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An Interdiction Median Model for Hierarchical Capacitated Facilities

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A R T I C L E I N F O

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A B S T R A C T

In this paper a partial interdiction problem on a capacitated hierarchical system is studied. We consider an attacker who can interdict facilities at different levels and each interdiction level causes a specified reduction in the capacity of a facility depending upon its service level in the hierarchy. First, the interdictor identifies her interdiction strategy whose aim is to cause the most demand satisfaction cost subject to her budgetary limitation. Subsequently, the defender tries to optimize the objective function which is similar to the attacker's one but in the opposite direction. The defender is responsible for choosing the least cost strategy in order to satisfy all customers' demand. She can achieve this goal by two ways: (1) allocating their demand to the hierarchical facilities subject to their residual capacity, (2) benefiting from outsourcing option. This problem can be regarded as a static Stackelberg game between a malicious interdictor as the leader and a system defender as the follower. In this paper we propose a bi-level mathematical formulation in order to model the problem. To solve this problem with exhaustive enumeration, CPLEX has been used.

1. Introduction and Overview

Introduced in 1960 [1], interdiction problem has a variety of applications. An intentional strike against a system is called interdiction [2]. According to [3], two of the most common types of interdiction used are disruption and destruction. Disruption involves ‘‘upsetting the flow of information, operational tempo, effective interaction, or cohesion of the enemy force or those systems’’ while destruction means ‘‘damage the structure, function, or condition of a target so that it can neither perform as intended nor be restored to a usable condition, rendering it ineffective or useless’’ [4]

In this paper we study a partial interdiction problem on a hierarchical system. The concept of partial interdiction first introduced in 1970 [5]. Adding partial option in contrast to the full version of interdiction, demonstrates the ability to plan more freely for spending interdiction budget on interdiction strategies. In [6], a reasonably comprehensive survey of partial

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interdiction problems is conducted. Our work can be considered as a development of [6] on the problem that was proposed in [7].

In our problem, we consider a situation which each attack causes a specified reduction in the capacity of a facility regarding to the service level of the facility and the level of interdiction. In this paper, inspiring of many real service systems we consider a nested hierarchical system with different service levels. Hierarchical systems have multiple layers of interacting facilities. A system is classified as nested or non-nested according to the service availability at the levels of hierarchy. In a nested hierarchy, a higher-level facility provides all the services provided by a lower level facility and at least one additional service. In a non-nested hierarchy, facilities on each level offer different services.

This problem could be considered as a two-player game. The attacker, as the leader, determines the most destructive interdiction strategy with respect to her budgetary limitation. Later, the defender tries to choose the best strategy to satisfy all customers' demand. The objective function of these two players is the same. To optimize the objective, the interdictor tries to maximize the total cost of demand satisfaction and the defender tries to minimize it regarding available facilities. As the two-player game nature of this problem, we use bi-level programming to model it. For solving the model, we implement a comprehensive enumeration code in CPLEX software [8]. In this way, it needs to call CPLEX to solve the second level in the exact way.

The rest of the paper is organized as follows. A bi-level mathematical formulation is given in section 2. In section 3, we present an example with computational result analysis to illustrate the problem that is proposed in this paper. Finally, section 4 concludes the paper with a brief summary of the findings.

2. Problem definition

In this problem we consider a nested hierarchical system with two levels of facilities. Due to its nested nature, the facilities at level 2 can serve the customers who require the first and second level of services. Each facility has a specific capacity for serving each level of services. For a facility at level one, the capacity for the second level service is zero.

By experience, it is known that the specific percentages of demand of each demand point are required particular service levels. Moreover, as a result of the hierarchical nature of the system, a distinct percentage of customers' demand first is considered as the demand that required the first service level but after served by a facility at level one, this facility refers the customer to the second level facility in order to complete demand satisfaction. In this problem the defender faces outsourcing option. In order to serve a customer, the defender can choose the best strategy between outsourcing and/or allocating the demand to her facilities with sufficient capacity, whichever is more cost-efficient. As an example, see Fig. 1. This example provides readers with a symbolic demand satisfaction strategy for a demand point.

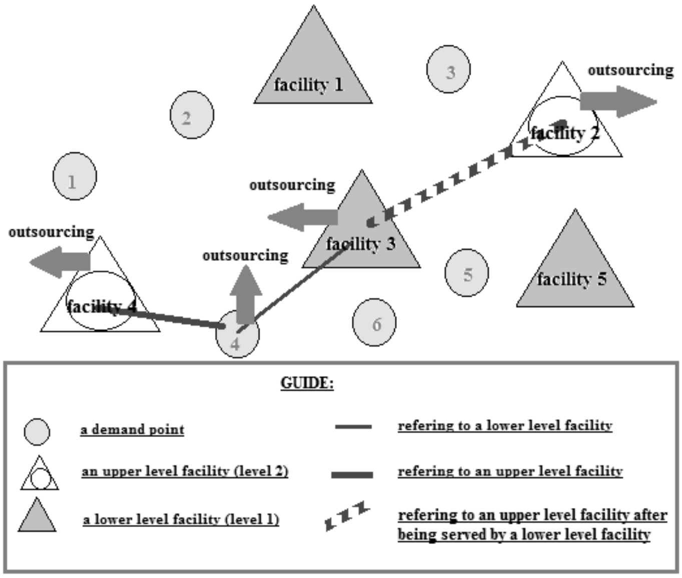

Figure 1- A symbolic demand satisfaction strategy for a demand point

3. Notations

To give a formal description of the developed model, some notations and decision variables are introduced as follows.

A. Parameters

l_{ij}	Distance between demand node i and facility j
l'_{jf}	Distance between facility j (level 1) and facility f (level 2)
z_i	Demand of demand point i
α	Cost of transporting a unit of demand to a facility at level 1 per unit of distance
β	Cost of transporting a unit of demand to a facility at level 2 per unit of distance
γ	Cost of transporting a unit of demand from a facility at level 1 to a facility at level 2 per unit of distance
α'	Cost of outsourcing a unit of demand for the first service level
β'	Cost of outsourcing a unit of demand for the second level service
γ'	Cost of outsourcing a unit of demand that is at first outsourced for service at level 1, but it needs to be served at level 2
ω'	Cost of outsourcing a unit of demand that is at first outsourced for service at level 1, but after serving at that level to complete demand satisfaction it also requires serving at level 2.
h_k^1	Cost of attack on a facility of the first level at interdiction level k

h_k^2	Cost of attack on a facility of the second level at interdiction level k
θ	Fraction of demand of each point that is referred to a facility in order to be served at first service level
$1-\theta$	Fraction of demand of each node that is referred to a facility at level two in order to be served at the second service level
σ	Fraction of demand in a facility at level one that require being referred to a facility at level two in order to receive second service level
c_j^1	Initial capacity of facility j for the first service level
c_j^2	Initial capacity of facility j for the second service level
d_k^1	Reduction ratio in capacity of a facility at level one after interdiction at level k
d_k^2	Reduction ratio in capacity of a facility at level two after interdiction at level k
B	Total budget for interdiction
S_1	Set of facilities for service level 1
S_2	Set of facilities for service level 2
K	Set of interdiction levels

B. Decision variables:

u_{ij}^1	Amount of demand of point i that is allocated to facility j to be served at level 1
u_{ij}^2	Amount of demand of point i that is allocated to facility j to be served at level 2
u_{jf}^3	Amount of demand of point i that is first allocated to facility j to be served at level 1 and then this facility refer the demand to facility f in order to be served at level 2
o_i^1	Amount of demand of point i that is outsourced to be served at level 1
o_i^2	Amount of demand of point i that is outsourced to be served at level 2
o_j^3	Amount of demand of point i that at first is outsourced to be served at level 1, but then it needs to be outsourced for second service level.
x_{jk}	Binary variable, equal to one if facility j is interdicted at level k

4. A bi-level model

$$\text{leader (attacker)}: \; Max \quad H(Z) \tag{1}$$

$$where\; H(Z) = \sum_{i\in I}\sum_{j\in S_1} l_{ij}.\alpha u_{ij}^1 + \sum_{i\in I}\sum_{j\in S_2} l_{ij}.\beta u_{ij}^1 + \sum_{i\in I}\sum_{j\in S_2} l_{ij}.\beta u_{ij}^2 + \sum_{j\in S_1}\sum_{f\in S_2} l'_{jf}.\gamma u_{jf}^3 + \sum_{i\in I}\alpha'.o_i^1 + \sum_{i\in I}\beta'.o_i^2 + \sum_{j\in S_1}\gamma'.o_j^3 + \sigma\times\sum_{i\in I}\omega'.o_i^1$$

Subject to:

$$\sum_{k=0}^{k_{\max}} x_{jk} = 1 \tag{2}$$

$$\sum_{j\in S_1}\sum_{k\in K} x_{jk}.h_k^1 + \sum_{j\in S_2}\sum_{k\in K} x_{jk}.h_k^2 \le B \tag{3}$$

$$x_{jk} \in \{0,1\} \tag{4}$$

$$\text{Follower (Defender)}: \; Min \quad H(z) \tag{5}$$

Subject to:

$$z_i.\theta = \sum_{j\in S_1, S_2} u_{ij}^1 + o_i^1 \qquad \forall i \tag{6}$$

$$z_i.(1-\theta) = \sum_{j\in s_2} u_{ij}^2 + o_i^2 \qquad \forall i \tag{7}$$

$$\sigma.\sum_{i\in I} u_{ij}^1 = \sum_{f\in S_2} u_{jf}^3 + o_j^3 \qquad \forall j \tag{8}$$

$$\sum_{i\in I} u_{ij}^1 \le c_j^1 - c_j^1.\sum_{k=1}^{k_{\max}} x_{jk}.d_k^1 \qquad \forall j \tag{9}$$

$$\sum_{i\in I} u_{ij}^1 \le c_j^1 - c_j^1.\sum_{k=1}^{k_{\max}} x_{jk}.d_k^2 \qquad \forall j \tag{10}$$

$$\sum_{i\in I} u_{ij}^2 + \sum_{f\in S_1} u_{fj}^3 \le c_j^2 - c_j^2.\sum_{k=1}^{k_{\max}} x_{jk}.d_k^2 \qquad \forall j \tag{11}$$

$$u_{ij}^1, u_{ij}^2, u_{fj}^3, o_i^1, o_i^2, o_j^3 \ge 0 \tag{12}$$

(6)

This model consists of two levels. At the upper level (1)-(4) the interdiction strategy for each facility is identified and at the lower level (5)-(12) the demand satisfaction strategy is optimized. The interdictor objective function, as shown in (1), states the goal of the attacker that is to maximize the total demand satisfaction cost. In (2), choosing exactly one interdiction level, including the zero level (i.e. no interdiction), for each facility is enforced. Constraint (3) restricts the budget of interdiction. (4) assures the binary characteristic of the interdiction decision variables. The objective function of the defender is presented in (5) is the same of the attacker's objective functions but in the opposite direction. Constraints (6)-(8) impose all customers' demand to be satisfied by allocating to the facilities and/or by outsourcing. Constraints (6) refer to the amount of demand that require to be served at level one and constraints (7) refer to the one that need to be served at level two. The two-stage demand satisfaction strategy is considered in (8). Constraints (9)-(11) enforce the facilities to serve customers' demand only if their residual capacities after interdiction are sufficient. In (9) and (10), the capacity for the first service level of a facility at level one and two, respectively and in (11), the capacity of the second service level for a facility at level two are considered. In (12), the continuous variables of the defender are identified.

5. Illustrative example

In this section we present the numerical example that has been run to illustrate the performance of the model. First, we provide some generic information about the solver software and the parameter setting.
The Code of the model has been written in C++ and compiled using Microsoft Visual 2010. To solve the MIP problem we used the generic MIP solver ILOG CPLEX 12.3.

A. Parameter setting

To set parameters we generate instance, numerical examples, which its size is small due to make it possible to use a comprehensive enumeration code to identify all interdiction strategies within a reasonable running time. In Table III, parameters of the example are presented.

B. Solution procedure

Bi-level programming problems are mathematical optimization problems where the set of all variables is partitioned between two vectors x and y, and x is to be chosen as an optimal solution of a second mathematical programming problem parameterized in y. Thus, the bi-level programming problem is hierarchical in the sense that its constraints are defined in part by a second optimization problem [9].
To solve this bi-level problem we use a combination of exact linear programming optimization technique and comprehensive enumeration. We code this combinational solution procedure in CPLEX. At the first level of the model all possible interdiction strategies subject to the attacker's budget are identified and CPLEX imports these strategies as the decision variables from the upper level into the lower level as the parameters. The lower level is a linear programming problem. This combinational solving approach is efficient only when the size of the problem is small or medium. For larger one, heuristics and metaheuristics methods are suggested (for more information on metaheuristic methods, see [10]).

Table 1. Parameter setting for the illustrative example

Parameters	**Values**
$\|I\|$	30
$\|J\|$	$9 \rightarrow \|S_1\| = 6,\ \|S_2\| = 3$
$\|k\|$	$4 \rightarrow \{0,1,2,3\}$
Allocating cost	$\alpha = 7, \beta = 11, \gamma = 12$
Outsourcing cost	$\alpha' = 460, \beta' = 650, \gamma' = 700, \omega' = 220$
Demand ratios	$\theta = 0.7, \sigma = 0.2$
B	2000
Coordination(i)	(15,15), (12,4), (5,14), (11,10), (6,1), (12,10), (10,5), (6,7), (0,0), (2,7), (3,2), (4,12), (12,3), (15,11), (0,3), (3,7), (6,9), (11,6), (1,12), (2,14), (15,10), (6,14), (0,11), (3,13), (12,0), (5,7), (14,5), (6,0), (5,13), (15,7)
Coordination(j)	$S_1 \rightarrow$ (4,4), (2,12), (6,12), (14,2), (13,8), (5,8) $S_2 \rightarrow$ (8,8), (2,10), (13,6)
h_k^1	$K = \{0,1,2,3\} \rightarrow \{0,700,1050,1500\}$
h_k^2	$K = \{0,1,2,3\} \rightarrow \{0,800,1200,1700\}$
d_k^1	$K = \{0,1,2,3\} \rightarrow \{0,0.6,0.9,1\}$
d_k^2	$K = \{0,1,2,3\} \rightarrow \{0,0.5,0.8,1\}$
z_i	$I \rightarrow$ {103, 59, 23, 39, 92, 46, 84, 95, 19, 32, 44, 47, 30, 26, 81, 90, 103, 14, 70, 83, 52, 61, 36, 42, 32, 73, 66, 11, 6, 40}
c_j^1	$J \in S_1 \rightarrow$ {70, 150, 60, 70, 225, 120} $J \in S_2 \rightarrow$ {78, 168, 170}
c_j^2	$J \in S_1 \rightarrow$ {0, 0, 0, 0, 0, 0} $J \in S_2 \rightarrow$ {280, 265, 160}

C. Computational results

For the upper level of this illustrative problem, CPLEX identifies 136 feasible interdiction strategies. As it is mentioned earlier in (1), the objective of the interdictor is to choose a strategy that causes the most demand satisfaction cost. In Table I, we only report 15 strategies that their objective functions are higher than 200,000 (unit of money).

In this example the 5th interdiction strategy is the best one from the attacker's point of view (see Figure 2).

Reporting several strategies versus a single optimal strategy has some beneficial points. It helps the decision-makers to identify the most critical facilities in the system. Furthermore, in real world owing to the uncertainty of estimating some parameters, other strategies even may be better than the optimal solution.

Table 2. The results of illustrative example

NO.	Strategy	Objective function	Residual budget
1	$x_{81} = 1, x_{92} = 1$	258,006	0
2	$x_{82} = 1, x_{91} = 1$	277,089	0
3	$x_{83} = 1$	240,803	300
4	$x_{71} = 1, x_{92} = 1$	244,746	0
5	$x_{71} = 1, x_{82} = 1$	293,765	0
6	$x_{72} = 1, x_{91} = 1$	253,847	0
7	$x_{72} = 1, x_{81} = 1$	285,406	0
8	$x_{73} = 1$	209,428	300
9	$x_{61} = 1, x_{82} = 1$	217,842	100
10	$x_{51} = 1, x_{82} = 1$	236,768	100
11	$x_{51} = 1, x_{72} = 1$	211,547	100
12	$x_{41} = 1, x_{82} = 1$	205,029	100
13	$x_{31} = 1, x_{82} = 1$	204,486	100
14	$x_{11} = 1, x_{82} = 1$	206,182	100
15	$x_{21} = 1, x_{82} = 1$	223,906	100

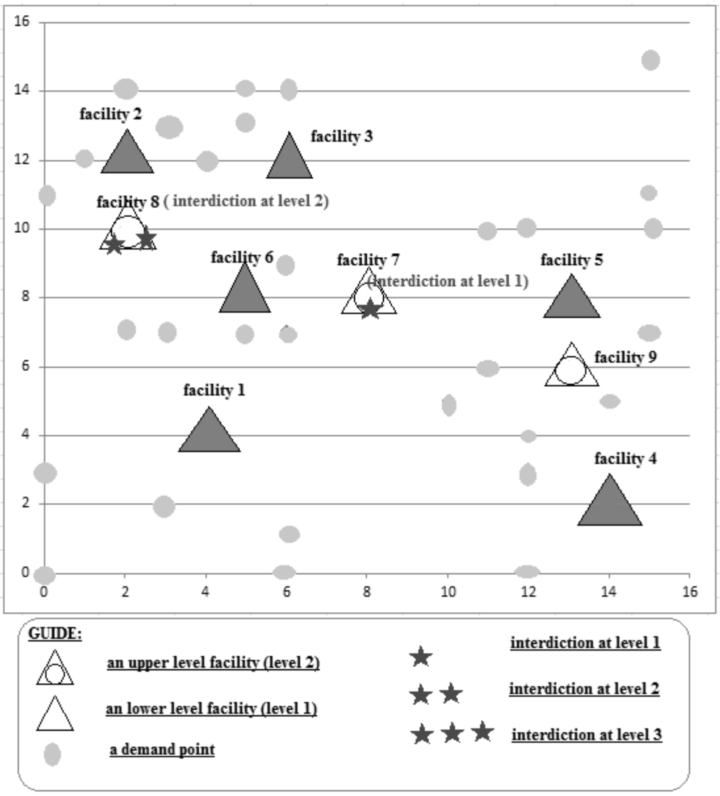

Figure 1. The optimal interdiction strategy for the illustrative example

6. Summary and Conclusions

The purpose of this study is to propose a mathematical model for a hierarchical system in a situation that there is an interdictor who is able to attack the facilities at different levels. This model is a beneficial one for an organization which is at risk of being interdicted or is interested in identifying its critical facilities and choosing the cheapest strategy for satisfying all customers' demand in the worst case scenario. The most important features of this problem are summarized as follows.

- The service system is hierarchical and capacitated.
- All customers' demand must be satisfied.
- The defender faces two options to serve customers' demand: allocating the demand to her facilities and/or outsourcing
- A bi-level formulation is proposed to model this problem. The interdictor chooses the most destructive interdiction strategy as the leader and the defender tries to minimize the total demand satisfaction cost.
- The interdiction may occur at different levels (partial interdiction).
- The attacker faces a budgetary limitation upon interdicting the facilities.
- The capacity reduction occurs with regard to interdiction level.
- The presented model can be used by the service organizations as well as defensive agents in such a way that they can find the most pessimistic incurred costs in case of losing some of their facilities,
- Within a review of interdiction problems, we identify several areas for future research. A brief report on these areas is presented in this subsection.
- Developing the model by adding capacity expansion option for the defender.
- Revising the model as a multi-objective one to seek Pareto frontier versus a single solution. This may provide the decision-maker with a wider perspective on the problem.
- Formulating the problem as a tri-level model and adding fortification ability to protect the most critical facilities.
- Proposing heuristics and metaheuristics procedures to reduce the computational effort.

References

[1] Rocco, C. and Ramirez- Marquez, J. E. (2009). "Deterministic network interdiction optimization via an evolutionary approach", *Reliability Engineering & System Safety,* Vol. 94, pp. 568-578.

[2] Church, R. L., Scaparra, M. P. and Middleton, R. S. (2004). "Identifying critical infrastructure: the median and covering facility interdiction problems", *Annals of the Association of American Geographers,* Vol. 94, pp. 491-502.

[3] Kennedy, K. T., Deckro, R. F., Moore, J. T. and Hopkinson, K. M., (2011). "Nodal interdiction", *Mathematical and Computer Modeling*, Vol. 54, pp. 3116-3125.

[4] Joint Publication 3-03, "Doctrine for joint interdiction", 03 May, (2007). Available at: www.fas.org/irp/doddir/dod/jp3_03.pdf, Accessed on 20, Sep. 2013.

[5] McMasters, A. W. and Mustin, T. M., (1970). "Optimal interdiction of supply network", *Naval Research Logistics Quarterly*, Vol. 17, No.3 pp. 261-268.

[6] Aksen, D., Aksa, S. S. and Aras, N., (2014). "A bilevel partial interdiction problem with capacitated facilities and demand outsourcing", *Computers & Operations Research*, Vol. 41, pp. 346-358.

[7] Aliakbarian, N. and Dehghanian, F. (2013). "A bi-level programming for protection planning of critical hierarchical facilities", 9th International Industrial Engineering Conference, Tehran, Iran, pp. 20-21.

[8] IBM ILOG Cplex, http://www.ilog.com.

[9] Dempe, S. (2002). "*Nonconvex optimization and its applications: foundations of bilevel programming"*, Kluwer Academic Publisher,

[10] Talbi, E. G. (2009). "*Metaheuristics: from design to implementation"*, John Wiley & Sons.