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# A New Method for Solving Fully Fuzzy Multi Objective Supplier Selection Problem

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**A B S T R A C T**

Supplier selection is one of the most critical activities of purchasing management in a supply chain. Because selecting right suppliers helps reduce purchasing costs, improve quality of final products and services, etc. In a real situation, for a supplier selection problem, most of the input information is not known precisely, since decision making deal with human judgment and comprehension and its nature includes ambiguity. In fact, on the one hand, deterministic models cannot easily take this vagueness into account. In these cases, the theory of fuzzy sets is one of the best tools to handle uncertainty. On the other hand, Kumar et al. proposed a new approach to find the fuzzy optimal solution of fully fuzzy linear programming problem. So, using this approach in this paper, we present a new mixed integer multi objective linear programming model for supplier selection problem. Due to uncertainty of the data, in continuation, we present a new method to solve multi objective fully fuzzy mixed integer linear programming and implement the method to supplier selection problem. Computational results present the application of the method and the proposal solving method.

**Keywords**: Supplier selection, fuzzy mcdm, fully fuzzy mixed integer linear programming problem, quantity discount, weighted additive, supply chain.

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## 1. Introduction

In 60th decay (1960−1970), the companies were forced to improve their market strategies, which is based on manufacturing, acquisition and customer retention. They also should get involved in network management of all companies which prepare input (whether directly or indirectly) before them and those who perform delivering and services after them. Hence, the term “supply chain” appeared. A supply chain consists of all stages that are directly or indirectly involved in fulfilling customer demand. Supply chain not only includes the manufacturer and suppliers, but also the transportation parts, depots, retailers and even customers themselves. The

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supply chain management involves integration of supply chain activities through improvement of chain relations to achieve competition privilege. In the context of supply chain management, the supplier selection decision plays a key role. Since usually in most industries, the primary cost of producing is the cost of buying raw materials and components, however in some cases, it is 70% of producing costs. Thus, the purchasing section can play a key role in reducing costs by selecting a proper supplier. In today's globally competitive environment, firms give great attention for selecting right suppliers because it helps reduce purchasing costs, improve quality of final products and services, etc. Supplier selection problem is a multi-criteria decision making problem, which includes both qualitative and quantitative factors like unit cost, delivery on time, service quality, etc. In the problem, many criteria may conflict with each other, so the selection process becomes complicated and it contains two major problems: (i) which supplier (s) should be chosen? And (ii) how much should be purchased from each selected supplier [4]? In the last several years, supplier selection problem has gained great importance and is handled by academic researchers and also practitioners in business environment. The literature on this problem exist some researches (i) focused on supplier selection problem criteria, and (ii) proposed methods for supplier selection process. Several methods have been appeared in literature for supplier selection problem (see in [22-26]). In real situation, for supplier selection problems, the weights of criteria are different and depend on purchasing strategies in a supply chain [26]. It is a common practice for suppliers to offer quantity discounts to encourage the buyer towards larger order. In this case, the buyer must decide what order quantities to assign to each supplier. In a real situation, for a supplier selection problem, most of the input information is not known precisely, since the decision making deal with human judgment and comprehension and its nature includes ambiguity. Deterministic models cannot easily take this vagueness into account. In these cases, the theory of fuzzy sets is one of the best tools to handle uncertainty. Fuzzy set theories are employed due to the presence of vagueness and imprecision of information in the supplier selection problem [1]. In fuzzy programming, the DM is no longer forced to formulate the problem in precise and rigid form. Based on fuzzy logic approaches, Kumar et al. [14] proposed fuzzy goal programming for supplier selection problems with multiple sourcing that include three primary goals: minimizing the net cost, minimizing the net rejections and minimizing the net late deliveries subject to realistic constraints regarding buyers demand and vendors capacity. In their proposed model, Zimmermann's weightless technique is used, in which there is no difference between objective functions. Also, Kumar et al. [13] proposed a new method, named as Mehars method, for solving the same type of fuzzy linear programming problems and it is shown that it is easy to apply the Mehar method as compared to the existing method for solving the same type of fuzzy linear programming problems. Ersa Aytac et al. [9] proposed an alternative version of the fuzzy PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations) method. Differently from other studies, preference functions used in PROMETHEE method are handled in terms of fuzzy distances between alternatives with respect to each criterion. Atakan Yucel et al. [2] developed a new weighted additive fuzzy programming approach to handle ambiguity and fuzziness in supplier selection problem effectively. In which firstly, linguistic values expressed as trapezoidal fuzzy numbers are used to assess the weights of the factors. By applying the distances of each factor between Fuzzy Positive Ideal Rating and Fuzzy Negative Ideal Rating, weights are obtained. Then applying suppliers' constraints, goals and weights of the factors, a fuzzy multi-objective linear model is developed to overcome the selection problem and assign optimum order quantities to each supplier. Manuel Diaz-Madronero et al. [18] developed the Vendor Selection (VS) problem with fuzzy goals. In which an interactive method is developed for solving multi-objective VS problems where fuzzy data are represented by using S-curve membership functions. The proposed method attempts simultaneously to minimize the total order costs, the number of rejected items and the number of late delivered items with reference to several constraints such as meeting buyers' demand, vendors' capacity, vendors' quota flexibility, vendors' allocated budget, etc. In this paper, a fully fuzzy multi objective model has been developed for the supplier selection problem under price breaks that depend on the sizes of order quantities. Through this model, purchase managers can assign different weights for numbers of criteria in order to manage flow of supply materials, components and finished products to improve quality, service and reduced cost, in order to make improvement in supply chain performance. This model can be used as a decision support system by the purchasing manager to decide what order quantities to place with each supplier in the case of multiple sourcing.

## 2. Preliminaries

### *2.1. Arithmetic on Fuzzy Numbers*

Here, we first give some fundamental concepts of fuzzy sets which are directly related to our discussion in the paper and which are taken from [7, 10, 19, 22, 23, 24]:

**Definition 1.** Let $X$ denotes a universal set. A fuzzy subset $\tilde{a}$ of $X$ is defined as a set of ordered pairs of element $x$ and grade $\mu_{\tilde{a}}(x)$ and is written:

$$\tilde{a} = \{(x, \mu_{\tilde{a}}(x));\ x \in X\} \tag{1}$$

where $\mu_{\tilde{A}}(x)$ is the membership function from $X$ to [0,1].

**Definition 2.** A fuzzy number is a fuzzy set $\tilde{a}$ on the real line $\mathbb{R}$ whose membership function $\mu_{\tilde{a}}$ is upper semi-continuous (we will suppose that it is continuous) and such that:

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, & \forall x \in (-\infty, a^L] \\ f_a(x), & \forall x \in [a^L, a^m] \\ 1, & \forall x \in [a^m, a^n] \\ g_a(x), & \forall x \in [a^n, a^R] \\ 0, & \forall x \in [a^R, +\infty) \end{cases} \tag{2}$$

Where the continuous functions $f_a(x)$ and $g_a(x)$ are respectively increasing and decreasing on their related intervals.

**Remark 3.** A fuzzy number is trapezoidal if $f_a$ and $g_a$ are linear functions. Then, we denote it by $\tilde{a} = (a^L, a^m, a^n, a^R)$. If $a^m = a^n$, the fuzzy number will be reduced to a triangular fuzzy number. In throughout the paper we focus on the triangular fuzzy numbers as is well known one and denote the set of all triangular fuzzy number by $F(\mathbb{R})$.

**Remark 4.** Consider any zero fuzzy number by $\tilde{0} = (0,0,0)$.

**Definition 5.** A ranking function is a function $\mathcal{R}: F(\mathbb{R}) \to \mathbb{R}$, which maps each fuzzy number into the real line, where a natural order exists. For example, if we use the linear ranking function entitled Yager for a triangular fuzzy number $\tilde{a} = (a^L, a^m, a^R)$ then $\mathcal{R}(\tilde{a}) = \frac{1}{4}(a^L + 2a^m + a^R)$ (for more detail see in [15-17]).

**Remark 6.** In this paper, all parameters and variables (except binary variables) are fuzzy numbers with the triangular possibility distribution as follows:

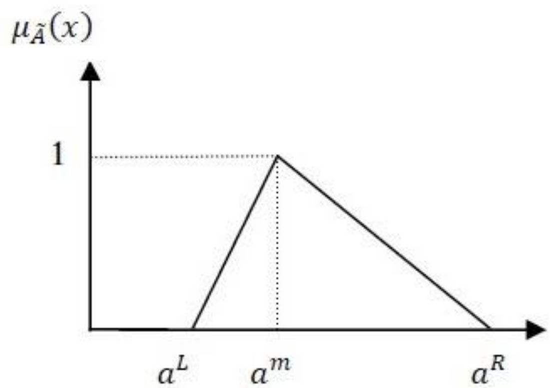


**Figure 1.** Triangular possibility distribution of fuzzy number $\tilde{A} = (a^L, a^m, a^R)$.

**Definition 7.** A triangular fuzzy number $(a^L, a^m, a^R)$ is said to be non-negative fuzzy number $iff\ a^L \geq 0$.We show any non-negative fuzzy number by $\tilde{a} \geq \tilde{0}$.

**Definition 8.** Let $\tilde{A} = (a^L, a^m, a^R)$ and $\tilde{B} = (b^L, b^m, b^R)$ be two triangular fuzzy numbers. Then, arithmetic operation on these fuzzy numbers can be defined as follows:

**(1) Addition:** $\tilde{A} \oplus \tilde{B} = (a^L + b^L, a^m + b^m, a^R + b^R)$.

**(2) Symmetry:** $-\tilde{A} = (-a^R, -a^m, -a^L)$.

**(3) Subtraction:** $\tilde{A} \ominus \tilde{B} = (a^L - b^R, a^m - b^m, a^R - b^L)$.

**(4) Equality:** $\tilde{A} = \tilde{B}$ *iff* $a^L = b^L, a^m = b^m, a^R = b^R$.

**(5) Multiplication:** Suppose $\tilde{A}$ be any triangular fuzzy number and $\tilde{B}$ be non-negative triangular fuzzy number, and then we define:

$$\mu_{\tilde{a}}(x) = \begin{cases} (a^L a^L, a^m b^m, a^R b^R), & a^L \geq 0 \\ (a^L b^R, a^m b^m, a^R b^R), & a^L < 0, a^R \geq 0 \\ (a^L b^R, a^m a^m, a^R a^L), & a^R < 0 \end{cases} \quad (3)$$

## 3. Problem Formulation

In this section, we present a new Fuzzy Integer Multi Objective Linear Programming (FIMOLP) model for the supplier selection problem under price breaks in a supply chain.

### *3.1. Problem Assumption*

I. All parameters, coefficients, and variables are triangle fuzzy number.
II. Only one item is purchased from one vendor.
III. Capacity of each supplier is finite.
IV. Demand of each item is determined from all retailers.
V. Quantity discount is offered by each supplier.

### *3.2. Notations*

Before presenting the model, it is necessary to introduce the notations including parameters, indices, and variables used in our model. The following indices are used in the model formulation:

1) $i$: Index for suppliers ($i \in 0,1,2,\ldots,n$).
2) $j$: Index for price levels of suppliers ($j \in 0,1,2,\ldots,m_i$).
3) $\tilde{x}_{ij}$: The fuzzy number of units purchased from the $i$-th supplier at price level $j$.
4) $\widetilde{P}_{ij}$: Fuzzy price of the $i$-th supplier at level $j$.
5) $\widetilde{V}_{ij}$: Maximum purchased fuzzy volume from the $i$-th supplier at $j$-th price level.
6) $\widetilde{D}$: Fuzzy demand over the period.
7) $m_i$: Number of price level of the $i$-th supplier.
8) $\widetilde{V}_{ij}^*$: Slightly less than $V_{ij}$.
9) $\widetilde{C}_i$: Fuzzy capacity of the $i$-th supplier.
10) $\widetilde{F}_i$: Fuzzy percentage of items delivered late for the $i$-th supplier.
11) $\widetilde{S}_i$: Fuzzy percentage of rejected units for the $i$-th supplier.
12) $\widetilde{Y}_{ij} = \begin{cases} 1, & \text{If the } i-\text{th supplier is selected at price level } j \\ 0, & \text{Other wise} \end{cases}.$
13) Number of suppliers.

### *3.3. Mathematical Model*

What is novel in this model and differ it from other researches is considering all the parameters and decision variables fuzzy numbers through the order allocation process to cope with the uncertainty which always govern the decision making processes. This model could be formulated as follows and we refer it to our model by FIMOLP.

$$\tilde{Z}_1 = \min \sum_{i=1}^{n} \sum_{j=1}^{m_i} \tilde{P}_{ij}\tilde{x}_{ij} \tag{4}$$

$$\tilde{Z}_2 = \min \sum_{i=1}^{n} \sum_{j=1}^{m_i} \tilde{S}_i\tilde{x}_{ij} \tag{5}$$

$$\tilde{Z}_3 = \min \sum_{i=1}^{n} \sum_{j=1}^{m_i} \tilde{F}_i\tilde{x}_{ij} \tag{6}$$

$$s.t: \quad \sum_{i=1}^{n} \sum_{j=1}^{m_i} \tilde{x}_{ij} \geq \tilde{D} \tag{7}$$

$$\sum_{j=1}^{m_i} \tilde{x}_{ij} \leq \tilde{C}_i, \quad i = 1, \dots, n_i \tag{8}$$

$$\tilde{V}_{ij-1}Y_{ij} \leq \tilde{x}_{ij}, \quad i = 1, \dots, n_i; \; j = 1, \dots, m_i$$
$$\tilde{V}^*_{ij-1}Y_{ij} \geq \tilde{x}_{ij}, \quad i = 1, \dots, n_i; \; j = 1, \dots, m_i$$

$$\sum_{j=1}^{m_i} \tilde{Y}_{ij} \leq 1, \qquad i = 1, \dots, n_i \tag{9}$$

$$Y_{ij} = 0, 1, \quad i = 1, \dots, n_i; \; j = 1, \dots, m_i$$
$$\tilde{x}_{ij} \geq \tilde{0}, \quad i = 1, \dots, n_i; \; j = 1, \dots, m_i$$

## 4. Shortcomings of The Existing Methods

In this section, the shortcomings of the existing methods [7, 10] for solving FFLP problems with equality constraints are pointed out (as well as discussed by Kumar et al. [12]).

Lotfi et al. [10] proposed a new method to find the fuzzy optimal solution of FFLP problems with equality constraints. This method can be applied only if the elements of the coefficient matrix are symmetric fuzzy numbers. To solve an FFLP problem, in which the elements of coefficient matrix are not symmetric triangular fuzzy numbers, by using this method first it is required to approximate the non-symmetric fuzzy number into a nearest symmetric fuzzy number. Due to this conversion, the obtained solutions are not exact.

Dehghan et al. [7] proposed a fuzzy linear programming approach for finding the exact solution of FFLS of equations. The existing method [7] is applicable only if all the elements of the coefficient matrix are non-negative fuzzy numbers, e.g., it is not possible to find the solution of FFLS, chosen in Example 4.1, by using the existing method [7] due to the existence of $(-5, 1, 2)$ and $(-1, 14, 20)$ which are not the non-negative fuzzy numbers.

### *4.1. Example*

Consider the following fully fuzzy linear system of equations:

$$(2, 3, 4)\otimes\tilde{x}_1\oplus(1, 2, 4)\otimes\tilde{x}_2 = (5, 19, 43) \tag{10}$$

$$(-5, 1, 2)\otimes\tilde{x}_1\oplus(1, 3, 4)\otimes\tilde{x}_2 = (-1, 14, 20) \tag{11}$$

$\tilde{x}_1, \tilde{x}_2$ are non-negative triangular fuzzy numbers.

Based on the above discussion, in the next section, we define a new model and a novel approach for solving the proposed model.

## 5. A New Model and Solving Approach

In this section, the proposed method by Kumar et al. [12] is extended to a multi objective problem. An FIMOLP problem with $m$ fuzzy constraints and $n$ variables may be formulated as follows:

$$\min(\max)\sum_{j=1}^{n}\tilde{c}_j\otimes\tilde{x}_j \tag{12}$$

$$\min(\max)\sum_{j=1}^{n}\tilde{c}_j{}'\otimes\tilde{x}_j \tag{13}$$

$$min(max)\sum_{j=1}^{n}\tilde{c}_j{}''\otimes\tilde{x}_j \tag{14}$$

$$s.t:\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j \le=\ge \tilde{b}_i \tag{15}$$

$$\forall i=1,\dots,m$$

$$\tilde{x}_j\ge\tilde{0},\quad j=1,\dots,n$$

Where $\tilde{C}^T=[\tilde{c}_j]_{1\times n}$, $\tilde{C}'^T=[\tilde{c}_j{}']_{1\times n}$, $\tilde{C}''^T=[\tilde{c}_j{}'']_{1\times n}$, $\tilde{X}=[\tilde{x}_j]_{n\times 1}$, $\tilde{A}=[\tilde{a}_{ij}]_{m\times n}$, $\tilde{b}=[\tilde{b}_i]_{m\times 1}$ so that $\tilde{a}_{ij},\tilde{c}_j,\tilde{c}_j{}',\tilde{c}_j{}'',\tilde{x}_j,\tilde{b}_i\in F(\mathbb{R})$. In this problem, the fuzzy arithmetic which is mentioned in Section 2 is used and also the fuzzy numbers are considered as the triangular form. The steps of our proposed solution algorithm are as follows:

**Initialization Step:** If all $\tilde{c}_j,\tilde{c}_j{}',\tilde{c}_j{}'',\tilde{x}_j,\tilde{a}_{ij}$ and $\tilde{b}_i$ are represented by triangular fuzzy numbers $(p_j,q_j,r_j)$, $(p_j',q_j',r_j')$, $(p_j'',q_j'',r_j'')$, $(a_{ij},b_{ij},c_{ij})$, $(b_i,g_i,h_i)$ and $(x_j,y_j,z_j)$ respectively, then by substituting these values, the FIMOLP problem, obtained in (5.1), may be written as follows:

$$\min(max)\sum_{j=1}^{n}(p_j,q_j,r_j)\otimes(x_j,y_j,z_j) \tag{16}$$

$$\min(max)\sum_{j=1}^{n}(p_j',q_j',r_j')\otimes(x_j,y_j,z_j) \tag{17}$$

$$\min(max)\sum_{j=1}^{n}(p_j'',q_j'',r_j'')\otimes(x_j,y_j,z_j) \tag{18}$$

$$s.t: \sum_{j=1}^{n} \left(a_{ij}, b_{ij}, c_{ij}\right) \otimes \left(x_{ij}, y_{ij}, z_{ij}\right) \leq = \geq \left(b_i, g_i, h_i\right) \tag{19}$$
$$\forall i = 1, \dots, m$$
$$\left(x_j, y_j, z_j\right) \geq 0, \quad \forall j = 1, \dots, n$$

**Step 2:** Using arithmetic operations defined in Section 2 and ranking function, the fuzzy integer multi objective linear programming problem of Step 1, converted into the following equivalent problem:

$$\min(\max)\, Z_1 = \mathcal{R}\left(\sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j)\right) = \mathcal{R}\left(\sum_{j=1}^{n} (\alpha_j, \beta_j, \gamma_j)\right) \tag{20}$$

$$\min(\max)\, Z_2 = \mathcal{R}\left(\sum_{j=1}^{n} (p'_j, q'_j, r'_j) \otimes (x_j, y_j, z_j)\right) = \mathcal{R}\left(\sum_{j=1}^{n} (\alpha'_j, \beta'_j, \gamma'_j)\right) \tag{21}$$

$$\min(max)\, Z_3 = \mathcal{R}\left(\sum_{j=1}^{n} (p''_j, q''_j, r''_j) \otimes (x_j, y_j, z_j)\right) = \mathcal{R}\left(\sum_{j=1}^{n} (\alpha''_j, \beta''_j, \gamma''_j)\right) \tag{22}$$

$$\sum_{j=1}^{n} m_{ij} \leq = \geq b_i, \quad \forall i = 1, \dots, m \tag{23}$$

$$\sum_{j=1}^{n} n_{ij} \leq = \geq b_i, \quad \forall i = 1, \dots, m \tag{24}$$

$$\sum_{j=1}^{n} 0_{ij} \leq = \geq g_i, \quad \forall i = 1, \dots, m \tag{25}$$

$(x_j, y_j, z_j) \geq 0, \quad \forall j = 1, \dots, n$
$y_j - x_j \geq 0, \quad \forall j = 1, \dots, n$
$z_j - y_j \geq 0, \quad \forall j = 1, \dots, n$
Where,

$$(a_{ij}, b_{ij}, c_{ij}) \otimes (x_j, y_j, z_j) \leq = \geq (m_{ij}, n_{ij}, o_{ij})$$
$$(p_j, q_j, r_j) \otimes (x_j, y_j, z_j) = (\alpha_j, \beta_j, \gamma_j)$$
$$(p'_j, q'_j, r'_j) \otimes (x_j, y_j, z_j) = (\alpha'_j, \beta'_j, \gamma'_j)$$
$$(p''_j, q''_j, r''_j) \otimes (x_j, y_j, z_j) = (\alpha''_j, \beta''_j, \gamma''_j)$$

**Step 3:** Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for each objective function as well as defined in the below.

Suppose the problem is in minimizing form, so the PIS is determined for each objective function by solving the minimizing form of each objective as a single objective problem with the corresponding constraints. In addition, the NIS is calculated by solving the maximizing form of each objective as a single objective problem. Hence, the positive ideal value and negative ideal value of problem is obtained by solving the corresponding model as follows:

$$Z_1^{PIS} = min\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha_j, \beta_j, \gamma_j)\right) \qquad Z_2^{NIS} = max\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha'_j, \beta'_j, \gamma'_j)\right) \tag{26}$$
$$v \in F(v) \qquad\qquad v \in F(v)$$

$$Z_1^{NIS} = max\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha_j, \beta_j, \gamma_j)\right) \qquad Z_3^{PIS} = min\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha''_j, \beta''_j, \gamma''_j)\right) \tag{27}$$
$$v \in F(v) \qquad\qquad v \in F(v)$$

$$Z_2^{PIS} = min\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha'_j, \beta'_j, \gamma'_j)\right) \qquad Z_3^{NIS} = max\, \mathcal{R}\left(\sum_{j=1}^{n} (\alpha''_j, \beta''_j, \gamma''_j)\right) \tag{28}$$
$$v \in F(v) \qquad\qquad v \in F(v)$$

Assume that, *F* is the set of all constraints. To reduce the computational time, the negative ideal solutions can be estimated as follows. Let $v_i^*$ and $Z_i(v_i^*)$ denote the decision vector associated with the PIS of *i*-th objective function and the corresponding value of *i*-th objective function, respectively. Therefore, we can estimate the related NIS as follows:

$$Z_i^{NIS} = max_{k=1,2,3}\{Z_i(v_k^*)\};\ i = 1, 2, 3 \tag{29}$$

**Step 4:** Determine a linear membership function for each objective function according to positive and negative ideal points. The linear membership functions for three objective functions of problem are given as follows:

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_1 < Z_1^{PIS} \\ \dfrac{Z_1^{NIS} - Z_1}{Z_1^{NIS} - Z_1^{PIS}}, & \text{if } Z_1^{PIS} \le Z_1 \le Z_1^{NIS} \\ 0, & \text{if } Z_1 > Z_1^{NIS} \end{cases} \tag{31}$$

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_2 < Z_2^{PIS} \\ \dfrac{Z_2^{NIS} - Z_2}{Z_2^{NIS} - Z_2^{PIS}}, & \text{if } Z_2^{PIS} \le Z_2 \le Z_{12}^{NIS} \\ 0, & \text{if} Z_2 > Z_2^{NIS} \end{cases} \tag{32}$$

$$\mu_1(v) = \begin{cases} 1, & \text{if } Z_3 < Z_3^{PIS} \\ \dfrac{Z_3^{NIS} - Z_3}{Z_3^{NIS} - Z_3^{PIS}} & \text{if } Z_3^{PIS} \le Z_3 \le Z_3^{NIS} \\ 0\,, & \text{if } Z_3 > Z_3^{NIS} \end{cases} \tag{33}$$

In practice, $\mu_i(v);\ i = 1, 2, 3$ presents the satisfaction level of $i$-th objective function for the given solution vector $v$. The graphs of these membership functions were represented in Figure 2.

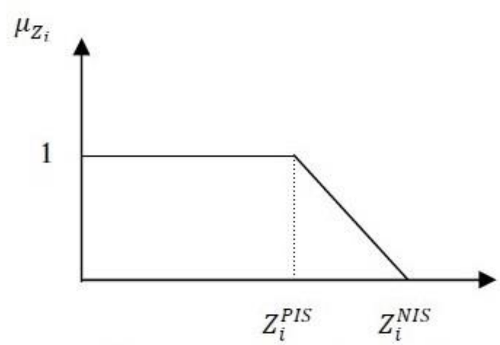


**Figure 2.** Linear membership function for each objective ($i = 1, 2, 3$).

**Step 5:** Convert the auxiliary MIMOLP model into an equivalent single-objective MILP by using the following auxiliary crisp formulation:

$$Z = max \sum_{j=1}^{3} w_i \lambda_i \tag{34}$$

$$s.t\colon \begin{cases} 0 \le \lambda_i \le 1 \\ \lambda_i \le \mu_i(v); \qquad i = 1, 2, 3 \\ v \in F(v) \end{cases} \tag{35}$$

where $\mu_i(v); i = 1, 2, 3$ presents the satisfaction level of $i$-th objective function for the given solution vector $v$ and $\lambda_i(i = 1, 2, 3)$ denotes the minimum satisfaction degree of each objectives. Moreover, $w_i$ indicate the relative importance of the $i$-thobjective function. The selection of $w_i$depends to the aims and opinion of decision maker.

**Step 6:** After solving problem (5.2), the solutions must be put into the objective function of primal FFMOLP problem in order to find the fuzzy objective value of problem.

## 6. Numerical examples

To demonstrate the applicability and the usefulness of the proposed supplier selection order allocation approach, one numerical example is designed. Suppose that three suppliers should be managed for one product. The fuzzy prices are in the three price levels ($\tilde{P}_{ij}$) for each supplier, and the fuzzy percentage of rejected items ($\tilde{S}_i$), the fuzzy percentage of late delivery ($\tilde{F}_i$) and suppliers fuzzy capacity ($\tilde{C}_i$) are presented the same as in Table 1. In order to find the optimal order quantities allocated to suppliers, the multi-objective programming model in this paper can be solved by the Software, AIMMS 3.13.

**Table 1.** The data set for supplier selection parameters.

| Supplier | Quantity level | Price($) | %Rejected | %Late deliveries | Capacity |
|---|---|---|---|---|---|
| S.1 | Q<(3999,4000,4001)<br>(3999,4000,4001)≤Q<(7999,8000,8001)<br>(7999,8000,8001) ≤Q | (15,15,17)<br>(14.5,14.5,16.5)<br>(14,14,16) | (0.1,0.3,0.5) | (0.1,0.1,0.3) | (15400,16000,16700) |
| S.2 | Q<(2999,3000,3002)<br>(2999,3000,3002)≤Q<(9998,10000,10002)<br>(9998,10000,10002) ≤Q | (17,17,19)<br>(16.5,16.5,18.5)<br>(16,16,18) | (0.1,0.1,0.3) | (0.13,0.15,0.17) | (14800,15000,15200) |
| S.3 | Q<(4999,5000,5001)<br>(4999,5000,5001)≤Q<(10999,11000,11001)<br>(10999,11000,11001) ≤Q | (13,13,15)<br>(12.5,12.5,14.5)<br>(12,12,14) | (0.3,0.4,0.7) | (0.1,0.3,0.5) | (16500,17000,17300) |

**Table 2.** The data set for membership functions.

| | $\mu = 0$ | $\mu = 1$ | $\mu = 0$ |
|---|---|---|---|
| $Z_1$(net cost) | 324699 | 259940 | …. |
| $Z_2$(rejected items) | 86.5 | 38.5 | …. |
| $Z_3$(late deliveries) | 56.3 | 30.3 | …. |
| Demand | 19500 | 20000 | 21000 |

The three objective functions $Z_1, Z_2$ and $Z_3$ are cost, net rejections and net late deliveries, respectively, and $\tilde{x}_{ij}$ is the fuzzy number of units purchased from the $i$-th supplier at price level $j$, $w_i$ $(i = 1, 2, 3)$ are the weights associated with the $i$-th objective.

**Case 1:** The DMs relative importance or weights of the fuzzy goals are given as $w_1 = 0.18$ , $w_2 = 0.54$ and $w_3 = 0.28$ are weights of net cost, net rejections and net late deliveries objective functions, respectively. Based on the convex fuzzy decision making (5.2) and the weights that are given by DM (decision maker), the crisp single objective formulation for the numerical example is simply written. The software AIMMS is used to solve this problem. The fuzzy optimal solution for the above formulation is obtained as follows:

$$\tilde{x}_{13} = (5001, 5001, 5801) \quad \text{, and} \quad \tilde{x}_{23} = (14499, 14999, 15199).$$

**Table 3.** Solution of numerical example ($w_1 = 0.18,\ w_2 = 0.54,\ w_2 = 0.28$).

| $Z_1$(net cost) | $Z_z$(rejected items) | $Z_3$(late deliveries) | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Z |
|---|---|---|---|---|---|---|
| 32098 | 38.5 | 30.5 | 0.040 | 1 | 0.99 | 0.82 |

Corresponding to DMs preferences ($w_1 = 0.18,\ w_2 = 0.54, w_3 = 0.28$), in this solution, maximum capacity items are assigned to supplier 2, because of the high-quality level of supplier 2. In other words, the allocated order to each supplier is correspondent with the priority of purchasing criteria (based on DMs preferences).

**Case 2**: In this case, net cost is the most important criterion for the DM in comparison with case 1; hence, the relative importance or weights of the fuzzy goals are assumed as $w_1 = 0.54,\ w_2 = 0.28,\ w_3 = 0.18$ are weights of net cost, net rejections and net late deliveries objective functions, respectively. Then, the ordered quantities and the value of objectives vary as follows:

$$\tilde{x}_{13} = (14500, 15000, 15999) \quad \text{, and} \quad \tilde{x}_{33} = (5000, 5000, 5001).$$

**Table 4.** Solution of numerical example ($w_1 = 0.54,\ w_2 = 0.28,\ w_2 = 0.18$).

| $Z_1$(net cost) | $Z_z$(rejected items) | $Z_3$(late deliveries) | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Z |
|---|---|---|---|---|---|---|
| 282250 | 68.6 | 38.1 | 0.66 | 0.37 | 0.70 | 0.58 |

Due to heavy weight on the net cost criterion, the cost performance is improved in comparison to case1 from \$322098 to \$282250.

**Case 3:** If the DMs relative importance or weight of late delivery criterion changes from 0.18 to 0.54, the priority of cost changes from 0.54 to 0.28 and weight of rejected items changes from 0.28 to 0.18 . The optimal solution for the above formulation is obtained as follows:

$$\tilde{x}_{13} = (14399, 15899, 15899) \quad \text{, and} \quad \tilde{x}_{23} = (4101, 4101, 5101).$$

**Table 5.** Solution of numerical example ($w_1 = 0.28,\ w_2 = 0.18,\ w_2 = 0.54$).

| $Z_1$(net cost) | $Z_z$(rejected items) | $Z_3$(late deliveries) | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | Z |
|---|---|---|---|---|---|---|
| 300952 | 54.5 | 30.3 | 0.37 | 0.67 | 1 | 0.67 |

In this solution, maximum capacity items are assigned to supplier 1, because of the high performance of supplier 1 on delivery criterion, and the remaining items are assigned to supplier 2. In Case 1, due to the DMs preference, quality is the most important criterion and the quality performance is the best value of three rejected items in comparison to other solutions. In Case 2, the cost performance is improved from $322098 to $282250 in comparison with case 1. Based on the DMs preference, the proposed model has a competence to improve the value of objectives function or performance on the objectives. It is shown that variation in priority of criteria will cause variation in ordered quantities to each supplier. This model enables the purchasing managers to calculate order quantities to each supplier based on the priority of criteria in a supply chain.

## 7. Conclusion

This paper introduced a multi-objective mixed integer programming model to support supplier selection decision between conflicting tangible and intangible factors. Simultaneously, in this model vagueness of input data and relative importance of criteria are considered. The proposed model can help the DM to find out the appropriate order to each supplier, and allows purchasing manager(s) to manage supply chain performance on service, quality, cost, etc. The selection process is influenced by the suppliers' price breaks, which depend on the sizes of order quantities. This approach represents deferent aspects of the supplier selection problem in the real world. What is novel in this model and differ it from other researches is considering all the parameters and decision variables as fuzzy numbers through the order allocation process to cope with the uncertainty which always govern the decision making processes. Besides, in this article the proposed method by Kumar et al. [12] is extended and applied to multi objective problem.

Moreover, the fuzzy multi objective supplier selection problem is transformed into a convex (weighted additive) fuzzy programming model and its equivalent crisp single-objective LP programming. This transformation reduces the dimension of the system, giving less computational complexity, and makes the application of fuzzy methodology more understandable.

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