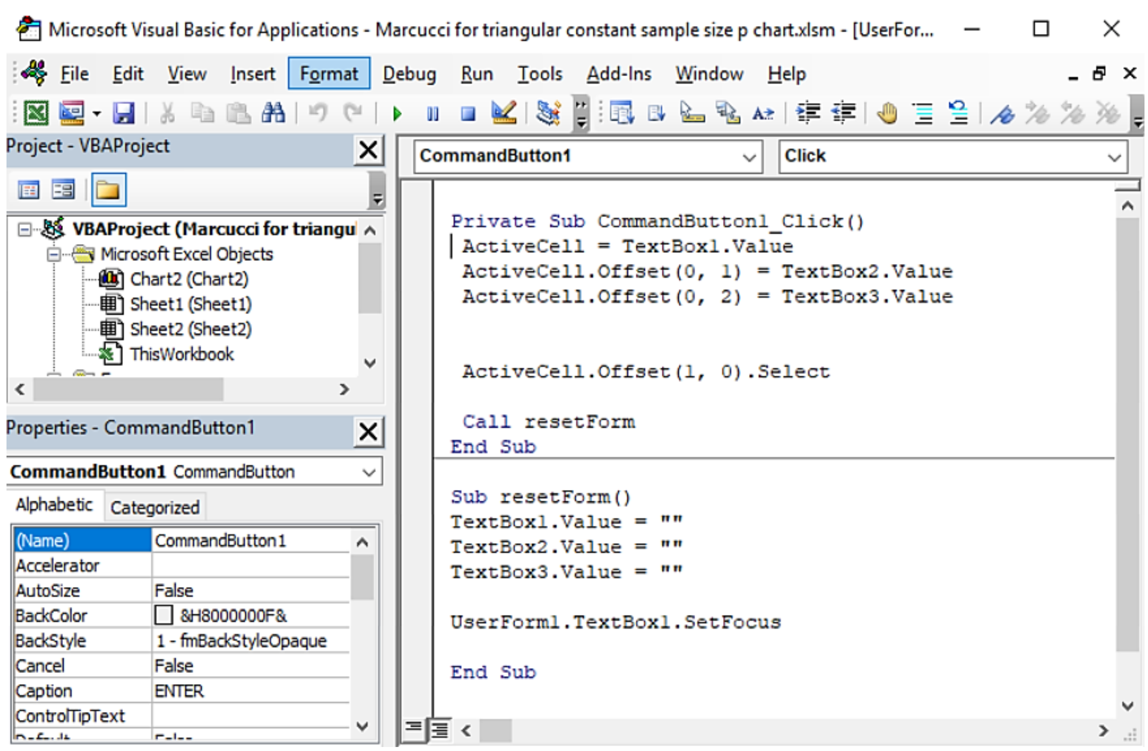


Figure 9. Visual Basic Coding Against the Enter Command Button.

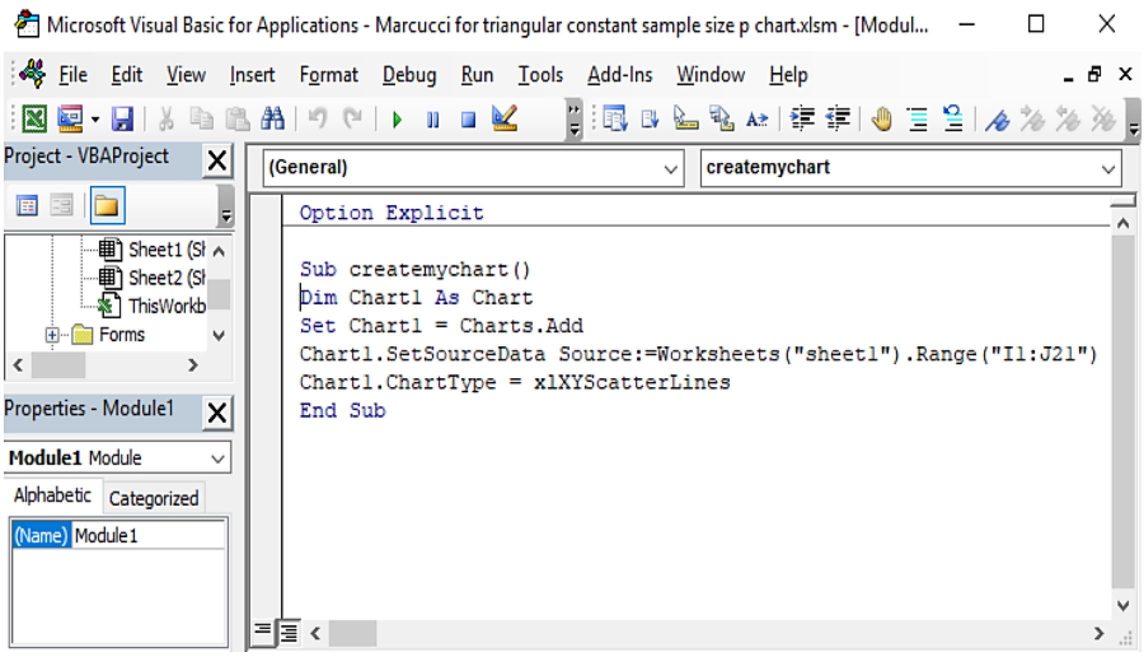


Figure 10. Visual Basic Coding for Creating P-Chart Automatically.

Table 3. Fuzzy P-Control Chart Based on Variable Sample Size TFN Case.

Sample no	Sample Size	Grade 1 (Accepted)	Grade2 (Rejected)	Grade 3 (Rejected)	P01	P02	P03	Zi^2	Upper Control Limit	Button
1	50	40	7	3	0.8	0.14	0.06	2.094117647	3.434444	
2	50	45	3	2	0.9	0.06	0.04	0	3.434444	Input Data
3	50	37	9	4	0.74	0.18	0.08	4.447154472	3.434444	
4	50	45	3	2	0.9	0.06	0.04	0	3.434444	
5	50	42	6	2	0.84	0.12	0.04	1.103448276	3.434444	
6	50	35	10	5	0.7	0.2	0.1	6.304945055	3.434444	
7	50	39	7	4	0.78	0.14	0.08	2.695238095	3.434444	Click here to chart
8	50	40	8	2	0.8	0.16	0.04	2.56684492	3.434444	
9	50	44	5	1	0.88	0.1	0.02	0.844569288	3.434444	
10	50	43	5	2	0.86	0.1	0.04	0.545454545	3.434444	
11	50	44	5	1	0.88	0.1	0.02	0.844569288	3.434444	
12	50	46	3	1	0.92	0.06	0.02	0.344322344	3.434444	
13	50	47	2	1	0.94	0.04	0.02	0.576811594	3.434444	
14	50	42	6	2	0.84	0.12	0.04	1.103448276	3.434444	
15	50	42	7	1	0.84	0.14	0.02	2.036781609	3.434444	
16	50	41	8	1	0.82	0.16	0.02	2.792107118	3.434444	
17	50	45	4	1	0.9	0.08	0.02	0.476190476	3.434444	
18	50	45	4	1	0.9	0.08	0.02	0.476190476	3.434444	
19	50	40	8	2	0.8	0.16	0.04	2.56684492	3.434444	
20	50	40	7	3	0.8	0.14	0.06	2.094117647	3.434444	

Sample no	Sample Size	Grade 1 (Accepted)	Grade2 (Rejected)	Grade 3 (Rejected)	P01	P02	P03	Zi^2	Upper Control Limit	Button
1	50	40	8	2	0.8	0.16	0.04	3.15	3.434444	
2	55	47	5	3	0.85455	0.091	0.055	0.79	3.434444	Click here to create chart
3	60	55	3	2	0.91667	0.05	0.033	0.02	3.434444	
4	55	50	3	2	0.90909	0.055	0.036	0	3.434444	
5	50	45	3	2	0.9	0.06	0.04	0.03	3.434444	
6	45	38	5	2	0.84444	0.111	0.044	1.15	3.434444	
7	34	26	5	3	0.76471	0.147	0.088	3.52	3.434444	
8	45	28	13	4	0.62222	0.289	0.089	12.2	3.434444	
9	65	45	15	5	0.69231	0.231	0.077	8.78	3.434444	
10	54	44	7	3	0.81481	0.13	0.056	2.17	3.434444	
11	52	48	3	1	0.92308	0.058	0.019	0.29	3.434444	
12	44	37	5	2	0.84091	0.114	0.045	1.24	3.434444	
13	47	39	5	3	0.82979	0.106	0.064	1.44	3.434444	
14	54	50	3	1	0.92593	0.056	0.019	0.32	3.434444	
15	55	51	3	1	0.92727	0.055	0.018	0.34	3.434444	
16	48	43	3	2	0.89583	0.063	0.042	0.05	3.434444	
17	49	31	15	3	0.63265	0.306	0.061	12.4	3.434444	
18	50	29	15	6	0.58	0.3	0.12	15.4	3.434444	
19	56	46	8	2	0.82143	0.143	0.036	2.43	3.434444	
20	55	45	4	1	0.81818	0.073	0.018	0.74	3.434444	

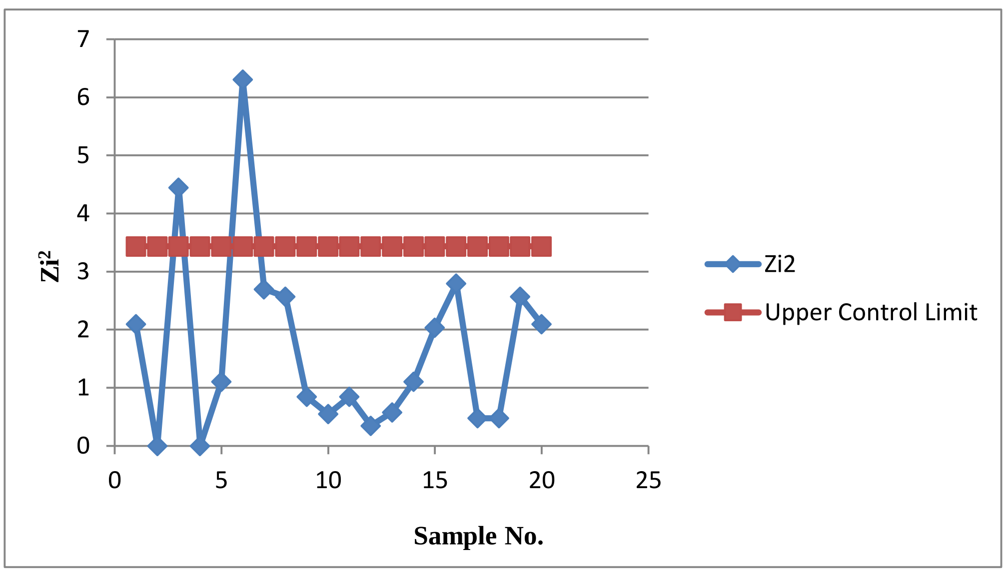


Figure 11. Generalized P-Chart from Marcucci Approach for Constant Sample Size.

5.3 Formation of Fuzzy P-Control Chart Based on a Variable Sample Size for a TFN Case

All the algorithms have been solved the fuzzy p-control chart based on a variable sample size for a TFN case and the control chart has been drawn in the following as it was done in the constant sample size case.

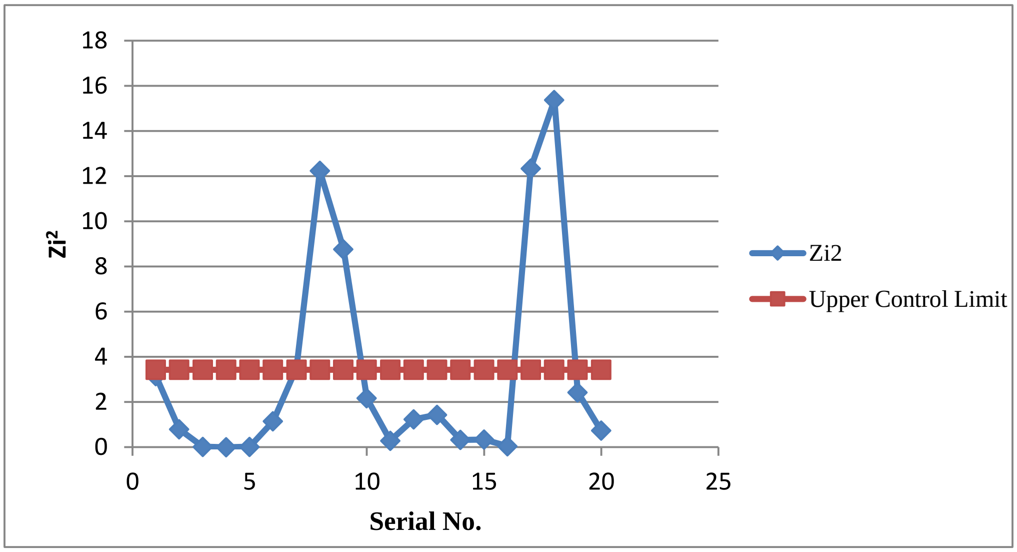


Figure 12. Generalized P Chart for Variable Sample Size in Marcucci Approach.

6. Results and Discussion

6.1 Fuzzy P-Chart Based on Constant Sample Size for TFN Case and Marcucci Approach Test

20 samples have been taken from the 20 production lines of the industry. The sample size is 50 for each of the sample. The number of non-conforming has been found 5-15 in those samples. After formulating all those data, it has been found out that the 3rd and 6th sample give the 'rather in control' and 'out of control' decision where all the other samples give the 'in control' decision. Again, the samples have been analyzed in the Marcucci approach and have found out the generalized p control chart. The generalized p control chart graph also shows that the 3rd and 6th sample graph just goes out of the control limit. That makes the decision that the fuzzy p chart based on constant sample size for TFN is reliable as it has given nearly same result with the goodness-of-fit test in Marcucci approach.

Table 4. Marcucci Approch for Variable Sample Size P-Chart.

Sample No	Sample Size	Number of non-conforming units	a	b	c	Pa	Pb	Pc	Process Condition	Button
1	50	10	6	8	10	0.12	0.16	0.2	in control	
2	55	8	5	6	8	0.090909	0.10909	0.1455	in control	Input Data
3	60	5	3	4	5	0.05	0.06667	0.0833	in control	
4	54	5	3	4	5	0.055556	0.07407	0.0926	in control	
5	50	5	3	4	5	0.06	0.08	0.1	in control	
6	45	7	5	6	7	0.111111	0.13333	0.1556	in control	Program
7	34	8	6	7	8	0.176471	0.20588	0.2353	in control	
8	45	17	13	15	17	0.288889	0.33333	0.3778	rather out of control	
9	65	20	16	18	20	0.246154	0.27692	0.3077	rather out of control	
10	54	10	8	9	10	0.148148	0.16667	0.1852	in control	
11	52	4	2	3	4	0.038462	0.05769	0.0769	rather out of control	
12	44	7	5	6	7	0.113636	0.13636	0.1591	in control	
13	47	8	5	6	8	0.106383	0.12766	0.1702	in control	
14	54	4	1	3	4	0.018519	0.05556	0.0741	rather out of control	
15	55	4	2	3	4	0.036364	0.05455	0.0727	rather out of control	
16	48	5	3	4	5	0.0625	0.08333	0.1042	in control	
17	49	18	14	16	18	0.285714	0.32653	0.3673	rather out of control	
18	50	21	19	20	21	0.38	0.4	0.42	out of control	
19	56	10	6	8	10	0.107143	0.14286	0.1786	in control	
20	55	5	3	4	5	0.054545	0.07273	0.0909	in control	

$\sum n$	UCLpa	UCLpb	UCLpc	$\sum Pa$	$\sum Pb$	$\sum Pc$
1022	0.267511	0	0.341	2.550503	3.06324	3.5969
$\bar{n}$	Clpa	CLpb	CLpc	$\bar{P}a$	$\bar{P}b$	$\bar{P}c$
51.1	0.127525	0	0.18	0.127525	0.15316	0.1798
	LCLpa	LCLpb	LCLpc			
	-0.03365	0	0.044			

6.2 Fuzzy P-Chart Based on Variable Sample Size for TFN Case and Marcucci Approach Test

Here, the sample sizes are variable for this reason that the approximate sample size is used here. The sample sizes vary from 34 to 60 and the number of non-conforming from 5 to 21. The unexpected results come from the 8, 9, 11, 14, 15, 17, and 18th sample number. So, the process state is not so stable and the indicated sample no. gives 'rather out of control' or 'in control' decision. Marcucci approach implementation graph peaks on that points that are the main concern. As the peaks testify the validity of the fuzzy approach, so the process seems to be out of control.

7. Conclusion

Fuzzy control charts based on TFN deals with the huge number of data. The main problem that can be faced from the fuzzy control charts is the control limits shift and that can be understood from the unusual decision from the small value non-conforming numbers. On that cases, the small non-conforming units must give 'in control' decisions but they would give 'rather in control' or 'rather out of control' decisions; because the control limits have been shifted and the second condition of 'in control' decision algorithms does not satisfy the sample. The LCL_d\ LCL_c value becomes larger than P_a value. But to give 'in control' decision, the P_a value must have been larger than the LCL_d\ LCL_c value. That happens only in the cases when the standard deviations among the samples are very high. So, during the analysis stage of the fuzzy control chart, the unexpected decisions and their non-conforming units should be checked properly. Actually, the random variation can easily be understood and detected from the fuzzy chart. In the 20 samples, two or three 'out of control' decisions were acceptable but the trend of increasing of that decisions was not acceptable in the production process. The fuzzy control chart was very sensitive so shat the trend and the unusual variation can easily be detected. The Marcucci approach gived the generalized chart by comparing the other samples from the base samples. Therefore, the base sample selection was a prime factor in the graph and decision making. The base sample was an average non-conforming value contained sample in the production. Experience and intuition were integrated on the formation and decision about the fuzzy control chart.

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