Volume 3, Number 2, 2014

International Journal of Research in Industrial Engineering

www.nvlscience.com

Open Vehicle Routing Problem Optimization under Realistic Assumptions

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A R T I C L E I N F O

Article history :
Received:
March 14, 2014
Revised:
April 10, 2014
Accepted:
May 03, 2014

Keywords :

Multi-Depot, Open Vehicle Routing, Simulated Annealing

A B S T R A C T

MDOVRP is a variant of classical VRP, which seeks to find a set of non-depot returning vehicle routes dispatching form several depots. A new integer linear programming model is presented considering limited number of vehicles at each depot. Randomly generated small-sized MDOVRPs are solved for validation and verification of the proposed model. Due to the complexity of the problem, a simulated annealing algorithm (SA) is developed for medium and large-sized MDVRPs' benchmarks.

1. Introduction

Distribution network design is a crucial problem that companies care about much more than the past. Open VRP (OVRP) is a variant of VRP in which each route is a sequence of customers with a deterministic demand and predefined geographical location, that starts at depot and ends at one of the customers (in contrast to classical VRP where it returns to the depot after servicing period). Practically, OVRP is applied when suppliers does not have a vehicle fleet and prefer outsource their transportation and distribution affairs. One should be noted that it is much more economical for suppliers to outsource the distribution of the goods or materials. In the OVRP, it is supposed that each customer is visited once by a single vehicle and the total demand of all customers allocated to a vehicle (route) does not exceed the vehicle capacity and the objective function is usually minimizing the overall traveling cost (Liu et al. [1]).

In contrast to the classical VRP, the OVRP has only been studied by very few people. Repoussis et al. [2] proposed a population based hybrid meta-heuristic that utilizes the basic solution framework of evolutionary algorithm (EA) combined with a memory-based trajectory local search algorithm. Chiang et al. [3] considered a similar case in a stochastic environment in which stochastic phenomena involve start time delivery, production rate,

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loading and unloading times and travel times. Mirhassani and Abolghasemi [4] presented continues version of Particle Swarm Optimization (PSO) methodology to deal with OVRP.
Yu et al. [5] applied a hybrid genetic algorithm (GA) and TS to minimize the number of vehicles and total distance. There are other researches that investigated OVRP and presented heuristics and meta-heuristics to solve this problem such as SA (Banos et al. [6]), and Ant Colony Optimization (ACO) (Tang et al. [7]).
In the conventional OVRP, only one central depot exists and all vehicles start their route from this depot. On the other hand, in real-life problems for companies, there is more than one depot to deal with a large vehicle fleet (Liu et al. [1]). Furthermore, using one depot may result in vehicle queues and delay in satisfaction of costumers' demand. As a result, Multi-depot VRP (MDVRP) is introduced for VRP with more than one depot. MDVRP is also a complex combinatorial problem that is addressed in the literature (Kuo and Wang [8]).
OVRP accompanied with more than one depot lead to Multi-Depot OVRP (MDOVRP) which is a more difficult problem. However, little attention has been paid to MDOVRP so far and to the best knowledge of the authors, there are only two studies in the literature that deal with the MDOVRP. As an early study, Tarantilis and Kiranoudis [9] studied MDOVRP for a real case of fresh meat industry in Greek. As the second study, Liu et al. [1] provided a mixed integer linear programming (MILP) mathematical formulation for MDOVRP and presented a hybrid genetic algorithm (HGA) to find best routes that minimize total traveling cost. They assumed that there is an unlimited identical vehicle fleet available in each depot and did not reflect any fixed cost for activated vehicles.
A new ILP mathematical model is developed in this paper. In the presented MILP, besides minimizing total traveling costs (Liu et al. [1]), the activation cost of engaging vehicles in depots (fixed costs) are considered in the objective function. Contrary to Liu et al. [1], limited number of vehicles can be assigned to a particular depot and also, different decision variables are applied. Due to validate presented MILP, the model is coded in LINGO 11 software and randomly generated small sized MDOVRPs are solved and optimum solutions are obtained.

2. Problem Description

Major assumptions of the proposed MDOVRP are described and a new ILP mathematical model is developed and explained in details in this section. Major assumptions of the MDOVRP are as follows.

1. Vehicles start at a particular depot and finish at the last customer to which the goods are delivered;
2. The total demand of customers allocated to a route should not surpass the vehicle capacity;
3. A limited number of vehicles can be activated to service in a certain depot;
4. The distance between customers and depots and the demand of each customer are deterministic;
5. Each customer can be visited once by a specific vehicle;
6. Demand of each customer can be satisfied by any vehicle from any depot;
7. Vehicle fleet is homogenous.

In the rest of this section, the new ILP mathematical model for MDOVRP is described in details.

Indices

D	Number of depots
N	Number of customers
I, J	Set of Depots and customers (i,j= 1, 2, 3,…, D, D+1,…,D+N)
H	Set of vehicles (h=1, 2, 3,…,K)

Parameters

Q	Capacity of a vehicle
V	Maximum number of vehicles which can be activated at each depot
K	Maximum number of vehicles which can be activated at all depots (K=V×D)
FC	Vehicle activation cost (Fixed Cost)
Cij	Traveling cost from customer/depot i to customer j
qi	Demand of customer i

Decision variables

$$x_{ijh} = \begin{cases} 1, \text{if vehicle } h \text{ travels directly from } i \text{ to } j, i \neq j \\ 0, \text{otherwise} \end{cases}$$

$$y_{ih} = \begin{cases} 1, \text{if customer } i \text{ is visited by vehicle } h \\ 0, \text{otherwise} \end{cases}$$

2.1.Proposed ILP model

The objective of proposed ILP is to minimize overall cost which consists of two parts: 1) vehicles traveling costs (variable cost), and 2) vehicles activation cost (fixed cost). Vehicles traveling cost is formulated as follows (Eqn. (1)).

$$\sum_{h=1}^{K} \sum_{i=1}^{D+N} \sum_{j=D+1}^{D+N} X_{ijh} \times C_{ij} \tag{1}$$

Vehicles activation cost is shown in Eqn. (2).

$$FC \times \left(\sum_{i=1}^{D} \sum_{h=1}^{K} Y_{ih} \right) \tag{2}$$

Overall cost of the presented problem is the summation of variable and fixed costs. It should be noted that variable and fixed costs are obtained in the planning time horizon e.g. a non-periodic MDOVRP is considered. Therefore, the proposed ILP model is developed as follows:

$$\text{Min} \quad \sum_{h=1}^{K} \sum_{i=1}^{D+N} \sum_{j=D+1}^{D+N} X_{ijh} \times C_{ij} + FC \times \left(\sum_{i=1}^{D} \sum_{h=1}^{K} Y_{ih} \right) \tag{3}$$

S.T.

$$\sum_{h=1}^{K} Y_{ih} = 1 \qquad (i = D+1,\dots,D+N), \tag{4}$$

$$1 \le \sum_{h=1}^{K}\sum_{i=1}^{D} Y_{ih} \le K, \tag{5}$$

$$\sum_{i=1}^{D} Y_{ih} \le 1 \qquad \forall h, \tag{6}$$

$$\sum_{h=1}^{K} Y_{ih} \le V \qquad (i = 1,\dots,D), \tag{7}$$

$$\sum_{i=1}^{D+N} X_{ijh} = Y_{ih} \qquad (j = D+1,\dots,D+N), \forall h, \tag{8}$$

$$\sum_{j=D+1}^{D+N} X_{ijh} \le Y_{ih} \qquad (i = 1,\dots,D+N), \forall h, \tag{9}$$

$$\sum_{i=1}^{D+N} Y_{ih} \times q_i \le Q \qquad \forall h \tag{10}$$

$$X_{ijh} = 0 \ if \ : i = j \qquad \forall i, j, h, \tag{11}$$

$$X_{ijh} = \{0,1\} \qquad \forall i, j, h, \tag{12}$$

$$Y_{ih} = \{0,1\} \qquad \forall i, h, \tag{13}$$

Constraint (4) guarantees that each customer is assigned to only one vehicle from all depots. In Constraint (5), total number of vehicles activated at all depots should be equal or less than maximum number of vehicles which can be activated at all depots. Constraint (6) indicates that each vehicle cannot be activated at more than one depot. Constraint (7) demonstrates that a limited number of vehicles (V) can be activated at a depot. Constraint (8) ensures that only one customer/depot can be selected as the predecessor of each customer in a route. Similarly, Constraint (9) indicates that the successor of a customer/depot in a route should be only one customer. Constraint (10) is the vehicles capacity constraint and constraints (11), (12) and (13) are integrity constraints.

2.2.Meta-Heuristic

The MDOVRP is in the NP-hard class of combinatorial problems, in which exact methods are incapable of solving the problem in a reasonable time. In such problems, heuristics and meta-heuristics are efficient and effective tools that provide optimal/near-optimal solutions within a short period of time. In this study, a simulated annealing algorithm (SA) is developed for the MDOVRP, which is explained in details in the following subsections.

Simulated annealing (SA)

SA of Aarts and Korst [10], is a heuristic search method based on statistical physics, has been found to be useful in many combinatorial optimization problems. The algorithm initiates with a randomly generated initial point or a first solution. This initial solution is the ‘‘current’’ solution. A neighbor which can be created by different structures of this current solution is then generated. If the neighbor has a better objective function than the current point, it is absolutely accepted as the new current point. Conversely, if the neighbor is found to be worse, it is not forbidden entirely, but accepted with a certain probability. The algorithm proceeds by iterating certain number of times over the transition from the current point to the adjacent point (which becomes the next current point). At the first steps of an SA run, the probability of accepting a worse point is kept high. Therefore it reduces the chance of the SA algorithm getting trapped in a local optimum. When the number of iteration increases, this probability is reduced according to a particular procedure. It is regular to use a control parameter, called the temperature, to modify the probability of acceptance/rejection. Generally, the temperature begins at a high value and gradually come down according to a schedule known as the annealing schedule. This annealing schedule determines how the probability of accepting a worse point decreases with time.

In the following subsections, an encoding procedure of feasible solutions for the presented MDOVRP and the methods used to generate neighbors are explained.

Solution representation

In order to solve the proposed MDOVRP, the problem should be encoded. Hence, a 3×N matrix is developed. The first row is a sequence of costumers, which should be served. The second row allocates each costumer to one of the vehicles available at each depot. Similarly, the last row allocates each costumer to one of depots. To illustrate the solution representation, it is assumed that there are 9 costumers, two depots, and two vehicles could be activated at each depot. Figure. ۱ shows a random feasible solution for this problem.

	Costumers								
	#1	*#2*	*#3*	*#4*	*#5*	*#6*	*#7*	*#8*	*#9*
Costumers Sequence	**1**	**7**	**8**	**3**	**5**	**4**	**2**	**9**	**6**
Vehicle	**1**	**1**	**2**	**1**	**2**	**2**	**2**	**1**	**2**
Depot	**2**	**1**	**2**	**2**	**1**	**2**	**1**	**1**	**2**

Figure. ١. Random feasible solution

As presented in Figure. ٢, Costumer #1 is first served by the first vehicle triggered at the second depot (highlighted in red color). Costumer #2 is served by the second vehicle triggered at the first depot (highlighted in blue color). This solution is demonstrated graphically in Figure. ٢.

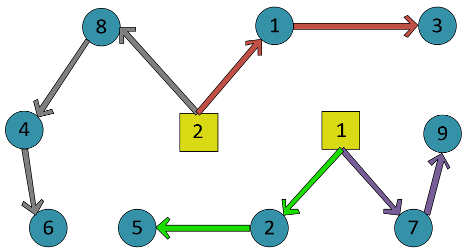

Figure. ٢. Graphical representation of the random feasible solution

In Figure. ٢, squares specify depots, circles are representing costumers and each vehicle route is represented using arrows in different colors.

Neighborhood generation methods

In this subsection, four procedures are proposed to generate neighbors for a solution. These procedures are described in details as follows.

Swap

In this method, two arrays of a row are selected randomly and their values are swapped.

Insertion

In the insertion method, an array is selected from a row randomly. Then, its value is inserted to the beginning, end or in a place middle of the row.

Inversion

This method selects two arrays of a row and reverses the sequence of the numbers between them.

Perturbation

In the perturbation, an element from a row (corresponding to vehicles and depots assignment rows) is selected and replaced by a logical random number. This random number should be between 1 and V for the second row and between 1 and D for the third row to maintain the feasibility of the solution. This method enables the search engine to escape from local optimum.

3. Computational results

In this section, the computational experiments of the proposed MDOVRP are presented. The ILP mathematical model, developed in Section 2, is encoded into the optimization software to be solved for small-sized problems. SA meta-heuristic is encoded in the MATLAB software and the results of applying these meta-heuristics to small-sized problems are attained and compared to the optimum solutions determined by the branch-and-bound (B/B) method in the optimization software tool. Since there are no standard benchmark test problems for the MDOVRP, the available benchmark test instances for the classical MDVRP is modified and utilized to validate the proposed meta-heuristics and to analyze them through comparing their results to other presented algorithms in the literature.

3.1. Small size MDOVRPs

In this section, seven test problems are generated randomly to evaluate the accuracy and effectiveness of the proposed meta-heuristics as follows. The number of customers are 4, 6 and 8, the number of depots are 1, 2, 3 and 4, and 1 or 2 vehicles can be triggered at each depot. Customers' and depots' coordinates are generated with uniform distributions, U(0,100) and U(25,75), respectively. Customers' demands are random numbers between 0 and 100, activation cost for each vehicle is between 50 and 150, and traveling cost is obtained based on the Euclidean distance. The vehicles' capacity is generated using the relation (14).

$$Q = \left\{ \frac{\text{Total Demand} \times 1.15}{\text{Total Number of Vehicles}} \right\} \tag{14}$$

In order to validate and verify the proposed meta-heuristic, these test problems are solved utilizing the optimization software tool, SA as well as the related results are compared and analyzed. Deviation between optimum values (obtained by the B/B method) and proposed meta-heuristic (SA) are calculated by the formulation (15):

$$\text{Deviation}_{\text{small}} = \frac{\text{Average of SA solutions - Optimum solution (LINGO11)}}{\text{Optimum solution (LINGO11)}} \times 100 \quad (15)$$

One should be noted that the proposed SA is applied ten times to a test problem and the average value is used to calculate deviations. Table ١ shows characteristics of random test problems and the results of the B/B and SA. As illustrated in this table, SA solve about 43% of random problems optimally (i.e., test problems 1, 2, and 3) and the average deviation between SA and the optimal solution is about 4.4 percent.

Table 1. Results for small-size problems.

Test Problem	*N*	*D*	*V*	Optimum OFV* from LINGO 11	SA	
					Average OFV	Deviation
1	4	1	2	425.35	425.35	0.00
2	4	2	1	325.28	325.28	0.00
3	6	2	2	359.81	359.81	0.00
4	6	3	1	278.69	295.18	0.059
5	8	2	2	451.97	457.98	0.013
6	8	3	1	308.37	343.66	0.114
7	8	4	1	350.62	392.10	0.118
					Average	0.044

3.2.Medium and large-sized MDOVRPs

In this section, 14 MDVRPs from Cordeau et al. [11] are modified and solved using SA.

Table ٢ demonstrates the results of solving 14 modified standard problems with SA. As shown in this Table ٢.

Table 2. Large-sized MDOVRPs solved by the proposed SA

Test problem	*N*	*D*	*V*	*Q*	SA	
					Best	Average
P01	50	4	4	80	1169.55	1334.46
P02	50	4	2	160	719.56	778.15
P03	75	5	3	140	1459.47	1662.65
P04	100	2	8	100	2163.52	2458.98
P05	100	2	5	200	1377.25	1501.14
P06	100	3	6	100	2188.45	2482.26
P07	100	4	4	100	2230.85	2500.65
P08	249	2	14	500	2101.75	2304.31
P09	249	3	12	500	8436.39	9323.87

Table 2. continued

P10	249	4	8	500	8797.58	9623.65
P11	249	5	6	500	8338.56	9410.35
P12	80	2	5	60	1974.31	2222.16
P15	160	4	5	60	5886.77	6522.59
P18	240	6	5	60	9715.20	11258.86

4. Conclusions

The multi-depot open vehicle routing problem (MDOVRP) is an extension of the conventional VRP, in which more than one depot exists and vehicles travel an open route. In this paper, a new integer linear programming (ILP) model has been developed for the MDOVRP and solved by a branch-and-bound method using the optimization software tool for randomly generated small-sized MDOVRPs to validate the presented ILP model. In order to solve large-sized MDOVRPs which are NP-hard problems, simulated annealing (SA) have been used. In order to validate the SA, the results of small-sized MDOVRPs obtained from a branch-and bound (B/B) method has been compared to the results of the proposed SA. 14 large-sized standard MDVRPs have been selected and modified and solved using the proposed SA.

Future research directions can be considered some operational constraints, such as time windows, travel distance limitations, heterogeneous fleet, and the like. Moreover, multi-criteria MDOVRPs can be considered and solved using proper multi-criteria decision making approaches. This research investigates the MDOVRP with a predetermined planning time horizon. However, a multi-period MDOVRP can be studied for future.

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