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Failure Modes and Effects Analysis under Fuzzy Environment Using Fuzzy Axiomatic Design Approach

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A B S T R A C T

Failure Modes and Effects Analysis (FMEA) is being widely used to detect and eliminate known and/or potential failures, problems, errors and so on from system design, process, and/or service, before they reach the customer. It can be done by calculating the risk priority number which is the product of three factors: occurrence, severity and detectability. A lot of efforts have been made to overcome the shortcomings of the crisp RPN calculation and extend it to fuzzy environment. In this study, the presented fuzzy approach allows experts to describe the variables of risk priority number using linguistic terms by applying the method of fuzzy axiomatic design (FAD). At the final part of this paper a hypothetical case study demonstrated the applicability of the FMEA model under fuzzy environment.

Keywords: *Fuzzy failure modes, Effects analysis, Fuzzy axiomatic design, Fuzzy AHP.*

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1. Introduction

Failure modes and effects analysis (FMEA) is a technique which is widely used to identify prioritize and eliminate or reduce the potential modes of failures, errors, problems and so on from system, process and design before reaching the customer[1- 5].

This structured method provides essential information for predicting reliability and design of a product or process. According to Chapter5 of BritishStandard5760 [6], FMEA is a reliability analysis technique that tries to identify the failures affecting on the functionality of a system in its defined range [7].

A key objective of FMEA is to identify, evaluate and rank potential failure modes using risk priority number (RPN) computed by multiplication of three risk factors such as occurrence O), severity (S) and detectability (D) [4].

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In other words:

$$RPN = O \times S \times D \tag{1}$$

Where O represents the frequency of failures, S shows the severity of the failures and D indicates difficulty of detection of the failure mode.

These three risk factors are evaluated on a scale of 10 units, described in Tables1-3.

Table 1. Crisp rating for occurrence of a failure

Probability of failure	Possible failure rates	Rank
Extremely high: failure almost inevitable	≥ 1 in 2	10
Very high	1 in 3	9
Repeated failures	1 in 8	8
High	1 in 20	7
Moderately high	I in 80	6
Moderate	1 in 400	5
Relatively low	1 in 2000	4
Low	1 in 15000	3
Remote	1 in 150000	2
Nearly impossible	≤ 1 in 1,500000	1

Table 2. Crisp rating for severity of a failure

Effect	Criteria: severity of effect	Rank
Hazardous	Failure is hazardous, and occurs without warning. It suspends operation of the system and/or involves non compliance with government regulations	10
Serious	Failure involves hazardous out comes and/or non compliance with government regulations or standards	9
Extreme	Product is in operable with loss of primary function. The system is inoperable	8
Major	Product performance is severely affected but functions. The system may not operate	7
Significant	Product performance is degraded. Comfort or convince functions may not operate	6
Moderate	Moderate effect on product performance. The product requires repair	5
Low	Small effect on product performance. The product does not require repair	4
Minor	Minor effect on product or system performance	3
Very minor	Very minor effect on product or system performance	2
None	No effect	1

Table 3. Crisp rating for detectability of a failure

Detection	Criteria: likelihood of detection by design control	Rank
Absolute uncertainty	Design control does not detect a potential cause of failure or subsequent failure mode; or there is no design control	10
Very remote	Very remote chance the design control will detect a potential cause of failure or subsequent failure mode	9
Remote	Remote chance the design control will detect a potential cause of failure or subsequent failure mode	8
Very low	Very low chance the design control will detect a potential cause of failure or subsequent failure mode	7
Low	Low chance the design control will detect a potential cause of failure or subsequent failure mode	6
Moderate	Moderate chance the design control will detect a potential cause of failure or subsequent failure mode	5
Moderately high	Moderately high chance the design control will detect a potential cause of failure or subsequent failure mode	4
High	High chance the design control will detect a potential cause of failure or subsequent failure mode	3
Very high	Very high chance the design control will detect a potential cause of failure or subsequent failure mode	2
Almost certain	Design control will almost certainly detect a potential cause of failure or subsequent failure mode	1

A design system of a process or product can have multiple failure modes or multiple causations. In these situations, any failure mode or cause needs to be evaluated and prioritize based on its risk in such a way that the failure modes with the highest risk (most risky), must have the highest priority (the highest RPN).

Crisp RPNs used in this technique has been criticized considerably for various reasons, some of these reasons are mentioned as follows [1-5].

- Different combinations of O, S and D may produce exactly the same value of RPN, although their hidden risk implications may be totally different. For instance, two different failures with the O, S and D values of 2, 3, 2 and 4, 1, 3, respectively, have the same RPN value of 12.
- The relative importance among the three risk factors occurrence, severity, and detection is not considered as they are accepted equally important.
- It is mostly difficult for O, S and D to be precisely evaluated. Usually, information in FMEA is expressed with the aid of some linguistic terms like very high, possible and etc.

To address these deficiencies, fuzzy logic has been applied widely in FMEA [4]. In addition, we cited the following reasons for applying fuzzy logic [1]:

- Since all the data associated with FMEA techniques are based on human language and can be promoted by experienced professionals, working with them on fuzzy logic are more acceptable and comfortable.
- This logic also allows the use of inaccurate data and this enables it to include many different cases.
- Fuzzy FMEA applies and manages both quantitative and qualitative data compatibly and allows us to combines occurrence, severity and detection of failures in a flexible structure.

In this paper, firstly we present a brief review on failure modes and effects analysis in fuzzy environment. Then we express a brief introduction of axiomatic design (AD) and its applications in ranking problems. Later in this section, we state the Buckley fuzzy AHP to determine the weight of the risk factors i.e. occurrence, severity and detectability.
Applying FAD to determine the failure modes have been explained in methodology section.
To clarify the proposed method, we present a hypothetical case study with sensitivity analysis and we compare the obtained results with the results of TOPSIS method. In the fourth section, the discussion and conclusion on this approach are provided.

1.2. Common Evaluation Methods

Conventional methods of fuzzy risk assessment algorithms can be classified into five following methods [5]:

1) Multi-Criteria Decision Making (MCDM)
2) Mathematical programming (MP),
3) Artificial intelligence (AI)
4) Hybrid Approaches
5) Other approaches

Some of them are mentioned below.

1.2.1. MCDM

Braglia et al. [8] proposed an alternative method of multi-criteria decision making technique called fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for FMECA. In This method, causes of failure are considered as the alternatives that must be ranked and the risk factors O, S and D related to a failure mode is also considered as the criteria for ranking. In The proposed fuzzy TOPSIS method, the corresponding importance weights of risk factors are considered as triangular fuzzy numbers, rather than crisp numbers, and this helps us to rank the failure causes with the simple interpretation.

Liu, Liu, Liu, and Mao [3] applied a method called “VIKOR “to rank acceptable priority of failure modes based on risk factors in FMEA. In this model, triangular or trapezoidal fuzzy numbers were used to express linguistic variables that are the basis for assessing the rates and weights of risk factors O, S and D.

1.2.2. Combined Methods

Liu et al [2] proposed a model of risk priority for FMEA with the aid of Fuzzy Evidential Reasoning (FER) and the gray theory. FER method was used for modeling variation and uncertainty assessment data of FMEA team members and gray analysis was applied for the determination of risk priorities of failure modes.

Kutlu and Ekmekçiog˘lu [1] proposed a fuzzy approach that allows experts to use linguistic terms for evaluation of O, S, and D for FMEA by applying combined fuzzy TOPSIS and fuzzy AHP. In this study, a fuzzy AHP method was utilized to determine the weight vector of risk factors, then with the help of language scores for all failure modes of risk factors and the weight vector of risk factors, fuzzy TOPSIS was used to get scores of potential failure modes, which had been ranked for prioritization of failure modes.

2. Axiomatic Design

Axiomatic design was proposed in 1990 by Su [9] based on the scientific basis of rational and logical processes in order to improve design activities. The main purpose of axiomatic design is creating a thought process to make a new design or improvement of existing projects.

Axiomatic design uses two axioms to improve a design. Axioms are facts that are true for all observations and for them there is no such violation.

1. Independence axiom
2. Information axiom

Independence axiom: this axiom implies that the independence of Functional Requirements (FR) should always be kept. Functional requirements are features that are expected from the designed product.

Information axiom: As mentioned above, if there are more than one alternative which can satisfy the functional requirements and the first axiom, the best alternative is the one which have the least possible information content. Here information is meant satisfying the desired functional requirements. The alternative which has the greatest chance to satisfy these requirements is selected as the best.

If the information related to functional requirement of the ith criteria “FR_i” is shown by I_i, its value is expressed by Equation (2) that p_i is the probability of satisfying “FR_i” [10 and 11].

$$I_i = \log_2(\frac{1}{p_i}) = -\log_2(p_i) \tag{2}$$

Equation (3) presents the sum of them for all functional requirements of a specific plan:

$$I_{system} = \sum_{i=1}^{n} \log_2 \ (\frac{1}{p_i}) = -\sum_{i=1}^{n} \log_2 \ (p_i) \tag{3}$$

In this case, if the amount of I_{system} becomes unlimited, then the system or plan will never work.

This probability is actually the chance that the system can achieve what it is intended to reach as tolerances by designers (design range).

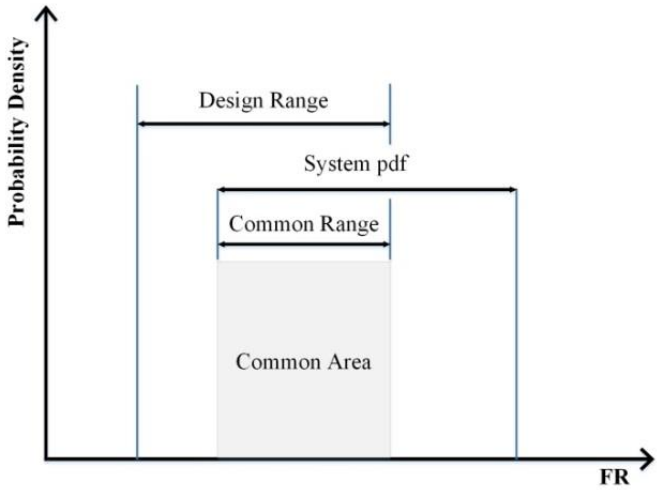

Figure 1. Assumption of uniform probability density function for design range and system range

Hence, Equation (4) can be obtained for uniform distribution. And the amount of information can be calculated from Equation (5):

$$p_i = \frac{\text{common range}}{\text{system range}} \tag{4}$$

$$I_i = \log_2 \frac{\text{common range}}{\text{system range}} \tag{5}$$

And if the variable is continuous, the probability of p_i is obtained as follows [10], [11];

$$p_i = \int_{dr^l}^{dr^u} p_s \, (FR_i) dFR_i \tag{6}$$

Figure 2 indicates the desired level of design and the amount of information can be obtained by taking logarithm in base 2 of the reverse value of the shaded area.

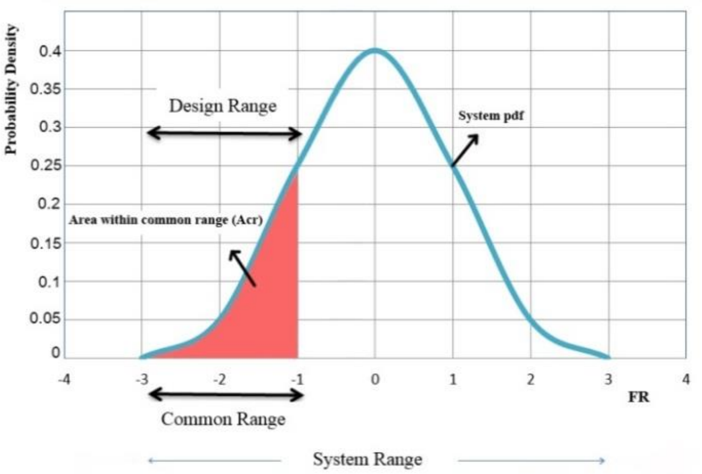

Figure2. Continuous density function for the system range and the uniform function for design range

2.1. The Fuzzy Second Axiom

When the information available is inaccurate, then, the second axiom can be extended to fuzzy sets. In this case, system range and design range can be presented by triangular or trapezoidal fuzzy numbers.

In this case, the amount of information is calculated by taking logarithm in base of 2 from the area of the fuzzy triangular number corresponding to the system range divided by common area of the triangular numbers of system and design range. Equation (7) shows how to perform calculations [10-15].

$$I = \log_2 \frac{\text{TFN of system design}}{\text{common area}} \tag{7}$$

If we specify the weight of w_i to criterion number 'i' and the informational value of an option related to criterion number 'i' is equal toI_i, then the informational value of this option related to all criteria is given by Equation(8)[11 ,15].

$$I = \sum_{i=1}^{j} I_i \times w_i \tag{8}$$

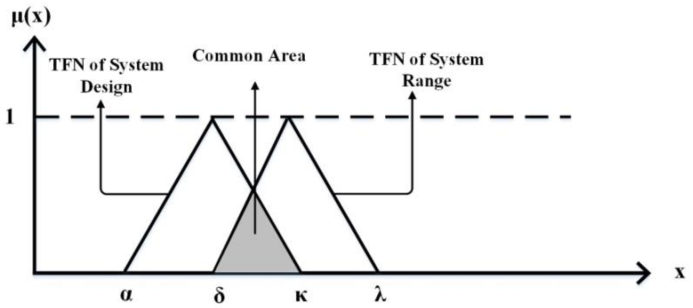

Figure 3. The common area of system and design ranges

Now all of the above description on the second axiom (information axiom) can be used in the multi-criteria decision-making, such that an alternative of a system and design range can be considered as a criterion and the best plan is the one that has the least amount of possible information. Some studies such as comparison of advanced manufacturing systems [12], support system for material handling equipment selection [13], selection of equipment [14], and etc., have been done in this area of research.

These kinds of multi-criteria decision makings have been developed and generalized to the case of hierarchical fuzzy multi-attribute decision making method [11] and design a support system based on information axiom [15].

2.2. Ranking Problems

We can use the second axiom as TOPSIS multi-criteria decision making in ranking problems [11 and 15]. In this case, the attributes are divided into two categories, "cost" and "benefit". Benefit is considered as a fuzzy number with $\alpha = 0$, $\mu(\alpha) = 0$.

Upper bound of benefit is defined as $\mu(\theta) = 1, \theta = \beta = X_{max}$ where X_{max} represents the upper bound of benefit among the alternatives; for cost also, we can define $\alpha = 0, \mu(\alpha) = 1, \theta = \beta = X_{max}, \mu(\theta) = 0$ where X_{max} indicates the upper bound of costs related to alternatives. In this case, by calculating information content of decision making area and alternatives using Equation(7)we can rank alternatives. Figures 4 and 5 represent this matter.

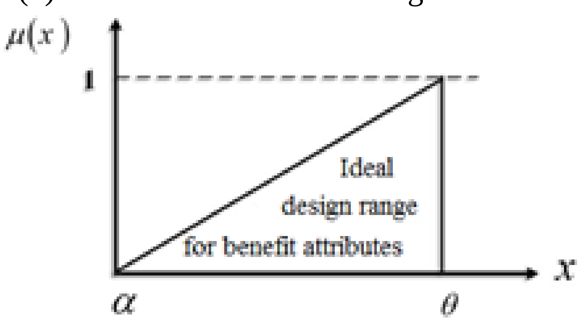

Figure 4. Ideal design range for benefit attributes

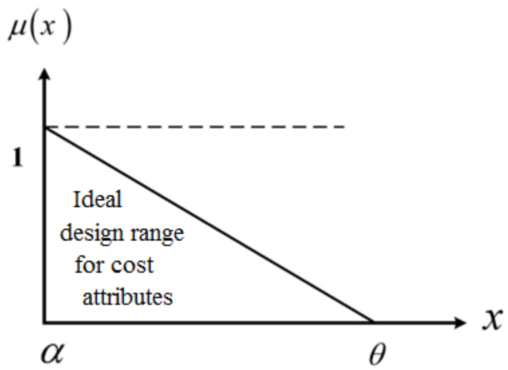

Figure 5. Ideal design range for cost attributes

For example suppose that the design range and system range are as shown in Figure 6.

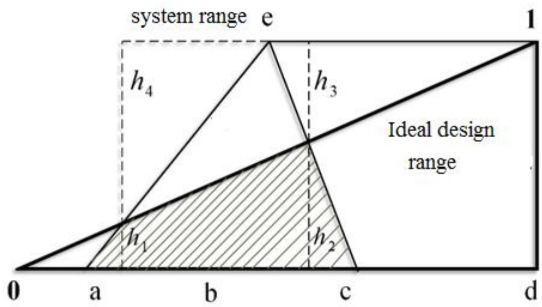

Figure 6. Ideal design range and system range

We calculate the common area of design range and system range to obtain amount of its information using Equation (7).

The common area shown in Equation (11) can be obtained using Equations (9) and (10).

$$\frac{h_2}{h_3} = \frac{c}{d-e} \rightarrow \frac{h_2}{h_3 + h_2} = \frac{c}{d-e+c} \rightarrow h_2 = \frac{c}{d-e+c} \tag{9}$$

$$\frac{h_1}{h_4} = \frac{a}{d-e} \rightarrow \frac{h_1}{h_4 + h_1} = \frac{a}{d-e+a} \rightarrow h_1 = \frac{a}{d-e+a} \tag{10}$$

$$\text{common area} = \frac{1}{2}\left(\frac{c^2}{d-e+c} - \frac{a^2}{d-e+a}\right) \tag{11}$$

If the design and system range is considered as shown in Figure6, then the amount of information can be calculated as follows:

$$I = \log_2 \frac{c-a}{\left(\frac{c^2}{d-e+c} - \frac{a^2}{d-e+a}\right)} \tag{12}$$

When using AD method, assessment of alternatives can be defined using fuzzy or crisp sets. The functional requirement of attributes must be defined with fuzzy or crisp sets, along with evaluation of alternatives. If the assessment of alternatives or criteria all have the same property, i.e. all are fuzzy or all are crisp, the problem must be solved with classic or fuzzy axiomatic design. If the problem includes both fuzzy and crisp assessment, then neither the AD nor the FAD can help us to solve it. In this case, the ratio which is called the information content is obtained using Equation (13).

$$I = \log_2 \frac{1}{\mu(x_i)} \quad , \quad \mu(x_i) = \begin{cases} \dfrac{x_i - \alpha}{\theta - \alpha} & \text{benefit} \\ \dfrac{\theta - x_i}{\theta - \alpha} & \text{cost} \end{cases} \tag{13}$$

In fact, for a system whose assessment is accurate, a fuzzy number is an imaginary concept and its information content is calculated by using Equation (13). Figure 7 illustrates this matter.

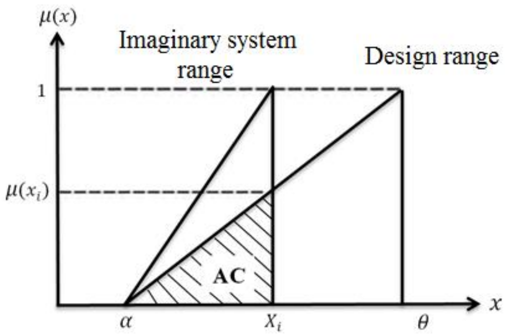

Figure 7. Triangular fuzzy range design and crisp range system

2.3. Fuzzy AHP to Determine Risk Factors for the Occurrence, Severity and Detectability

To determine the weights of three criteria of occurrence, severity, and detectability, analytic hierarchy process can be applied. This technique is a multi-criteria decision making method based on pairwise comparisons that have the capability of using both qualitative and quantitative data and its use does not require complex calculations. The purpose of this method is to take advantage of expert knowledge for decision making. But the traditional methods, cannot be utilized when the knowledge of experts is in linguistic terms. Hence, it is generalized to the fuzzy environment with the help of some methods introduced by Laarhoven, Pedrycz's and Buckley. In this study, because of the simplicity of the method introduced by Buckley, this method was used in which the steps will be as follows [11, 15, and 16];

Step1: The idea of FMEA team members about the importance of criteria weights of occurrence, severity and detectability is obtained to create a decision matrix and the weight vector.

Step 2: Since decision making is a team work, aggregated decision matrix is obtained.

In order to obtain the aggregate weights of criteria and to rank them, Equations (14) and (15) can be used (Equations (14) and (15) can also be replaced by any other equation with the use of aggregation).

$$\tilde{S}_{ij} = \frac{1}{K}(\tilde{S}^1{}_{ij} + \tilde{S}^2{}_{ij} + \ldots + \tilde{S}^t{}_{ij} + \ldots + \tilde{S}^k{}_{ij}), \quad \tilde{S}^t{}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \tag{14}$$

$$\tilde{w}_j = \frac{1}{K}(\tilde{w}^1{}_j + \tilde{w}^2{}_j + \ldots + \tilde{w}^t{}_j + \ldots + \tilde{w}^k{}_j), \quad \tilde{w}^t{}_j = (w_{jl}, w_{jm}, w_{ju}) \tag{15}$$

Where k represents the number of decision makers, $\tilde{S}_{ij}$ indicates the rank of i'th alternative in terms of j'th criterion. So a fuzzy multi-criteria decision making problem with m alternatives and n criteria can be stated with the following matrix:

$$\tilde{D} = \begin{vmatrix} \tilde{S}_{11} & \tilde{S}_{12} & . & . & . & \tilde{S}_{1n} \\ \tilde{S}_{21} & \tilde{S}_{22} & . & . & . & \tilde{S}_{2n} \\ . & . & & \cdots & & . \\ . & . & & \cdots & & . \\ . & . & & \cdots & & . \\ \tilde{S}_{m1} & \tilde{S}_{m2} & & \ldots & & \tilde{S}_{mn} \end{vmatrix} \tag{16}$$

And the matrix of fuzzy weights is as follows;

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_r, \ldots, \tilde{w}_n] \tag{17}$$

where l, m and u are respectively the lower, middle and upper limits of $\tilde{w}_r$ which is a fuzzy number.

Step 3: The relative importance of weights can be directly determined by decision makers or a pairwise comparison, as described in Step 1.

$$\tilde{C} = \begin{vmatrix} 1 & \tilde{C}_{12} & . & . & . & \tilde{C}_{1n} \\ \tilde{C}_{21} & 1 & . & . & . & \tilde{C}_{2n} \\ . & . & & \cdots & & . \\ . & . & & \cdots & & . \\ . & . & & \cdots & & . \\ \tilde{C}_{m1} & \tilde{C}_{m2} & & \ldots & & 1 \end{vmatrix} \tag{18}$$

If we consider the matrix presented in Equation (18) as a pairwise comparison matrix, then Equation (19) can be resulted:

$$\mathrm{Cij} = \begin{cases} i > j, & (1,1,3), (1,3,5), (3,5,7), (5,7,9), (7,9,9) \\ i = j, & 1 \\ i < j, & (1,1,3)^{-1}, (1,3,5)^{-1}, (3,5,7)^{-1}, (5,7,9)^{-1}, (7,9,9)^{-1} \end{cases} \tag{19}$$

Linguistic scales for triangular fuzzy numbers are given in Table4. Now, we calculate fuzzy weight matrix using Buckley method with the help of Equations (20) and (21):

$$\tilde{r}_i = (\tilde{C}_{i1} \otimes \tilde{C}_{i2} \otimes \ldots \otimes \tilde{C}_{in})^{1/n} \tag{20}$$

$$\widetilde{w}_i = \tilde{r}_i \otimes (\tilde{r}_1 + \tilde{r}_2 + \cdots + \tilde{r}_n)^{-1} \quad (21)$$

In above Equations, c_{in} is the fuzzy comparison of criterion'i' related to the criterion 'n' and r_i is the comparison value of criterion'i' related to the other criteria. After obtaining importance weight matrix, we use a defuzzification process to convert fuzzy numbers to crisp ones. Therefore, firstly fuzzy numbers are converted to crisp numbers, and then normalizing process is done. To defuzzify fuzzy numbers, we use centroid method, which is the most commonly used method in this field.

Equation (22) represents both defuzzification and normalizing process.

$$w_r = \frac{\widetilde{w}_r}{\sum_{j=1}^{n} \widetilde{w}_j} = \frac{w_{rl} + w_{rm} + w_{ru}}{\sum_{j=1}^{n} \widetilde{w}_j} \quad (22)$$

where n indicates the number of criteria andw_r is the weight of r'th criterion which is a crisp number.

Table 4. Linguistic scale for weight matrix

Linguistic scales	**Scale of fuzzy number**	
(1,1,3)	Equally important	Eq
(1,3,5)	Weakly important	Wk
(3,5,7)	Essentially important	Es
(5,7,9)	Very strongly important	Vs
(7,9,9)	absolutely important	Ab

3. Methodology

Fuzzy logic is a tool to convert ambiguous sense of human decision making capabilities to mathematical formulas. In addition, a significant demonstration of the size of uncertainty and vague concepts that are expressed in natural language, so fuzzy multi-criteria decision making methods are preferred to overcome FMEA procedure rather than a definitive decision method. To determine importance of one failure mode, we propose FAD approach as follows:

Firstly, potential failure modes are detected by a group of experts (FMEA team), then the comparison matrix for risk factors is made and Buckley Fuzzy AHP is used to determine the weight vector of risk factors.

Then, language assessment of experts for all failure modes associated with risk factors are aggregated to obtain their mean value and run FAD methodology.

Next, using the weight vector of risk factors and the system range, the common area using a proposed design range (see figure 8) is calculated and the area of the system range is obtained and the amount of information content of the failure mode is calculated. Finally, failure mode ranking will be done.

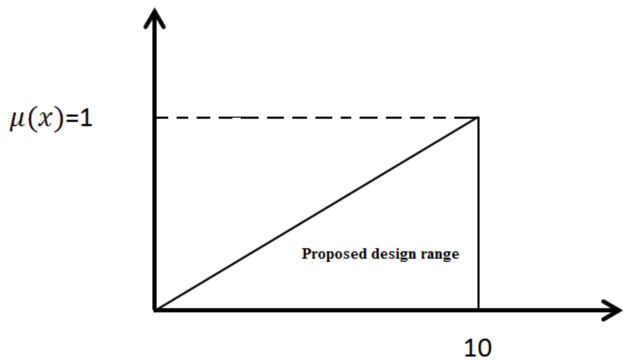

Figure 8. Proposed design range for ranking the failure modes

Figure 9 represents the proposed fuzzy FMEA model. Overall, the most important failure modes are determined by the following steps:

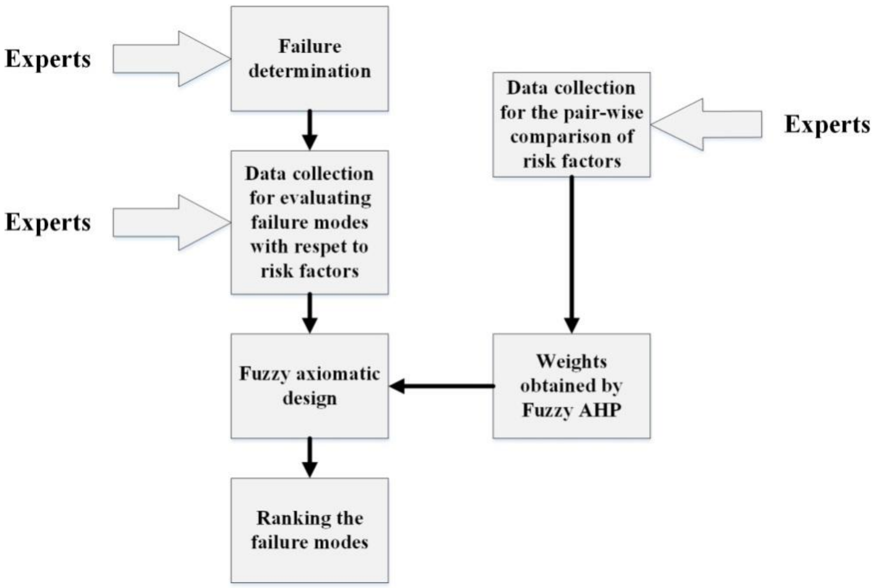

Figure 9. Flowchart of the proposed method

Step1: Detect potential failure modes by experts (FMEA team).

Step2: Evaluate failure modes based on risk factors and according to Table 5 and aggregate them.

Step3: Obtain the decisions of experts about the relative importance of risk factors and aggregate them.

Step4: Use fuzzy AHP Buckley method to determine the weight of risk factors.

Step5: Apply FAD method and Equations (8) and (12) to calculate the information content and value of each failure mode according to the proposed design range presented in Figure 8.

Step 6: Rank the failure modes increasingly based on their information contents.

We provide a hypothetical case study for further explain which is taken from reference [1].

4. An Illustrative Example

The proposed methodology is applied to the production line of an SME in an automobile industry. Important potential failure modes (PFMs) are identified by a group of experts in an assembly process at the manufacturing facility as a non-conforming material (A), wrong die (B), wrong program (C), excessive cycle time (D), wrong process (E),damaged goods (F),wrong part (G), and incorrect forms (H). After determining the PFMs by using FAHP method (the weights for the risk factors are calculated as (0.468 0.2010.331), see [1]), experts linguistic evaluations for the risk factors in respect of each failure modes are obtained as indicated in Table 6 and the aggregated matrix is shown in Table 7.

The fuzzy scores corresponding to these linguistic terms (system ranges) are presented in table 5.

In the next step, using weight vector of the risk factors and the fuzzy evaluations of each risk factor with respect to PFMs, FAD is utilized as illustrated in Table 9. Finally, as shown in Table 9, the scores are ranked and results show that the most important failure mode is ‘‘wrong process’’ (E).

Table 5. Fuzzy evaluation scores for alternatives (system ranges)

Linguistic terms	Fuzzy score
Very poor (VP)	(0,0,1)
Poor (P)	(0,1,3)
Medium poor (MP)	(1,3,5)
Fair (F)	(3,5,7)
Medium good (MG)	(5,7,9)
Good (G)	(7,9,10)
Very good (VG)	(9,10,10)

4.1. Sensitivity Analysis

To evaluate the sensitivity of the results to the weights of risk factors, some other hypothetical case studies of different weights are considered and a sensitivity analysis was performed where the results are shown in Figure 10 and Table 10. Table 9 shows the risk factor weights of the case studies.

Table 6. Fuzzy experts' evaluations of risk factors associated with the potential failure modes

Potential failure modes	Occurrence	Severity	Detection
(A) Non-conforming material	MG ،MG ،F	F,F,MP	G ،MG ،G
(B) Wrong die	VG ،G ،VG	P.MP,MP	P ،MP ،MP
(C) Wrong program	VG ،G ،G	MP ،P ،MP	P ،MP ،VP
(D) Excessive cycle time	F ،MG ،MG	MP ،F ،MP	G ،MG ،G
(E) Wrong process	MG ،MG ،G	MP ،F ،F	G ،V ،G
(F) Damaged goods	MG ،G ،MG	F ،MG ،MG	F ،MP ،MP
(G) Wrong part	VG ،VG ،VG	VP ،MP ،P	P ،MP ،VP
(H) Incorrect forms	VP ،VP ،VP	P ،VP ،VP	VP ،VP ،VP

Table 7. Aggregated matrix

Potential failure modes	Severity	Occurrence	Detection
(A)	(2.33,4.33,6.33)	(4,33,6.33,8.33)	(6,33,8.33,9.66)
(B)	(0.67,2.33,4.33)	(8.33,9.67,10)	(0.67,2.33,4.33)
(C)	(0.67,2.33,4.33)	(7.67,9.33,10)	(0.33,1,2.33)
(D)	(1.67,3.67,5.67)	(4.33,6,33,8.33)	(6,33,8,33,9.66)
(E)	(2,33,4,33,6.33)	(5,67,7.67,9.33)	(7.67,9.33,10)
(F)	(4,33,6.33,8.33)	(5.67,7,67,9.33)	(1,67,3,67,5.66)
(G)	(0.33,1.67,3.67)	(9,10,10)	(0.33,1.67,3.66)
(H)	(0,0.33,1.67)	(0.0,1)	(0,0,1)

4.2. Comparison with the Results of TOPSIS Method

To verify the correctness and accuracy of the method, the results with the results of the implementation method in reference [1] were compared. Comparing results, it is observed that, except in one case which is shown in Table 11, the results are exactly the same as the results obtained from the fuzzy TOPSIS method.

Table 8. Weights of risk factors corresponding to the hypothetical case studies

Weight of risk factors	Case 0	Case 1	Case 2	Case 3	Case 4
Occurrence	0.468	0.6	0.5	0.4	0.4
Severity	0.201	0.2	0.25	0.3	0.2
Detection	0.331	0.2	0.25	0.3	0.4

Table 9. Fuzzy FMEA analysis using FAD

Potential failure modes	Fuzzy axiomatic design			Total information content
	Detection **W_D** **0.331**	**Occurrence** **W_O** **0.201**	**Severity** **W_S** **0.468**	
(A)	(6.33,8.33,9.66)	(4.33,6.33,8.33)	(2.33,4.33,6.33)	0.334053
(B)	(0.67,2.33,4.33)	(8.33,9.67,10)	(0.67,2.33,4.33)	1.0211994
(C)	(0.33,1,2.33)	(7.67,9.33,10)	(0.67,2.33,4.33)	1.293979
(D)	(6.33,8.33,9.66)	(4.33,6.33,8.33)	(1.67,3.67,5.67)	0.423513
(E)	(7.67,9.33,10)	(5.67,7.67,9.33)	(2.33,4.33,6.33)	0.295138
(F)	(1.67,3.67,5.66)	(5.67,7.67,9.33)	(4.33,6.33,8.33)	0.377719
(G)	(0.33,1.67,3.66)	(9,10,10)	(0.33,1.67,3.67)	1.270816
(H)	(0,0,1)	(0,0,1)	(0.33,0,1.67)	3.133736

Table 10. Fuzzy FMEA analysis using FAD for all hypothetical case studies

Failure modes	Case 0	Case 1	Case 2	Case 3	Case 4
			I_{Total}		
A	0.334	0.4056	0.36	0.3142	0.2969
B	1.022	1.0233	0.9594	0.8956	1.0233
C	1.294	1.188	1.1654	1.1427	1.3517
D	0.4235	0.5203	0.4555	0.3907	0.3734
E	0.2951	0.3718	0.3176	0.2635	0.2555
F	0.3778	0.3042	0.3257	0.3473	0.4166
G	1.2708	1.2721	1.1927	1.1134	1.2726
H	3.1337	3.0419	3.1115	3.1811	3.1811

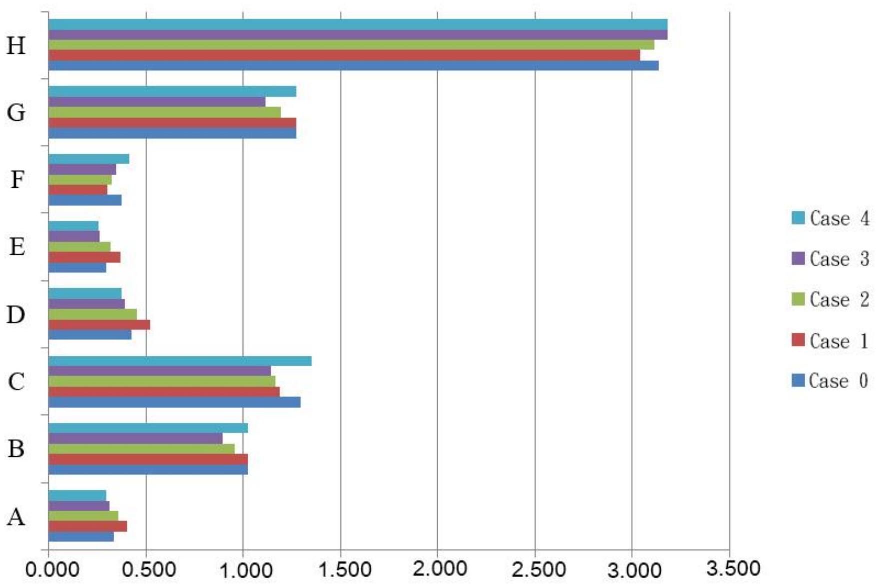

Figure10. Sensitivity analysis results

Table 11. Comparison of fuzzy TOPSIS and FAD

Failure mode	Case 0		Case 1		Case 2		Case 3		Case 4	
	TOPSIS	FAD	TOPSIS	FAD	TOPSIS	FAD	TOPSIS	FAD	TOPSIS	FAD
A	2	2	3	3	3	3	2	2	2	2
B	5	5	5	5	5	5	5	5	5	5
C	7	7	6	6	7	6	7	7	7	7
D	4	4	4	4	4	4	4	4	3	3
E	1	1	2	2	1	1	1	1	1	1
F	3	3	1	1	2	2	3	3	4	4
G	6	6	7	7	6	7	6	6	6	6

5. Conclusion

FMEA, designed to provide information for risk management decision-making, is a widely used technique in industries. In FMEA, potential failure modes are determined by three factors named occurrence, severity and detection. In traditional method of risk priorities, risk was estimated by multiplying crisp numbers, although this traditional method was criticized in the literature for many reasons including lack of consideration of the relative importance of risk factors and imprecise evaluation. Because of these criticisms, in this study, a fuzzy approach that is superior to the traditional approach has been considered. Fuzzy approach based on fuzzy axiomatic design approach is employed and used for prioritizing failure modes and also this method combined with Fuzzy AHP to consider the relative importance of risk factors. In addition, it is possible for experts to assess the risk factors for each potential failure modes with linguistic variables. Among the advantages of this method, considering the relative importance of risk factors and evaluation of these factors, either crisp or fuzzy can be noted. This model may be useful for providing information for decision-making in the context of risk management in industrial and service organizations. For further research, this study can be generalized to epistemic uncertainty.

References

[1] Kutlu, A. C. and Ekmekçiog˘lu, M. (2012). Fuzzy failure modes and effects analysis by using fuzzy TOPSIS-based fuzzy AHP. *Expert Systems with Applications*, Vol. 39, pp. 61-67.

[2] Liu, H. C., Liu, L., Bian, Q. H., Lin, Q. L., Dong, N. and Xu, P. C. (2011). Failure mode and effects analysis using fuzzy evidential reasoning approach and grey theory. *Expert Systems with Applications*, Vol. 38, pp. 4403-4415.

[3] Liu, H. C., Liu, L., Liu, N. and Mao, L. X. (2012). Risk evaluation in failure mode and effects analysis with extended VIKOR method under fuzzy environment. *Expert Systems with Applications*, Vol. 39, pp. 12926-12934.

[4] Wang, Y. M., Chin, K. S., Poon, G. K. K. and Yang, J. B. (2009). Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean. *Expert Systems with Applications*, Vol. 36, pp. 1195-1207.

[5] Liu, H. C., Liu, L. and Liu, N. (2013). Risk evaluation approaches in failure mode and effects analysis: A literature review. *Expert Systems with Applications*, Vol. 40, pp. 828-838.

[6] BS 5760 Part 5, Reliability of systems, equipment and components. Guide to failure modes, effects and criticality analysis (FMEA and FMECA), (1991).

[7] Teoh, P. C. and Case, K. (2004). Failure modes and effects analysis through knowledge modelling. *Journal of Materials Processing Technolog*.

[8] Braglia, M., Frosolini, M. and Montanari, R. (2003). Fuzzy TOPSIS approach for failure mode, effects and criticality analysis. *Quality and Reliability Engineering International*, Vol. 19, pp. 425-443.

[9] Suh, N. P. (1990). *The principles of design*. Oxford University Press, New York.

[10] Kulak, O. and Kahraman, C. (2005a). Fuzzy multi-attribute selection among transportation companies using axiomatic design and analytic hierarchy process. *Information Sciences*, Vol. 170, pp. 191-210.

[11] Kahraman, C. and Çebı, C. (2009). A new multi-attribute decision making method Hierarchical fuzzy axiomatic design. *Expert Systems with Applications*, Vol. 36, pp. 4848-4861.

[12] Kulak, O. and Kahraman, C. (2005b). Multi-attribute comparison of advanced manufacturing systems using fuzzy vs. crisp axiomatic design approach. *International Journal of Production Economics*, Vol. 95, pp. 415-424.

[13] Kulak, O. (2005). A decision support system for fuzzy multi-attribute selection of material handling equipments. *Expert Systems with Applications*, Vol. 29, No. 2, pp. 310-319.

[14] Kulak, O., Durmusoglu, M. B. and Kahraman, C. (2005). Fuzzy multi-attribute equipment selection based on information axiom. *Journal of Materials Processing Technology*, Vol. 169, pp. 337-345.

[15] Çebı, C. and Kahraman, C. (2010). Developing a group decision support system based on fuzzy information axiom. *Knowledge-Based Systems*, Vol. 23, pp. 3-16.

[16] Buckley, J. J. (1985). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, Vol. 17, pp. 233-247.