



Dynamic Pricing Decisions for Substitutable Products

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ABSTRACT

This paper considers a dynamic pricing decision problem, in which two different manufacturers compete to distribute substitutable products through a single retailer under two presented scenarios. In the first scenario, the pricing policy is determined via a centralized decision-making, while the second scenario manages the policy in a decentralized one. Utilizing the game-theory-based modeling approaches, the pricing decision problem is achieved under with two different structures. Numerical experiments are also given to examine the effects of the presented scenarios and provide further managerial insights on the solutions.

Keywords: *Game theory, supply chain, pricing decisions, substitutable product.*

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1. Introduction

In today's ever-increasing competitiveness of companies, a decentralized policy has become more highlighted due to its advantages and flexibility [1]. Therefore, considerable attentions from practitioners have been paid on the dominant downstream members. These competing supply chains often contain many upstream manufacturers, who may offer substitutable products while only few huge downstream outlets, whose major role is distribution of such products [2]. In this field, Anderson and Bao [3] presented pricing decisions in centralized and decentralized supply chains with a linear demand function and comparing two cases. Gao et al. [4] proposed a pricing decision-making policy in a closed-loop supply chain for different channel power structures. Rodríguez and Aydın [5] focused on pricing decisions for a manufacturer selling through dual channels considering fluctuations in demands.

Among the variety of efforts in this field, a new trend has become considering the dynamic pricing in supply chains. Liu and Tang [1] presented strategic transfer pricing in a marketing-operations interface considering quality level and advertising dependent goodwill in a dynamic environment. Zhou et al. [6] presented a two period pricing model for new fashion style launching strategy and introduced three scenarios, namely N-Strategy, when the firm

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does not launch a new style; S-Strategy, when the firm launches a new style and stops selling the previous one; and D-Strategy, when the firm sells the new and old style simultaneously. Jia and Zhang [7] proposed dynamic ordering and pricing strategies in a multi-generation supply chain of durable products, in which demands of the products are dependent to both price and quality. Chen and Chang [8] presented a dynamic pricing policy in a closed-loop supply chain. The presented analytical models were handled using dynamic programming and Lagrangean relaxation schemes. Jia and Hu [9] proposed a dynamic pricing decision-making policy for two types of perishable products (i.e., fresh and non-fresh) in a finite horizon.

Among a variety of papers addressed pricing models in supply chains, a few papers have considered only the substitutability of products. In this field, Huang and Ke [2] addressed the pricing decisions for substitutable products under a uncertain demand function. Chen et al. [10] presented a pricing policy for substitutable products in both an internet-based and traditional supply chain.

In this paper, a pricing decision in a two-echelon supply chain in a dynamic environment is investigated. To this end, we consider two manufacturers who produce two substitutable products at different wholesale price, and the existing retailer sells the produced product at the retail price in a dynamic environment.

The remainder of this paper is organized as follows. Section 2 is dedicated to the problem framework and mathematical formulations. Numerical experiments are addressed in Section 3. And finally, Section 4 presents the conclusion and some future research avenues.

2. Problem framework

This problem deals with two manufacturers, who produces a different product that can be substituted with another, and products are then being sold via a retailer. The manufacturer sells its product with at the wholesale cost to the retailer, and the product is sold with a retail price to the potential customer. The interactions between the manufacturers and retailer is depicted in Fig. 1. The pricing decisions sets the unit wholesale price and retail price in a dynamic situation under two separate scenarios. Description of the scenarios is discussed in detail in their corresponding subsections.

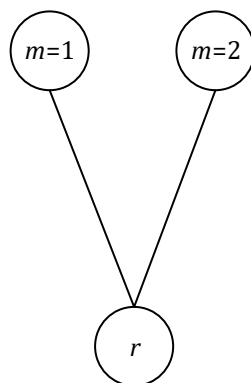


Fig. 1. Interaction between wholesalers and retailer.

In what follows, the notations and assumptions of the problem are addressed.

Notations

Sets

| | |
|-----|-----------------------------------------------------|
| i | index of the introduced product from a manufacturer |
| t | index of time periods |

Parameters

| | |
|----------------|-----------------------------------------------------------|
| v | substitutability of two products |
| β | price sensitivity of the consumer |
| $C_i(t)$ | unit manufacturing cost of product type i at period t |
| $\varphi_i(t)$ | market base of product type i at period t |

Decision variables

| | |
|----------|--------------------------------------------------------|
| $P_i(t)$ | unit retail price of product type i at period t |
| $w_i(t)$ | unit wholesale price of product type i at period t |

State variables

| | |
|----------|--------------------------------------------------------|
| $D_i(t)$ | customer demand rate of product type i at period t |
|----------|--------------------------------------------------------|

Assumption 1. The demand function of the customer for product type i at time period t is assumed as follows:

$$D_i(t) = \varphi_i(t) - \beta P_i(t) + v \sum_{j=1, j \neq i}^I (P_j(t) - P_i(t)) \quad (1)$$

Since the considered problem deals with two manufacturers with a single product each, we have $I=2$, and consequently, the demand function can be simplified as follows:

$$D_i(t) = \varphi_i(t) - (\beta + v)P_i(t) + vP_{3-i}(t) \quad (2)$$

By defining the demand function of each product type, the profit function for two manufacturers and retailer can be expressed as follows.

$$\text{Max } \pi_{M_1} = \sum_t (w_1(t) - C_1(t))(\varphi_1(t) - \alpha P_1(t) + vP_2(t)), \quad (3)$$

$$\text{Max } \pi_{M_2} = \sum_t (w_2(t) - C_2(t))(\varphi_2(t) - \alpha P_2(t) + vP_1(t)), \quad (4)$$

$$\text{Max } \pi_R = \sum_t \sum_{i=1}^2 (P_i(t) - w_i(t))(\varphi_i(t) - \alpha P_i(t) + vP_{3-i}(t)), \tag{5}$$

where $\alpha = (\beta + v)$.

For simplicity and without losing the generality of the model, from now on the subscription of each notation is omitted.

2.1. Centralized scenario

In the centralized scenario, the manufacturers and retailer pursue a negotiation-based prices, by which both sides are benefitted (see Fig. 2).

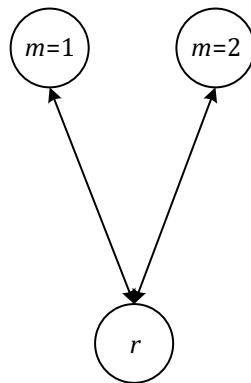


Fig. 2. Interaction between the wholesalers and retailer in the centralized scenario.

In this way, the integrated profit function can be obtained using Eq. (6). As it revealed, the integrated profit function is independent of the wholesale price, while the optimum value of the retail prices should be determined.

$$H_T = \sum_{i=1}^2 (P_i - C_i)(\varphi_i - \alpha P_i + vP_{3-i}), \tag{6}$$

Proposition 1. For every wholesale price, the retail prices of each product type are as follows:

$$P_1^* = \frac{2\alpha\varphi_1 + v\varphi_2 + \alpha vC_2 + 2\alpha^2C_1}{(4\alpha^2 - v^2)}, \tag{7}$$

$$P_2^* = \frac{2\alpha\varphi_2 + v\varphi_1 + \alpha vC_1 + 2\alpha^2C_2}{(4\alpha^2 - v^2)}. \tag{8}$$

Proof. By separating the product types from Eq. (6), we have:

$$H_{T1} = (P_1 - C_1)(\varphi_1 - \alpha P_1 + vP_2) = P_1\varphi_1 - \alpha P_1^2 + vP_1P_2 - \varphi_1C_1 + \alpha C_1P_1 - vC_1P_2, \tag{9}$$

$$\begin{aligned}
 H_{T2} &= (P_2 - C_2)(\varphi_2 - \alpha P_2 + vP_1) \\
 &= P_2\varphi_2 - \alpha P_2^2 + vP_2P_1 - \varphi_2C_2 + \alpha C_2P_2 - vC_2P_1,
 \end{aligned}
 \tag{10}$$

The optimal values of variables can be obtained by setting the first-order derivations.

$$\frac{\partial H_T}{\partial P_1} = \varphi_1 - 2\alpha P_1 + vP_2 + \alpha C_1 = 0,
 \tag{11}$$

$$\frac{\partial H_T}{\partial P_2} = \varphi_2 - 2\alpha P_2 + vP_1 + \alpha C_2 = 0,
 \tag{12}$$

The retail prices can easily be obtained using following equations.

$$P_1 = \frac{\varphi_1 + vP_2 + \alpha C_1}{2\alpha},
 \tag{13}$$

$$P_2 = \frac{\varphi_2 + vP_1 + \alpha C_2}{2\alpha}.
 \tag{14}$$

Proposition 1 is then proved. \square

2.2. Decentralized scenario

In this scenario, two manufacturers are dominant; hence, for the given retail price, the optimum wholesale prices are then obtained (see Fig. 3). Using the non-cooperative game theory approach, a Nash–Stackelberg game approach is utilized for this scenario. The profit function of manufacturers and retailer is indicated in Eqs. (15) to (17).

$$H_{M_1} = (w_1 - C_1)(\varphi_1 - \alpha P_1 + vP_2),
 \tag{15}$$

$$H_{M_2} = (w_2 - C_2)(\varphi_2 - \alpha P_2 + vP_1),
 \tag{16}$$

$$H_R = \sum_{i=1}^2 (P_i - w_i)(\varphi_i - \alpha P_i + vP_{3-i}),
 \tag{17}$$

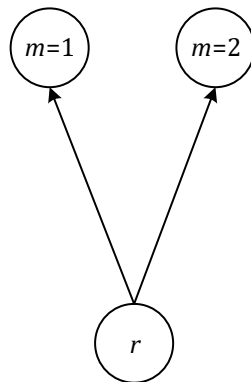


Fig. 3. Interaction between the wholesalers and retailer in the decentralized scenario.

Proposition 2. For the given wholesale prices, the optimal retail prices are:

$$P_1^* = \frac{2\alpha\varphi_1 + v\varphi_2 + \alpha v w_2 + 2\alpha^2 w_1}{4\alpha^2 - v}, \tag{18}$$

$$P_2^* = \frac{2\alpha\varphi_2 + v\varphi_1 + \alpha v w_1 + 2\alpha^2 w_2}{4\alpha^2 - v}. \tag{19}$$

Proof. Using the profit function of Eq. (17) and it separated product types equations, we have:

$$H_{T1} = (P_1 - w_1)(\varphi_1 - \alpha P_1 + v P_2) = (P_1\varphi_1 - \alpha P_1^2 + v P_1 P_2 - \varphi_1 w_1 + \alpha w_1 P_1 - v w_1 P_2), \tag{20}$$

$$H_{T2} = (P_2 - w_2)(\varphi_2 - \alpha P_2 + v P_1) = (P_2\varphi_2 - \alpha P_2^2 + v P_2 P_1 - \varphi_2 w_2 + \alpha w_2 P_2 - v w_2 P_1), \tag{21}$$

The optimal values of variables can be obtained setting the first-order derivations as follows:

$$\frac{\partial H_T}{\partial P_1} = \varphi_1 - 2\alpha P_1 + v P_2 + \alpha w_1 = 0, \tag{22}$$

$$\frac{\partial H_T}{\partial P_2} = \varphi_2 - 2\alpha P_2 + v P_1 + \alpha w_2 = 0, \tag{23}$$

The retail prices can easily be obtained by using the following equations.

$$P_1 = \frac{\varphi_1 + v P_2 + \alpha w_1}{2\alpha}, \tag{24}$$

$$P_2 = \frac{\varphi_2 + v P_1 + \alpha w_2}{2\alpha}. \tag{25}$$

Proposition 2 is thus proved. □

Proposition 3. For the given retailer prices, the optimal wholesale prices are:

$$w_1^* = \frac{2\alpha^2\varphi_1 K - v\varphi_1 K + \alpha v\varphi_2 K + v^2\varphi_1 K + 2v\alpha^4\varphi_2 - v^2\alpha^2\varphi_2 + v^2\alpha^3\varphi_1 + v^3\alpha^2\varphi_2 + 2v\alpha^5 C_2 - \alpha^3 v^3 C_2 + 2\alpha^3 C_1 K - \alpha v^2 C_1 K}{(K - v^2\alpha^4)}, \tag{26}$$

$$w_2^* = \frac{2\alpha^2\varphi_2 K - v\varphi_2 K + \alpha v\varphi_1 K + v^2\varphi_2 K + 2v\alpha^4\varphi_1 - v^2\alpha^2\varphi_1 + v^2\alpha^3\varphi_2 + v^3\alpha^2\varphi_1 + 2v\alpha^5 C_1 - \alpha^3 v^3 C_1 + 2\alpha^3 C_2 K - \alpha v^2 C_2 K}{(K - v^2\alpha^4)}, \tag{27}$$

Proof. Inserting the optimal retail prices in Eqs. (15) and (16), we have:

$$H_{M1} = (w_1 - C_1) \left(\frac{2\alpha^2\varphi_1 - v\varphi_1 - 2\alpha^3 w_1 + \alpha v\varphi_2 + v^2\varphi_1 + \alpha v^2 w_1 + v\alpha^2 w_2}{4\alpha^2 - v} \right), \tag{28}$$

$$H_{M2} = (w_2 - C_2) \left(\frac{2\alpha^2\varphi_2 - v\varphi_2 - 2\alpha^3 w_2 + \alpha v\varphi_1 + v^2\varphi_2 + \alpha v^2 w_2 + v\alpha^2 w_1}{4\alpha^2 - v} \right), \tag{29}$$

The first derivation of Eqs. (28) and (29) results to:

$$\begin{aligned} \frac{\partial H_{M_1}}{\partial w_1} &= 2\alpha^2\varphi_1 - v\varphi_1 - 4\alpha^3w_1 + \alpha v\varphi_2 + v^2\varphi_1 + 2\alpha v^2w_1 + v\alpha^2w_2 + 2\alpha^3C_1 \\ &\quad - \alpha v^2C_1 = 0, \end{aligned} \tag{30}$$

$$\begin{aligned} \frac{\partial H_{M_2}}{\partial w_2} &= 2\alpha^2\varphi_2 - v\varphi_2 - 4\alpha^3w_2 + \alpha v\varphi_1 + v^2\varphi_2 + 2\alpha v^2w_2 + v\alpha^2w_1 + 2\alpha^3C_2 \\ &\quad - \alpha v^2C_2 = 0, \end{aligned} \tag{31}$$

The optimal wholesale prices can easily be obtained by using the following equations.

$$w_1 = \frac{2\alpha^2\varphi_1 - v\varphi_1 + \alpha v\varphi_2 + v^2\varphi_1 + v\alpha^2w_2 + 2\alpha^3C_1 - \alpha v^2C_1}{K}, \tag{32}$$

$$w_2 = \frac{2\alpha^2\varphi_2 - v\varphi_2 + \alpha v\varphi_1 + v^2\varphi_2 + v\alpha^2w_1 + 2\alpha^3C_2 - \alpha v^2C_2}{K}, \tag{33}$$

where $K = (4\alpha^3 - 2\alpha v^2)$.

Proposition 3 is then proved. \square

3. Numerical examples

In this section, some numerical examples are provided to show the applicability of the model as well as some sensitivity analyses on the derived solutions to provide some insights. It should be noted that all computational results are implemented on a Corei-5 system with 4GB RAM. The input parameters of the model are given in Table 1.

Figs. 3 to 9 show some sensitivity analyses under different values of input parameters and different scenarios. As can be seen, by increasing the values of C and φ for both scenarios (i.e., centralized and decentralized), both retail price (P) and wholesale price (W) pursue an increasing behavior. The same behavior can be seen with increasing the values of v . However, with increasing the values of β , we see a decreasing trend in both P and W for both scenarios. Detailed data used in the figures are addressed in Table 2.

Table 1. Initial values of input parameters.

| Parameters | Values |
|-------------|--------|
| C_1 | 100 |
| C_2 | 100 |
| φ_1 | 3000 |
| φ_2 | 3000 |
| β | 20 |
| v | 15 |

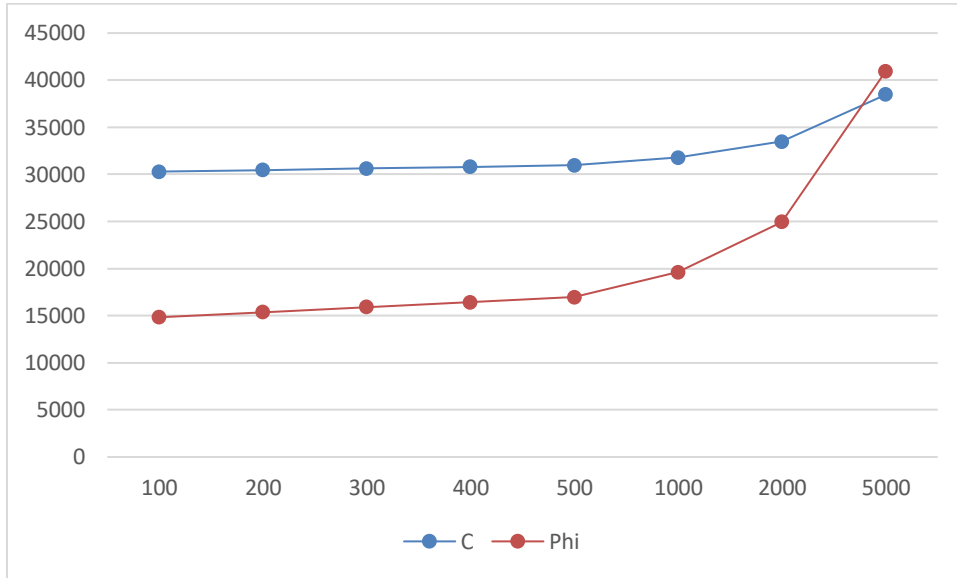


Fig. 4. Values of P under alteration of C and φ for the decentralized scenario.

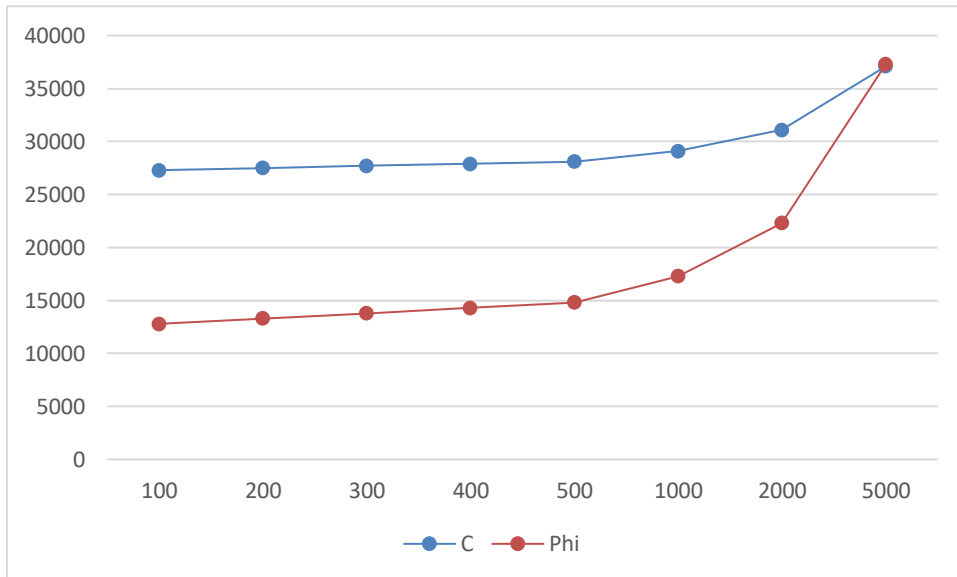


Fig. 5. Values of W under alteration of C and φ for the decentralized scenario.

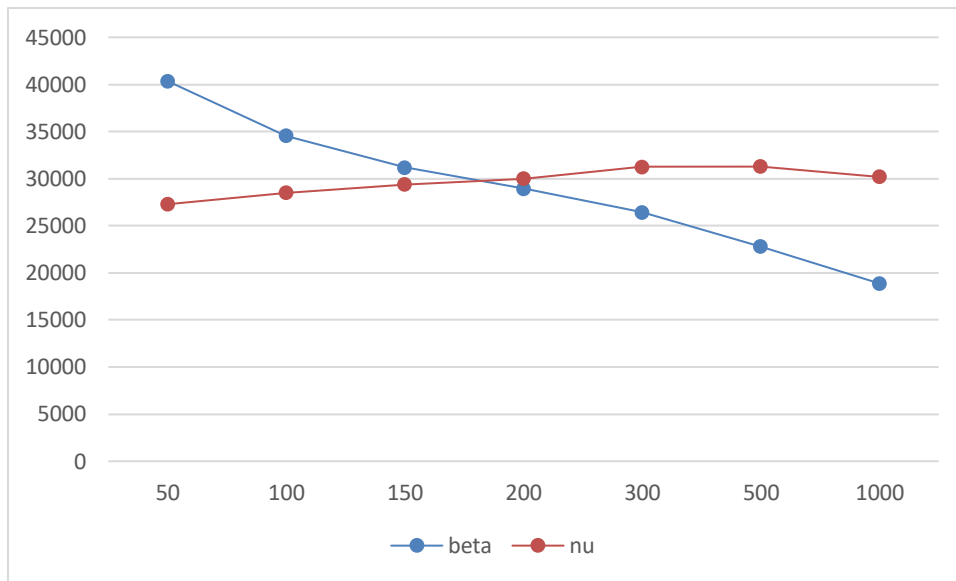


Fig. 6. Values of P under alteration of β and ν for decentralized scenario.

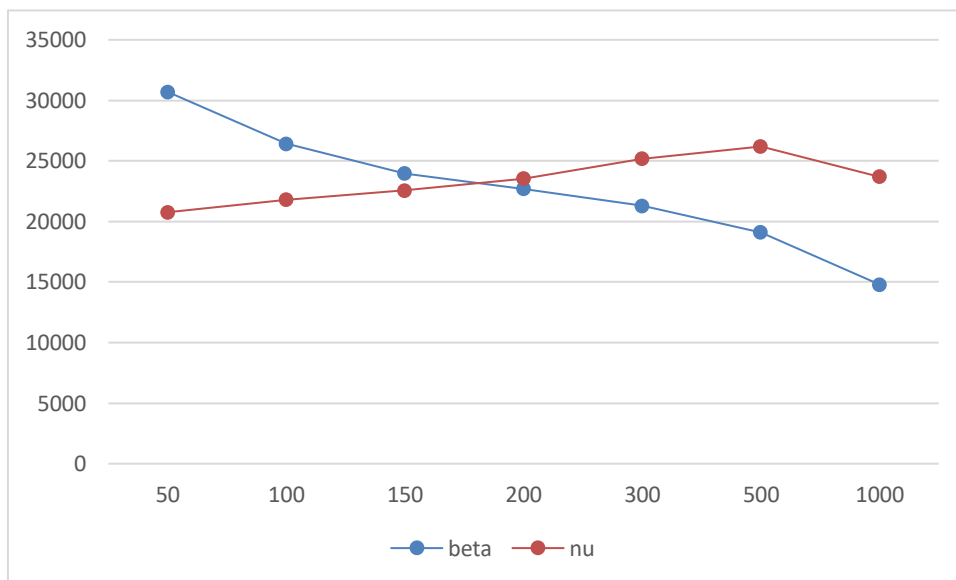


Fig. 7. Values of W under alteration of β and ν for the decentralized scenario.

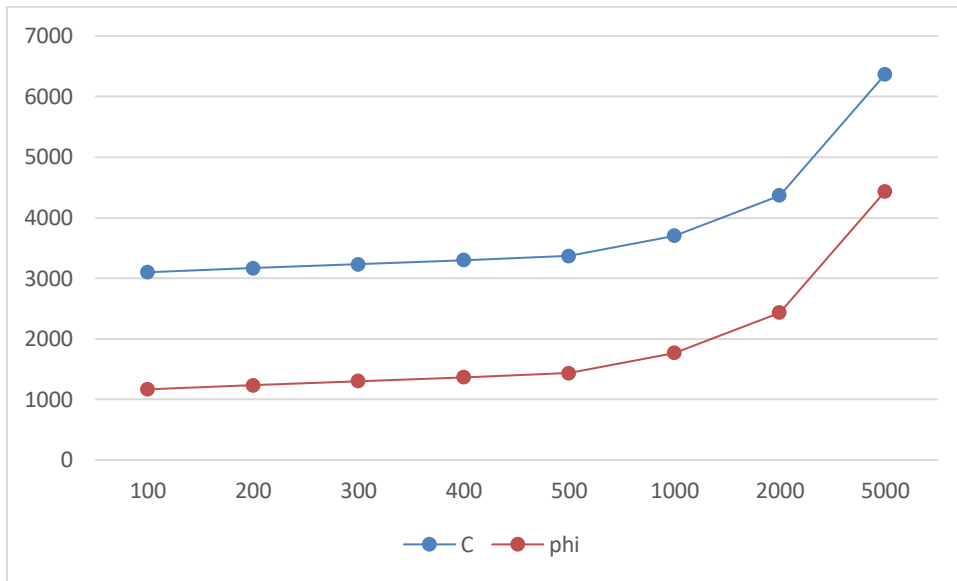


Fig. 8. Values of P under alteration of C and φ for the centralized scenario.

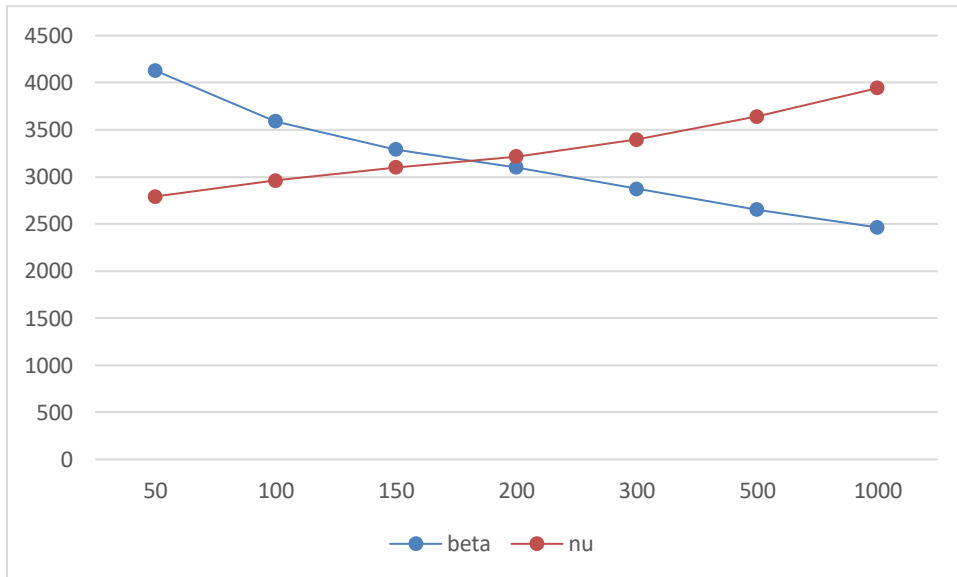


Fig. 9. Values of P under alteration of β and ν for the centralized scenario.

Table 3 shows the different profit levels for each manufacturer and retailer. As can be seen, the centralized scenario results to a more profitable policy for both manufacturers, while the retailer is more benefitted when the decentralized scenario is applied.

Table 2. Detailed derived data for different sensitivity analyses.

| <i>C</i> | <i>P</i> | <i>W</i> | φ | <i>P</i> | <i>W</i> |
|----------|----------|----------|-----------|----------|----------|
| 100 | 30300 | 27300 | 100 | 14833.33 | 12800 |
| 200 | 30466.67 | 27500 | 200 | 15366.67 | 13300 |
| 300 | 30633.33 | 27700 | 300 | 15900 | 13800 |
| 400 | 30800 | 27900 | 400 | 16433.33 | 14300 |
| 500 | 30966.67 | 28100 | 500 | 16966.67 | 14800 |
| 1000 | 31800 | 29100 | 1000 | 19633.33 | 17300 |
| 2000 | 33466.67 | 31100 | 2000 | 24966.67 | 22300 |
| 5000 | 38466.67 | 37100 | 5000 | 40966.67 | 37300 |
| β | <i>P</i> | <i>W</i> | <i>v</i> | <i>P</i> | <i>W</i> |
| 50 | 40346.79 | 30700.73 | 50 | 27287.59 | 20763.7 |
| 100 | 34534.47 | 26414.91 | 100 | 28490.93 | 21792.3 |
| 150 | 31173.35 | 23960.14 | 150 | 29370.11 | 22574.15 |
| 200 | 28933.33 | 22692.25 | 200 | 30012.09 | 23538.31 |
| 300 | 26412.8 | 21285.18 | 300 | 31247.17 | 25181.04 |
| 500 | 22802.03 | 19094.28 | 500 | 31278.46 | 26192.39 |
| 1000 | 18873.32 | 14802.24 | 1000 | 30211.97 | 23695.08 |

Table 3. Different profits under different scenarios.

| | Centralized | Decentralized |
|-------------|-------------|---------------|
| π_{M_1} | 82040658.68 | 81772257.6 |
| π_{M_2} | 82040658.68 | 81772257.6 |
| π_R | 17839009.59 | 18037998 |

4. Conclusion

In this paper, a dynamic pricing decision-making policy for substitutable products has been proposed. The considered supply chain consists of two manufacturers who produce differentiated but substitutable products, and a common retailer, by which the products have been sold to the customers. Introducing two scenarios (i.e., centralized and decentralized), the optimal wholesale prices and retail price have been achieved by utilizing a game-based approach. Then, the model has been discussed in-depth and some sensitivity analyses have been provided. Considering an epistemic uncertainty of the input data and the advertising aspect of each product can be considered as the main future research.

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