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Evaluation of the Efficiency by DEA: A Case Study of Hospital Sectors

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A B S T R A C T

Data Envelopment Analysis (DEA) is a nonparametric technique used to determine the relative efficiency of Decision Making Units (DMUs). Results from DEA analysis yield important information regarding the optimal operating capabilities of each unit. This paper studies DEA to hospital sectors and identifies their rankings during a period of six years. Data for the study comes from a hospital in the North of Iran. This article compares the performance of different parts of the hospital over the years. It can also aid improve hospital performance.

Keywords: DEA, Efficiency, Super-efficiency method, Hospital sector.

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1. Introduction

The analysis of hospital industry provides an important contribution to the comparison of health care system performance. Hospital efficiency is one of the key indicators of hospital performance. Manifold efforts have been undertaken to evaluate and compare the hospital efficiency within a particular country. The analysis of the efficiency of health production, without the focus on hospital sectors, using parametric and nonparametric approaches has been attempted previously. The available studies performed the efficiency analysis using nonparametric approaches such as Data Envelopment Analysis (DEA) and directional distance function. DEA was developed to measure the performance of decision making units in multiple inputs and output setting. The seminal paper of [4] introduced DEA a linear programming approach to performance evaluation when production is characterized by constant return to scale and developed in [2] by variable return to scale. In other words, DEA extends the theoretical discussion of technical efficiency of [8] into direct measurability by developing the observed data to determine a best-practice frontier. This technique assigns a

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score of one to a unit only when comparisons with other relevant units do not provide evidence of inefficiency in the use of any input or output. DEA assigns an efficiency score less than one to (relatively) inefficient units. A score less than one means that a linear combination of other units from the sample could produce the same vector of outputs using a smaller vector of inputs. This score reflects the radial distance from the estimated production frontier to the DMU under estimation. Although in previous research efforts, some non-radial performance evaluation were proposed such as Russell measure of technical efficiency proposed by [8] and Slacks-Based Measure of Efficiency (SBM) approached by [9]. Furthermore, the application domain for DEA has grown to real world application. Banking, education, health care, and hospital efficiency were found to be most popular application areas. In an effort to hospital performance measurement, [8] presented a non-parametric model most closely related to efficiency studies in the hospital sectors which focused on the production of intermediate hospital outputs over the period from 1974 to 1989. However, their inputs did not completely discriminate between the input and output sectors, and their output variables did not include any control for case-mix differences. As [3] argued, a medical sector can be considered as block box. The principal consideration in that paper was data such as number of doctors as inputs to treat number of patient per month. Although in most application of DEA presented in the literature, the models presented are designed to obtain a measure of efficiency in two three-year periods .Since the health system performance brings the topic to the policymakers around the world are paying attention to this a contribution to provide an assessment of efficiency measurement. As a result, the empirical measurement of economic efficiency centers on determining the extent of technical efficiency in a given organization or a given industry. Most recently, economists have employed frontier efficiency measurement techniques to measure the productive performance of health care services. DEA techniques use a production possibility frontier to map a locus of technically, the potential can be to produce a combination of outputs that an organization can produce at one point in time. Accordingly, if we can determine production frontiers that represent total economic efficiency using the best currently known production techniques, then we can use this idealized yardstick to evaluate the economic performance of actual organizations and industries. By comparing the actual behavior of organizations against the idealized benchmark of economic efficiency, we can determine the degree of efficiency exhibited by some real-world agency. This study concentrates on efficiency measurement of an Iranian hospital in North region of Iran using DEA techniques. However, this study share a common step by step empirical procedure that determines [5] the choice of inputs and outputs to be used in the selected approach [6] the method used to explain efficiency differences and the factors thought to be associated with these differences and finally the approach which specifies the ranking of efficient units in the sample. This study is aimed to expand the previous attempts to compare the efficiency of a hospital sector and to examine the differences in hospital sectors through using cross-efficiency method. Moreover, this study aims to enhance the understanding of the use of efficiency methods for the purpose of international comparison of the hospital sector performance. As the DEA sample shows uncovers the greatest potential efficiency gains, while allocating resources more effectively.

Organizational structure of health system in Iran includes ministry of health and medical education, social security organization, government hospitals, private hospitals (non-profit), private hospitals (for-profit) and private teaching hospitals. The University of medical sciences plays an important role both in medical education and the provision of health services in province level in Iran. In addition, Social Security Organization (SSO) provides the medical services to its beneficiaries through their hospitals. Furthermore, the private sector provides 10 to 20% of the health care services

In 2000, the World Health Organization (WHO) reported that Iranian health system performance was ranked ninety-three among other countries [6]. Mismanagement of resources can be a reason for this issue [5]. In 2014, Iranian Health Sector Evolution Plan (HSEP) was initiated by the ministry of health. This plan included some packages to the following: increasing coverage of basic health insurance, increasing quality of care, reducing Out-Of-Pocket (OOP) payments for inpatient services, and increasing quality of primary healthcare [11]. Healthcare providers concerns potentially about the sustainability and efficiency of HSEP in Iran [11].

Low hospital efficiency is deemed as one of the most important problems of Iranian health system. The studies conducted through many Iranian researchers were conducted in hospital efficiency evolution. In most of these studies, the efficiency indicator was reported low [10]. According to a study conducted in Iran, the hospital efficiency was better than that of before the implementation of HSEP [5]. Research results in some countries indicated that the hospital efficiency was low [5, 6]. In some studies, high hospital efficiency has been reported [5].

Hospitals are important economic enterprises. Therefore, measuring its financial performance is a key action toward improving resource management [5]. Efficiency means the maximum use of resources to generate returns. Two non-parametric and parametric approaches were introduced to measure the efficiency. DEA is an applied and frequent nonparametric technique used to measure the efficiency of units. Inefficient units and reason of the inefficiency can be assessed through the DEA methods [7]. Considering the importance of hospital performance measurement, this study is aimed to analyze the efficiency of public hospitals through DEA technique.

2. Methods

DEA is used to find the relative efficiency for medical systems. Since DEA allows multiple inputs and outputs simultneously, the consideration here will be the data manner. The usual output per input definition of efficiency may be used to judge the relative efficiency for medical sectors. DEA was developed on the seminal work of [8] which quantifys an efficiency score as the ratio of single output to a single input. However, hospitals are not single input-outputs systems. There are a number of equivalent formulation for DEA. The mosr direct formulation of DEA are as follows. Let X_i b the vector of inputs into DMU_j. let Y_i be the vector of outputs into DMU_j. Let X_o be the inputs into DMU_o for which we want to determine its efficiency and Y_o be the outputs. The measure of efficiency for DMU_o is given by the following linear programming:

$$
\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t.} \\
&\quad \sum \lambda_i X_i \le \theta X_o \\
&\quad \sum \lambda_i Y_i \ge Y_o \\
&\quad \lambda \ge 0.
\end{aligned} \tag{1}
$$

Where λ_i is the weight given to DMU_j in its ffort to dominate DMU_o and θ is the efficiency of DMU_o. The optimal θ cannot possibly be more than one. Performance units 1 is called efficient, otherwise the units are inefficient. The formulation above are called CCR input-oriented formulation. One important assumption to make when performing DEA is whether to use an input or output orientation. An input-oriented models holds the current level of output constant and minimize inputs, whereas an output-oriented model maximizes output keeping the amount of inputs constant. [7] did not specify a formal definition of the contemporary “Farrell measure” of the technical efficiency of production and did not standardize the two differentmeasures of technical efficiency. Deprins and Simar () defined the input technical efficiency as a measure between zero and one, whereas output technical efficiency as a measure greater than one. Another important theoretical assumption in DEA is whether it applys constant or variable returnt to scale. The CCR model for efficiency measurement assumed the Constant Return to Scale (CRS). Later on, the [12] incorporated the Variable Return to Scale (VRS) to account for firms, which do not operate at their optimal scale. The VRS model is as follows:

$$
\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t.} \\
&\quad \sum \lambda_i X_i \le \theta X_o \\
&\quad \sum \lambda_i Y_i \ge Y_o \\
&\quad \sum \lambda_i = 1 \\
&\quad \lambda \ge 0.
\end{aligned} \tag{2}
$$

Eq. (2) is called BCC input-oriented formulation. These models are basic models of DEA. Usually as the number of input and output increases, more DMUs tend to get an efficiency rating one as they become too specialized to be evaluated with respect to other units. So the ranking of efficient units take an important place in DEA literature. The first model which was suggested for ranking efficient DMUs introduced by [1] with development of CCR model. This model is formulated as follows:

$$
\begin{aligned}
&\text{Min} \quad \theta \\
&\text{s.t.} \\
&\quad \sum_{i \ne o} \lambda_i X_i \le \theta X_o \\
&\quad \sum_{i \ne o} \lambda_i Y_i \ge Y_o \\
&\quad \lambda_i \ge 0.
\end{aligned} \tag{3}
$$

We see that the difference between the super-efficiency and the original DEA models is that the DMU_o under evaluation is excluded from the reference set, i.e. the super efficiency DEA models are based on a reference technology constructed from all other units. The super-efficiency DEA models are always feasible and equivalent to the original DEA models when under evaluation DMU belongs to efficient points which are not extreme or belongs to weakly efficient reference set or inefficient units. Thus we only need to consider the situation when under evaluated unit belongs to extreme efficient DMUs. Study [12] showed that if the super-efficiency CCR model is infeasible, the under evaluated unit belongs to extreme efficient DMUs. However, he failed to recognize that the output-based super-efficient CCR model is always feasible for the trivial solution which has all variables set equal to zero. Moreover, [12] showed that the input-based super-efficient CCR model is infeasible if and only if a certain pattern of zero data occurs in the inputs and outputs, e.g. DMU_o has some zero inputs which are positive for all other DMUs or DMU_o has some positive outputs which are equal to zero for all other DMUs.

3. Variables and Data Specification

Data for analysis are driven from an Iranian hospital, in North region of Iran in years 2010 and 2019. The hospital consists of fourteen units in 2010 and fifteen units in 2019. In this paper, the performance of these fourteen units and fifteen units are interpreted as the activity efficiency to increase the satisfaction of patients. ***Table 1*** and ***Table 2*** present the specified units of this hospital.

Table 1. Hospital sectors in 2010**.**

Number of sectors	Name of Sector(DMU)
1	Emergency
2	Orthopedic Surgery
3	CCU
4	Otorhinolaryngology
5	P-CCU
6	Surgery
7	Eye
8	ICU
9	Urology
10	Obstetrics
11	NICU
12	Pediatrics
13	Internal Medicine
14	Infants

Table 2. Hospital sectors in 2019.

Number of sectors	Name of Sector(DMU)
1	Emergency
2	Orthopedic Surgery
3	CCU
4	Otorhinolaryngology
5	P-CCU
6	Surgery
7	Eye
8	ICU
9	Urology
10	Obstetrics
11	NICU
12	Pediatrics
13	Internal Medicine
14	Infants
15	Neurology

The ideal measure of final output in hospital care would be some measures of the health gain of the individual patients. However, these data are not readily available. So, the output variables involve percentage of occupied beds and the number of patients. The first variable is calculated by the following formula:

$$100 \times \frac{\text{number of beds-occupied days}}{\text{number of days-total days}}.$$

The ratio records the indicated total days that each bed was occupied by each patient during a determined period of time. Also, total days express as multiply of total number of available beds in each section and the number of determined period of time. As for input variables, the number of active beds in each section, full or empty, represents a measure of resources, which are available for providing service to inpatients in hospitals. Moreover, the number of physician and nurses can be treated as representing the variation in service technology among different sectors in hospitals. The next section argues the results.

4. Results and Discussion

While the real advantage of DEA is its ability to estimate a multiple inputs and outputs, two important consideration should be taken into account. First, CCR model imposes no convex combination and CRS model in which both input and output oriented models are applied. For more details, input or output oriented BCC models are argued to have a closer look to DMUs. Since this nature of standard DEA model creates difficulties in discriminating efficient units in those evaluated models, the super-efficiency models are used to rank the efficient units in the sample. In doing so, the evaluated unit is taken out of the reference set then the relevant CCR model is applied. The resulting score shows the ranking of the unit under evaluation. The same procedure in ranking units is done for BCC models. It is clear that we have ignored some factors due to lack of information, though, the selected outputs are less controlled by hospital administrative. As a result, the input-oriented DEA models are recommended. Also. DEA-solver software is used to investigate the results of information is collected. Two basic DEA models were selected for calculating efficiency scores: CCR-I and BCC-I. A brief description about the efficiency scores in three years of applied input-oriented models were recorded in ***Table 3***.

Table 3. Efficiency score and ranking in case study application in year 1389.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR -Ranking
1	Internal Medicine	0.46	0.48	0.46(12)
2	CCU	1	1	1.56(3)
3	Infants	0.38	0.46	0.38(14)
4		0.47	0.52	0.47(11)
5	NICU	0.76	1	0.76(9)
6		1	1	1.22(6)
7	Surgery	0.69	0.71	0.61(10)
8	Urology	0.85	1	0.85(7)
9		1	1	1.23(4)
10	Eye	1	1	1.23(5)
11	Orthopedic Surgery	1	1	2.52(2)
12	Emergency	1	1	3.38(1)
13	P-CCU	0.82	0.84	0.82(8)
14	ICU	0.46	0.78	0.46(13)

Table 3 shows that six units of fourteen units are evaluated as efficient units in CCR input oriented evaluations. It can be seen that BCC models presents more than six units as efficient ones. It can be seen that there is six MPSS in the set of determined sectors. Recall that MPSS is a DMU, which exhibits the most productive scale size. In order to make a closer look to results and for more investigation between efficient units, the super-efficiency method is applied for ranking DMUs. The last column shows that unit # has the first rank in the sample. For the next two years, 1390 and 1391, ***Table 4*** and ***Table 5*** depict the efficiency score and ranking of units, respectively.

Table 4. *Efficiency score and ranking in case study application in year 1390*.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR -Ranking
1	Internal Medicine	0.53	0.55	0.53(12)
2	CCU	1	1	1.63(3)
3	Infants	0.33	0.50	0.33(14)
4		0.37	0.50	0.37(13)
5	NICU	0.86	1	0.86(7)
6		0.84	0.85	0.84(8)
7	Surgery	0.65	0.70	0.65(11)
8	Urology	0.74	1	0.74(9)
9		1	1	1.36(4)
10	Eye	1	1	1.19(5)
11	Orthopedic Surgery	1	1	1.86(2)
12	Emergency	1	1	3.42(1)
13	P-CCU	1	1	1.03(6)
14	ICU	0.72	1	0.72(10)

Table 4 represents six efficient units in CCR model and nine efficient units in BCC model. MPSS units are units #2, #9, #10, #11, #12, and #13. Obviously, BCC model demonstrates more efficient units. Top of all, the unit #12 has the highest rank between the data set evaluated in this year. Testing the data set of third year, ***Table 4*** records the relative efficiency scores and ranking orders in this year.

Table 5. Efficiency score and ranking in case study application in year 1391.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR - Ranking
1	Internal Medicine	0.45	0.55	0.45(13)
2	CCU	1	1	1.45(4)
3	Infants	0.26	0.44	0.26(14)
4		0.52	0.54	0.52(12)
5	NICU	0.54	0.56	0.54(11)
6		0.68	0.78	0.68(10)
7	Surgery	0.80	0.84	0.80(6)
8	Urology	0.69	1	0.69(9)
9		1	1	1.90(3)
10	Eye	0.79	1	0.79(7)
11	Orthopedic Surgery	1	1	2.26(2)
12	Emergency	1	1	4.56(1)
13	P-CCU	0.90	0.95	0.90(5)
14	ICU	0.75	1	0.75(8)

As the ***Table 5*** shows four units are efficient in this year evaluated by CCR model, although the number of efficient units increases to seven units in BBC model. As ***Table 5*** shows, the unit #12 has the highest order in the data set. However, a sample of a hospital is not a sufficiently good basis for performing such kind of studies. The present research should be extended either by more hospitals or by analyzing the longer period of time to get more observations in regress or progress. Besides, the regrouping of hospitals could be executed in order to form clusters with an approximately equal number of objects. What's more, surveying the RTS situations of efficient units and considering the data set used in this study from an inverse DEA view can be worth studying. Many factors can influence the relative rankings of such a wide variety of hospital sectors.

Table 6. Efficiency score and ranking in case study application in year 1395.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR -Ranking
1	Internal Medicine	0.71	0.72	0.71(11)
2	CCU	1	1	1.70(4)
3	Infants	0.59	0.67	0.59(12)
4		0.50	0.55	0.50(13)
5	NICU	0.80	1	0.80(10)
6		1	1	1.34(6)
7	Surgery	0.51	0.53	0.48(14)
8	Urology	0.85	1	0.85(9)
9		1	1	1.35(5)
10	Eye	1	1	2.53(3)
11	Orthopedic Surgery	1	1	3.28(1)
12	Emergency	0.98	0.94	0.98(7)
13	P-CCU	0.91	0.92	0.91(8)
14	ICU	0.40	0.73	0.40(15)
15	Neurology	1	1	3.25(2)

Table 6 shows six out of fourteen units are evaluated as efficient units in CCR input oriented evaluations. It can be seen that BCC models present more than six units as efficient ones. It can be seen that there is six MPSS in the set of determined sectors. Recall that MPSS is a DMU which exhibits the most productive scale size. In order to make a closer look to results and for more investigation between efficient units, the super-efficiency method is applied for ranking DMUs. The last column shows that unit # has the first rank in the sample. For the next two years, 1396 and 1397, ***Table 7*** and ***Table 8*** depict the efficiency score and ranking of units, respectively.

Table 7. Efficiency score and ranking in case study application in year 1396.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR -Ranking
1	Internal Medicine	0.98	0.99	0.98(8)
2	CCU	1	1	2.50(5)
3	Infants	0.61	0.69	0.61(14)
4		0.52	0.59	0.52(15)
5	NICU	1	1	1.2(7)
6		0.70	0.81	0.70(13)
7	Surgery	0.80	0.84	0.80(11)
8	Urology	0.84	1	0.84(10)
9		1	1	2.99(4)
10	Eye	1	1	1.14(6)
11	Orthopedic Surgery	1	1	3.38(2)
12	Emergency	1	1	4.56(1)
13	P-CCU	0.90	0.95	0.90(9)
14	ICU	0.75	1	0.75(12)
15	Neurology	1	1	3.18(3)

Table 7 represents seven efficient units in CCR model and nine efficient units in BCC model. MPSS units are units #2, #5, #9, #10, #11, #12, and #15. Obviously, BCC model demonstrates

more efficient units. Top of all, the unit #12 has the highest rank between the data set evaluated in this year. Testing the data set for the third year.

Table 8. Efficiency score and ranking in case study application in year 1397.

	DMU	CCR Efficiency Score	BCC Efficiency Score	CCR -Ranking
1	Internal Medicine	0.98	0.99	0.98(10)
2	CCU	1	1	2.19(6)
3	Infants	0.83	0.88	0.83(14)
4		0.71	0.78	0.71(15)
5	NICU	1	1	2.46(5)
6		0.87	0.91	0.87(13)
7	Surgery	0.93	0.97	0.93(11)
8	Urology	0.99	1	0.99(9)
9		1	1	3.12(3)
10	Eye	1	1	2.15(7)
11	Orthopedic Surgery	1	1	3.45(2)
12	Emergency	1	1	4.9(1)
13	P-CCU	0.92	0.95	0.92(12)
14	ICU	1	1	1.24(8)
15	Neurology	1	1	3.11(4)

As the ***Table 8*** records, this year they were made 8 sectors efficient by the CCR model, although the number of efficient units increases to nine units in BBC model. As ***Table 8*** shows, unit #12 has the highest order in the data set. However, a sample of a hospital is not a sufficiently good basis for performing such kind of studies. The present research should be extended either by more hospitals or by analyzing the longer period of time to get more observations in regress or progress. Besides, the regrouping of hospitals could be executed in order to form clusters with an approximately equal number of objects. What's more, surveying the RTS situations of efficient units and considering the data set used in this study from an inverse DEA view can be worth studying in addition, the nature of productivity is important when designing policies to improve resource allocation. Many factors can influence the relative rankings of such a wide variety of hospital sectors.

5. Conclusion

The aim of this paper was not only to estimate the efficiency of hospital sectors but also to enhance the understanding of using efficiency methods for sectors comparison. A real world application of DEA for assessing the hospital sectors performance of an Iranian hospital has been addressed. The results depicted that unit #12 takes the first order in the ranking in theses six years. Although using the nonparametric techniques reveals the efficiency score of each evaluated units and encourages to investigate the ranking of efficient units. For future research finding, some approaches for inverse DEA are not difficult. On the other hand, the findings about the sector's regress and progress takes another research aspect in the literature.

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