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A Multi-Objective Model for Location-Allocation Problem in a Supply Chain

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ABSTRACT

The fast changing and dynamic global business environment require companies to plan their entire supply chain from the raw material supplier to the end customer. In this paper, we design an integrated supply chain including multiple suppliers, multiple factories, multiple distributors, multiple customers, multiple products, and multiple transportation alternatives. A new multi-objective mixed-integer nonlinear programming model is proposed to deal with this facility location-allocation problem. It considers two conflicting objectives simultaneously, and then the problem is transformed into a multi-objective linear one. The first objective function aims to minimize total losses of the supply chain including raw material purchasing costs, transportation costs and establishment costs of factories and distributions. The second objective function is to minimize the sum deterioration rate of end products and raw materials incurred by transportation alternatives. Finally, the proposed model is solved as a single-objective, mixed-integer, programming model applying the Global Criteria Method. We test their model with numerical example and the results indicate that the proposed model can provide a promising approach to fulfill customer demand and design an efficient supply chain.

Keywords: ***Supply chain design, Facility location-allocation, Optimization, Global Criteria Method, multi-objective programming.***

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1. Introduction

Supply chain management (SCM) involves the planning of the entire supply chain from the raw material supplier to the end customer. Since SCM has become the core of the organization's management in the 21st century, there is a high interest to exploit the full potential of SCM in increasing organizational competitiveness. SCM has a tremendous influence on organizational performance in terms of competing based on price, quality,

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responsiveness, and flexibility in the global market. So, this requires more defined organizational structures, performance measures. The SCM has made managers and analysts to shift their focus from simply the manufacturing plant to the entities the plant interacts with. For example suppliers, warehouses, distribution centers, and customers [1], [2]. Demand changed over time while the facilities are built once at a given time. Once a new facility is built, some of the customers will use its services and other customers will patronize an existing facility. At any given time, customers look for the best facility to meet their objective such as cost, speed, time, or flexibility. Finding the best locations for the new facilities is the problem [3].

Cooper presented the calculational aspects of solving certain classes of location-allocation problems. Exact extremal equations and a heuristic method are presented to decide locations of warehouses and the allocation of customer demand given the locations and demand of customers [4]. After that time, this problem has received much attention from other researchers and it has been considered in a number of various ways. Many facilities location–allocation problems were studied, such as, the dynamic multi-period location–allocation problem[5,6]; the continuous site location problem, [7,8,9]; joint facility location–allocation and production problem[10,11,12], the capacitated facility location–allocation problem [13,6] and the multi-objective facility location–allocation problem [14,15,16,17]. For a detailed review of the literature on the facility location–allocation problem, readers are referred to [18,19,20, 21].

Facility location allocation problem in SCM is become popular and practical in real world. Many articles solved these problems in different manners. There are some of this articles to illustrate by focus on solving manner and then the practical aspect in continue. Minimizing production costs, transportation cost, inventory holding and shortage costs solved by a hybrid approach combining mathematical programming and simulation model. The simulation-based optimization strategy uses an agent-based system to model the supply chain network [22]. To determine the best short-term operational planning to meet all customer requests at minimum total cost in SCM that include multiple factories, distribution centers, retailers and end user located in many different places. The proposed model solved by MILP-base framework [23]. In supply chain with three levels, model integrates three decisions: the distribution centers location, flows allocation, and shipment sizes. The proposed model is a nonlinear continuous formulation, including transportation, fixed, handling and holding costs, which decomposes into a closed form equation and a linear program when the DC flows are fixed. We thus develop an iterative heuristic that estimates the DC flows a priori, solves the linear program, and then improves the DC flow estimations. Proposed model is a simple heuristic that able to design large supply chains and reveal the benefits of good inventory management [24]. The problem of minimizing the expected cost of locating a number of single product facilities and allocating uncertain customer demand to these facilities was model with tow objective function. The first one is transportation cost and the second one is the costs of investing in a facility as well as maintaining and operating it. The problem was formulated as a two-stage stochastic programming model where both demand and short-run costs may be uncertain at the investment time and used a solution method based on Lagrangean relaxation [25]. Total operating cost of an organization is reduced by optimizations various constraints through generic model validation were considered; that supply chain involve four levels: suppliers, plants, distribution centers and retailers [26]. Article [19] summarized the types of location models, mixed-integer programming models, and applications.

The location-allocation problem of parking facilities in Mobarakeh Steel Company is aim to find the optimal location for the parking facilities and allocate travels between departments to each parking facility to optimize the cost and facing the transportation demands of all departments [27]. A conceptual model, according to rising demand in fuelwood supply chain over the years because of the price of fuelwood is relative low, comparing to oil or gas was presented. Due to the sensitive nature of the forest, which is the “production plant” in the examined supply chain, certain restrictions concerning the production and distribution of fuelwood should be taken into account. For this purpose a mixed integer linear programming (MILP) model is considered in modeling uncertainty for fuelwood demand [28].The potential future use of hydrogen in fuel cell electrical vehicles to face problems such as global warming, air pollution, energy security and competitiveness was considered by focused on the design of a hydrogen supply chain.[29]

Nearly 80% of the investigated papers refer to one or two location layers and among these, about two thirds model location decisions in only one Layer. In addition, in core location problems it is usually assumed that customers can only be supplied from the nearest layer. This presumption is not valid in many SCND (supply chain network design) problems, where it might be feasible to have direct transport from upper layer facilities to customer zones or to facilities not in the layer instantly below (e.g., due to very big deliveries). These perspectives were considered in [24, 25, 26, 27, 28, 29, 30, 31, 32].

In this paper, we study the facilities location-allocation problem to design a supply chain under customer demands and cost parameters. We consider several suppliers, several factories, and several customer zones with different transportation alternatives (TA). The supply chain produces three kinds of different products to fulfill customer demand and the information is given for one period (i.e. planning period). Two contradictory objectives are considered, simultaneously. The first objective is to minimize the total cost of the supply chain including, raw material purchasing costs, transportation costs and establishment costs of factories and distribution centers. This objective determines which factories at which capacity level should be opened, which distribution centers should be opened, allocation of customer demand to the factories and distributors, and the supplier selection and order allocation problem. The second objective goal tries to minimize the deterioration rate caused by different transportation alternatives. Using the LP-metric method, these two objectives are then combined into a single objective and solved. Additionally, to enable the model to deal with real situations, different transportation alternatives are considered in the whole supply chain. Results show that the proposed model enables decision makers to design an effective supply chain and provide them a global view to plan for a whole supply chain.

2. Problem statement

Nowadays transportation is a major problem in a supply chain. That means, is safety and suitable costs of transportation are very important. We aim to reduce the transportation costs and have safe transportation by select the best location for factories and distribution centers between the candidate locations in a supply chain. In the proposed model the locations of suppliers and customer zones are identified. Raw materials supply in suppliers and end products produce in factories and transfer to distribution centers or customer zones. The customers could receive end product in two ways directly from factories both from distribution centers (see Fig.1).

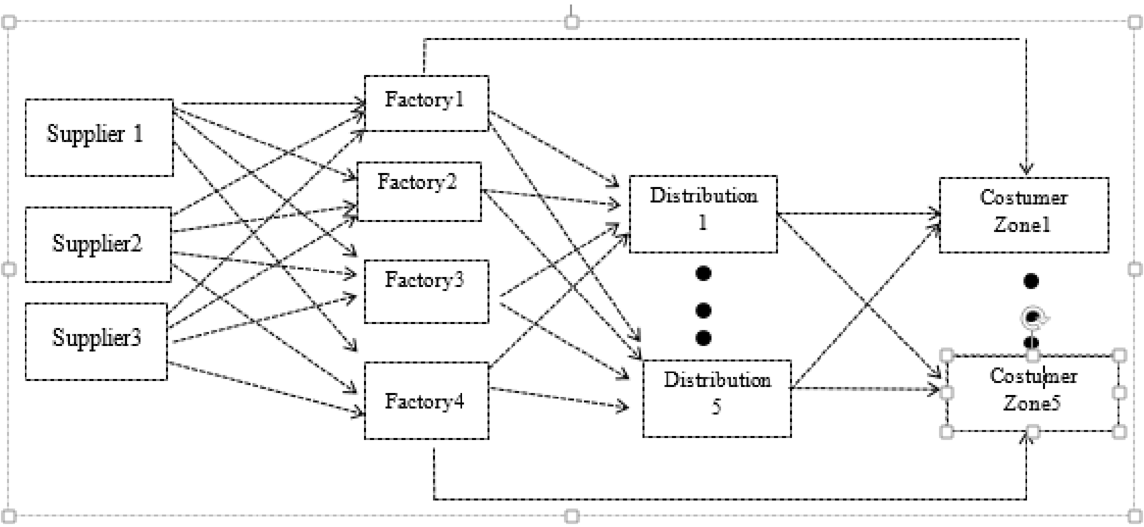

Fig1. Schematic example supply chain

3. Model description

The proposed multi objective mathematical model can be described as follows:

This model is a single period model because it is decided about factory’s locations and distribution’s locations according to customer’s location and supplier’s location and customer’s demand and they may change in another period but that locations couldn't change. The model considered for multi products. So this model is Single period and multi products model. Customer demand should be covered completely. The cost of producing a particular item in various factories and the cost of raw materials in various suppliers and the price of end product in various customer zones can be different. Suppliers and customer zones are located in different geographical areas. Products produced by factories are from raw material suppliers which are supplied according to the consumption rates. The problem is to determine the set of distribution centers *d* and factories *f* to be opened and the capacity level *n* of these factories. Also the quantity of raw materials *r* provided by supplier *s* to fulfill requirement of factory *f* and quantity of end products m transfer to customers- from factory *f* or distribution *d*- are determined in a way that total cost and deterioration rate of transportation *t* are minimized simultaneously. It is worth note that different transportation alternatives are allowed in the whole supply chain network.

- ***Indices***

f	Index for factories *(1,2,…,F)*
s	Index for suppliers*(1,2,…,S)*
d	Index for distributions*(1,2,…,D)*
c	Index for customer zones*(1,2,…,C)*
r	Index for raw materials *(1,2,…,R)*
m	Index for products*(1,2,…,M)*
t	Index for Index for transportation alternative*(1,2,…,T)*
n	Index for capacity level of plants*(1,2,…,N)*

- ***Parameters***

p_{mc}	Selling price of end product *m* in customer zone *c*
TS_{sfrt}	transportation cost of raw material *r* from supplier *s* to factory *f* by using transportation alternative *t*
TD_{fdmt}	transportation cost of end product *m* from factory *f* to distribution *d* by using transportation alternative *t*
TC_{dcmt}	transportation cost of end product *m* from distribution *d* to customer zone *c* by using transportation alternative *t*
TC_{fcmt}	transportation cost of end product *m* from factory *f* to customer zone *c* by using transportation alternative *t*
CM_{rs}	Cost of purchasing raw material *r* from supplier *s*

ES_{fn}	Establish cost of factory f in capacity level n
ES_d	Establish cost of distribution d
D_{mc}	Customer demand for end product m in Customer zone c
U_{rm}	Amount of units of raw material r needed per unit of end product m
MX_{sr}	The maximum amount of the raw material r that the supplier s could produce
MX_{dt}	The maximum number of the end product m that the distributor d which could be distribute
MX_f	The maximum number of the end product m from factory f to the customer zone c could send directly
b_{fn}	capacity level n for factory f
α_{rt}	deterioration rates for raw material r using of transportation alternative t
β_{mtfc}	deterioration rates for end product m using of transportation alternative t from factory f to customer zone c
θ_{mtdc}	deterioration rates for end product m using of transportation alternative t from distribution d to customer zone c
γ_{mtfd}	deterioration rates for end product m using of transportation alternative t from factory f to distribution d
n_t	available quantities of vehicles t
a_t	capacity of each vehicles t
t_m	production time of end product m
$\mathcal{P}_r$	Space occupied for the transport of raw material r
l_m	Space occupied for the transport of end product m

- ***Decision variables***

DIS_{fdmt}	The number of units of products *m* transported from factory *f* to distribution *d* by using transportation alternative *t*
CZD_{dcmt}	The number of units of products *m* transported from distribution *d* to customer zone *c* by using transportation alternative *t*
CUS_{fcmt}	The number of units of products *m* transported from factory *f* to customer zone *c* by using transportation alternative *t*
SUP_{sfrt}	The amount of units of raw material *r* transported from supplier *s* to factory *f* by using transportation alternative *t*
X_{mf}	Number of product *m* produced in the factory *f*
S_{fn}	$\begin{cases} 1 & \text{if factory } f \text{ established in capacity level } n. \\ 0 & \text{else} \end{cases}$
g	$\begin{cases} 1 & \text{if distribution } d \text{ established .} \\ 0 & \text{else} \end{cases}$

Mathematical model

$$
\begin{aligned}
MinZ1 = {} & \sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{t=1}^{T} CM_{rs}.SUP_{sfrt} + \sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{t=1}^{T} TS_{sfrt}.SUP_{sfrt} + \\
& \sum_{f=1}^{F}\sum_{c=1}^{C}\sum_{q=1}^{Q}\sum_{m=1}^{M} TC_{fcmt}.CUS_{fcmt} + \sum_{f=1}^{F}\sum_{d=1}^{D}\sum_{t=1}^{T}\sum_{m=1}^{M} TD_{fdtm}.DIS_{fdmt} + \\
& \sum_{d=1}^{D}\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{m=1}^{M} TC_{dcmt}.CZD_{dcmt} + \sum_{d=1}^{D} ES_{d}.g + \sum_{f=1}^{F} ES_{fn}.S_{fn} - \sum_{f=1}^{F}\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{m=1}^{M} P_{mc}.CUS_{fcmt} \\
& - \sum_{d=1}^{D}\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{m=1}^{M} P_{mc}.CZD_{dcmt}
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
MinZ2 = {} & \sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{r=1}^{R}\sum_{t=1}^{T} \alpha_{rt}.SUP_{sfrt} + \sum_{f=1}^{F}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{t=1}^{T} \beta_{mt}.CUS_{fcmt} \\
& + \sum_{f=1}^{F}\sum_{d=1}^{D}\sum_{m=1}^{M}\sum_{t=1}^{T} \gamma_{mt}.DIS_{fdmt} + \sum_{d=1}^{D}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{t=1}^{T} \theta_{mt}.CZD_{dcmt}
\end{aligned}
\tag{2}
$$

Subject to:

$$\sum_{s=1}^{S}\sum_{t=1}^{T} SUP_{sfrt} = \sum_{m=1}^{M} X_{mf}.Y_{rm} \qquad \forall f,r \qquad (3)$$

$$\sum_{f=1}^{F}\sum_{t=1}^{T} CUS_{fcmq} + \sum_{d=1}^{D}\sum_{t=1}^{T} CZD_{dcmt} = D_{mc} \qquad \forall c,m \qquad (4)$$

$$X_{mi} = \sum_{c=1}^{C}\sum_{t=1}^{T} CUS_{fcmt} + \sum_{d=1}^{D}\sum_{t=1}^{T} DIS_{fdmt} \qquad \forall f,m \qquad (5)$$

$$\sum_{f=1}^{F}\sum_{t=1}^{T} SUP_{sfrt} \le MX_{sr} \qquad \forall s,r \qquad (6)$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} DIS_{fdmt} \le MX_{dt} \qquad \forall d,m \qquad (7)$$

$$\sum_{f=1}^{F}\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{d=1}^{D} DIS_{fdmt} \ge \sum_{d=1}^{D}\sum_{t=1}^{T} CUS_{dcmt} \qquad \forall m,c \qquad (8)$$

$$\sum_{m=1}^{M} T_m.X_{mf} \le \sum_{n=1}^{N} B_{fn}.S_{fn} \qquad \forall f \qquad (9)$$

$$\sum_{m=1}^{M} X_{mf} \ge \sum_{n=1}^{N} S_{fn} \qquad \forall f \qquad (10)$$

$$\sum_{n=1}^{N} S_{fn} \le 1 \qquad \forall f \qquad (11)$$

$$\begin{aligned}&\sum_{f=1}^{F}\sum_{s=1}^{S}\sum_{r=1}^{R} P_r.SUP_{sfrt} + \sum_{f=1}^{F}\sum_{c=1}^{C}\sum_{m=1}^{M} l_m CUS_{fcmt} + \\ &\sum_{c=1}^{C}\sum_{d=1}^{D}\sum_{m=1}^{M} l_m.CZD_{dcmt} + \sum_{f=1}^{F}\sum_{d=1}^{D}\sum_{m=1}^{M} l_m.DIS_{fdmt} \le n_t.a_t \qquad \forall t\end{aligned} \qquad (12)$$

$$\sum_{m=1}^{M}\sum_{c=1}^{C}\sum_{t=1}^{T} CUS_{fcmt} \le H_f \qquad \forall f \qquad (13)$$

$$\sum_{f=1}^{F}\sum_{t=1}^{T} DIS_{fdmt} \le \sum_{c=1}^{C}\sum_{t=1}^{T} g.CZD_{dcmt} \qquad \forall m,d \qquad (14)$$

First objective function (Eq. 1) aims to minimize costs of four-level supply chain , These costs include raw material purchasing cost, raw material transportation cost, end product transportation cost from factories to customer zones directly, end product transportation cost from distribution centers to customer zones, and establishment cost of factories and distribution centers, from which the total sell is deducted. Second objective function (Eq. 2) attempts to minimize total deterioration rates of different transportation alternatives.

Constraint 3 checked that the factories just use raw materials that are supplied from supply chain participants. Constraint 4 is checking to fulfill customer's demand through a distributor or through a factory. Constraint 5 checked total customer's receive is equal number of products that factories and distribution centers sent to the customer. Constraint 6 MX_{st} is maximum amount of raw materials that suppliers can be supplied. Constraint 7 MX_{dt} is maximum number of end products that distribution centers can be distributed.

Constraint 8 ensures, the amount of products distributed by distribution centers is less than the amount sent from factories to the distribution centers. Constraint 9 Checked Factories do not produced more than their capacity's time. Constraint 10 considering if factories are established, that it products at least one product. Constraint 11 is for checking capacity levels of factories that if it is selected to establish (low or high).

Constraint 12, the amount of space required for end products or raw materials that must be transferred are not more than the amount of space vehicles capable of carrying them. Constraint 13, sending products from the factory to the customer directly have limitation in amounts. We adjusted it to 1000 so in this case study (Hi= 1000). Constraint 14, If g=1 then Distributors selected and it can send product to customer otherwise g=0.

4. The global solution

Global Criteria Method is a way to solve the problem which aims to minimize the summation of normalized differences between each objective and the optimal values of them. Problems with *p* linear objective function which all of Constraints are linear, presented as follow:

$$MaxZ = [z_1, z_2, ..., z_p] \tag{15}$$

$$st:$$

$$g_i(x_j) \leq or (\geq =) b_i \quad i = 1,2,...,m \tag{16}$$

$$x_j \geq 0 \quad j = 1,2,...,n \tag{17}$$

Solving this problem requires solving *P* linear Planning problem that any problem must be solved with a single objective function. If the optimal solution of each of problems Is shown with that t = p ,...,1,2 , then the optimal value will be calculated from solving problem as below:

$$MinZ = \sum_{t=1}^{p} \left[\frac{Z_t^* - Z_t(x_j)}{Z_t^*} \right] \tag{18}$$

$$st:$$

$$g_i(x_j) \leq \text{or} (\geq =) b_i \qquad i = 1,2,\ldots,m \qquad (19)$$

$$x_j \geq 0 \qquad j = 1,2,\ldots,n \qquad (20)$$

Solving Problem by using Global Criteria Method requires doing following three steps:

Step 1: solving P linear Planning problems, which each of them contains only one objective function of a problem with P objective function.

Step 2: create a table of optimal value that obtained in Step 1. If there were an optimal value in obtained optimal values, it is your final answer. Otherwise, go to Step 3.

Step 3: to obtain the preferable answer, the final value is obtained by solving as the following model:

$$MinZ = \sum_{t=1}^{p} \left[\frac{Z_t^* - Z_t(x_j)}{Z_t^*} \right] \qquad (21)$$

$$st:$$

$$g_i(x_j) \leq or (\geq =) b_i \qquad i = 1,2,\ldots,m \qquad (22)$$

$$x_j \geq 0 \qquad j = 1,2,\ldots,n \qquad (23)$$

5. Solution procedure

Since the proposed model is a multi-objective mixed integer nonlinear programming (MOMINLP) whose objective functions are completely inconsistent, we used the Global Criteria Method which is one of the well-known MCDM methods for solving multi-objective problems with conflicting objectives simultaneously. According to this method, a multi-objective problem is solved by considering each objective function separately and then a single objective is reformulated which aims to minimize the summation of normalized differences between each objective and the optimal values of them. IN the proposed model, two objective functions are named Z_1 and Z_2, each objective function is solved once separately. We are done the steps of Global Criteria Method that explain it above. Notice this is used for small problems with limit variables. If it is applied in a problem with so many variables then we have to use another solution ways like Metaheuristic Methods.

6. Numerical example

This example is four levels supply chain that contains three suppliers, four factories, five distribution centers, and five costumer zones. The suppliers supplied two row materials and the factories produce three products that transported by two kinds of vehicles (see figure1).

All computations were run using the branch and bound algorithm accessed via LINGO64_13 on cpu intel core i3 under windows 7. Tables 1-6 present data of Numerical example.

Table 1- the Costs of purchase and transportation of raw materials, the maximum amount of supply in each supplier, the deterioration rate of raw materials

raw material	Suppliers(s)	The Cost of transportation of raw materials (TS_{sfrt})								The Cost of purchase of raw materials (CM_{rs})	Maximum amount of Raw material production (MX_{sr})	Space required for Transferring raw Material ($\mathcal{P}_r$)	deterioration rate of raw materials (α_{rt})	
		Factory (F1)		Factory (F2)		Factory (F3)		Factory (F4)						
		T1	*T2*	*T1*	*T2*	*T1*	*T2*	*T1*	*T2*				*T1*	*T2*
r1	1	200	500	350	750	1700	3000	1500	2500	1.5	$10*10^8$	0.0005	0.05	0.1
	2	150	400	300	700	2000	3700	1000	2000	1.7	$20*10^8$			
	3	380	3500	500	1100	800	1500	2000	3500	1.2	$70*10^8$			
r2	1	100	400	200	600	1200	2000	1000	2000	0.85	$30*10^8$	0.0005	0.06	0.04
	2	80	200	100	500	1000	1800	800	1700	0.9	$50*10^8$			
	3	300	2500	300	900	600	1000	1500	3000	0.7	$130*10^8$			

Table 2- The costs of transportation end product from factory to customer and raw materials needed per unit of end product

Products (*m*)	Factories(f)	customer zone										units of raw materials required for products (Urm)	
		C1		C2		C3		C4		C5			
		(*T1*)	(*T2*)	(T1)	(*T2*)	(*T1*)	(*T2*)	(T1)	(*T2*)	(T1)	(*T2*)	(*r1*)	(*r3*)
M1	1	1000	3000	400	800	500	1000	900	2000	200	500	5	1
	2	800	2500	600	1000	500	950	1000	2500	400	800		
	3	300	700	400	800	450	900	700	1500	1500	3000		
	4	3000	4500	1500	3000	2000	3500	800	1700	300	700		
M2	1	1500	3500	500	1000	550	1100	1000	2300	400	700	0	3
	2	1000	2800	100	1200	600	1000	1200	2700	600	1000		
	3	400	1000	600	1000	500	900	1000	2000	1700	3200		
	4	3200	4700	1700	3200	2300	3700	1000	2000	500	800		
M3	1	800	2500	200	500	300	700	700	1500	100	300	2	0
	2	500	2000	500	900	400	800	900	2000	300	700		
	3	200	500	300	500	400	700	500	1000	1000	2500		
	4	2500	4000	1000	2000	1500	3000	500	1500	250	600		

Table 3- The Price of each product, construction time, and space required for end products

Products (m)	Price of Products (p_{mc}) Customer Zones(c_n)					Demand for Products (D_{mc}) Customer Zones(c_n)					Production time per product (t_m)	Space required for Transferring end Products (l_m)
	C1	C2	C3	C4	C5	C1	C2	C3	C4	C5		
M1	0.7	0.65	0.6	0.75	0.8	1000000	7000000	2000000	5000000	3000000	40	1
M2	0.5	0.57	0.4	0.55	0.6	5000000	10000000	4000000	7000000	6000000	10	1
M3	0.975	1	0.958	1.3	1.5	900000	5000000	1000000	10000000	4000000	15	1

Table 4 - The transportation costs of products from distribution center (TC_{dcmt}) to customer zones by different vehicles

Products (m)	distribution center	Customer Zones(c_n) transportation alternative (Tn)									
		C1		*C2*		*C3*		*C4*		*C5*	
		(*T1*)	(*T2*)	(*T1*)	(*T2*)	(*T1*)	(*T2*)	(*T1*)	(*T2*)	(*T1*)	(*T2*)
M1	1	2000	4500	1000	3000	900	2500	600	1500	100	500
	2	1500	3200	1000	2500	1100	2600	400	1200	500	1000
	3	500	1500	700	2000	600	1700	1500	3500	400	1000
	4	300	700	500	1500	200	500	800	1500	1500	3500
	5	1500	3500	500	1000	800	1500	200	500	1000	3000
M2	1	2500	5000	1200	3500	1100	3000	1000	2000	300	700
	2	2000	3000	1000	3000	1500	3000	800	1500	600	1200
	3	800	1800	1000	2500	1000	2000	1700	3700	600	1200
	4	500	1000	700	2000	300	700	1000	2000	2000	4000
	5	2000	4000	700	1500	1000	1800	300	700	1200	3200
M3	1	1500	4000	700	2500	700	2000	500	1000	100	300
	2	1000	3000	800	200	1000	2200	300	1000	300	700
	3	300	1000	500	1500	400	1200	1000	3000	300	700
	4	200	500	300	1200	100	300	500	1000	1000	3000
	5	1200	3000	300	800	500	1000	100	400	800	2500

Table 5 - The costs of transportation end products from the factory to the distribution centers and maximum amount spread by any distribution centers.

Products (*m*)	Factories(f)	distribution centers (*TD_{fdmt}*)										Maximum amount spreaded by distribution centers. (*MX_{dt}*)
		distribution center D1		distribution center D2		distribution center D3		distribution center D4		distribution center D5		
		T1	*T2*	*T1*	*T2*	*T1*	*T2*	*T1*	*T2*	*T1*	*T2*	
M1	1	210	510	250	620	600	2010	500	1510	800	2030	7000000
	2	270	750	450	1010	450	1400	300	1010	900	2400	
	3	850	3020	650	2200	1500	3600	250	710	500	1250	
	4	310	1020	380	810	1000	2510	800	3020	300	800	
M2	1	220	520	270	620	620	2020	520	1520	820	2020	10000000
	2	270	770	420	1020	470	1520	320	1020	920	2520	
	3	820	3020	670	2400	1500	3520	300	750	560	1300	
	4	320	1020	370	850	1200	2500	900	3500	350	850	
M3	1	200	500	250	600	600	2000	500	1500	800	2000	10000000
	2	250	750	400	1000	450	1500	300	1000	900	2500	
	3	800	3000	650	2300	1500	3500	250	700	500	1200	
	4	300	1000	350	800	1000	2500	800	3000	300	800	

Table 6- deterioration rate

Products (*m*)	deterioration rate From Products Factory To Customer (β_{mtfc})		deterioration rate Products from factory to distributor (γ_{mtfd})		deterioration rate Products From distributor to the customer (θ_{mtdc})	
	T1	T2	T1	T2	T1	T2
M1	0.05	0.02	0.01	0	0.03	0.01
M2	0.06	0.04	0.05	0.03	0.04	0.02
M3	0.03	0.01	0.01	0	0.02	0

The results shown the proposed model really worked and we can use it for all four level supply chains. For more considering you can refer to follow tables. Tables 7-10 present data of Numerical example.

Table 7 shown the amounts of raw materials purchased from each supplier by factories. These numbers determine by solving the model. So by using the proposed model suppliers can supply raw materials as much as the factories needed. Therefore suppliers can omit the inventory or, no need to have inventory and the costs related it. Table 8 shown the amount end products produce by potential factories that selected to open by solving the model. They don’t need to inventory too.

Table7- Amounts of raw materials purchased from various suppliers

Suppliers (s)	raw material	factories							
		F1		F2		F3		F4	
		T1	T2	T1	T2	T1	T2	T1	T2
1	R1	9.02	1.14	31.08	1.14	$8.3*10^{7}$	1.17	$4.7*10^{7}$	1.16
	R2	236.89	1.19	26.94	1.20	$8.3*10^{7}$	1.21	$3.1*10^{7}$	1.21
2	R1	1.24	1.14	1.24	1.15	1.25	1.20	1.23	1.16
	R2	1.24	1.19	1.23	1.19	1.23	1.20	1.23	1.21
3	R1	1.23	1.22	1.23	1.15	1.20	1.12	1.24	1.18
	R2	1.24	1.25	1.23	1.20	1.21	1.17	1.24	1.24

Table 8- Amount production factory

Products (m)	Number of Products in each factory			
	F1	F2	F3	F4
M1	3	3	8412508	9587486
M2	80	10	$2.50*10^{5}$	7000001
M3		11	$2.09*10^{5}$	

Table 9- Amounts of end product which factory sends to each of costumer zones

Products (m)	Customers (s)	F1		F2		F3		F4	
		T1	T2	T1	T2	T1	T2	T1	T2
M1	1								
	2					141.74			
	3								
	4								
	5								
M2	1								
	2					104.68	76.63		
	3								
	4								
	5								
M3	1								
	2								
	3								
	4								
	5								

Table 10- amounts of end product which factory sends to each of distribution centers

Products (m)	Factory(f)	Distributor Centers									
		D1		D2		D3		D4		D5	
		T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
M1	1		1.21		1.78						
	2		1.25	1.25	0.50						
	3		1.54	1.48	1.50		$6*10^6$			$24.2*10^5$	
	4		1.45	1.44	$65.9*10^6$				$3*10^6$		
M2	1	78.81	1.19								
	2		1.21			6.94	1.85				
	3		$9.9*10^6$		$5*10^6$		$9.9*10^6$				
	4		1.15						$6.9*10^6$		3.70
M3	1										
	2										11
	3		$5.8*10^6$		$5*10^6$		$10*10^6$				
	4										

Table 11- Amounts of products sent from the distribution centers to the customer zones

Products (m)	Customers(s)	distribution centers									
		D1		D2		D3		D4		D5	
		T1	T2	T1	T2	T1	T2	T1	T2	T1	T2
M1	1						$1*10^6$				
	2		5.47		$6.5*10^6$	0.15	0.15			$4.1*10^5$	
	3					0.15	0.14			$2*10^6$	
	4					0.17	$5*10^6$				
	5								$3*10^6$		
M2	1				$5*10^6$						
	2		$9.9*10^6$	0.15		0.003		0.02	0.01	0.13	3.57
	3					$4*10^6$					
	4						$6*10^6$	$9.9*10^6$			
	5								$6*10^6$		
M3	1		$9*10^5$								
	2			$5*10^6$							
	3		$9*10^6$							11	
	4						$1*10^7$				
	5		$4*10^6$								

Table 9 shown amounts of end product which factories send to each of costumer zones. in this table you can see, Despite of the transportation costs from factories to customer zones directly less than the transportation costs from factories to distribution centers and then customer zones, the propose model limit it to support distribution centers. Amounts of end products which factories send to each of distribution centers and amounts of products send from the distribution centers to the customer zones are shown in tables 10 and 11 respectively.

7. Concluding remarks

Facilities location-allocation decisions have a critical role in the strategic design of supply chain. Most of supply chain model analyze each level of supply chain separately. In cases which the relationship between the levels is checked, all components of the problem are considered as black boxes.

In this paper, the proposed approach is not only about establishing distribution centers and factories with different capacities at the same level of the supply chain decides, but also, considered supply chain as an integrated and find the most affordable and safest way to transport products. The proposed model is a mixed integer non-linear Programming model for the facilities location-allocation problem in a four-level supply chain. It present the best result numerical example and same studies on appropriate site selection for factories and distributors and also choose a safe mode of transportation by the suitable vehicle offers. The benefits and advantages that can be achieved by using the proposed model are noted as following:

- Reducing transportation costs
- Reducing deterioration and damage of end products and raw materials
- Establishment of location factories and distribution centers in the suitable places
- Establishment of factories with proper production capacity
- Support of distribution centers with the restrictions sent from factories to customers directly
- fulfil the customer demands, and ultimately their satisfaction
- selecting proper suppliers and purchasing from selected suppliers

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