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Fixed Point Theorems of LW-Type Lipschitz Cyclic Mappings in Complete B-Metric-Like Spaces

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A B S T R A C T

The research of Fixed Point Theorems (FPTs) plays an important role in nonlinear analysis. It is of great theoretical significance and profound application value to study general abstract theory in a B-Metric-Like Space (BMLS). So, it is an important topic to study the Fixed Point Theory (FPT) in a BMLS. In this paper, for the purpose of introducing some new FPTs, a LW-type Lipschitz Cyclic Mapping (LTLCM) is first proposed by us in a complete BMLS, and we proved the existence theorem of fixed points of LW-type Lipschitz Cyclic Mappings (LTLCMs) in complete BLMS. Next, we also construct an example in a discrete complete BMLS to show and illustrate the effectiveness of LTLCM. In the end, on the basis of the example, we show the calculation of the range of the parameter s that appears in B-Metric-Like Spaces (BMLSs). Our main theorems extend and develop existing results in the recent literature.

Keywords: LW-Type Lipschitz Cyclic Mapping (LTLCM), B-Metric-Like Space (BMLS), Fixed Point Theorem (FPT), Nonlinear analysis.

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1. Introduction

The FPT is an interesting research direction, it has extremely strong vitality. Many mathematical researchers have never stopped research in FPT. In 1922, Banach [1] proved the existence theorem of fixed point of a contractive mapping in metric spaces that is Banach contraction principle; its expression is as follows. Let (X, d) be a metric space and let $T: X \to X$ be a mapping. Then T is called a Banach contraction mapping if there exists $k \in [0,1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$. Then, many mathematicians found many mappings and they proved the

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existence theorems of fixed points of these mappings. Let A and B be nonempty subsets of a metric space (X,d) and $T: A \cup B \to A \cup B$ be a cyclic mapping. Then T is called a cyclic map if $T(A) \subseteq B$ and $T(B) \subseteq A$. In 2003, Kirk et al. [2] introduced cyclic contraction as follows. A cyclic mapping $T: A \cup B \to A \cup B$ is said to be a cyclic contraction if there exists $a \in [0,1)$ such that $d(Tx, Ty) \leq ad(x,y)$ for all $x \in A$ and $y \in B$.

From here, we are going to break the narrative of the development of mapping, move into some new concepts of spaces, and discuss the development of spaces. There exist many generalizations of the concept of metric spaces in the literature. Matthews [3] introduced the notion of partial metric space and proved that the Banach contraction mapping theorem can be generalized to the partial metric context for applications in program verification. In 2003, Hitzler and Seda [4] introduced dislocated metric spaces as a generalization of metric spaces; Zeyada et al. [5] generalized the result of Hitzler and Seda, Wilson, and introduced the concept of dislocated quasi-metric space. The concept of b-metric space was introduced and studied by Bakhtin [6] and Czerwik [7]. In 2012, Amini-Harandi [8, 9] introduced the notion of metric-like space, which is an interesting generalization of partial metric space and dislocated metric space. In 2013, Alghamdi et al. [10] introduced the concept of b-metric-like space, which is a new generalization of metric-like space and partial metric space. Here, we are going to introduce two specific Spaces that is metric-like space and partial metric space, respectively. Therefore, we will focus on the development of relevant theories in some generalized metric spaces.

Definition 1.1 A mapping $p: X \times X \to R_+$, where X is a nonempty set, is said to be a partial metric on X if for any $x, y, z \in X$ the following four conditions hold true: (i) $x = y$ if and only if $p(x,x) = p(y,y) = p(x,y)$; (ii) $p(x,x) \leq p(x,y)$; (iii) $p(x,y) = p(y,x)$; (iv) $p(x,z) \leq p(x,y) + p(y,z) - p(y,y)$. The pair (X,p) is then called a partial metric space.

Definition 1.2 A mapping $\psi: X \times X \to R_+$, where X is a nonempty set, is said to be a metric-like on X if for any $x, y, z \in X$ the following three conditions hold true: (i) $\psi(x,y) = 0 \Rightarrow x = y$; (ii) $\psi(x,y) = \psi(y,x)$; (iii) $\psi(x,z) \leq \psi(x,y) + \psi(y,z)$. The pair (X,ψ) is then called a metric-like space. A metric-like on X satisfies all the conditions of a metric except that $\psi(x,x)$ may be positive for $x \in X$. In fact, BMLS can be seen to produce from Definition 1.1 and Definition 1.2. Recently, many research results involving FPT are obtained by lots of researchers.

In 2015, Klin-eam and Suanoom [11] introduced the notion of dislocated quasi-b-metric spaces, and they proposed the concept of dqb-cyclic-Banach contraction and dqb-cyclic-Kannan mapping, and obtained some FPTs in dislocated quasi-b-metric spaces. At the same year, Van An et al. [12] summarized some generalized metric space of kinds of metric space and studied FPTs involving these generalized metric spaces. In 2016, Aydi et al. [13] established some best proximity results for Kannan-Chatterjea-Ciri´c type contractions in the setting of metric-like spaces. They also provided some concrete examples illustrating the obtained results. At the same year, Wu and Wu [14] studied FPTs of cyclic mappings in complete dislocated quasi-b-metric space. Next, Alsulami [15] defined a class of general type α-admissible contraction mappings on quasi-*b*-metric-like spaces and discussed the existence and uniqueness of fixed points for this class of mappings and applied these results to Ulam stability problems. At the same year, Fan [16] introduced the concepts of *qpb*-cyclic-Banach contraction mapping, *qpb*-cyclic-Kannan mapping, and *qpb*-cyclic β-quasi- contraction mapping; they established the existence and uniqueness of FPTs for these mappings in quasi-partial b-metric spaces.

In 2017, Zoto [17] introduced a new class of α_(qs^p-) admissible mappings and provide some FTPs involving this class of mappings satisfying some new conditions of contractivity in the setting of BMLSs. At the same year, Nashine [18] introduced the notion of rational (α-β-FG)-contraction mapping in BLMSs and produced relevant fixed point and periodic point results for weakly α-admissible mappings. Ulam-Hyers stability of this problem was also investigated. In 2018, Aydi and Czerwik [19] introduced some FPTs involving the mapping of linear quasi-contractions and nonlinear contractions in quasi-b-metric-like spaces. At the same year, Afshari et al. [20] introduced the concept of generalized α-Ψ-Suzuki-contractions in the content of quasi-b-metric-like spaces and they established some related FPTs. Some concrete examples were also provided illustrating the obtained results. Zoto et al. [21] introduced the notions of (s,p, α)-quasi-contractions and (s, p)-weak contractions and deduced some fixed point results concerning such contractions in the setting of BMLS. Their results extended and generalized some recent known results in literatures to more general metric spaces. In this paper, motivated and inspired by the above work, in order to further advance, the theoretical research work in BMLSs show its value and develop FTP; we define a new cyclic mapping and discuss the existence and uniqueness of fixed points of it in BMLSs. In addition, we give a real example to show the effectiveness of the new mapping.

2. Preliminaries

In this section, we collect some definitions, which will be used in next section.

Definition 2.1 [10] A b-metric-like on a nonempty set X is a function $r: X \times X \to [0, \infty)$ such that for all $z_1, z_2, z \in X$ and a constant $s \geq 1$ the following three conditions hold true: (i) $r(z_1, z_2) = 0 \Rightarrow z_1 = z_2$; (ii) $r(z_1, z_2) = r(z_2, z_1)$; (iii) $r(z_1, z_2) \leq s\big(r(z_1, z) + r(z, z_2)\big); \forall z_1, z_2, z \in X$.

The pair (X, r) is called a b-metric-like space.

Definition 2.2 [10] Let (X, r) be a b-metric-like space and let $\{z_n\}$ be a sequence of points of X. A point $z \in X$ is said to be the limit of the sequence $\{z_n\}$ if $r(z, z) = \lim_{n\to\infty} r(z, z_n)$; we say that the sequence $\{z_n\}$ is convergent to z and denote it by $z_n \to z (n \to \infty)$.

Definition 2.3 [10] Let (X, r) be a b-metric-like space. A sequence $\{z_n\}$ is called Cauchy if and only if $\lim_{m,n\to\infty} r(z_m, z_n)$ exists and is finite.

Definition 2.4 [10] A b-metric-like space (X, r) is said to be complete if and only if every Cauchy sequence $\{z_n\}$ in X converges to $z \in X$ so that

$$\lim_{m,n\to\infty} r(z_n, z_m) = r(z, z) = \lim_{n\to\infty} r(z_n, z).$$

Definition 2.5 [22] Let G_1, G_2 be nonempty sets of metric space, if $B(G_1) \subset G_2$, and $S(G_2) \subset G_1$, then the mapping $(B, S): G_1 \times G_2 \to G_2 \times G_1$ is called as a pair semi-cyclic mapping, where B is said to be a lower semi-cyclic mapping, S is said to be a upper semi-cyclic mapping. If $B = S$, then B is said to be a cyclic mapping.

Definition 2.6 [22] If $\prec$ is a partially ordered in b-metric-like spaces (X, r), then $(X, r, \prec)$ is a partially ordered b-metric-like space.

In 2017, Lei and Wu [23] proposed a definition, which is for LW-type cyclic mapping in a complete b-metric-like space.

Definition 2.7 Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real constants γ, δ, t such that for all $u \in G_1, v \in G_2$ satisfy the following condition:

$$\gamma r(u, Bu) + \delta r(v, Sv) + t r(Bu, Sv) \leq r(u, v).$$

Then, (B, S) is called as a LW-type cyclic mapping.

Next, according to Definition 2.7 of LW-type cyclic mappings, we propose a new definition as the LTLCM in a complete BMLS and we give the proof for the existence theorems of fixed points of LTLCM in complete BMLSs.

3. Main results

In this section, we give a definition for LTLCM and prove the existence theorems of fixed points of LTLCM in complete BLMSs.

Definition 3.1 Let G_1, G_2 be nonempty closed sets in (X, r). If (B, S) is a pair semi-cyclic mapping in $G_1 \times G_2$ and exists some nonnegative real constants γ, δ, t, L and $L < \gamma + \delta + t$ such that for all $x \in G_1, y \in G_2$ satisfy the following condition:

$$\gamma r(x, Bx) + \delta r(y, Sy) + t r(Bx, Sy) \leq L r(x, y).$$

Then, (B, S) is called as a LW-type Lipschitz cyclic mapping.

It is easy to know that LTLCM is more general than LW-type cyclic mapping. In fact, it is obvious that a LTLCM is a LW- type cyclic mapping when $L = 1$.

Theorem 3.2 Let (X, r) be a complete b-metric-like space, and (B, S) be a LW-type Lipschitz cyclic mapping in X. Suppose that G_1, G_2 are nonempty closed sets in metric space X and $G_1 \cap$

$G_2 \neq \emptyset$, if $L \leq t$, $L = \max\{\gamma, \delta\}$, $\gamma \neq \delta$, and $s \in [1, \frac{L+t}{|\gamma-\delta|})$. Then there exists a unique $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Proof. Define the sequence $\{z_n\}$ as follows:

$z_0 \in G_1$, $z_1 = Bz_0$, $z_2 = Sz_1$, $z_3 = Bz_2$, and $z_4 = Sz_3, \ldots, n \geq 0$.

Step 1. Prove that the sequence $\{z_n\}$ is a cauchy sequence. Because (B, S) be a LW-type Lipschitz cyclic mapping, thus we have

$$\begin{aligned} Lr(z_0, z_1) &\geq \gamma r(z_0, Bz_0) + \delta r(z_1, Sz_1) + tr(Bz_0, Sz_1) \\ &= \gamma r(z_0, z_1) + \delta r(z_1, z_2) + tr(z_1, z_2) \\ &= \gamma r(z_0, z_1) + (\delta + t) tr(z_1, z_2), \end{aligned}$$

so we obtain

$$r(z_1, z_2) \leq \frac{L-\gamma}{\delta + t}\, r(z_0, z_1). \tag{1}$$

Summary it and we get

$$\begin{aligned} Lr(z_2, z_1) &\geq \gamma r(z_2, Bz_2) + \delta r(z_1, Sz_1) + tr(Bz_2, Sz_1) \\ &= \gamma r(z_2, z_3) + \delta r(z_1, z_2) + tr(z_3, z_2) \\ &= \delta r(z_1, z_2) + (\gamma + t) r(z_2, z_3) \end{aligned}$$

and we have

$$r(z_2, z_3) \leq \frac{L-\delta}{\gamma + t}\, r(z_1, z_2). \tag{2}$$

Let $P = \max\{\frac{L-\delta}{\gamma+t}, \frac{L-\gamma}{\delta+t}\}$ and $Q = r(z_0, z_1)$. The above, in combination with Eqs. (1) and (2), it shows that

$$r(z_2, z_3) \leq Pr(z_1, z_2) \leq P^2 r(z_0, z_1) = P^2 Q.$$

Repeating the above work, we have

$$r(z_n, z_{n+1}) \leq P^n Q, \qquad \forall n \in N.$$

Since $L < \gamma + \delta + t, L = \max\{\gamma, \delta\}$, so we have $0 < P < 1$. Because of $s \in [1, \frac{L+t}{|\gamma-\delta|})$, then $0 < sP < 1$. Let $m, n \in N$ and $n < m$, this shows

$$\begin{aligned} r(z_n, z_m) &\leq s\big(r(z_n, z_{n+1}) + r(z_{n+1}, z_m)\big) \\ &= sr(z_n, z_{n+1}) + sr(z_{n+1}, z_m) \\ &\leq sr(z_n, z_{n+1}) + s^2 r(z_{n+1}, z_{n+2}) + \cdots + s^{m-n} r(z_{m-1}, z_m) \end{aligned}$$

$$\leq sP^n r(z_0, z_1) + s^2 P^{n+1} r(z_0, z_1) + \cdots + s^{m-n} P^{m-1} r(z_0, z_1) \tag{3}$$

$$= [sP^n + s^2 P^{n+1} + \cdots + s^{m-n} P^{m-1}] r(z_0, z_1)$$

$$= [(sP) + (sP)^2 + \cdots + (sP)^{m-n}] P^{n-1} Q$$

$$= \frac{1-(sP)^{m-n}}{1-(sP)} sP^n Q$$

$$\leq \frac{1}{1-(sP)} P^{n-1} Q.$$

From Eq. (3), let $n \to \infty$, we get that

$$\lim_{n\to\infty} r(z_n, z_m) = 0. \tag{4}$$

This implies from (4) that the sequence $\{z_n\}$ is a Cauchy sequence. Due to, (X, r) is a complete b-metric-like space, then there exists an element $z \in X$ such that

$$z_n \to z \ (n \to \infty).$$

Therefore

$$z_{2n} \to z; \ \ z_{2n+1} \to z \ (n \to \infty).$$

Because $\{z_{2n}\} \subseteq G_1$, $\{z_{2n+1}\} \subseteq G_2$ and G_1, G_2 is closed, then

$$z \in G_1 \cap G_2.$$

Step 2. Prove that z is a fixed point of the mapping B and S, that is, $Bz = z = Sz$. Because (B, S) be a LW-type Lipschitz cyclic mapping, then we get that

$$\gamma r(z, Bz) + \delta r(z, Sz) + t r(Bz, Sz) \leq L r(z, z).$$

Due to $r(z, z) = \lim_{m,n\to\infty} r(z_n, z_m)$, then by (4) we have

$$r(z, Bz) = r(z, Sz) = 0.$$

It implies that

$$Bz = z = Sz.$$

Step 3. Prove that the mappings B and S have a unique common fixed point. Now, let $z, z^* \in X$ are the common fixed points of mappings B and S in X. Then, by Definition 3.1, we have

$$Lr(z, z^*) \geq \gamma r(z, Bz) + \delta r(z^*, Sz^*) + t r(Bz, Sz^*).$$

That is,

$$Lr(z, z^*) \geq \gamma r(z, z) + \delta r(z^*, z^*) + t r(z, z^*).$$

Due to $L \leq t$, then we obtain that

$$r(z, z^*) = 0, that\ is, z = z^*.$$

This complete the proof.

Corollary 3.3 Let (X, r) be a complete b-metric-like space and (B, S) be a LW-type Lipschitz cyclic mapping in X. Suppose that G_1, G_2 are nonempty closed sets in metric space X, $G_1 \cap G_2 \neq \emptyset$, if $L \leq t$, $L = \max\{\gamma, \delta\}$, $\gamma = \delta$, and $s \in [1, \infty)$, then there exists a unique point $x^* \in G_1 \cap G_2$ such that $Bx^* = x^* = Sx^*$, that is, B and S have a unique common fixed point.

Proof. This proof is trivial and it is easy to obtain from Theorem 3.2.

Corollary 3.4 Let $(X, r, \prec)$ is a complete partially ordered b-metric-like space; (B, S) is a LW-type Lipschitz cyclic mapping in X. G_1, G_2 are nonempty closed sets in metric space X and $G_1 \cap G_2 \neq \emptyset$. If $L \leq t$ and $L = \max\{\gamma, \delta\}$, then there exists a unique point $z \in G_1 \cap G_2$ such that $Bz = z = Sz$, that is, B and S have a unique common fixed point.

Proof. First of all, define the partially ordered as $u \prec Bu \Leftrightarrow u \in G_1$ or $v \prec Sv \Leftrightarrow v \in G_2$. Then, the next proof follows Theorem 3. 2. This completes the proof.

4. An Example

In this section, we will give an example to show the validity of the LTLCM.

Let $X = \{0,1,2\}$, $G_1 = \{0,1\}$, $G_2 = \{1,2\}$, and $r: X \times X \to [0, \infty)$ is a function, defined as follows:

$r(0,0) = r(1,1) = 0,\quad r(2,2) = 1,\quad r(0,1) = r(1,0) = 4,\quad r(0,2) = r(2,0) = 2.5,$ and $r(1,2) = r(2,1) = 1.$

It is easy to verify that (X, r) is a complete b-metric-like space and s = 2. In fact, according to Definition 2.1, we go to verify the condition (iii) of Definition 2.1. Since

$$4 = r(0,1) \leq s\big(r(0,2) + r(2,1)\big) = s(2.1 + 1).$$

This shows that s $\geq \frac{8}{7}$ is fine.

Now, define the mapping B and S in the following manner:

$$B(0) = 2, B(1) = 1, S(1) = 1, S(2) = 1.$$

This implies that

$$B(G_1) \subseteq G_2, S(G_2) \subseteq G_1.$$

Thus, (B, S) is a pair semi-cyclic mapping. If we choose $0 \in G_1$, $1 \in G_2$ then we have

$$Lr(0,1) \geq \gamma r\big(0, B(0)\big) + \delta r\big(1, S(1)\big) + tr\big(B(0), S(1)\big),$$

it implies that

$$\gamma r(0,2) + tr(2,1) \leq Lr(0,1),$$

that is,

$$2.5\gamma + t \leq 4L.$$

If we choose $0 \in G_1$, $2 \in G_2$, then we have

$$Lr(0,2) \geq \gamma r\big(0, B(0)\big) + \delta r\big(2, S(2)\big) + tr\big(B(0), S(2)\big),$$

it implies that

$$Lr(0,2) \geq \gamma r(0,2) + \delta r(2,1) + tr(2,1).$$

That is,

$$L \geq \gamma + \frac{2}{5}\delta + \frac{2}{5}t.$$

If we choose $1 \in G_1$ and $1 \in G_2$ then we have

$$Lr(1,1) \geq \gamma r\big(1, B(1)\big) + \delta r\big(1, S(1)\big) + tr\big(B(1), S(1)\big),$$

it implies that

$$0 = Lr(1,1) \geq \gamma r(1,1) + \delta r(1,1) + tr(1,1) = 0.$$

If we choose $1 \in G_1$ and $2 \in G_2$ then we have

$$Lr(1,2) \geq \gamma r\big(1, B(1)\big) + \delta r\big(2, S(2)\big) + tr\big(B(1), S(2)\big),$$

it implies that

$$Lr(1,2) \geq \gamma r(1,1) + \delta r(2,1) + tr(1,1).$$

That is,

$$L \geq \delta.$$

Because of the conditions of a LW-type Lipschitz cyclic mapping, let us summarize the above contents; we obtain the following (remark it as (4.1)):

$$\begin{aligned} &L = \max\{\gamma, \delta\}, \\ &L < \gamma + \delta + t\ , \\ &L \geq \frac{5}{8}\gamma + \frac{1}{4}t\ , \\ &L \geq \gamma + \frac{2}{5}\delta + \frac{2}{5}t, \\ &L \geq \delta, \\ &L \leq t. \end{aligned}$$

If we choose that

$$\gamma = 0.1, \delta = 1, t = 1.1, L = 1,$$

they satisfy the condition (4.1) and because of $s < \frac{L+t}{|\gamma-\delta|}$, so $s < 2.33$. Thus, the cyclic mapping B and S have a unique common fixed point. Indeed, since $L = 1$ and (B, S) is a LW-type cyclic mapping here.

In addition, if we choose that

$$\gamma = 0.3, \delta = 1.6, t = 1.7, L = 1.6,$$

they also satisfy the condition (4.1); in the same way, $s < 2.33$. Therefore, the cyclic mapping B and S have a unique common fixed point.

5. The Calculation for the Range of *s*

In this section, we show the calculation of the range of *s* for the purpose to obtain the effectiveness of the mapping.

In order to satisfy the condition $sP < 1$, we need to discuss the range of s. Since $P = \max\left\{\frac{L-\gamma}{\delta+t}, \frac{L-\delta}{\gamma+t}\right\}$, if $P = \frac{L-\delta}{\gamma+t}$, this shows that

$$\frac{L-\delta}{\gamma+t} \geq \frac{L-\gamma}{\delta+t}.$$

In fact, we have

$$(L-\delta)(\delta+t) > (L-\gamma)(\gamma+t)$$

$$\Leftrightarrow L(\delta+t) - \delta^2 - \delta t > L(\gamma+t) - \gamma^2 - \gamma t$$

$$\Leftrightarrow L(\delta-\gamma) > \delta^2 - \gamma^2 + \delta t - \gamma t$$

$$\Leftrightarrow L(\delta-\gamma) > (\delta+t)(\delta-\gamma) + t(\delta-\gamma) = (\delta+\gamma+t)(\delta-\gamma).$$

Since $L < \gamma + \delta + t$, then $\delta - \gamma < 0$, that is, $\delta < \gamma$. Since $L = \max\{\gamma, \delta\}$, thus $L = \gamma$, then

$$P = \frac{\gamma-\delta}{\gamma+t}.$$

Hence,

$$s < \frac{1}{P} = \frac{\gamma+t}{\gamma-\delta}.$$

In the same way, if $P = \frac{L-\gamma}{\delta+t}$, we also obtain

$$s < \frac{1}{P} = \frac{\delta+t}{\delta-\gamma}.$$

From the above, this shows

$$s < \frac{L+t}{|\gamma-\delta|}.$$

In addition, since $s \geq 1$, so we obtain

$$1 \leq s < \frac{L+t}{|\gamma-\delta|}.$$

6. Conclusion

In this article, we first proposed a new mapping in a BMLS, called it as LW-type Lipschitz Cyclic Mapping (LTLCM). We proved the existence and uniqueness theorem of fixed points of the LTLCM. In the end, we gave a concrete real example to show the effectiveness of a LTLCM that was defined by us in BMLSs and we showed the calculation process of the range of s involving the Definition 2.1 for BMLS. All the above work supported the effectiveness of a LTLCM and showed the LTLCM was more general than the LW-type cyclic mapping defined in reference [23]. Through this study, we advanced the work of researching FPT in BMLSs; this is beneficial to complete the structure of BMLS.

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