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Determine the Efficiency of Time Depended Units by Using Data Envelopment Analysis

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A B S T R A C T

In the last years, several techniques have been reported for managing a system and recognition of the related decision making units. One of them is based on mathematical modeling. Efficiency of any system is very important for all decision makings. Often applied data have time dependent inputs/ outputs. To calculate the efficiency of time dependent data, a new calculation method has been developed and reported here. By this method, the efficiency has been calculated, with minimum errors and minimum mathematical solving model. The data are often time dependent, therefore Spline function has been estimated as a function of time, without using any particular time. Based on this developed function, the efficiency of time dependent data of a numerical example has been calculated and reported.

Keywords: Data envelopment analysis, efficiency, spline.

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1. Introduction

Data envelopment analysis (DEA) is a linear programming methodology to measure the efficiency of Decision Making Unites (DMU).By DEA it is possible to use mathematical models for efficiency evaluation, that the managers may assess the unit without personal consideration and more exactly. As the efficiency is an original part of productivity, managers are interested in determining it. It used to be using parametric method, since Farrel [1] proposed non-parametric model and an activity analysis approach, but his suggestion was for one input and one output DMUs. Multiple inputs and outputs are satisfied to measure the efficiency. Twenty years after, Charnes, Cooper and Rhodes [2] respond to the need for satisfactory procedures to assess the relative efficiencies of multi-input multi-output production units, by introducing a powerful methodology which has subsequently been titled DEA, then it was extended by Banker et al. (BCC) [3]. Originally, DEA identifies an efficient frontier where all DMUs have a unique score.

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In recent years, many DEA researchers have studied the sensitivity of efficiency and inefficiency units [4-16]. The original DEA models assume that inputs and outputs are measured by exact values on a ratio scale.

In applied problems, researchers attitude special data like fuzzy, interval and ... numbers which have been shown inputs and outputs of DMUs. They demonstrated several models to measure the efficiency by using special generated data. Efficiency for fuzzy numbers and interval numbers [20] have been calculated before. More investigations result to changing special data to fixed data and then using basic method for calculating the efficiency. Current research will determine the efficiency of DMUs without changing data, it calculate time dependent data by mathematical function.

The purpose of the current paper is to report the results of studies on the time dependents data. It was found that the most inputs and outputs of DMUs are time dependent. As efficiency is a very important item related to any unit, time dependent efficiency will be important as well.

The paper will be started with basic definitions of efficiency with fixed data will be illustrated [17, 18] and will be continued to present the efficiency of time depended data. Finally, an application operation will be obtained and reported.

2. Background

Manager prominence efficiency can be calculated by its productivity. In any system, productivity is a function of efficiency and impression. By applying linear programming to the Farrell`s theory [1], Charnes et al. [2] succeed to estimate an empirical production technology frontier for the first time. Other than comparing efficiency across DMUs within an organization, DEA has also been used to compare efficiency across firms.

The original idea behind DEA was to provide a methodology whereby, within a set of comparable decision making units (DMUs) can be identify those exhibiting the best practice and can form an efficient frontier. Furthermore, the methodology enables one to measure the level of efficiency of non-frontier units, and to identify benchmarks against which such inefficient units can be compared. Since the advent of DEA, there has been an impressive growth both in theoretical developments and applications of the ideas to practical situations. Any system is a set of decision making units. A DMU is efficient, if it products maximum interest of available possibility. Suppose a system includes *n* DMUs and each DMU includes *m* inputs and *s* outputs. All inputs and outputs are non-negative and at least one of them has to be positive. Production possibility set T_c for CCR model is obtained as non-empty, possibility, constant return to scale and convexity as below:

$$T_c = \left\{ (X,Y)/X \geq \sum_{j=1}^{n} \lambda_J X_j \& \, Y \leq \sum_{j=1}^{n} \lambda_J X_j \, \& \lambda_j \geq 0, \qquad j = 1, \dots, n \right\} \tag{1}$$

The frontier of this set is named efficiency frontier and it is a piecewise linier. One DMU is efficient if and only if it is on this frontier.

CCR model for calculating of theDMU_p efficiency in the input oriented with $T_c$is as:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \quad \sum_{j=1}^{n} \lambda_j X_j \le \theta X_p \\
& \qquad \sum_{j=1}^{n} \lambda_j Y_j \ge Y_p \\
& \qquad \lambda_j \ge 0 \ , \qquad j = 1, \dots, n
\end{aligned} \tag{2}
$$

Unknowns in this model are $\theta, \lambda_1, \lambda_2, \dots, \lambda_n$and θ is the relative efficiency of DMU_p.

In the mentioned model, inputs and outputs have exact value. Distinct data determine an exact efficiency. When time depended data was available, current model cannot calculate the efficiency; so a more advanced model will be needed.

3. Efficiency of time depended data

Applied researches show that more data are parametric, as in bank branches most inputs and outputs are time dependent. When t in interval [a, b] is changing continuously, the inputs and outputs are mutable.

In this Section, investigators will describe DMUs efficiency with time dependent inputs and outputs. It shows the possible way of calculating the efficiency when some or all data are dependent. It is more practical to find the efficiency ($\theta_i(t)$) based on the time. Different ideas are proposed; one is to solve the classic models. In that, models are parametric in objective function, right hand side vector and technological coefficient matrix (model 3). Inputs and outputs are depended to parameter *t*; as $x_{ip}(t), y_{rp}(t)$ are i-th input and *r*-th output forDMU_p. Main object is to compute the efficiency of DMU_p:

$$
\begin{aligned}
& Min \ \theta(t) \\
& s.t. \qquad \sum_{j=1}^{n} \lambda_j x_{ij}(t) \le \ \theta(t) x_{ip}(t) \qquad \forall i \ , a \le t \le b \\
& \sum_{j=1}^{n} \lambda_j y_{rj}(t) \ge \ y_{rp}(t) \qquad \forall r \ , a \le t \le b \\
& \lambda \ge 0
\end{aligned} \tag{3}
$$

Solving these models is either impossible or very difficult, because the parameters will be multiplicities, however the basic matrix may be available or not. Then models will be infeasible or calculating procedure will be complex. This way is a theory and of course exact; so estimating method is purposed. The aim is to keep some credibility knots efficiency, or estimate linear or nonlinear$\theta_j(t)$. The numbers of these knots are important; choosing a little or more number of the knots has advantages or fault. If it is a little, errors are increased but calculates are decreased, otherwise on the contrary. In this research, it is tried to decrease the errors.

Suppose a system includes *n* DMUs and each DMU includes *m* inputs and *s* outputs. Each inputs and outputs may be a function related to time. Then assume that for DMU_pi-th inputs and o-th output are impressed by $f_{ip}(t)$ and $g_{op}(t)$. Therefore, DMU_p is represented as:

$$DMU_p = \left(f_{1p}(t), f_{2p}(t), \dots, f_{mp}(t), g_{1p}(t), g_{2p}(t), \dots, g_{sp}(t)\right)^t \tag{4}$$

The mention functions can be linear or nonlinear. To find the efficiency of DMU_p,in case of known function, for some *t* the inputs and outputs will be in hand. This parameter may assume random numbers with uniform distribution in some interval *[a, b]* may be suppose that:

$$t_i = a + (b-a)d_i \ \ , i = 1, \dots, r \tag{5}$$

Where, r is the number of numeral used in this interval and d_i are random numbers (without lose of generality, we assume that t_i are distinct). At first, the time points t_i will be sorted in an increasing order then named w_i and indeed a=w_0 and b=w_{r+1}. Because of this, in the fixed amount of *r*, all inputs and outputs for any DMUs are fixed, also. For each *t*, all DMUs are fixed; then by CCR model efficiency of DMU_pwill be distinguished ratio other DMUs. To calculate the efficiency of DMUs, it is necessary to solve *n* linear programs (LPs). Therefore, always the number of LPs is more. In supposed model, only distinguished number models are solved and by estimating the function of efficiency, efficiency of ideal *t* is known.

As a result, the efficiencies for DMU_p are examined. The aim is to indicate it in a distinguish function. It is obvious that this function is depended on *t*. In this paper, the function will be approximate by cubic spline interpolation function. By this method the efficiencies for DMU_i are computed, which are showed by $\theta_i(t)$ as:

$$\theta_i(t) = \alpha_i + \beta_i(t - w_{i-1}) + \gamma_i(t - w_{i-1})^2 + \delta_i(t - w_{i-1})^3, i = 1, \dots, r+1. \tag{6}$$

Then it can be assigned the efficiency function for DMU_i such as:

$$\theta_{DMU_p}(t) = \text{Min}\{\theta_i(t), 1\} \ for \ t \ \in [w_{i-1}, w_i], i = 1, \dots, r+1. \tag{7}$$

In distinct data, the value of efficiency will be determined by $\theta_i(t)$ or otherwise the cubic spline function will assess a function for calculating the efficiency. If the efficiency is more than one, view DMU will be supposed efficient and it`s efficiency value will be hypothesis one; because the efficiency value is changing in interval $(0,1]$. Efficiency for orient t is computed by this function that is a simple calculation and so with less error.

4. Numerical Example

In this section, a numerical example is illustrated as a case study. Inputs and outputs of five DMUs are as:

DMU_1=($(t-4)^2+1$,2$(t-4)^2$+1)DMU_2=($-(t-4)^2$+30,- 3$(t-4)^2$+15)

DMU_3= (7-t,18-2t) DMU_4=(2t+1,-t+7) DMU_5=(6,-2t+20)

In this example, the values are supposed uniform numbers in interval $[2,6]$ and the number of the knots are 50. The diagram of DMU_1efficiencies with these knots is shown in Figure 1.

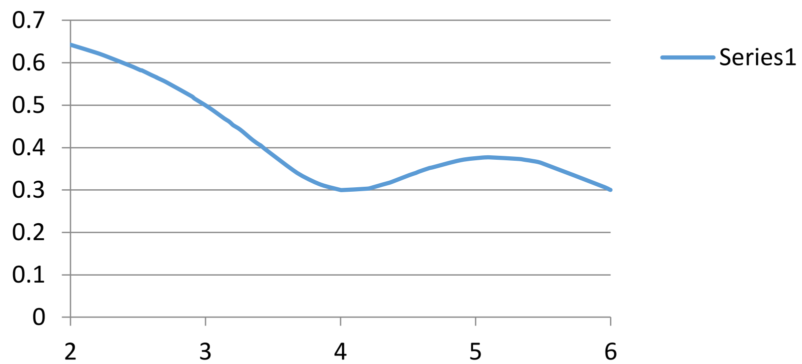

Figure 1. The diagram of DMU1.

The Cubic Spline function to evaluate the $DMU_1$is as follows:

$\theta_1(t)$=-0.112489085t^3+0.668638553t^2-1.418007054t+1.704232573 2.00$\leq t<2.19$

$\theta_2(t)$ =-0.112489085t^3+0.668638553t^2-1.418007054t+1.704232573 2.19$\leq t<2.27$

$\theta_3(t)$=0.4254278t^3-2.994575433t^2+6.897488696t-4.587825877 2.27 $\leq t<2.44$

$\theta_4(t)$=-15.95716348t^3+116.9259927t^2-285.7086976t+233.398539 2.44 $\leq t<2.47$

$\theta_5(t)$=30.3595913t^3+-226.2811602t^2+562.01297t-464.5589673 2.47$\leq t<2.52$

$\theta_6(t)$=-199.4202439t^3+1510.854394t^2+-3815.*568627+t3212*.609574 2.52 $\leq t<2.53$

$\theta_7(t)$=18.27259192t^3-141.43423t^2+364.7215922t-312.7685107 2.53$\leq t<2.60$

$\theta_8(t)$=-12.15000978t^3+95.86206329t^2-252.2487703t+221.9391368 2.60 $\leq t<2.64$

$\theta_9(t)$ =0.920502933t^3-7.65639737t^2+21.03996585t-18.55495104 $2.64\leq t<2.69$

$\theta_{10}(t)$=4.697337948t^3-38.13545594t^2+103.0286334t-92.07145629 $2.69\leq t<2.73$

$\theta_{11}(t)$=-2.420783249t^3+20.16195666t^2-56.123303t+52.75680585 $2.73\leq t<2.90$

$\theta_{12}(t)$=36.98224449t^3-322.6443847t^2+938.0150869t-908.2436377 $2.90\leq t<2.92$

$\theta_{13}(t)$=-7.611688808t^3+67.99847102t^2-202.6620517t+202.0154439 $2.92\leq t<3.00$

$\theta_{14}(t)$=1.08161489t^3-10.24126227t^2+32.0571481t-32.70375596 $3.00\leq t<3.06$

$\theta_{15}(t)$=4.634751095t^3-42.85905262t^2+131.8675866t-134.5104032 $3.06\leq t<3.14$

$\theta_{16}(t)$=-26.43061739t^3+249.7767185t^2-787.0087348t+827.2468132 $3.14\leq t<3.17$

$\theta_{17}(t)$=-25.49561315t^3+240.8848282t^2-758.8214425t+797.4622409 $3.17\leq t<3.19$

$\theta_{18}(t)$=310.4744465t^3-2974.348642t^2+9497.773329t-10108.71687 $3.19\leq t<3.20$

$\theta_{19}(t)$=-180.7342997t^3+1741.25532t^2-5592.159352t+5987.211328 $3.20\leq t<3.21$

$\theta_{20}(t)$=-11.56572577t^3+112.1619539t^2-362.7696457t+391.7643412 $3.21\leq t<3.26$

$\theta_{21}(t)$=5.089640147t^3-50.72752476t^2+168.2500547t-185.2770665 $3.26\leq t<3.35$

$\theta_{22}(t)$=9.333457122t^3-93.37788536t^2+311.1287627t-344.8249571 $3.35\leq t<3.40$

$\theta_{23}(t)$=-216.6899422t^3+2212.060788t^2-7527.362725t+8538.798729 $3.40\leq t<3.41$

$\theta_{24}(t)$=57.50797599t^3-592.9839154t^2+2037.839712t-2333.648041 $3.41\leq t<3.44$

$\theta_{25}(t)$=-0.764517239t^3+8.388214761t^2-30.88041591t+38.48437224 $3.44\leq t<3.65$

$\theta_{26}(t)$=1.029488367t^3-11.25614662t^2+40.82150312t-48.75296258 $3.65\leq t<3.74$

$\theta_{27}(t)$=-0.389333301t^3+4.663032488t^2-18.71622673t+25.47074063 $3.74\leq t<3.86$

$\theta_{28}(t)$=0.684905132t^3-7.776648556t^2+29.3009421t-36.31134993 $3.86\leq t<4.00$

$\theta_{29}(t)$=-6.220985826t^3+75.09404293t^2-302.1818238t+405.6656713 $4.00\leq t<4.01$

$\theta_{30}(t)$=-0.032128556t^3+0.642089972t^2-3.629492481t+6.600721744 $4.01\leq t<4.20$

$\theta_{31}(t)$=0.430670202t^3-5.189174368t^2+20.86181775t-27.68711258 $4.20\leq t<4.24$

$\theta_{32}(t)$=-2.585086077t^3+33.1712455t^2-141.7863625t+202.1889822 $4.24\leq t<4.31$

$\theta_{33}(t)$=4.789841781t^3-62.18657171t^2+269.2058297t-388.2698006 $4.31\leq t<4.36$

$\theta_{34}(t)$=-2.861276536t^3+37.89005587t^2-167.1282666t+245.869086 $4.36 \leq t < 4.38$

$\theta_{35}(t)$=-1.652248144t^3+22.00342281t^2-97.54481377t+144.2772449 $4.38 \leq t < 4.51$

$\theta_{36}(t)$=7.836873971t^3-106.3843994t^2+481.4842644t-726.1964693 $4.51 \leq t < 4.56$

$\theta_{37}(t)$=-18.7259276t^3+256.9947261t^2-1175.524548t+1792.456925 $4.56 \leq t < 4.58$

$\theta_{38}(t)$=-1.080227628t^3+14.54280846t^2-65.09476511t+97.20079033 $4.58 \leq t < 4.66$

$\theta_{39}(t)$=13.34730118t^3-187.1540443t^2+874.8125687t-1362.788602 $4.66 \leq t < 4.68$

$\theta_{40}(t)$=-0.756995172t^3+10.87027652t^2-51.94125266t+82.9473598 $4.68 \leq t < 4.89$

$\theta_{41}(t)$=0.365953766t^3-5.603384403t^2+28.61494925t-48.35924932 $4.89 \leq t < 5.05$

$\theta_{42}(t)$=-0.591438698t^3+8.901111426t^2-44.63275469t+74.94105231 $5.05 \leq t < 5.08$

$\theta_{43}(t)$=-0.195611735t^3+2.868708503t^2-13.98814784t+23.04951805 $5.08 \leq t < 5.10$

$\theta_{44}(t)$=0.015773956t^3-0.36549257t^2+2.506277634t-4.991005257 $5.10 \leq t < 5.32$

$\theta_{45}(t)$=1.171146709t^3-18.80524171t^2+100.605743t-178.9540573 $5.32 \leq t < 5.37$

$\theta_{46}(t)$=-2.05128576t^3+33.10814536t^2-178.1691455t+320.0529933 $5.37 \leq t < 5.43$

$\theta_{47}(t)$=1.293888072t^3-21.38473635t^2+117.7272022t-215.5193961 $5.43 \leq t < 5.49$

$\theta_{48}(t)$=-0.016673405t^3+0.200211174t^2-0.774159728t+1.338096233 $5.49 \leq t < 5.96$

$\theta_{49}(t)$=11.16060916t^3-199.649601t^2+1190.330721t-2364.990266 $5.96 \leq t < 5.99$

$\theta_{50}(t)$=11.16060916t^3-199.649601t^2+1190.330721t-2364.990266 $5.99 \leq t \leq 6.00$

This function shows that the approximate Cubic Spline function are near to the efficiency diagram (Figure 2).

In specific time with substituting *t* by relative interval, efficiency is calculated without solving model. To be more precise, the researchers have the authority to choose the number of knots. This method is instrumental in time dependent and can be calculated by a facile solution. Consequently, with solving n linear programs and simple calculations, efficiency is available.

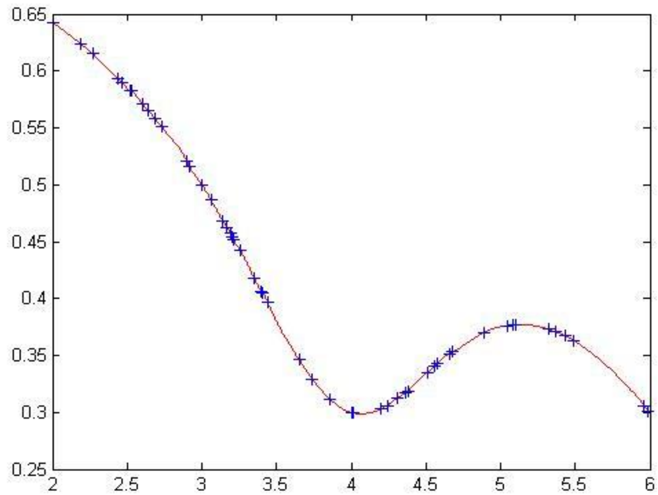

Figure 2. The Cubic Spline approximate of DMU_1.

5. Conclusion

By using this method, and regarding the evaluation of the data in random time efficiency can be determined by mentioned function, so there would be no need to have them in other time. Usually for calculating any efficiency in a particular time, it is necessary to calculate n linier programs; for the rest of the time substituting the value of time in the mentioned function is enough to calculate the efficiency. This method can be used to find outliers if there would be any, and will distinguish whether the diagram efficiency is smooth all the time or not. In addition to have less error and calculations, this method will be suggested.

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